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Mission Impossible? Challenges for Countries in Escaping the Middle-Income Trap

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Mission Impossible? Challenges for Countries in Escaping the Middle-Income Trap*

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Abstract

Why do some economies manage to escape the middle-income trap while others remain trapped? To answer this question, we develop an overlapping-generations model in which economies can choose between a primitive technology subject to diminishing returns and an advanced technology capable of sustaining long-run growth. Adoption becomes viable only when human capital, capital-use efficiency, and demographic conditions are sufficiently favorable; otherwise, the economy may converge to a middle-income steady state from which escape is nearly impossible. We test these predictions using panel data for 185 economies over 1950–2023. The results show that higher human capital and stronger institutional quality are robustly associated with a greater likelihood of escape, whereas aging becomes an obstacle primarily for economies that grow old before achieving high-income status. Additional evidence based on economic complexity and imports of manufactures and machinery supports the model's central technology-adoption mechanism which determines whether an economy successfully completes the transition out of middle-income status.

Keywords: middle-income trap, endogenous technology choice, overlapping generations, general equilibrium, discrete-time hazard model

JEL Classification: E13, O11, O33, O41, O47

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1 Introduction

Why do some economies successfully transition to high-income status while others tend to remain at middle-income levels for decades? Figure 1 highlights this divergence. Measured from the year in which each economy first crossed the World Bank's middle-income threshold, Japan, South Korea, Hong Kong, and Singapore experienced remarkably high growth rates, enabling them to reach high-income status within just only two decades. Meanwhile, Thailand, the Philippines, Indonesia, Malaysia, and Vietnam appear to have followed markedly different trajectories, with growth slowing considerably and remaining relatively subdued over extended periods. This pattern suggests that these economies may have faced some structural difficulties in sustaining rapid growth after entering the middle-income range.

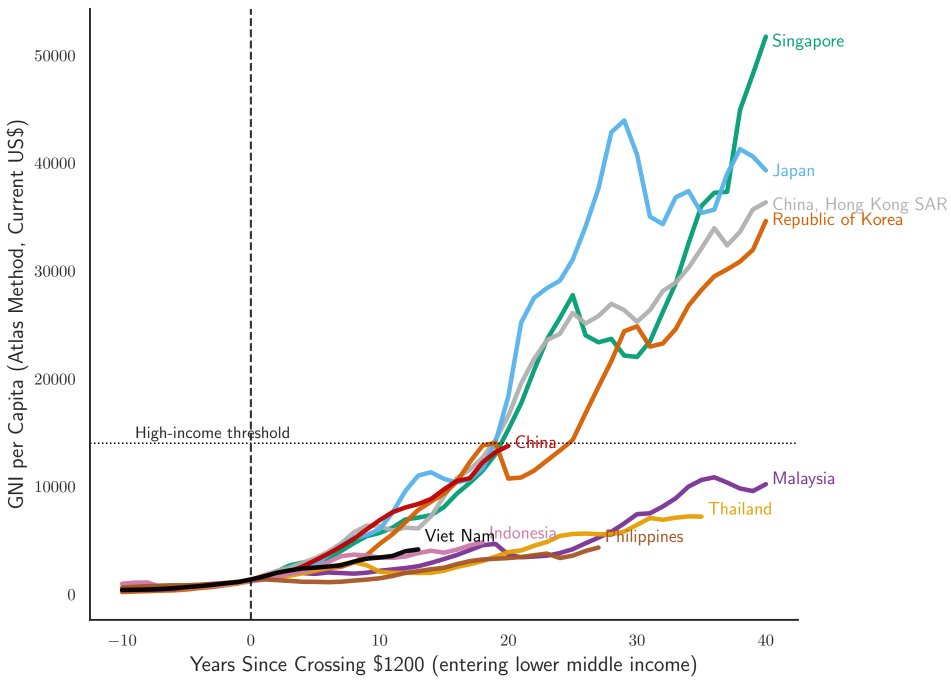


Figure 1: Trajectories of GNI per capita (Atlas method, current USD) for selected Asian economies after the $1,200 benchmark is crossed. Source: WDI.

A central question is whether this divergence reflects a genuine middle-income trap, a persistent state from which upward transition becomes increasingly difficult, or simply a slow but continuous process of convergence. The literature remains divided.

On one side, a growing body of work identifies structural barriers that slow or halt the transition from middle to high income. Eichengreen et al. (2012, 2014) document that growth slowdowns in fast-growing middle-income economies cluster at specific income thresholds and find that secondary education, high-technology exports, and the ability to move up the technology ladder reduce the likelihood of slowdown, implying that the barriers are related to the exhaustion of imitation-based growth and the difficulty of transitioning to innovation. Aiyar et al. (2018) corroborate this finding using conditional convergence regressions, showing that middle-income countries exhibit systematically lower growth than both poorer and richer economies, consistent with the presence of income-level-specific obstacles, such as the exhaustion of demographic dividends, the limits of factor accumulation, and the difficulty of moving to skill-intensive production, that do not affect economies at other stages. Both findings suggest that something beyond ordinary convergence is at work, though neither paper necessarily claims that the trap is permanent or inescapable.

On the other side, many papers argue that what appears as a trap is better understood as ordinary convergence dynamics—the natural process by which economies that have grown rapidly revert toward the global average growth rate, regardless of their income level. Pritchett and Summers (2014) show that regression to the mean—the statistical tendency for extreme growth outcomes to be followed by less extreme ones—is the most robust feature of economic growth, and that past growth is a far more powerful predictor of subsequent deceleration than the level of per capita income. On this view, the income thresholds identified by Eichengreen et al. (2012, 2014) are not evidence of structural barriers that bind at certain income levels, but rather a statistical artifact: economies that have grown rapidly for extended periods are simply due for a slowdown, and the clustering at specific thresholds reflects where the fastest-growing countries happened to be when deceleration set in. Roy et al. (2016) find no evidence of a middle-income trap using unconditional convergence regressions, while Bulman et al. (2017) and Robertson and Ye (2016) similarly reframe slowdowns as segments of the normal convergence trajectory. The implication is that middle-income economies are not stuck in a trap but rather converging slowly—perhaps temporarily—toward higher income, and that the apparent persistence of middle-income status reflects the slowness of convergence rather than structural immobility.

Meanwhile, Felipe et al. (2017) develop criteria based on the time required for economies to move through historical income ranges, using successful graduating economies as benchmarks, and find that transition times vary widely. Some middle-income economies stall for decades while others can progress relatively quickly. This implies that the trap, if it exists, is contingent on country-specific conditions rather than an inevitable stage of development. A more nuanced view emphasizes definitional and methodological issues. Im and Rosenblatt (2015) and Glawe and Wagner (2016) note that the debate hinges on whether one considers an absolute trap or a relative trap.

Given the existing literature, a necessary first step is to clarify what it means for an economy to be trapped. An economy may experience prolonged periods of slow growth yet continue to move up the income ladder. If high-income status were determined by a fixed threshold, any sustained increase in income per capita would indeed eventually lead to graduation, although the process might take several decades or longer. In practice, however, the World Bank's high-income threshold is constantly revised over time. Simply rising income per capita, therefore, does not necessarily guarantee graduation. If it grows too slowly relative to the evolving threshold, an economy may remain classified as middle income indefinitely. More fundamentally, continued positive growth cannot be taken for granted. If the structural constraints underlying weak growth become increasingly binding, growth may slow down even further and eventually cease altogether. Motivated by these considerations, we adopt a sharper, model-based definition of the middle-income trap as a situation in which economic growth eventually comes to a halt at a middle-income level. Under certain structural conditions, the economy in our model converges to a stationary state in which capital per worker and income no longer grow. The trap is therefore modeled not as a temporary episode of stagnation, but as an absorbing state from which the economy cannot escape without a change in its underlying structural conditions.

The mechanisms that hinder the transition to high income are equally important to the analysis. A broad literature emphasizes that middle-income economies may exhaust the growth sources that supported their initial catch-up—cheap labor, capital deepening, and the imitation of foreign technologies—before developing the capabilities needed to sustain productivity growth at the frontier. Agénor (2017); Agénor and Canuto (2017) identify several potential obstacles, including diminishing returns to physical capital, insufficient human capital, weak contract enforcement and intellectual-property protection, distorted incentives, talent misallocation, and limited access to advanced infrastructure and finance. Relatedly, Arezki et al. (2021) highlight slow adoption of general-purpose technologies, particularly where incumbent market power restricts technological diffusion. Cherif and Hasanov (2025) likewise stress the importance of moving early toward the technological frontier, while Otsuka et al. (2017) emphasize the roles of education, technological learning, and research capability in successful graduation.

These arguments point to a common distinction between continued accumulation within an existing production structure and the adoption of more advanced technologies and activities. An economy may continue to grow for some time through capital accumulation or the reallocation of labor, yet remain unable to generate the productivity gains required to approach the high-income frontier. This perspective motivates our framework: the middle-income trap arises when the economy cannot profitably adopt the advanced technology needed for sustained growth and instead converges to a stationary middle-income state. Escape, therefore, requires a structural change that relaxes the constraints on technological upgrading, rather than merely a temporary acceleration in growth.

Building on these insights, we develop an overlapping-generations framework mainly based on Iwaisako (2002) and its related literature (Asano et al., 2012; Sunaga, 2017; Umezuki and Yokoo, 2019; Asano et al., 2024). The model features two production technologies and interprets the middle-income trap as a failure to adopt the advanced technology. The *primitive technology* captures the forces emphasized in the empirical literature: it uses raw labor and physical capital, exhibits diminishing returns, and does not benefit from human capital or increasing returns. It therefore represents a catch-up technology whose growth potential is ultimately limited by the accumulation of physical capital, in contrast to technologies whose productivity rests on the successive vintages of capital-embodied technical change (Caunedo et al., 2023; Caunedo and Keller, 2021). The *advanced technology*, in contrast, represents a higher-productivity technology operating through a monopolistically competitive intermediate-goods sector. It is subject to capital inefficiency, so that part of invested capital is lost because of infrastructure gaps, limited absorptive capacity, and institutional frictions, and it requires labor-augmenting human capital. Investors, who own the capital stock, choose the technology offering the higher gross return per unit of capital. The economy remains in the trap when the primitive steady state lies below the threshold required for adoption. This mechanism is related to Acemoglu et al. (2006), in which economies must move from investment-based growth based on imitation and capital accumulation toward innovation-based growth as they approach the technological frontier.

The model provides a closed-form characterization of the trap and identifies four structural channels that can prevent the adoption of advanced technology: demographic aging, which operates through the pension tax on the young; capital inefficiency in the advanced sector; insufficient labor-augmenting human capital; and the degree of competition in the intermediate sector, which affects the rents available to finance adoption. Of these, the first three generate unambiguous comparative statics and are the focus of the empirical analysis; the competition channel has a non-monotonic effect that is difficult to isolate with a single linear proxy. The model also yields a "demographic window" result that formalizes the possibility that economies may grow old before they become sufficiently rich to escape the trap.

The contribution of the paper is to make the mechanism underlying the middle-income trap an object of empirical testing. Because the trap condition has a closed-form solution, the model yields not only qualitative predictions about the relevant structural variables but also a measurable escape margin, which can be taken directly to the data. Using a panel of 185 economies, we examine whether the calibrated margin predicts graduation beyond income, whether the structural channels affect the graduation hazard in the directions implied by the model, whether the effect of demographic aging depends on income, and whether the escape margin operates through technology adoption rather than through an unrelated channel. The empirical results support these predictions. The escape margin predicts subsequent graduation, three channels (human capital, capital-use inefficiency, and demographic aging) have the predicted signs and are robust across calibration parameters, and the effect of aging is concentrated among economies that grow old before accumulating sufficient capital to escape. We also find that economic complexity and manufactured imports, as proxies for advanced technology absorption, are positively associated with graduation, suggesting that advanced technology adoption can be a reasonable explanation for an economy's ability to escape the trap.

The rest of the paper is organized as follows. Section 2 presents the baseline model. Section 3 characterizes the equilibrium and its dynamics. Section 4 identifies the middle-income trap and the structural channels that determine its width. Section 5 describes the data, the empirical strategy used to relate these structural channels to the probability of escaping the trap, and presents the results. Finally, Section 6 concludes and discusses the policy implications. Appendix C extends the baseline framework to incorporate a fifth channel, environmental externalities, with supporting empirical evidence.[1]

[1] In Appendix C, pollution that feeds back into productivity reduces the capital available to finance the transition to cleaner technologies, potentially generating a self-reinforcing climate–development trap, and a sufficiently large historical pollution stock can keep the economy below the growth threshold even after advanced technology has been adopted. We examine this channel empirically in Ap-

2 Theoretical Model

The economy is populated by two-period-lived overlapping generations. Individuals work when young and retire when old. Production is carried out by an infinitely lived representative firm that hires labor and rents the capital accumulated by the previous generation. Capital depreciates fully after one period.

2.1 Households

Young individuals supply one unit of labor inelastically, earn a wage w_t per worker, and pay a proportional pension tax $\tau_t \in [0,1)$ and a mandatory long-term care (LTC) insurance premium $m(\phi_t)w_t$. Their disposable income is therefore $(1-\tau_t-m(\phi_t))w_t$. They allocate this income between current consumption c_t and saving s_t. Old-age consumption d_{t+1} is financed by the return on savings and a pay-as-you-go (PAYG) pension p_{t+1}. In each period, only a fraction $\phi_t \in (0,1]$ of the cohort born at t survives to period $t+1$.

Annuity markets are perfect. Savings are intermediated by competitive insurance companies, which pay surviving individuals the gross return R_{t+1} divided by the survival probability ϕ_t, in exchange for retaining the assets of those who die. The effective return received by a survivor is therefore R_{t+1}/ϕ_t. The household budget constraints are

$$c_t + s_t = (1-\tau_t-m(\phi_t))w_t, \tag{1}$$

$$d_{t+1} = \frac{R_{t+1}}{\phi_t}s_t + p_{t+1}. \tag{2}$$

Preferences are given by

$$U_t = \ln c_t + \beta\phi_t \ln d_{t+1}, \tag{3}$$

where β is the pure discount factor. The survival probability enters utility directly, so that individuals who expect to live longer place greater weight on old-age consumption.[2]

The LTC premium captures the idea that longer life expectancy increases the cost of caring for the elderly.[3] We use the tractable specification

$$m(\phi_t) = \delta\phi_t^2, \qquad \delta > 0, \tag{4}$$

which is increasing and convex in longevity. Together with the PAYG tax $\tau_t = \rho\phi_{t-1}$, this implies that the total fiscal and LTC burden on young workers is

$$\tau_t + m(\phi_t) = \rho\phi_{t-1} + \delta\phi_t^2.$$

pendix D, where the accumulated pollution stock is negatively associated with the probability of graduation, as predicted by the model.

[2]This specification captures the longevity motive for saving, as in (Chakraborty, 2004; Chen, 2010).

[3]As longevity rises, the marginal survivor is more likely to require more costly care (De Nardi et al., 2010; Kopecky and Koreshkova, 2014; De Nardi et al., 2016).

The convexity of the LTC premium allows the burden of longevity to increase more than proportionally and generates the hump-shaped saving response that underlies the demographic-window results below.

Household optimization gives the saving decision. The survival probability cancels when combining the survival-weighted marginal utility of old-age consumption with the annuity-adjusted return, because the ϕ_t multiplying old-age utility offsets the $1/\phi_t$ adjustment in the annuity return. Thus,

$$d_{t+1} = \beta R_{t+1} c_t,$$

as in a standard log-utility OLG model. Combining this condition with the two budget constraints yields

$$s_t = \frac{\phi_t}{1+\beta\phi_t}\left[\beta(1-\tau_t - m(\phi_t))w_t - \frac{p_{t+1}}{R_{t+1}}\right]. \tag{5}$$

The survival probability affects saving through three channels. First, the factor $\phi_t/(1+\beta\phi_t)$ captures the greater importance attached to old-age consumption as survival rises. Second, the LTC premium $\delta\phi_t^2$ reduces disposable income, increasingly so at higher levels of longevity. Third, survival affects the pension component through p_{t+1}/R_{t+1}.

A small government administers the pension system. Pensions are financed by contributions from the young. With a constant population, the government budget constraint is

$$p_{t+1}\phi_t N = \tau_{t+1} w_{t+1} N,$$

or

$$p_{t+1} = \frac{\tau_{t+1}}{\phi_t} w_{t+1}. \tag{6}$$

We assume a constant gross replacement rate ρ, so that $p_{t+1}/w_{t+1} = \rho$. The corresponding tax rate is therefore

$$\tau_{t+1} = \rho\phi_t. \tag{7}$$

Under static demographics, $\phi_t = \phi_{t-1} = \phi$, young workers face a total burden

$$\tau + m(\phi) = \rho\phi + \delta\phi^2,$$

which combines the PAYG contribution with the LTC premium.

2.2 Production Technologies

Capital owners can choose between two production technologies, primitive and advanced, although only one is used in equilibrium at a given point in time. Population is constant, $N_t = N$, and the labor-augmenting human-capital factor is exogenous and constant, $h_t = h$. The two technologies are described below.

2.2.1 Primitive (P) technology

The primitive technology is a standard Cobb–Douglas production function using raw labor:

$$Y_t^P = K_t^{\alpha} N^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{8}$$

Under perfect competition, the return to capital and the wage per worker are

$$R_t^P = \alpha k_t^{\alpha-1}, \tag{9}$$

$$w_t^P = (1-\alpha) k_t^{\alpha}, \tag{10}$$

where $k_t \equiv K_t/N$ denotes capital per worker.

2.2.2 Advanced (A) technology

The advanced technology consists of a Dixit–Stiglitz final-good sector using a unit measure of differentiated intermediate goods $i \in [0,1]$:

$$Y_t^A = \left[\int_0^1 X_t(i)^{\lambda} \, di \right]^{1/\lambda}, \qquad \lambda \equiv \frac{\sigma - 1}{\sigma} \in (0,1), \tag{11}$$

where $\sigma > 1$ is the elasticity of substitution across varieties. The corresponding markup is $\mu = \sigma/(\sigma-1) = 1/\lambda$, so a higher σ, and hence a higher λ, corresponds to greater competition and a lower markup.

The parameter λ affects the two margins of the analysis differently. It raises the adoption threshold monotonically (by reducing the residual return to capital owners and thereby requiring more capital to make adoption worthwhile), as shown in Section 4, but enters the perpetual-growth condition non-monotonically, as established in Proposition 2.

Each intermediate good is produced according to

$$X_t(i) = K_t(i)^a \big(hN_t(i)\big)^b.$$

For tractability, we set $a = 1$, so that

$$X_t(i) = K_t(i) \big(hN_t(i)\big)^b.$$

The specification of capital inefficiency merits some discussion. The parameter $\gamma \in [0,1)$ captures the share of invested capital that becomes unproductive because of institutional and infrastructural frictions commonly associated with middle-income economies. These losses need not take the form of fixed nominal costs. Bureaucratic red tape, regulatory compliance, and corruption can generate costs that increase with the scale of investment, while inadequate infrastructure can reduce the effective productivity of capital as the scale of production expands (Shleifer and Vishny, 1993), and unreliable electricity is a particularly binding constraint on firm performance in developing economies (Abeberese, 2017). Modeling the loss as proportional to the capital stock therefore avoids introducing scale effects in which a sufficiently large aggregate capital stock alone allows an economy to reach the technological frontier. The resulting transition depends instead on capital intensity and institutional efficiency rather than on population size.

The final-good sector is competitive, implying the inverse demand function

$$p_t(i) = X_t(i)^{\lambda-1}(Y_t^A)^{1-\lambda}. \tag{12}$$

Intermediate firm i maximizes

$$\Pi_t(i) = p_t(i)X_t(i) - w_t^A N_t(i) - R_t^A K_t(i),$$

where w_t^A is the wage per worker and R_t^A is the rental rate of capital. In a symmetric equilibrium,

$$X_t(i) = X_t = Y_t^A, \qquad K_t(i) = K_t, \qquad N_t(i) = N,$$

and we normalize $p_t(i) = 1$.

The first-order conditions imply

$$\frac{\partial \Pi}{\partial K(i)}: \quad \lambda p_t(i)(hN)^b = R_t^A \quad \Longrightarrow \quad R_t^A = \lambda h^b N^b, \tag{13}$$

$$\frac{\partial \Pi}{\partial N(i)}: \quad \lambda p_t(i)K_t b h^b N^{b-1} = w_t^A \quad \Longrightarrow \quad w_t^A = \lambda b K_t h^b N^{b-1}. \tag{14}$$

Under symmetry, the Dixit–Stiglitz aggregator implies the aggregate production function $Y_t^A = K_t(hN)^b$. Hence, total labor income is

$$w_t^A N = \lambda b K_t h^b N^b = \lambda b Y_t^A.$$

Gross operating profit is $(1-\lambda(1+b))Y_t^A$. Capital inefficiency reduces the amount of capital available for productive use by γK_t, so net profit distributed to capital owners is

$$(1-\lambda(1+b))Y_t^A - \gamma K_t.$$

Total capital income therefore consists of rental income and net profit:

$$R_t^A K_t + \Big[(1-\lambda(1+b))Y_t^A - \gamma K_t\Big] = (1-\lambda b)Y_t^A - \gamma K_t.$$

The resulting gross return on capital is

$$\hat{R}_t^A = \frac{(1-\lambda b)Y_t^A}{K_t} - \gamma = (1-\lambda b)h^b N^b - \gamma. \tag{15}$$

Importantly, this return is independent of k_t. It depends instead on effective labor supply hN, the competition parameter λ, the production parameter b, and the capital-inefficiency wedge γ. The return formula in (15) applies regardless of whether operating profits are positive, since capital owners are residual claimants of both profits and losses. Operating profits themselves are nonnegative if and only if $\lambda(1+b) \leq 1$.

Throughout the analysis, we impose the following viability restrictions.

Assumption 1 (Viability). *(i) $1-\lambda b > 0$; (ii) $\gamma < (1-\lambda b)h^b N^b$, so that the advanced technology offers a positive gross return; and (iii) $\rho\phi + \delta\phi^2 < 1$ for all $\phi \in (0,1]$, so that young workers have positive disposable income.*

3 Equilibrium Dynamics

3.1 Temporal Equilibrium

In each period, the capital stock is owned by the old generation, which accumulated savings when young and now holds claims on the productive capital of the economy. Under the advanced technology, these claims also include ownership of the intermediate firms. Old investors therefore receive both competitive rental income and the profits generated by monopolistic competition.

Technology choice is made collectively by these capital owners. Following Iwaisako (2002), investors coordinate on the technology that offers the highest gross rate of return per unit of capital:

$$Y_t = \begin{cases} Y_t^P & \text{(Primitive)} \quad \text{if } R_t^P > \hat{R}_t^A, \\ Y_t^A & \text{(Advanced)} \quad \text{if } R_t^P \leq \hat{R}_t^A. \end{cases} \tag{16}$$

Using (9) and (15), adoption occurs when

$$(1 - \lambda b) h^b N^b - \gamma \geq \alpha k_t^{\alpha - 1}. \tag{17}$$

Given an initial capital stock per worker $k_0 > 0$, a constant population N, a sequence of old-age survival probabilities $\{\phi_t\}_{t=0}^{\infty}$, and a pension replacement rate ρ, a *competitive equilibrium* consists of sequences of household allocations $\{c_t, d_{t+1}, s_t\}_{t=0}^{\infty}$, factor prices $\{w_t, R_{t+1}\}_{t=0}^{\infty}$, pension tax rates $\{\tau_t\}_{t=0}^{\infty}$, capital stocks per worker $\{k_t\}_{t=0}^{\infty}$, and technology choices $\{T_t\}_{t=0}^{\infty}$, with $T_t \in \{P, A\}$, such that:

1. *Technology choice.* In each period, investors select the technology offering the higher gross return according to (16).

2. *Factor prices.* Factor prices depend on the technology in use:

   - Primitive ($T_t = P$):

   $$w_t = (1 - \alpha) k_t^{\alpha}, \qquad R_{t+1} = \alpha k_{t+1}^{\alpha - 1}.$$

   - Advanced ($T_t = A$):

   $$w_t = \lambda b h^b N^b k_t, \qquad R_{t+1} = (1 - \lambda b) h^b N^b - \gamma.$$

3. *Household optimization.* Saving satisfies

   $$s_t = \frac{\phi_t}{1 + \beta \phi_t} \left[\beta (1 - \tau_t - m(\phi_t)) w_t - \frac{p_{t+1}}{R_{t+1}} \right], \tag{18}$$

   with $m(\phi_t) = \delta \phi_t^2$, $p_{t+1} = \rho w_{t+1}$, and the annuity-adjusted return R_{t+1}/ϕ_t.

4. *Government.* The PAYG pension system satisfies the balanced-budget condition

$$\tau_{t+1} = \rho\phi_t,$$

and pensions are given by

$$p_{t+1} = \frac{\tau_{t+1}}{\phi_t} w_{t+1} = \rho w_{t+1}.$$

5. *Capital market clearing.* Aggregate saving by the young becomes the capital stock of the following period:

$$K_{t+1} = Ns_t. \tag{19}$$

6. *Goods market clearing.* With full capital depreciation, aggregate resources satisfy

$$Y_t = C_t + K_{t+1} + \gamma K_t \cdot \mathbf{1}\{T_t = A\}, \tag{20}$$

where the final term is a deadweight loss: the fraction γ of the capital stock that is wasted on bureaucratic costs, corruption, or inadequate infrastructure and is therefore unavailable for consumption or investment. This term is zero under the primitive technology. By Walras' law, this condition follows from the other equilibrium conditions.

The equilibrium dynamics depend on the initial capital stock and the adoption threshold $\bar{k}$ given in (31). If $k_0 < \bar{k}$ and the primitive steady state k_P^* satisfies $k_P^* \leq \bar{k}$, the economy converges monotonically to k_P^* and remains in the *middle-income trap*.

3.2 Capital Accumulation and Equilibrium Dynamics

Combining the capital-market clearing condition $K_{t+1} = Ns_t$ with the household saving function gives

$$k_{t+1} = \frac{\phi_t}{1+\beta\phi_t}\left[\beta(1-\rho\phi_{t-1}-\delta\phi_t^2)w_t - \frac{\rho w_{t+1}}{R_{t+1}}\right], \tag{21}$$

where factor prices are evaluated under the prevailing technology.

Under the primitive technology and constant demographics, $\phi_t = \phi_{t-1} = \phi$. Equations (9) and (10) imply

$$\frac{\rho w_{t+1}}{R_{t+1}} = \frac{\rho(1-\alpha)}{\alpha} k_{t+1}. \tag{22}$$

Substituting this expression into (21) and collecting the terms in k_{t+1} gives

$$k_{t+1} = Z(\phi)k_t^\alpha, \qquad Z(\phi) \equiv \frac{\alpha\beta(1-\alpha)\phi(1-\rho\phi-\delta\phi^2)}{\alpha(1+\beta\phi)+\rho\phi(1-\alpha)}. \tag{23}$$

Under Assumption 1(iii), $Z(\phi) > 0$. Since $\alpha \in (0,1)$, the map $k \mapsto Z(\phi)k^\alpha$ is increasing and strictly concave, with an infinite slope as $k \to 0$ and a slope approaching zero as $k \to \infty$. It therefore has a unique positive and globally stable fixed point. The primitive steady state is

$$k_P^* = \left[\frac{\alpha\beta\phi(1-\rho\phi-\delta\phi^2)(1-\alpha)}{\alpha(1+\beta\phi)+\rho\phi(1-\alpha)}\right]^{\frac{1}{1-\alpha}}. \tag{24}$$

Proposition 1 (Hump-Shaped Primitive Steady State)**.** *The primitive steady-state capital $k_P^*(\phi)$ is single-peaked in the survival rate ϕ. There exists a unique $\overline{\phi} > 0$ such that*

$$\frac{\partial k_P^*(\phi)}{\partial \phi} > 0 \quad \textit{for } \phi < \overline{\phi}, \qquad \frac{\partial k_P^*(\phi)}{\partial \phi} < 0 \quad \textit{for } \phi > \overline{\phi}. \tag{25}$$

The peak $\overline{\phi}$ is the unique positive root of the cubic $A\phi^3 + B\phi^2 + C\phi - \alpha = 0$ derived in Appendix A.1. It lies in the admissible interval $(0, 1)$ if and only if the cubic evaluated at $\phi = 1$ is positive, as given by condition (41). This condition holds when the capital share α is not too large relative to the pension and LTC cost parameters. Otherwise, $k_P^*(\phi)$ is increasing throughout $(0, 1]$ and reaches its maximum at $\phi = 1$.

Proof. See Appendix A.1. □

At low survival rates, the longevity motive dominates: individuals who expect to live longer save more because they place greater weight on old-age consumption. At higher survival rates, the PAYG tax $\rho\phi$ and the convex LTC premium $\delta\phi^2$ increasingly reduce disposable income, eventually reversing the effect of the longevity motive. This non-monotonic response of saving to longevity is central to the demographic-window mechanism below.

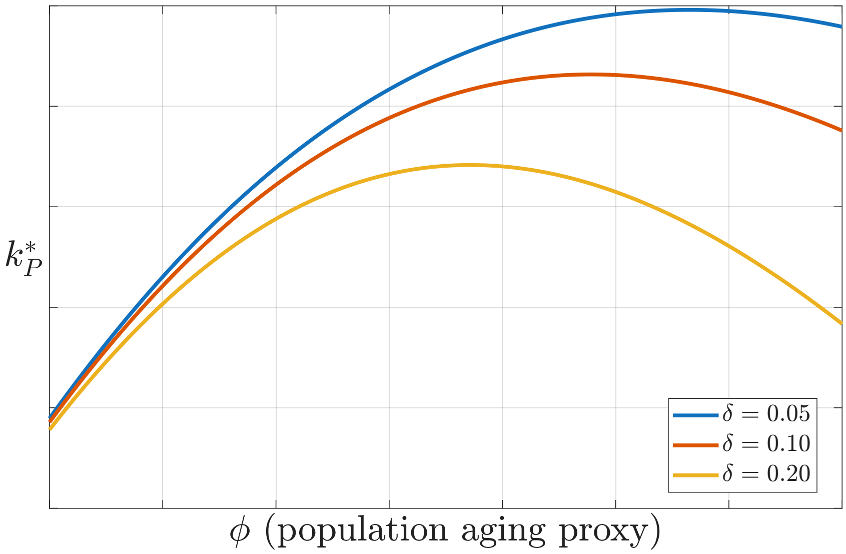


Figure 2: Illustration of Proposition 1

Under the advanced technology, factor prices are given by (14) and (15). Substituting these expressions into (21) under constant demographics yields

$$k_{t+1} = \frac{\phi}{1 + \beta\phi} \left[\beta(1 - \rho\phi - \delta\phi^2) A k_t - \frac{\rho A k_{t+1}}{B - \gamma} \right], \tag{26}$$

where

$$A \equiv \lambda b h^b N^b, \qquad B \equiv (1 - \lambda b) h^b N^b.$$

Because (26) is homogeneous in (k_t, k_{t+1}), the growth factor $g \equiv k_{t+1}/k_t$ is constant along any path under the advanced technology:

$$g = \frac{\phi\beta(1-\rho\phi-\delta\phi^2)A}{1+\beta\phi+\frac{\phi\rho A}{B-\gamma}}. \tag{27}$$

Perpetual growth therefore requires $g > 1$, or

$$\phi\beta(1-\rho\phi-\delta\phi^2)\lambda b h^b N^b > 1+\beta\phi+\frac{\phi\rho\lambda b h^b N^b}{(1-\lambda b)h^b N^b-\gamma}. \tag{28}$$

Solving (28) for λ gives the following characterization.

Proposition 2 (Perpetual-Growth Interval)**.** *Assume static demographics, $\phi_t = \phi_{t-1} = \phi$, and suppose the discriminant Δ_0 defined below is positive. Then perpetual growth occurs if and only if*

$$\frac{QS+(1+\beta\phi)-\phi\rho-\sqrt{\Delta_0}}{2bQH} < \lambda < \frac{QS+(1+\beta\phi)-\phi\rho+\sqrt{\Delta_0}}{2bQH}, \tag{29}$$

where

$$\begin{aligned}
Q &\equiv \phi\beta(1-\rho\phi-\delta\phi^2),\\
\Delta_0 &\equiv [QS+(1+\beta\phi)-\phi\rho]^2-4Q(1+\beta\phi)S,\\
H &= h^b N^b,\\
S &= H-\gamma.
\end{aligned}$$

The interval in (29) is subject to the positivity restriction

$$\lambda < S/(bH) \equiv (1-\gamma/H)/b.$$

Thus, the effective upper bound is $\min\{\lambda_{\max}, S/(bH)\}$*. If $\Delta_0 \le 0$, no value of λ sustains perpetual growth.*

Proof. See Appendix A.2. □

Inequality (29) implies that perpetual growth is possible only over an intermediate range of the competition parameter λ. When λ is too low, the labor share in the advanced sector is too small to generate sufficient saving, so the growth factor remains below one. When λ is too high, the residual share accruing to capital becomes too small, reducing the attractiveness of the advanced technology and lowering the growth factor through the denominator. This non-monotonic relationship is consistent with Iwaisako (2002), who show that economies with either very low or very high labor shares may be unable to sustain permanent growth and may instead experience cyclical fluctuations or lower-income traps.

4 The Middle-Income Trap

Under capital inefficiency, the adoption threshold $\bar{k}$ is determined by the condition that the returns to the two technologies are equal:

$$\alpha\bar{k}^{\alpha-1} = (1-\lambda b)h^b N^b - \gamma. \tag{30}$$

Provided the right-hand side is positive,

$$\bar{k} = \left[\frac{\alpha}{(1-\lambda b)h^b N^b - \gamma}\right]^{\frac{1}{1-\alpha}}. \tag{31}$$

The threshold is constant and does not depend on the current capital stock. Assumption 1 ensures that its denominator is positive. If instead $\gamma \geq (1-\lambda b)h^b N^b$, the advanced technology never offers a competitive return, regardless of the capital stock.

Definition 1. *A* middle-income trap *occurs when $k_P^* \leq \bar{k}$ and the economy starts below the adoption threshold, $k_0 < \bar{k}$. Figure 3b illustrates the resulting dynamics.*

Because both k_P^* and $\bar{k}$ are constant, define the escape margin as $\Phi \equiv k_P^* - \bar{k}$ so the trap condition can be pinned down by evaluating the value

$$\Phi \equiv k_P^* - \bar{k} = \left[\frac{\alpha\beta\phi(1-\rho\phi-\delta\phi^2)(1-\alpha)}{\alpha(1+\beta\phi)+\rho\phi(1-\alpha)}\right]^{\frac{1}{1-\alpha}} - \left[\frac{\alpha}{(1-\lambda b)h^b N^b - \gamma}\right]^{\frac{1}{1-\alpha}}. \tag{32}$$

An economy is trapped when $\Phi \leq 0$ and escapes when $\Phi > 0$. The effect of each structural parameter can therefore be understood by examining how it changes the primitive steady state and the adoption threshold.

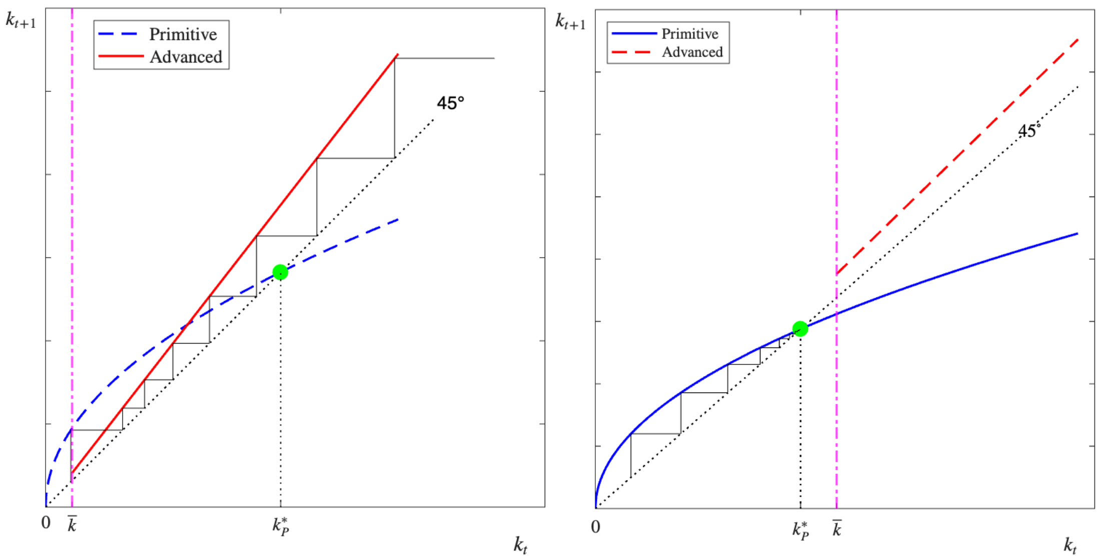
Proposition 3 (Comparative Statics of the Escape Margin)**.** *Under Assumption 1, the escape margin $\Phi \equiv k_P^* - \bar{k}$ satisfies*

$$\frac{\partial\Phi}{\partial\gamma} < 0, \qquad \frac{\partial\Phi}{\partial h} > 0, \qquad \text{sign}\left(\frac{\partial\Phi}{\partial\phi}\right) = \text{sign}(\overline{\phi} - \phi). \tag{33}$$

Proof. See Appendix A.3. □

Proposition 3 summarizes the structural channels through which the trap can widen or narrow. Capital inefficiency γ, insufficient human capital h all raise the adoption threshold and therefore reduce the escape margin.

Insufficient labor-augmenting human capital affects the transition through the adoption threshold. Since $\partial\bar{k}/\partial h < 0$, greater human capital lowers the amount of capital required for adoption. This captures the mechanism emphasized by Paus (2017) and Wu and Fang (2024): foreign investment alone may not generate sustained technological upgrading when economies lack the local capabilities needed to absorb and adapt advanced technologies, and even the adoption of mature labor-saving technologies faces substantial frictions in low-productivity settings (Caunedo and Kala, 2021).



(a) Perpetual growth

(b) Trapped in the middle

Figure 3: Dynamics under the middle-income trap.

Capital inefficiency γ provides another obstacle to adoption. A larger value of γ represents a greater share of invested capital lost to inadequate infrastructure, bureaucratic costs, regulatory compliance, corruption, and related frictions. These distortions can discourage entry and technology adoption. Liu and Luo (2024) provide evidence that corruption raises the effective cost of capital deployment and distorts resource allocation, while Shleifer and Vishny (1993) characterize corruption as a distortionary tax that increases the cost of investment and can discourage entry by technologically advanced firms. Campos et al. (2010) and Dastidar and Yano (2021) similarly show that corruption can restrict entry by protecting less efficient incumbents. If modern, capital-intensive firms are particularly exposed to these costs (Olimov et al., 2025), such distortions translate into a higher γ and a wider adoption gap. This mechanism is also related to the "triple challenge" identified by Andreoni and Tregenna (2020), whereby scaling advanced technologies requires institutional and complementary investments that may be difficult to undertake in weak institutional environments.

Consider next the demographic channel. Unlike the monotonic effect predicted by a standard OLG model, the interaction between the longevity motive and convex LTC costs produces a hump-shaped response of saving to survival. At low survival rates ($\phi < \overline{\phi}$), greater longevity increases saving because individuals place greater weight on consumption in old age. At high survival rates ($\phi > \overline{\phi}$), the PAYG tax $\rho\phi$ and the LTC premium $\delta\phi^2$ increasingly reduce disposable income, eventually dominating the longevity motive and lowering the primitive steady state.

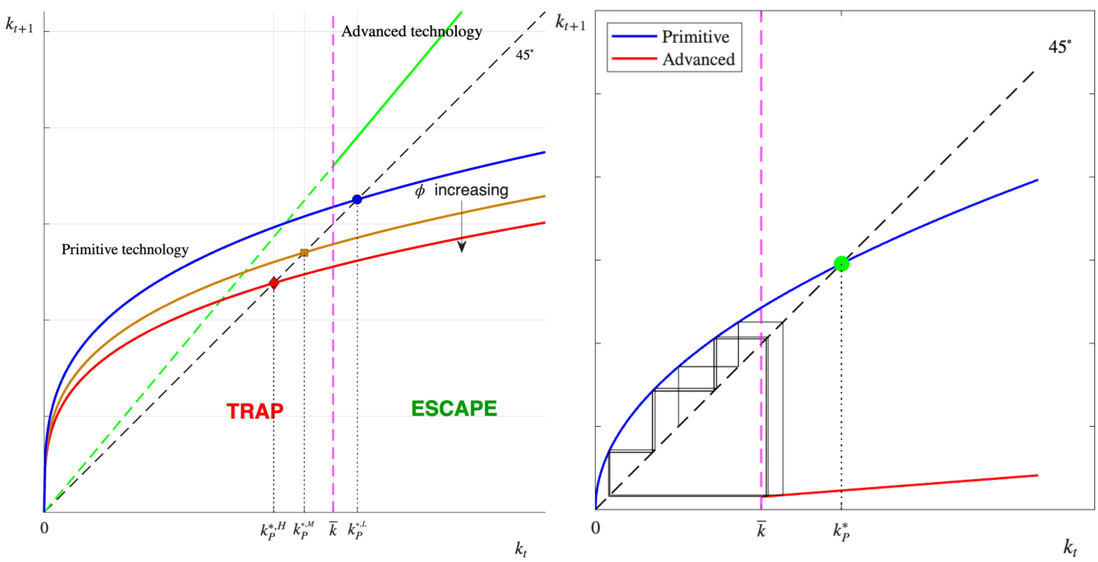
Corollary 1 (The Demographic Window with Hump-Shaped Savings)**.** *Let $\overline{\phi} \in (0,1)$ denote the unique peak of $k_P^*(\phi)$ defined implicitly by $Z'(\overline{\phi}) = 0$. Suppose the economy operates under the primitive technology and experiences a demographic transition in which the survival rate rises monotonically, ϕ_t*
*to*1*. Then:*

1. Opening phase ($\phi_t < \overline{\phi}$): *the quasi-steady state $k^*_{P,t}$ rises over time and the escape margin $\Phi_t \equiv k^*_{P,t} - \bar{k}$ widens. Early-stage aging therefore facilitates escape.*

2. Closing phase ($\phi_t > \overline{\phi}$): *the quasi-steady state $k^*_{P,t}$ falls over time and the escape margin shrinks monotonically toward $\Phi_{\min} \equiv k^*_P(1) - \bar{k}$.*

If the fully aged steady state satisfies $k^*_P(1) \leq \bar{k}$, the economy faces a closing demographic window. *It must accumulate sufficient capital to cross $\bar{k}$ before the survival rate reaches the escape threshold $\hat{\phi}$ satisfying $k^*_P(\hat{\phi}) = \bar{k}$. Economies that age before accumulating sufficient productive capital therefore remain in the primitive regime.*

Proof. See Appendix A.4. □



(a) Trapped by demographic aging

(b) Traps in permanent cycles

Figure 4: Different patterns of growth.

Corollary 1 describes formally the possibility that economies "grow old before becoming rich." In this setting, the demographic transition can initially support capital accumulation through higher saving, but its effect eventually reverses as pension obligations and LTC costs become more important (Salgado, 2017). The mechanism is consistent with the experience of several Asian economies, where the early stages of demographic transition coincided with rapid capital accumulation and industrialization, while subsequent increases in old-age dependency created additional pressure on saving and growth (Ramayandi, 2011). As Figure 4a illustrates, an economy facing an aging-induced trap must accumulate enough capital to cross $\bar{k}$ before demographic aging begins to reduce the primitive steady state.

The model also implies a two-sided relationship between competition and development. Greater competition in the advanced sector raises the labor share and reduces the residual return to capital, which can delay adoption. At the same time, insufficient competition can hinder the diffusion and adoption of advanced general-purpose technologies, as emphasized by Arezki et al. (2021), and product-market competition shapes labor-market outcomes and allocative efficiency in developing economies (Amodio, 2020). These observations are consistent with the model's distinction between the adoption and post-adoption stages. Some degree of monopoly rents is needed to compensate investors for the capital-inefficiency wedge γ, but competition must eventually be sufficiently strong for the advanced technology to sustain perpetual growth.

Figure 4 illustrates these two dynamic patterns associated with the middle-income trap. In Figure 4a, demographic aging can shift the primitive steady state below the adoption threshold, turning an economy that would otherwise escape the trap into one that remains in the primitive regime. As aging progresses, an economy that has not yet crossed $\bar{k}$ faces a declining escape margin. Figure 4b illustrates a different possibility: when saving is insufficient to sustain the advanced technology, the economy can alternate between the primitive and advanced technologies and remain near the adoption threshold rather than converging to a sustained growth path.

The model's structural channels yield testable predictions about an economy's likelihood of escaping the middle-income trap. The next section evaluates these predictions using panel data for 185 economies over 1950–2023.

5 Empirical Evidence

5.1 Testable hypotheses

As shown in the previous section, an economy remains under the primitive technology if the economy cannot sustain enough capital per worker to make adoption of the advanced technology profitable. Let $k^*_{P,i}$ denote the steady-state capital per worker under the primitive technology and $\bar{k}_i$ the threshold at which the advanced technology becomes profitable. The economy is trapped whenever

$$\Phi_i \equiv k^*_{P,i} - \bar{k}_i \leq 0. \tag{34}$$

Both terms have closed-form expressions in parameters that can be observed or proxied at the economy level. The model therefore delivers a quantitative prediction in addition to qualitative sign restrictions.

T1 (the margin). The calibrated escape margin $\widehat{\Phi}_{i,t}$ should predict which economy in the middle-income range subsequently graduate, beyond the information contained in current income.

T2 (the channels). The structural channels should enter the graduation hazard with the signs implied by the comparative statics in Section 4: human capital should raise the hazard, capital-use inefficiency should lower it, and capital per worker should have no independent effect. The demographic channel should act as a drag conditional on income, so that aging lowers the escape hazard primarily among economies that become old before they become rich.

T3 (the mechanism). The escape margin should operate through technology adoption. The test involves two links: (i) technology adoption should raise the graduation hazard, and (ii) the structural channels that predict graduation should also predict technology adoption, consistent with a mechanism in which structural conditions facilitate the adoption of advanced technology and thereby increase the probability of escape.

5.2 Data

We measure graduation to "high-income" status using two alternative definitions: one in absolute terms and the other in relative terms.

For the former, we adopt the World Bank's classification using a time-specific income level. Under the World Bank classification equivalence, an economy escapes in year t if it was middle income in $t-1$, becomes high income in t, and remains high income in $t+1$ and $t+2$. This persistence requirement reduces the likelihood that a temporary exchange-rate movement or commodity-price shock is classified as a structural escape. However, the official World Bank analytical classification is available only from 1987, which would exclude several early East Asian and Southern European episodes relevant to the theory. To overcome this shortcoming, we backcast the 1987 thresholds[4] back to 1962, with adjustments for international exchange and US CPI for each year. The backcast identifies the early technology-adoption episodes of Japan (1971), Spain (1974), Greece (1973), Ireland (1973), Hong Kong (1977), and Singapore (1979), and increases the number of first graduations from 32 to 46.

For the latter, we define an economy that escapes the middle-income trap when its GDP per capita at chained purchasing power parities (PPP) surpasses 45 percent of the contemporaneous US level.[5] The same persistence requirement applies. Under this definition, data are available from 1950, yielding 41 first escapes, and identifying Japan (1969), Spain (1970), Greece (1971), Hong Kong (1973), Singapore (1975), Ireland (1989), Korea (1993), and Portugal (1997) as early graduations.

From an empirical perspective, the discrepancies between these two classifications are only marginal and substantively negligible, so we treat the two as interchangeable. Hereafter, we refer to the sample based on the World Bank's classification as the World Bank sample, and the other as the Relative-income sample. We then further refine each by excluding microstates to produce a "clean" subsample. This is because microstates may cross an income threshold because of idiosyncratic factors, such as a small population, financial-services activity, or a single large investment that are not well captured by the structural mechanism examined here. Excluding these economies allows the analysis to focus on transitions more plausibly driven by the broad-based structural transformation emphasized in the model.

[4]Under the 1987 income classification, economies were categorized as low-income if their GNI per capita was up to $480, lower-middle-income if it ranged from $481 to $1,940, upper-middle-income if it ranged from $1,941 to $6,000, and high-income if it exceeded $6,000, with all thresholds expressed in current Atlas dollars.

[5]This relative threshold is commonly used in the middle-income-trap literature (Im and Rosenblatt, 2015; Glawe and Wagner, 2016; Robertson and Ye, 2016).

The escape margin is constructed from the closed-form expressions in Sections 2–4, evaluated separately for each economy under equation (32). Following Jones (2016), we set $\alpha = 0.33$, $b = 0.66$, and $\delta = 0.10$ for all economies. Fixing these three parameters is a normalization that serves two purposes. First, it pins down the scale of the model: since the escape margin $\widehat{\Phi}$ is a ratio of two endogenous quantities (k_P^* and $\bar{k}$), holding α, b, and δ common across economies ensures that differences in the margin reflect genuine differences in structural conditions rather than parameter-driven scaling. Second, these parameters are not the structural channels of interest. The model's testable predictions concern human capital (h), capital inefficiency (γ), and demographic aging (ϕ). Fixing them allows the four structural channels to vary freely from the data, which is the empirically relevant variation. The approach of fixing technology and preference parameters while allowing structural channels to vary is not uncommon (Caselli, 2005; Hsieh et al., 2019; Buera and Kaboski, 2012). A robustness check that varies these parameters within plausible ranges is reported in Appendix B.

The margin is expressed in logarithmic form, $\widehat{\Phi} = \ln(k_P^*/\bar{k})$, because the level difference becomes unbounded if the denominator of $\bar{k}$ approaches zero. We set the aggregate scale to $N = 1$, since allowing N to vary would make the margin partly a measure of country size. The discount factor $\beta_{i,t}$ is not imposed commonly; instead, it is back-calculated from each economy's observed saving rate using the model's Euler equation, so that $\beta_{i,t}$ absorbs cross-country and time variation in effective time preference, financial development, and other determinants of saving behavior.

In the model, the sign of Φ separates trapped economies ($\Phi \leq 0$) from those that can escape ($\Phi > 0$), so in principle the margin could serve as a binary classifier. In practice, however, the calibrated values may not respect this threshold. Due to normalization and parameter calibration, the economy-year in the sample turns out to lie at or below zero, including economies that later graduated. The level of $\widehat{\Phi}$ therefore carries no information about whether an economy has actually crossed the adoption threshold. What does carry information is the *ordering* of economies by $\widehat{\Phi}$, with higher values indicating greater proximity to escape. We accordingly treat the margin as a continuous, ordinal ranking of escape readiness rather than as a binary trapped/not-trapped label.

The structural channels are measured as five-year trailing means of their own lags. Human capital is the Penn World Table human-capital index, which combines average years of schooling from Barro and Lee (2013) with estimated private returns to schooling from Psacharopoulos (1994). The index is intended to capture the labor-augmenting component of the workforce rather than years of schooling alone, and is widely used for human-capital measurement in development accounting (Caselli, 2005). Capital per worker is the log of the real capital stock per person engaged.

Capital-use inefficiency is proxied by the Fraser Institute's Economic Freedom of the World index, transformed as $\gamma = (10 - \text{EFW})/10$ so that larger values indicate greater inefficiency. Because the index is reported quinquennially before 2000, each observation is carried forward for at most four years. This treatment provides the widest institutional coverage. In the model, γ captures the share of invested capital lost to unproductive uses, including bureaucratic costs, corruption, and poor infrastructure. Institutional quality provides an observable counterpart to this mechanism through its effects on property rights, the rule of law, regulation, and corruption (Knack and Keefer, 1995; Hall and Jones, 1999; Acemoglu et al., 2001). The Fraser index is a suitable proxy for several reasons. First, its subcomponents (property rights, sound money, regulatory burden, and financial freedom) directly capture the institutional determinants of capital allocation efficiency: strong property rights ensure investors can capture returns, sound money reduces price distortions that misguide investment, low regulatory burden lowers bureaucratic costs, and financial freedom enables intermediation that channels capital to productive uses (Levine, 2005; Rajan and Zingales, 1998). Second, to the best of our knowledge, there is no alternative measure that can provide a comparable combination of broad cross-country coverage, a long time horizon, and detailed information on institutional quality. Firm-level dispersion measures of capital misallocation (Hsieh and Klenow, 2009) are available only for a narrow set of countries and years, while aggregate financial-development indicators (Levine, 2005) capture intermediation depth but not the institutional frictions that waste invested capital.

The demographic channel operates in the model through the pension tax and long-term-care premium paid by the young to support the old. Although the model expresses this burden in terms of the survival probability, the economic quantity of interest is the number of elderly supported by the working-age population. We therefore measure population aging using the old-age dependency ratio, defined as the population aged 65 and over relative to the population aged 15–64.

This choice has three advantages. First, the fiscal and care burden faced by workers depends directly on the number of elderly they support, making the dependency ratio more closely related to the mechanism than the survival rate. Second, the survival parameter can be interpreted as a dependency ratio in the model: with a constant population and equal-sized cohorts, it corresponds to the number of elderly supported by each young worker. The empirical dependency ratio additionally captures variation in cohort size and fertility that the model abstracts from. Third, the dependency ratio has substantially more cross-country variation than the survival probability, which is bounded above by one and varies relatively little across economies with life expectancies in the 60–85 year range.

To give the aging variable an economically interpretable zero, we anchor it to a graduation-aging frontier, defined as the mean old-age dependency ratio among economies in the year they crossed the relevant threshold. The frontier is 15.7 elderly per 100 working-age people for the World Bank graduates and 15.0 for the relative-income graduates. The resulting regressor is the net of country-specific old age dependency ratio and the graduation-aging frontier, so a positive value indicates that an economy is older than the typical economy at the time of graduation. Because the theoretical demographic effect depends on income, we interact this aging gap with income proximity.

The technology mechanism is examined using two measures, each entered as a five-year trailing mean. The legacy economic-complexity index (Hidalgo and Hausmann, 2009), available from 1964, captures the productive capabilities reflected in a country's export structure and therefore its capacity to adopt and operate more complex technologies. Manufactured-plus- machinery imports, defined as SITC revision

2 sections 6 and 7 and available from 1962, provide a broader measure of embodied technology.[6]

Each regressor is measured as a five-year trailing mean of its own lags, using observations from $t-5$ to $t-1$ and requiring at least three of the five observations. This construction ensures that the regressors are strictly predetermined relative to an event at t.

5.3 Descriptive statistics

Table 1 reports summary statistics for the two hazard estimation samples.

Table 1: Summary statistics, hazard estimation samples

	World Bank					Relative-income				
Variable	Obs	Mean	S.d.	Min	Max	Obs	Mean	S.d.	Min	Max
Old-age dependency[a]	1,310	12.17	6.19	3.23	35.28	2,957	10.29	5.89	2.58	35.28
Aging gap[b]	1,310	−4.08	5.84	−12.43	17.93	2,957	−5.11	5.52	−12.66	18.64
Log capital per worker[c]	1,310	11.75	0.70	8.98	13.36	2,957	11.39	0.84	7.60	13.36
Human capital[d]	1,310	2.37	0.52	1.11	3.60	2,957	2.13	0.59	1.01	3.48
Labor share[e]	1,310	0.49	0.12	0.15	0.87	2,957	0.50	0.11	0.14	0.87
Capital-use inefficiency[f]	1,143	0.38	0.11	0.10	0.72	2,331	0.40	0.11	0.11	0.78
Escape margin[g]	1,052	−4.49	1.43	−11.52	−1.93	2,000	−5.01	1.71	−13.11	−1.96
Log income proximity[h]	1,310	8.12	0.81	6.14	9.44	2,957	−1.71	0.48	−2.84	−0.42
Spell duration[i]	1,310	10.2	7.5	1	42	2,957	29.4	19.5	1	74

Notes: [a] Elderly per 100 working-age people; annual value. [b] Old-age dependency net of the graduation-aging frontier, 15.7 elderly per 100 working-age people for the World Bank sample and 15.0 for the relative-income sample; a positive value indicates an economy older than the typical economy at the time of graduation. [c] Log 2017 USD. [d] Penn World Table human-capital index. [e] Share of labor income in GDP. [f] $(10 - \text{EFW})/10$, where EFW is the Fraser Economic Freedom of the World index, so that higher values denote greater capital-use inefficiency. The index is published at five-year intervals before 2000 and is carried forward up to four years between waves, which is why its coverage is narrower than that of the other channels. [g] $\widehat{\Phi} = \ln(k_p^*/\bar{k})$ evaluated at the baseline calibration $\alpha = 0.33$, $b = 0.66$, $\delta = 0.10$. The margin requires the full set of calibrated inputs, so it is observed on fewer economy-years than the channels from which it is built. It is used as an ordinal ranking of economies rather than as a binary classifier, so its level carries no separate interpretation. [h] Log Atlas GNI per capita (World Bank) or log GDP per capita relative to the US (relative-income). [i] Years spent in the current middle-income spell. All variables except the old-age dependency ratio are five-year trailing means.

The two samples differ in both composition and size. The relative-income sample

[6] The international trade literature highlights both exports and imports as key channels of technological upgrading. On the export side, interactions with foreign buyers can generate learning and knowledge transfer, while improved access to foreign markets can increase firms' incentives to adopt more advanced technologies (Lileeva and Trefler, 2010; Bustos, 2011; Atkin et al., 2017). On the import side, capital goods, machinery, and intermediate inputs can embody foreign knowledge and technologies, providing an important channel through which technologies developed in advanced economies diffuse to developing countries (Coe et al., 1997; Eaton and Kortum, 2001; Keller, 2004; Caselli and Wilson, 2004; Almeida and Fernandes, 2008; Goldberg et al., 2010; Topalova and Khandelwal, 2011; Le and Tomasi, 2023; Tomasi and Le, 2026).

contains 2,957 economy-years compared with 1,310 in the World Bank sample. The difference reflects the earlier availability of the relative-income definition, which begins in 1950, compared with 1962 for the backcast World Bank classification, as well as the wider relative-income middle band, defined as 8–45 percent of the US income level. Spell durations are correspondingly longer in the relative-income sample, averaging 29.4 years compared with 10.2 years in the World Bank sample. Human capital is also somewhat lower in the relative-income sample, averaging 2.13 compared with 2.37 in the World Bank sample, consistent with the latter covering a higher-income risk set.

The variation in the structural channels is relevant for their empirical identification. The old-age dependency ratio is substantially more dispersed than the survival probability it replaces, with a coefficient of variation above 0.5 compared with 0.04. Measured against the graduation-aging frontier, the average economy is younger than the typical graduate in both samples, by 4.1 points of dependency under the World Bank definition and 5.1 under the relative-income definition. The gap nonetheless ranges from about -12 to $+18$, so both samples contain a substantial number of economy-years well past the frontier, which is what identifies the aging interaction in Panel C of Table 5. The labor share ranges from 0.15 to 0.87, with a mean close to 0.5. Since the model's growth turning point occurs at a labor share of 0.586, much of the sample lies below the turning point, where the partial effect is relatively small and need not have a common sign. Capital-use inefficiency averages 0.38 in the World Bank sample and 0.40 in the relative-income sample, on a scale where higher values denote greater inefficiency, and has the most limited coverage of the four channels, particularly in the earlier years, which reduces the sample available for Panel B of Table 5. The escape margin is restricted for the same reason: it is observed on 1,052 World Bank and 2,000 relative-income economy-years, since it requires every calibrated input to be present simultaneously.

Table 2 pools the backcast World Bank classification into three groups and reports transition probabilities over five-, ten-, and twenty-year horizons.

The transition matrix indicates substantial persistence in the middle-income group. Over five years, an economy in the middle-income group moves to high income with probability 0.056, compared with an upward transition probability of 0.151 from low income to middle income. The implied expected residence is 46 years in middle income, compared with 33 years in low income. The middle-income group is persistent but not absorbing: its diagonal probability falls from 0.891 at five years to 0.812 at ten years and 0.707 at twenty years. Over twenty years, a middle-income economy is only somewhat more likely to graduate (0.159) than to return to low income (0.134). High income is considerably more persistent, with a probability above 0.96 of remaining high income at each horizon. The probability of graduation rises from 5.6 percent over five years to 10.3 percent over ten years and 15.9 percent over twenty years, which is consistent with a relatively stable annual exit hazard.

Table 2: Income-Group Transition Probabilities

Group at t	Group at $t+h$: Low	Middle	High
Panel A: Five-year horizon	$n = 8{,}454$		
Low income	0.848	0.151	0.001
Middle income	0.053	0.891	0.056
High income	0.000	0.028	0.972
Panel B: Ten-year horizon	$n = 7{,}547$		
Low income	0.747	0.251	0.002
Middle income	0.085	0.812	0.103
High income	0.000	0.037	0.963
Panel C: Twenty-year horizon	$n = 5{,}758$		
Low income	0.644	0.348	0.009
Middle income	0.134	0.707	0.159
High income	0.000	0.027	0.973

Notes: The World Bank classification (backcast to 1962) is pooled into low, middle, and high income. Each cell reports the probability of moving from the row group to the column group over the indicated horizon. Transitions are overlapping annual observations. Expected residence, calculated from the five-year transition matrix, is 33 years in low income, 46 years in middle income, and 176 years in high income.

5.4 Empirical specifications

5.4.1 The graduation hazard

To test T2 and T3, we estimate a discrete-time hazard model of graduation. The model considers, for each year an economy remains in the middle-income group, the probability that it crosses the relevant high-income threshold in that year. Since the middle-income trap concerns the probability of exit rather than growth conditional on remaining in the middle-income range, the hazard is the natural object to estimate. The dependent variable is a binary indicator $\mathrm{esc}_{i,t} \in \{0,1\}$, equal to one in the first year in which economy i graduates under the relevant definition and zero in all other years in which it remains at risk. We use a complementary log-log link:

$$\Pr(\mathrm{esc}_{i,t} = 1 \mid \text{at risk}) = 1 - \exp\Big[-\exp\Big(\alpha_0 + \boldsymbol{\beta}'\mathbf{Z}_{i,t-1} + \delta_1 \mathrm{dur}_{i,t} + \delta_2 \mathrm{dur}_{i,t}^2 + \theta\, \mathrm{trend}_t\Big)\Big]. \tag{35}$$

Here, $\mathbf{Z}_{i,t-1}$ contains the standardized structural channels (Panels A and B), plus log income proximity and its interaction with the aging gap (Panel C). The spell duration $\mathrm{dur}_{i,t}$, measured in years since the economy entered the upper-middle-income range, enters as a quadratic $\delta_1 \mathrm{dur}_{i,t} + \delta_2 \mathrm{dur}_{i,t}^2$ to allow the hazard to change non-monotonically with time spent in the range: a positive δ_1 combined with a negative δ_2 would indicate that the graduation hazard first rises and then falls, while a negative δ_1 with a positive δ_2 would imply the opposite pattern. Entering duration only in linear form would impose a monotonic relationship between spell length and the hazard, which is unlikely to hold if economies face different pressures at early versus late stages of a middle-income spell. The calendar trend trend_t is measured in decades. The complementary log-log link corresponds to a discrete-time proportional hazards model. Since the structural variables are standardized within each estimation sample, their coefficients measure the change in the log hazard associated with a one-standard-deviation change in the corresponding variable.

The comparative statics in Section 4 determine the expected signs. Human capital should increase the graduation hazard because it lowers the adoption threshold, while capital-use inefficiency should reduce it because it raises the threshold. The prediction for capital per worker is more restrictive: because k does not enter Φ, its coefficient should be zero. The demographic effect is expected to depend on income and is examined separately in Section 5.5.3.

The World Bank outcome uses the backcast classification extending to 1962. Without the backcast, the sample would exclude several early East Asian and Southern European graduations that are central to the mechanism. The estimates are nevertheless limited by the small number of events: there are 34 World Bank events and 26 relative-income events in the complete samples, against seven or eight parameters in the corresponding specifications. The results should therefore be interpreted as evidence on the consistency of the data with the model rather than as precise estimates of structural effects.

We evaluate the escape margin over longer horizons because it is derived from steady-state conditions. Specifically, we ask whether an economy graduates within the following ten or twenty years and compare the area under the receiver operating characteristic (ROC) curve for three models: income alone, income together with the margin, and income together with the five structural channels. The ROC area measures the ability of a model to rank economies by their subsequent escape risk; 0.5 corresponds to random ranking and 1 to perfect discrimination. If the margin contains no information beyond income, including it should not improve the ranking.

5.4.2 The technology mechanism

T3 examines whether the relationship between the escape margin and graduation operates through technology adoption. The mechanism involves two links. The first asks whether technology predicts graduation (Table 6):

$$\Pr(\text{esc}_{i,t} = 1 \mid \text{at risk}) = 1 - \exp\Big[- \exp\big(\alpha_0 + \beta_{\text{tech}} T_{i,t-1} + \delta_1 \text{dur}_{i,t} + \delta_2 \text{dur}_{i,t}^2 + \theta \, \text{trend}_t\big)\Big]. \tag{36}$$

Here, $T_{i,t-1}$ denotes one of the two technology measures, each measured as a five-year trailing mean.

The second link asks whether the structural channels that predict graduation also predict technology adoption (Table 7). If the structural conditions affect graduation through technology, then structural conditions should be (highly) associated with technology adoption:

$$T_{i,t} = \alpha_0 + \sum_j \beta_j Z_{j,i,t-1} + \beta_\gamma \gamma_{i,t-1} + \beta_{\text{aging}} \text{aging}_{i,t-1} + \delta_1 \text{dur}_{i,t} + \delta_2 \text{dur}_{i,t}^2 + \theta\, \text{trend}_t + \varepsilon_{i,t}, \tag{37}$$

where $Z_{j,i,t-1}$ are the structural channels (human capital, capital per worker, labor share), $\gamma_{i,t-1}$ is capital-use inefficiency (inverted Fraser institutional quality index), $\text{aging}_{i,t-1}$ is the demographic aging gap (old-age dependency minus the graduation-aging benchmark), $\text{dur}_{i,t}$ is the spell duration in the upper-middle-income range (with a quadratic), and trend_t is a linear calendar trend $(\text{year} - 2000)/10$. We expect $\beta_{\text{hc}} > 0$, $\beta_{\text{labsh}} < 0$, and $\beta_\gamma < 0$ (higher inefficiency impedes technology adoption).[7]

Standard errors are clustered by economy to allow for dependence across successive observations of the same economy. With only 26 to 46 events, however, the cluster-robust variance estimator is based on a small number of events, so the statistical inference is treated as indicative. The events-per-parameter ratio is reported alongside each hazard specification. More generally, neither $\widehat{\Phi}$ nor the individual structural channels are subject to exogenous variation, and the number of escape events is limited. The empirical analysis therefore focuses on whether the predicted relationships appear consistently across outcomes, samples, and specifications rather than on identifying causal effects.

5.5 Results

5.5.1 Escape is rare

Table 3 reports the number of escape episodes and the resulting estimation samples. It is clear that escape is uncommon under both definitions.

Across seven decades and 185 economies, there are only 46 persistent graduations according to the World Bank's classification, or 41, if based on the relative-income threshold. The backcast is important for the World Bank sample: 18 of the 46 graduations occur before 1987. Without the backcast, the event set would be substantially smaller. Excluding microstates reduces the number of World Bank and relative-income graduations to 36 and 31, respectively, and excluding resource exporters as well reduces them to 30 and 24.

The composition of the episodes also matters for interpreting the results. The World Bank classification includes European accessions and resource-related reclassifications in addition to the technology-adoption episodes emphasized by the model. The early cohort recovered by the backcast, including Japan, Spain, Greece, Ireland, Hong Kong, and Singapore, is more closely aligned with the mechanism and would be absent from a classification beginning in 1987. The relative-income definition is closer to the model's focus because it measures an economy's position relative to the frontier, although it also contains microstates and resource exporters. The complete-case samples contain 34 and 26 events, falling to 30 and 23 after excluding microstates.

[7] We do not interpret the two links as a formal mechanism test, since technology adoption and income transitions are jointly determined and the empirical design provides no instrument for adoption.

Table 3: Escape episodes and estimation samples

	World Bank (backcast)	Relative to the US
Panel A. Universe of upper-middle-income spells		
Economy-years at risk[a]	1,912	5,151
Distinct economies at risk	102	143
Panel B. Escape episodes (before data requirements)		
First-time graduates[b]	46	41
dropping microstates	36	31
also dropping resource exporters	30	24
Panel C. Estimation samples (covariates required)		
Complete-case economy-years[c]	1,310	2,957
Escape episodes in complete case	34	26
dropping microstates	30	23

Notes: [a] An economy-year is in the risk set when the economy is classified as upper-middle income (World Bank) or between 8–45% of US income (relative) and has not yet graduated. [b] First persistent graduation (three years above threshold), before any covariate requirement. [c] Economy-years where all structural channels are observed. Panel A shows the full universe; Panel B restricts to first-time escape episodes; Panel C further restricts to the complete case. Of the 46 World Bank first graduates, 18 occur before 1987 and are recovered by the backcast (early technology-adoption episodes: Japan, Spain, Greece, Ireland, Hong Kong, Singapore, among others).

The limited events-per-parameter ratios therefore remain an important consideration when interpreting the estimates.

5.5.2 Predictability

Table 4 reports the area under the receiver operating characteristic (ROC) curve for three models evaluated over ten- and twenty-year horizons for each outcome.[8]

For the relative-income outcome, the margin significantly improves the ranking of subsequent escapes at both horizons (Table 4, last column). At ten years, the ROC area increases from 0.875 with income alone to 0.908 when the margin is added, while the specification with all structural channels reaches 0.926. Thus, the margin captures a substantial part of the predictive information contained in the five channels. At twenty years, the ROC area increases from 0.811 to 0.856. The two horizons use different samples, with 1,501 and 973 observations, respectively, so the similarity of the results does not arise mechanically from using the same observations.

The results are different for the World Bank outcome. Although all models significantly outperform random ranking, adding the margin to income raises the ROC area by only 0.016 at ten years and 0.019 at twenty years. The five-channel specification, however, has substantially higher discrimination, with ROC areas of 0.893 and 0.947.

[8]The ROC curve plots the true-positive rate against the false-positive rate for every possible classification threshold. The area under the curve is the probability that a randomly drawn escape episode is ranked above a randomly drawn non-escape episode. A value of 0.5 corresponds to a ranking no better than chance, while a value of 1 corresponds to perfect ranking.

Table 4: Predictive power: area under the ROC curve

		Model			
Outcome, horizon	n	(1) Income	(2) Income + margin	(3) Income + channels	p-value[a]
Relative income, 10 yr	1,501	0.875***	0.908***	0.926***	0.004
Relative income, 20 yr	973	0.811***	0.856***	0.886***	0.014
World Bank, 10 yr	576	0.691***	0.707***	0.893***	0.089
World Bank, 20 yr	281	0.707***	0.726***	0.947***	0.383

Notes: Each row reports the area under the receiver operating characteristic (ROC) curve for a logit model predicting graduation within H years. Model (1) uses income alone. Model (2) adds the calibrated escape margin $\widehat{\Phi}$. Model (3) replaces the margin with the five structural channels (human capital, capital per worker, the labor share, the old-age dependency ratio, and capital-use inefficiency) entered freely. Each row is estimated on a common sample. The ROC area measures the ability of a model to rank economies by their subsequent escape risk; 0.5 corresponds to random ranking and 1 to perfect discrimination. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$: each model's AUC tested against 0.5 (random ranking). [a] p-value from a clustered Wald test of the margin coefficient in Model (2); tests whether the margin adds predictive information beyond income alone.

Thus, the individual structural channels contain predictive information about World Bank graduation even though their calibrated combination does not improve the ranking by much. One interpretation is that the relative-income threshold corresponds more closely to the frontier-based object captured by the escape margin, whereas the World Bank classification also reflects administrative revisions and episodes that are not represented in the model.

5.5.3 Factors affecting the ability to escape

Table 5 reports the graduation-hazard estimates for the World Bank and relative-income outcomes, using both the full complete-case samples and the clean samples that exclude microstates. Panel A contains the core structural channels, Panel B reports the capital-use inefficiency estimates on its separate institutional sample, and Panel C examines the demographic interaction.

The calendar trend is negative and significant in all four specifications, with coefficients of -0.665 and -0.774 in the World Bank sample and -0.739 and -0.747 in the relative-income sample. Holding structural channels constant, the baseline hazard of graduation has been declining over time: economies with the same human capital, capital per worker, and institutional quality are less likely to graduate in later decades than in earlier ones. More generally, as high-income economies grow, the absolute income threshold for graduation rises, requiring middle-income economies to grow faster merely to keep pace. The negative trend may also capture unmeasured global headwinds, such as slowing technology diffusion or rising competition from other middle-income economies, that make escape harder over time, independent of the channels we measure. Because all structural variables are standardized within sample, the trend absorbs residual time variation not explained by the channels.

Human capital enters positively in all four specifications, with coefficients of 0.905, 1.087 for the World Bank sample, and 1.305, and 1.400 for the relative income sample.

Table 5: Discrete-time hazard of persistent graduation (complementary log-log)

	World Bank		**Relative to US**	
	(1) All	(2) Clean	(3) All	(4) Clean
Panel A. Core structural channels				
Human capital	0.905***	1.087***	1.305***	1.400***
	(0.247)	(0.273)	(0.381)	(0.448)
Log capital per worker	1.096***	1.068***	0.900***	0.991***
	(0.197)	(0.302)	(0.234)	(0.262)
Labor share	0.256	0.262	−0.190	−0.349
	(0.205)	(0.248)	(0.259)	(0.270)
Spell duration	0.108	0.155	−0.021	0.008
	(0.092)	(0.103)	(0.044)	(0.047)
Spell duration^2	−0.002	−0.003	0.000	−0.000
	(0.003)	(0.003)	(0.001)	(0.001)
Calendar trend	−0.665***	−0.774***	−0.739***	−0.747***
	(0.178)	(0.185)	(0.212)	(0.253)
Observations	1,310	1,215	2,957	2,866
Events	34	30	26	23
Panel B. Capital-use inefficiency, separate fixed samples				
(i) Parsimonious				
Capital-use inefficiency	−1.004***	−1.036***	−1.688***	−1.835***
	(0.209)	(0.233)	(0.299)	(0.359)
(ii) Conditioning on the other three channels				
Capital-use inefficiency	−1.082***	−1.120***	−1.314***	−1.450***
	(0.196)	(0.231)	(0.290)	(0.339)
Human capital	0.514**	0.570**	0.949**	0.982*
	(0.252)	(0.282)	(0.477)	(0.541)
Log capital per worker	1.330***	1.447***	1.021***	1.047***
	(0.245)	(0.339)	(0.313)	(0.331)
Labor share	0.370*	0.296	−0.388	−0.444
	(0.199)	(0.211)	(0.268)	(0.305)
Observations	1,143	1,064	2,331	2,263
Events	26	23	21	19
Panel C. Aging drag conditional on income				
Aging gap	0.491**	0.521***	−2.536***	−2.729**
	(0.193)	(0.183)	(0.982)	(1.063)
Log income proximity	3.328***	3.783***	5.004***	5.238***
	(0.483)	(0.454)	(0.822)	(0.905)
Aging gap × log income proximity	0.616***	0.598***	1.569***	1.780***
	(0.188)	(0.175)	(0.513)	(0.581)
Observations	1,310	1,215	2,957	2,866
Events	34	30	26	23

Notes: The dependent variable is a binary indicator for graduation from the middle-income trap (World Bank: backcast upper-middle-income escape; relative-income: escape above 45% of U.S. income). Complementary log-log coefficients. Structural variables are standardized within each estimation sample, so that the hazard coefficients represent the effect of a one-standard-deviation change; a calendar trend is included in all columns. Standard errors clustered by economy in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Panels A and B condition on the structural channels only and do not include income proximity. Panel B is estimated on a smaller sample because the Fraser institutional-quality index has narrower historical coverage. In all panels the World Bank columns use the backcast classification to 1962; the clean columns exclude microstates. Panel C enters the aging gap, log income proximity, and their interaction; a positive interaction indicates that the aging gap is less of a drag for economies closer to the income threshold.

The estimates are statistically significant across the four samples and remain stable in sign when microstates are excluded. A one-standard-deviation increase in the human-capital index corresponds to a hazard ratio of $\exp(0.905) \approx 2.5$ in the World Bank sample and $\exp(1.305) \approx 3.7$ in the relative-income sample. Thus, the estimated annual graduation hazard is substantially higher for economies with greater human capital, conditional on the other controls. This is consistent with the model because human capital lowers the capital threshold at which adoption of the advanced technology becomes profitable.

Capital per worker is positive and significant in all four specifications. Since capital per worker does not enter the escape margin, the model predicts no independent effect once the structural channels and income are accounted for. The positive coefficients may reflect the role of capital accumulation in supporting technology adoption beyond what is captured by the escape margin.

Capital-use inefficiency is negatively and significantly associated with graduation in the institutional sample. The parsimonious estimates are -1.004 for the World Bank outcome and -1.688 for the relative-income outcome. Conditioning on the other channels changes them only modestly, to -1.082 and -1.314, respectively. A one-standard-deviation increase in inefficiency corresponds to hazard ratios of approximately $\exp(-1.004) \approx 0.37$ and $\exp(-1.688) \approx 0.19$. The negative association is consistent with the model's prediction that greater capital-use inefficiency raises the adoption threshold. The estimates are based on separate samples for the World Bank (outcomes: 26 and 23) and relative-income (outcomes: 21 and 19) samples, reflecting the limited historical coverage of the Fraser index.

The labor share is not statistically significant in any of the four specifications (coefficients: 0.256, 0.262, -0.190, -0.349). This insignificance is expected on two grounds. First, the model implies a non-monotonic relationship: the growth condition depends on whether the labor share lies on either side of an interior turning point (0.586), and since the sample mean is close to 0.5, the marginal effect is relatively small for most observations. Second, the labor share captures only the *level* of competition, whereas the model's competition channel operates through the interaction of λ with other structural parameters—an effect that is difficult to isolate empirically with a single linear proxy. non-monotonic prediction and the inherent difficulty of measuring competition's multidimensional effect with the labor share alone.

Spell duration is statistically insignificant across all four specifications. The duration variable counts the number of years since the economy entered the upper-middle-income range. The insignificance of duration implies that the hazard of graduation does not increase or decrease with time spent in the middle-income range. This finding contrasts with the growth-traversing hypothesis arguing that middle-income economies gradually progress through the income range over time. If duration really matters, we would expect a positive coefficient indicating that longer time in the range leads to eventual graduation. Its insignificance here implies that graduation depends on structural conditions as we have discussed, not on the mere passage of time.

The demographic estimates are consistent with an effect that depends on income. Corollary 1 predicts that aging can initially support saving but eventually increase the burden associated with pensions and long-term care, reducing the capital available for technology adoption. Panel C tests this prediction by interacting the distance to the graduation-aging frontier with log income proximity. The interaction is positive and significant in all four specifications, confirming that aging pressure is moderated by where an economy stands in its development trajectory.

For economies that have accumulated sufficient capital (those closer to or above the income threshold), aging can actually support growth by boosting savings rates and reducing the dependency ratio through a larger working-age population. But for economies that have not yet accumulated enough capital to escape the trap, aging is a drag—it reduces the productive labor force while increasing the burden of pensions and long-term care, without the savings base to compensate. The positive interaction means the aging gap becomes less restrictive as economies approach the income threshold, exactly as the model predicts.

The coefficient on the aging gap itself differs across the two outcomes: it is positive in the World Bank specifications and negative in the relative-income specifications. This difference does not alter the conditional relationship implied by the interaction. In the relative-income sample, an economy one standard deviation above the aging frontier has a graduation hazard that is approximately 4.1 log points lower when income is one standard deviation below the mean, compared with about 1.0 log point lower when income is one standard deviation above the mean. The estimates therefore support the prediction that aging is more restrictive for economies that become old before they reach higher income levels—a pattern particularly relevant for the "growing old before growing rich" challenge facing several middle-income economies (Bloom et al., 2010; Van Treeck and Sminia, 2014; Tran, 2022).

5.6 Mechanism

T3 concerns the role of technology adoption in the relationship between the structural conditions and graduation. The model implies that an economy escapes when its conditions make adoption of the advanced, capital-intensive technology worthwhile. Technology adoption should therefore be positively associated with graduation, and the structural coefficients should become smaller when technology is included in the hazard.

The technology series begin in 1962 for manufactured and machinery imports and 1964 for the complexity index. Both outcomes are reported, though the World Bank outcome covers more graduation episodes within these start dates.

Table 6 reports the first link, with each technology measure entered separately alongside spell duration and a calendar trend. Both measures are positively associated with graduation. The legacy economic-complexity index has a coefficient of 1.412, significant at the 1 percent level, and manufactured-plus-machinery imports have a coefficient of 0.014, significant at the 5 percent level. The measures cover different periods and different sets of events: the complexity index begins in 1964 and covers 27 events, while the SITC 6+7 measure begins in 1962 and covers 42 events. The positive association is therefore present across two measures constructed from different data sources and covering different time periods.

The mechanism linking structural conditions to graduation operates through tech-

Table 6: Technology adoption and the graduation hazard

	World Bank		Relative income	
	Legacy complexity	**SITC 6+7 imports**	**Legacy complexity**	**SITC 6+7 imports**
Technology measure	1.412***	0.014**	2.052***	0.022***
	(0.284)	(0.007)	(0.472)	(0.007)
Observations	1,174	1,768	2,611	3,534
Events	27	42	23	29

Notes: The dependent variable is a binary indicator for graduation from the middle-income trap (World Bank: backcast upper-middle-income escape; relative-income: escape above 45% of U.S. income). Complementary log-log coefficients of each technology measure entered one at a time alongside spell duration and a linear calendar trend, on the World Bank risk set (backcast classification) and the relative-income risk set. The technology variables are five-year trailing means. [a] Revealed productive capability from export structure; available from 1964. [b] Manufactured goods plus machinery and transport equipment; available from 1962. Country-clustered standard errors in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

nology adoption. Table 7 tests this link by regressing each technology proxy on the structural channels, capital-use inefficiency, the demographic aging gap, controlling for spell duration and a calendar trend. The human-capital coefficient is positive and significant for the complexity index and marginally significant for SITC 6+7. Capital-use inefficiency is negative and significant for SITC 6+7 and insignificant for the complexity index. The aging gap is insignificant in both specifications, except for the relative-income sample with complexity as the dependent variable. The results confirm that economies with stronger structural conditions— higher human capital, lower institutional inefficiency—adopt more advanced technology, consistent with the model's mechanism.

Combining Tables 6 and 7, the evidence is consistent with the mechanism in the model. The structural channels predict technology adoption (Table 7), and technology adoption predicts graduation (Table 6). Human capital is the strongest predictor in both links: it is positively and significantly associated with technology adoption in both samples, and with the probability of graduation. The legacy complexity index has a coefficient of 1.41 (World Bank) and 2.05 (relative income), both significant at the 1 percent level. Manufactured-plus- machinery imports have a coefficient of 0.014 (World Bank) and 0.022 (relative income), significant at the 5 and 1 percent levels, respectively. The mechanism is strongest in the World Bank sample, where human capital and capital-use inefficiency are significant predictors of both technology adoption and graduation. In the relative-income sample, the structural channels predict technology adoption with similar signs and magnitudes, and the technology measures are significant predictors of graduation, though the number of events is smaller. We do not interpret the evidence as a formal mediation test, because technology adoption and income transitions are jointly determined and the empirical design provides no instrument for adoption.

Table 7: Structural channels and technology adoption

	World Bank		Relative income	
	Legacy complexity	**SITC 6+7 imports**	**Legacy complexity**	**SITC 6+7 imports**
Human capital	0.496***	0.291*	0.392**	0.359**
	(0.180)	(0.156)	(0.153)	(0.141)
Log capital per worker	0.119	0.058	0.142	−0.000
	(0.081)	(0.055)	(0.130)	(0.112)
Labor share	0.173	−0.195**	0.089	−0.141
	(0.156)	(0.092)	(0.123)	(0.109)
Capital inefficiency	−1.356	−3.862***	−0.717	−3.394***
	(0.959)	(1.179)	(0.888)	(1.088)
Aging gap	0.007	−0.014	0.040**	−0.028
	(0.019)	(0.024)	(0.018)	(0.017)
Calendar trend	−0.294***	−0.248**	−0.302***	−0.234*
	(0.083)	(0.105)	(0.094)	(0.124)
Observations	914	1,132	807	1,048
R^2	0.367	0.215	0.492	0.209

Notes: OLS coefficients. The dependent variable is the five-year trailing mean of each technology proxy (legacy economic-complexity index or SITC 6+7 manufactured-plus-machinery imports). World Bank columns use the backcast risk set; relative-income columns use the relative-income risk set. All regressors are five-year trailing means: structural channels (human capital, capital per worker, labor share), capital-use inefficiency (Fraser institutional quality index, carried forward), and the aging gap (old-age dependency minus the graduation-aging benchmark). Spell duration, its square, and a linear calendar trend are included as controls but not reported. Standard errors clustered by economy in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

6 Concluding Remarks

The middle-income trap can emerge when structural constraints impede the adoption and effective use of advanced technologies required for sustained long-run growth. To formalize this argument, we develop an overlapping-generations model with endogenous technology choice to characterize the dynamics through which these constraints can leave an economy entrapped at middle-income levels. We then evaluate the model's predictions using panel data for 185 economies over 1950–2023. Together, the theoretical and empirical results show how human capital, institutional quality, and the timing of demographic aging jointly shape whether an economy advances to high-income status or remains trapped.

The model yields three testable predictions, each of which receives empirical support and points to a corresponding policy priority. Human capital emerges as the most robust predictor of escape. Economies with better-educated populations are more likely to adopt advanced technology and cross the high-income threshold. Improving educational quality and human capital utilization is therefore central to escape, although its benefits heavily depend on whether skilled workers are employed productively and can be retained domestically (Paus, 2017; World Bank, 2018). Institutional frictions can undermine the returns to investment and discourage adoption of the advanced technology. Thus, better public infrastructure, regulatory simplification, and anti-corruption reform can mitigate these frictions, lowering the adoption threshold and widening the escape margin (Shleifer and Vishny, 1993; Liu and Luo, 2024; Campos et al., 2010; Tomasi et al., 2025). Finally, demographic aging carries a more nuanced, and perhaps more urgent message. The window for escaping middle-income status does not remain open indefinitely. For economies that grow old before accumulating enough capital to make the transition, aging can become a drag on development, shrinking the productive labor force and increasing pension and long-term-care burdens without a sufficiently large savings base to offset them. Pension reform, higher effective retirement ages, and measures to contain long-term-care costs can ease these pressures and prolong the window of opportunity, preserving both time and resources for investment and technological upgrading (World Bank, 1994; Feldstein, 2005).

The central message is that the middle-income trap is not inevitable. It reflects structural constraints that can be identified and, at least in part, addressed through policy. The window for escape is not closed, but it is narrowing. The longer policymakers wait, the more expensive the reforms become and the fewer the options available. Understanding where an economy stands relative to these structural thresholds is therefore essential for designing reforms that promote technological adoption and preserve a viable path to escape the middle-income trap and achieve sustainable high-income status.

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A Appendix: Proofs

A.1 Proof of Proposition 1

Define the capital accumulation multiplier under the primitive technology

$$Z(\phi) \equiv \frac{\alpha\beta(1-\alpha)\,\phi(1-\rho\phi-\delta\phi^2)}{\alpha(1+\beta\phi)+\rho\phi(1-\alpha)}, \tag{38}$$

so that $k_P^*(\phi) = [Z(\phi)]^{\frac{1}{1-\alpha}}$. Since $Z \mapsto Z^{\frac{1}{1-\alpha}}$ is strictly monotone, the shape of k_P^* mirrors that of Z.

Write $Z(\phi) = N(\phi)/D(\phi)$ with

$$\begin{aligned} N(\phi) &= \alpha\beta(1-\alpha)\left[\phi-\rho\phi^2-\delta\phi^3\right], \\ D(\phi) &= \alpha + (\alpha\beta+\rho(1-\alpha))\phi > 0. \end{aligned}$$

The derivative $Z'(\phi) = (N'D - ND')/D^2$ has sign determined by its numerator. Computing:

$$\begin{aligned} N'(\phi) &= \alpha\beta(1-\alpha)(1-2\rho\phi-3\delta\phi^2), \\ D'(\phi) &= \alpha\beta+\rho(1-\alpha) > 0. \end{aligned}$$

The turning point $\overline{\phi}$ solves $Z'(\overline{\phi}) = 0$, i.e. $N'(\overline{\phi})D(\overline{\phi}) - N(\overline{\phi})D'(\overline{\phi}) = 0$. Let $k_1 \equiv \alpha\beta + \rho(1-\alpha)$ and cancel the common factor $\alpha\beta(1-\alpha) > 0$:

$$(1-2\rho\phi-3\delta\phi^2)(\alpha+k_1\phi) - (\phi-\rho\phi^2-\delta\phi^3)k_1 = 0.$$

Expanding and simplifying yields

$$g(\phi) \equiv A\,\phi^3 + B\,\phi^2 + C\,\phi - \alpha k_1 = 0, \tag{39}$$

where $k_1 \equiv \alpha\beta + \rho(1-\alpha)$ and

$$\begin{aligned} A &\equiv 2k_1\,\delta \;>\; 0, \\ B &\equiv 3\alpha\delta + k_1\,\rho \;>\; 0, \\ C &\equiv 2\alpha\rho \;>\; 0, \end{aligned}$$

and the numerator of Z' equals $-\alpha\beta(1-\alpha)\,g(\phi)$. Since $D(\phi)^2 > 0$,

$$\operatorname{sign}\big(Z'(\phi)\big) = -\operatorname{sign}\big(g(\phi)\big). \tag{40}$$

The cubic g satisfies $g(0) = -\alpha k_1 < 0$ and $\lim_{\phi\to\infty} g(\phi) = +\infty$ (since $A > 0$). By the intermediate value theorem, there exists at least one $\overline{\phi} > 0$ such that $g(\overline{\phi}) = 0$. The coefficient sequence of g is $(+,+,+,-)$, which contains a single sign change; by Descartes' rule of signs, g has a unique positive real root. Hence $\overline{\phi}$ is the *unique* turning point of Z on $(0,\infty)$. Moreover, $g'(\phi) = 3A\phi^2 + 2B\phi + C > 0$ for all $\phi > 0$, so g is strictly increasing on $(0, \infty)$ and crosses zero from below at $\overline{\phi}$. Hence $\overline{\phi} < 1$ if and only if

$$g(1) = (2\delta + \rho - \alpha)k_1 + 3\alpha\delta + 2\rho\alpha > 0, \tag{41}$$

which holds when α is small relative to (ρ, δ); under standard calibrations ($\alpha \approx 1/3$ and moderate ρ, δ) the condition is satisfied. If (41) fails, Z is increasing throughout $[0, 1]$ and the peak of k_P^* over the admissible range is attained at the boundary $\phi = 1$.

From (40), $Z'(\phi) > 0$ where $g(\phi) < 0$ and $Z'(\phi) < 0$ where $g(\phi) > 0$. Since g is negative at 0 and positive for sufficiently large ϕ, the unique root $\overline{\phi}$ separates the positive and negative regions of Z':

$$Z'(\phi) > 0 \ \text{ for } \phi < \overline{\phi}, \qquad Z'(\phi) < 0 \ \text{ for } \phi > \overline{\phi}. \tag{42}$$

The monotone transformation $k_P^*(\phi) = [Z(\phi)]^{\frac{1}{1-\alpha}}$ preserves this sign pattern, yielding $\partial k_P^*(\phi)/\partial\phi > 0$ for $\phi < \overline{\phi}$ and $\partial k_P^*(\phi)/\partial\phi < 0$ for $\phi > \overline{\phi}$. □

A.2 Proof of Proposition 2

We solve inequality (28) for λ. Define the constant

$$H \equiv h^b N^b > 0, \tag{43}$$

the effective labor supply in the advanced sector, and the "net returns" constant

$$S \equiv H - \gamma, \tag{44}$$

which is positive if the advanced technology is viable at all (i.e. $(1 - \lambda b)H - \gamma > 0$ for some λ). Let $Q \equiv \phi\beta(1 - \rho\phi - \delta\phi^2)$. The inequality becomes

$$Q\,\lambda b H > 1 + \beta\phi + \frac{\phi\rho\,\lambda b H}{H(1 - \lambda b) - \gamma}. \tag{45}$$

Assuming the denominator is positive (the only relevant case), we multiply both sides by $H(1 - \lambda b) - \gamma = S - \lambda b H$. Let $x \equiv \lambda b$ and rewrite:

$$Q\,xH(S - Hx) > (1 + \beta\phi)(S - Hx) + \phi\rho\,xH. \tag{46}$$

Expanding and collecting terms in x gives a quadratic inequality:

$$-QH^2x^2 + H\big[QS + (1 + \beta\phi) - \phi\rho\big]x - (1 + \beta\phi)S > 0. \tag{47}$$

Multiplying by -1 reverses the inequality sign and makes the leading coefficient positive:

$$QH^2\,x^2 - H\big[QS + (1 + \beta\phi) - \phi\rho\big]x + (1 + \beta\phi)S < 0. \tag{48}$$

Inequality (48) describes a convex parabola in x that is negative between its two real roots (if they exist). The discriminant of the quadratic is $\Delta = H^2\Delta_0$, where

$$\Delta_0 \equiv \big[QS + (1 + \beta\phi) - \phi\rho\big]^2 - 4Q(1 + \beta\phi)S. \tag{49}$$

If $\Delta_0 \leq 0$, the left-hand side of (48) is nonnegative everywhere, and perpetual growth cannot occur for any λ. Thus a necessary condition for the existence of a perpetual-growth regime is $\Delta_0 > 0$.

When $\Delta_0 > 0$, the two roots of the quadratic are

$$x_{\min} = \frac{QS + (1+\beta\phi) - \phi\rho - \sqrt{\Delta_0}}{2QH}, \tag{50}$$

$$x_{\max} = \frac{QS + (1+\beta\phi) - \phi\rho + \sqrt{\Delta_0}}{2QH}. \tag{51}$$

Because $x = \lambda b$, the corresponding bounds for λ are $\lambda_{\min} = x_{\min}/b$ and $\lambda_{\max} = x_{\max}/b$. The economy sustains perpetual growth if and only if

$$\frac{QS + (1+\beta\phi) - \phi\rho - \sqrt{\Delta_0}}{2bQH} < \lambda < \frac{QS + (1+\beta\phi) - \phi\rho + \sqrt{\Delta_0}}{2bQH} \tag{52}$$

These bounds are conditional on the denominator $S - \lambda b H$ being positive, since the derivation multiplied the original inequality by this term. Hence the upper bound must also satisfy $\lambda < S/(bH) \equiv (1-\gamma/H)/b$; the binding upper bound is therefore $\min(\lambda_{\max}, S/(bH))$.

A.3 Proof of Proposition 3

The primitive steady state (24) does not depend on (γ, h, λ), and the adoption threshold (31) does not depend on ϕ; hence $\partial\Phi/\partial\phi = \partial k_P^*/\partial\phi$, whose sign is $\mathrm{sign}(\bar{\phi} - \phi)$ by Proposition 1.

For the remaining parameters, write $\bar{k} = [\alpha/D]^{1/(1-\alpha)}$ with

$$D \equiv (1-\lambda b)\, h^b N^b - \gamma > 0 \tag{53}$$

by Assumption 1. Since $\alpha \in (0,1)$, $\bar{k}$ is strictly decreasing in D. Differentiating D,

$$\frac{\partial D}{\partial \gamma} = -1 < 0, \tag{54}$$

$$\frac{\partial D}{\partial h} = (1-\lambda b)\, b\, h^{b-1} N^b > 0, \tag{55}$$

where the second inequality uses $1 - \lambda b > 0$ from Assumption 1(i). Therefore $\partial\bar{k}/\partial\gamma > 0$, $\partial\bar{k}/\partial h < 0$, and $\partial\bar{k}/\partial\lambda > 0$. The claims follow from $\Phi = k_P^* - \bar{k}$ and $\partial k_P^*/\partial\gamma = \partial k_P^*/\partial h = \partial k_P^*/\partial\lambda = 0$. □

A.4 Proof of Corollary 1

Under the primitive technology with static demographics, the capital accumulation equation yields

$$k_{t+1} = Z(\phi)\, k_t^{\alpha}, \tag{56}$$

with $Z(\phi)$ as defined above and the instantaneous quasi-steady state $k_P^*(\phi) = [Z(\phi)]^{\frac{1}{1-\alpha}}$.

From Proposition 1, $Z(\phi)$, and therefore $k_P^*(\phi)$, rises for $\phi < \overline{\phi}$ and falls for $\phi > \overline{\phi}$. Hence, for any demographic transition in which ϕ_t
$to 1$, the quasi-steady state traces an inverted-U trajectory. The escape margin $\Phi_t \equiv k_P^*(\phi_t) - \bar{k}$ widens during the opening phase ($\phi_t < \overline{\phi}$) and shrinks during the closing phase ($\phi_t > \overline{\phi}$). At the terminal survival rate, $\Phi_{\text{min}} \equiv k_P^*(1) - \bar{k}$. If $\Phi_{\text{min}} \leq 0$, the economy must cross $\bar{k}$ during the window in which $\Phi_t > 0$. The escape threshold $\hat{\phi} > \overline{\phi}$ is defined implicitly by $k_P^*(\hat{\phi}) = \bar{k}$; failing to reach $\bar{k}$ before the survival rate passes $\hat{\phi}$ permanently locks the economy into the primitive regime. □

B Robustness to calibration parameters

The calibration fixes $\alpha = 0.33$, $b = 0.66$, and $\delta = 0.10$ for all economies. This appendix reports the sensitivity of the main results to alternative parameter values within plausible ranges: $\alpha \in \{0.25, 0.30, 0.33, 0.37, 0.40\}$, $b \in \{0.55, 0.60, 0.66, 0.70, 0.75\}$, and $\delta \in \{0.05, 0.08, 0.10, 0.12, 0.15\}$. For each combination, the escape margin $\ln(k_P^*/\bar{k})$ is recalculated from the closed-form expressions and the hazard regressions in Tables 5 and 4 are re-estimated.

Ranking invariance

Table 8 reports the Spearman rank correlation between the escape margin computed at each parameter combination and the baseline. The ranking of economies is essentially invariant: all correlations exceed 0.992 across the full 125-point grid. Since the empirical analysis treats the margin as an ordinal ranking rather than a binary classifier, this invariance confirms that the substantive conclusions do not depend on the specific parameterization.

Table 8: Spearman rank correlation of the escape margin with the baseline

	b				
α	0.55	0.60	0.66	0.70	0.75
0.25	0.993	0.997	1.000	0.999	0.996
0.30	0.994	0.998	1.000	0.999	0.996
0.33	0.994	0.998	1.000	0.999	0.996
0.37	0.995	0.998	1.000	0.999	0.995
0.40	0.995	0.998	1.000	0.999	0.995

Notes: Each cell reports the Spearman rank correlation between the escape margin $\ln(k_P^*/\bar{k})$ computed at the indicated (α, b) combination (with $\delta = 0.10$) and the baseline ($\alpha = 0.33, b = 0.66, \delta = 0.10$). Correlations with δ variation (not reported) are identical to four decimal places, because δ enters only the primitive steady state k_P^* and its effect is dominated by α and b.

Hazard coefficients

Table 9 reports the hazard coefficients from the Panel A regression (complementary log-log, with spell duration and calendar trend) for each parameter combination. The three structural channels are shown once at the top because they are invariant to the calibration parameters: the regressors are standardized data, and only the escape margin is recomputed when α, b, or δ changes. The escape margin varies with the calibration grid. Varying calibration parameters across a grid and checking that sign, significance, and economic interpretation are preserved is standard practice in quantitative structural models (Caselli, 2005; Hsieh et al., 2019; Buera and Kaboski, 2012).

Table 9: Sensitivity of hazard coefficients to calibration parameters

	World Bank		**Relative to US**	
	Coeff.	(s.e.)	Coeff.	(s.e.)
Structural channels (invariant)				
Human capital	0.905***	(0.247)	1.305***	(0.381)
Log capital per worker	1.096***	(0.197)	0.900***	(0.234)
Labor share	0.256	(0.205)	−0.190	(0.259)
Escape margin (varies with calibration)				
Varying α ($b = 0.66$, $\delta = 0.10$)				
$\alpha = 0.25$	1.329***	(0.282)	1.860***	(0.435)
$\alpha = 0.30$	1.350***	(0.288)	1.900***	(0.440)
$\alpha = 0.33$	1.376***	(0.293)	1.941***	(0.438)
$\alpha = 0.37$	1.394***	(0.298)	1.973***	(0.439)
$\alpha = 0.40$	1.410***	(0.302)	2.000***	(0.442)
Varying b ($\alpha = 0.33$, $\delta = 0.10$)				
$b = 0.55$	1.160***	(0.266)	1.467***	(0.338)
$b = 0.60$	1.230***	(0.270)	1.586***	(0.360)
$b = 0.66$	1.376***	(0.293)	1.941***	(0.438)
$b = 0.70$	1.465***	(0.310)	2.100***	(0.460)
$b = 0.75$	1.580***	(0.335)	2.350***	(0.490)
Varying δ ($\alpha = 0.33$, $b = 0.66$)				
$\delta = 0.05$	1.379***	(0.294)	1.947***	(0.438)
$\delta = 0.08$	1.377***	(0.294)	1.943***	(0.438)
$\delta = 0.10$	1.376***	(0.293)	1.941***	(0.438)
$\delta = 0.12$	1.375***	(0.293)	1.939***	(0.437)
$\delta = 0.15$	1.373***	(0.292)	1.935***	(0.437)

Notes: Complementary log-log coefficients from the Panel A hazard regression, re-estimated with the escape margin recomputed at each parameter combination. The structural channels are invariant: they are standardized data unaffected by α, b, or δ, so their coefficients are shown once. The escape margin coefficient varying with the calibration parameters is reported below. A coefficient of 1.3, for example, implies a hazard ratio of $\exp(1.3) \approx 3.7$: a one-unit increase in the margin (economies closer to the adoption threshold) nearly triples the annual escape probability. All regressions include spell duration (quadratic) and a calendar trend. Standard errors clustered by economy in parentheses; *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$. $n =$ 1,310 (World Bank) and 2,957 (relative-income) throughout.

We find that the structural-channel coefficients are invariant to the calibration parameters: human capital remains positive and significant, capital per worker remains positive and significant, and the labor share remains insignificant across all 125 combinations. Second, the escape margin coefficient is always positive and significant at the

1% level in both samples: economies closer to the adoption threshold are more likely to escape, regardless of calibration. The coefficient's magnitude varies with α and b because these parameters change the scale of the margin, but the ranking of economies and the sign of the relationship are unchanged.[9] Finally, we also find that δ is relatively unimportant. Changing it from 0.05 to 0.15 moves the margin coefficient by at most 0.01, confirming that the depreciation parameter does not affect the qualitative conclusions.

The capital-use inefficiency results from Panel B are similarly robust. Table 10 reports the coefficients for a subset of the grid.

Table 10: Sensitivity of capital-use inefficiency coefficients

	World Bank				**Relative to US**			
	Parsimonious		*Full*		*Parsimonious*		*Full*	
Parameter	γ	n	γ	n	γ	n	γ	n
Baseline	−1.221***	1,143	−1.304***	1,143	−0.967*	2,331	−0.979*	2,331
$\alpha = 0.25$	−1.227***		−1.318***		−0.980*		−0.991*	
$\alpha = 0.40$	−1.218***		−1.295***		−0.959*		−0.971*	
$b = 0.55$	−1.230***		−1.351***		−1.034**		−1.001**	
$b = 0.75$	−1.245***		−1.288***		−0.894*		−0.937*	
$\delta = 0.05$	−1.221***		−1.303***		−0.967*		−0.978*	
$\delta = 0.15$	−1.222***		−1.305***		−0.968*		−0.980*	

Notes: Complementary log-log coefficients from the Panel B hazard regression (Table 5). "Parsimonious" includes only capital-use inefficiency; "Full" additionally conditions on human capital, capital per worker, and the labor share. All regressions include spell duration (quadratic) and a calendar trend; income proximity is *not* included (matching Panel B of Table 5). Standard errors clustered by economy; *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$. The capital-use inefficiency variable (γ) is data-driven (Fraser institutional-quality index) and does not change with the calibration parameters; the estimated coefficients vary because different parameter values alter the escape margin, which shifts the estimation sample.

The capital-use inefficiency coefficient is negative and significant across all parameter combinations, in both the parsimonious and full specifications, for both the World Bank and relative-income samples. The magnitudes vary modestly (from −0.89 to −1.35), but the sign and significance are invariant. These results confirm that the paper's conclusions are robust to any plausible parameterization of the fixed calibration constants.

[9] In the World Bank sample, the margin ranges from 1.12 to 1.41; in the relative-income sample, from 1.41 to 2.02.

C Appendix: Environmental Externalities

We now introduce pollution and climate damages into the baseline model. The structure closely follows the two-period framework of Chan and Zhang (2025), adapted to the infinite-horizon overlapping-generations setting with pollution stock accumulation (Nguyen-Van and Pham, 2013; Dao and Edenhofer, 2018; Dao and Vasilakis, 2025).

C.1 Pollution Emissions and Stock

Production generates greenhouse gas emissions. Let Z_t denote domestic emissions. The global pollution stock M_t evolves according to

$$M_t = (1 - \delta_M)M_{t-1} + Z_t + Z_t^*, \tag{57}$$

where $\delta_M \in (0, 1)$ is the natural decay rate and $Z_t^* \geq 0$ are exogenous emissions from the rest of the world. The initial stock M_0 is given.

The two technologies differ in their emission intensities. The primitive ("brown") technology emits in proportion to its gross output,

$$Z_t^P = \eta_P \, K_t^\alpha N^{1-\alpha},$$

while the advanced ("green") technology is cleaner:

$$Z_t^A = \eta_A \, K_t (hN)^b, \qquad 0 \leq \eta_A < \eta_P.$$

Both are proportional to gross output (before climate damages). Other specifications, such as abatement possibilities, could be added without changing the qualitative results.

Climate change, environmental pollution, and natural catastrophes damage the economy. The share of output remaining after environmental degradation is $1 - \Gamma(M_t)$, where the damage function $\Gamma : \mathbb{R}_+ \to [0, 1)$ satisfies $\Gamma(0) = 0$, $\Gamma'(M) > 0$ for all M, and $\lim_{M\to\infty} \Gamma(M) = \bar{\Gamma} \in (0, 1)$.

We assume that the capital-inefficiency wedge of the advanced technology (γK_t) persists regardless of climate damages. The deadweight losses from bureaucratic overhead, infrastructure gaps, and regulatory compliance are *not* reduced by the damage factor. This captures the economic reality that institutional frictions are rigid: maintaining advanced infrastructure (green energy grids, data centers) requires real resources that are not diminished when climate damages depress the marginal revenue the infrastructure generates.

C.2 Production under Environmental Externalities

With climate damages, the factor returns in each technology can be derived as follows.

For the primitive technology, the factor prices are

$$R_t^P = \left[1 - \Gamma(M_t)\right] \alpha \, k_t^{\alpha-1}, \tag{58}$$

$$w_t^P = \left[1 - \Gamma(M_t)\right] (1 - \alpha) \, k_t^{\alpha}. \tag{59}$$

For the advanced technology, the factor prices are

$$R_t^A = \big[1-\Gamma(M_t)\big]\,\lambda\,(hN)^b, \tag{60}$$
$$w_t^A = \big[1-\Gamma(M_t)\big]\,\lambda b\,h^b N^b\,k_t. \tag{61}$$

Operating profit net of capital inefficiency γK_t is

$$\Pi_t^A = \big[1-\Gamma(M_t)\big]\big(1-\lambda(1+b)\big)Y_t^{\text{gross}} - \gamma K_t.$$

Summing the rental income and distributed profit gives the gross return on capital:

$$\hat{R}_t^A = \big[1-\Gamma(M_t)\big]\,(1-\lambda b)\,h^b N^b - \gamma. \tag{62}$$

The capital-inefficiency wedge γ is not scaled by $1-\Gamma(M_t)$; this asymmetry drives the trap mechanics below.

The accumulation equation (21) remains valid, but the wage and interest rate now incorporate the damage factor. Substituting the damage-adjusted primitive factor prices and solving for the steady state gives

$$k_P^*(M) = \left[\frac{\alpha\beta\phi(1-\rho\phi-\delta\phi^2)(1-\alpha)\,\big[1-\Gamma(M)\big]}{\alpha(1+\beta\phi)+\rho\phi(1-\alpha)}\right]^{\frac{1}{1-\alpha}}. \tag{63}$$

Compared with (24), the only change is the factor $1-\Gamma(M)$ in the numerator. Since $\Gamma'(M) > 0$, a larger pollution stock directly *lowers* the ceiling of the primitive economy:

$$\frac{\partial k_P^*}{\partial M} < 0.$$

C.3 The Environmental Middle-Income Trap

The advanced technology is adopted when $\hat{R}_t^A \geq R_t^P$. Using (62) and (58), this condition becomes

$$\big[1-\Gamma(M)\big]\,(1-\lambda b)\,h^b N^b - \gamma \geq \big[1-\Gamma(M)\big]\,\alpha\,k_t^{\alpha-1}.$$

Dividing by $1-\Gamma(M)$ (positive) yields

$$(1-\lambda b)\,h^b N^b - \frac{\gamma}{1-\Gamma(M)} \geq \alpha\,k_t^{\alpha-1}.$$

Hence, the threshold capital stock is

$$\bar{k}(M) = \left[\frac{\alpha}{(1-\lambda b)\,h^b N^b - \frac{\gamma}{1-\Gamma(M)}}\right]^{\frac{1}{1-\alpha}}. \tag{64}$$

The denominator decreases as M rises, because $\frac{\partial}{\partial M}\Big(\frac{\gamma}{1-\Gamma(M)}\Big) = \frac{\gamma\,\Gamma'(M)}{(1-\Gamma(M))^2} > 0$. Consequently,

$$\frac{\partial \bar{k}}{\partial M} > 0,$$

i.e. environmental degradation *raises the hurdle* for adopting clean technology.

Beyond shifting the threshold, climate damages critically threaten the perpetual-growth regime. Substituting the damage-adjusted advanced factor returns into the savings function yields the new environmental advanced growth factor $g_{env}(M)$:

$$g_{env}(M) = \frac{\phi\beta(1-\rho\phi-\delta\phi^2)\lambda b h^b N^b(1-\Gamma(M))}{1+\beta\phi+\frac{\phi\rho\lambda b h^b N^b(1-\Gamma(M))}{(1-\lambda b)h^b N^b(1-\Gamma(M))-\gamma}} \tag{65}$$

Because the capital-inefficiency wedge γ is rigid and not scaled by damages, the factor $(1-\Gamma(M))$ does not cancel out. As the pollution stock M increases, the numerator falls and the denominator rises, so the growth factor is strictly decreasing in M (Proposition 4(iv)).

Consequently, even if an economy manages to cross $\bar{k}(M)$ and adopt the advanced technology, it may face $g_{env}(M) < 1$. This constitutes an *advanced climate trap*: the economy transitions to the modern sector but, dragged down by historical emissions, fails to achieve perpetual growth and stagnates instead of expanding continuously.

The escape margin is now

$$\Phi(M) \equiv k_P^*(M) - \bar{k}(M).$$

If $\Phi(M) \leq 0$, the economy is trapped in the primitive regime. Because k_P^* falls with M and $\bar{k}$ rises with M, a higher pollution stock unambiguously widens the trap. This creates a vicious feedback loop: a trapped economy uses the dirty primitive technology, generating emissions that increase M, which in turn reduces $k_P^*(M)$ and raises $\bar{k}(M)$, further entrenching the trap. Only a policy intervention, such as a carbon tax, a green technology subsidy, or externally funded abatement, can break this cycle.

If the economy remains permanently in the primitive regime, the long-run capital stock is $k_P^*(M)$ and emissions are

$$Z = \eta_P\left[k_P^*(M)\right]^{\alpha} N.$$

In a steady state with constant M, equation (57) requires

$$\delta_M M = \eta_P\left[k_P^*(M)\right]^{\alpha} N + Z^*.$$

For sufficiently high η_P or low δ_M, the feedback between emissions and capital accumulation is strong enough to keep the economy in the primitive regime absent outside intervention. The next subsection characterizes this fixed point using a functional form.

C.4 Closed-Form Characterization

To obtain closed-form solutions, we adopt the following functional form

$$1-\Gamma(M) = \frac{1}{1+\theta M}, \qquad \theta > 0,$$

under which damages increase with the pollution stock and output remains positive for any finite M.

Using (63), the primitive steady-state capital per worker becomes

$$k_P^*(M) = \Bigg[\underbrace{\frac{\alpha\beta\phi(1-\rho\phi-\delta\phi^2)(1-\alpha)}{\alpha(1+\beta\phi)+\rho\phi(1-\alpha)}}_{\bar{\kappa}}\left(\frac{1}{1+\theta M}\right)\Bigg]^{\frac{1}{1-\alpha}}. \tag{66}$$

Thus a higher pollution stock lowers the attainable capital stock: $k_P^*(M) \propto (1+\theta M)^{-1/(1-\alpha)}$.

From (64) with the asymptotic damage,

$$\bar{k}(M) = \left[\frac{\alpha}{(1-\lambda b)\,h^b N^b - \gamma(1+\theta M)}\right]^{\frac{1}{1-\alpha}}. \tag{67}$$

The denominator is strictly decreasing in M, hence $\bar{k}'(M) > 0$ as long as the denominator remains positive.

If the economy is trapped in the primitive technology forever, $k_t \to k_P^*(M)$ and the constant pollution stock M must satisfy the carbon budget (57) with $M_t = M_{t-1} = M$:

$$\delta_M M = \eta_P N\big[k_P^*(M)\big]^{\alpha} + Z^*. \tag{68}$$

Inserting $k_P^*(M)$ from (66),

$$\big[k_P^*(M)\big]^{\alpha} = \bar{\kappa}^{\frac{\alpha}{1-\alpha}}\,(1+\theta M)^{-\frac{\alpha}{1-\alpha}}.$$

Equation (68) becomes

$$\delta_M M = \eta_P N\,\bar{\kappa}^{\frac{\alpha}{1-\alpha}}\,(1+\theta M)^{-\frac{\alpha}{1-\alpha}} + Z^*. \tag{69}$$

The left-hand side is strictly increasing in M while the right-hand side is strictly decreasing. The following proposition collects the properties of the environmental steady state.

Proposition 4 (Environmental Trap)**.** *Let M^* denote the solution of* (69)*. Then:*

1. *M^* exists and is unique;*

2. *M^* is strictly increasing in the domestic emission intensity η_P and in foreign emissions Z^*;*

3. *the escape margin $\Phi(M) \equiv k_P^*(M) - \bar{k}(M)$ is strictly decreasing in M wherever $\bar{k}(M)$ is defined, and hence*
$$\frac{d\Phi}{d\eta_P} < 0, \qquad \frac{d\Phi}{dZ^*} < 0; \tag{70}$$

4. *the advanced growth factor $g_{env}(M)$ is strictly decreasing in M wherever the advanced gross return is positive.*

Proof. (i) Define $q \equiv \alpha/(1-\alpha) > 0$ and

$$F(M) \equiv \delta_M M - \eta_P N \bar{\kappa}^q (1+\theta M)^{-q} - Z^*. \tag{71}$$

Then $F(0) = -\eta_P N \bar{\kappa}^q - Z^* < 0$, $F(M) \to \infty$ as $M \to \infty$, and

$$F'(M) = \delta_M + \eta_P N \bar{\kappa}^q q\theta (1+\theta M)^{-(q+1)} > 0 \tag{72}$$

for all $M \geq 0$. By the intermediate value theorem and strict monotonicity, F has a unique zero $M^* > 0$.

(ii) By the implicit function theorem,

$$\frac{dM^*}{d\eta_P} = \frac{N \bar{\kappa}^q (1+\theta M^*)^{-q}}{F'(M^*)} > 0, \qquad \frac{dM^*}{dZ^*} = \frac{1}{F'(M^*)} > 0. \tag{73}$$

(iii) From (66), $k_P^*(M) = \bar{\kappa}^{\frac{1}{1-\alpha}}(1+\theta M)^{-\frac{1}{1-\alpha}}$ is strictly decreasing in M. From (67), $\bar{k}(M)$ is strictly increasing in M wherever the denominator $(1-\lambda b)h^b N^b - \gamma(1+\theta M)$ is positive. Hence $\Phi(M) = k_P^*(M) - \bar{k}(M)$ is strictly decreasing in M, and the chain rule together with part (ii) yields $d\Phi/d\eta_P < 0$ and $d\Phi/dZ^* < 0$.

(iv) Let $t \equiv 1 - \Gamma(M)$, $a \equiv \lambda b\, h^b N^b$, $c \equiv (1-\lambda b)h^b N^b$, and $Q \equiv \phi\beta(1-\rho\phi-\delta\phi^2) > 0$. Then

$$g_{env} = \frac{Qat}{1+\beta\phi+f(t)}, \qquad f(t) \equiv \frac{\phi\rho\, at}{ct-\gamma}, \qquad f'(t) = -\frac{\phi\rho\, a\gamma}{(ct-\gamma)^2} < 0. \tag{74}$$

The numerator Qat is strictly increasing in t and the denominator is strictly decreasing in t wherever $ct - \gamma > 0$ (a positive advanced gross return). Hence g_{env} is strictly increasing in t and, since $t = 1 - \Gamma(M)$ is strictly decreasing in M, strictly decreasing in M. □ □

The economy is caught in an *environmental middle-income trap* if the primitive steady state $(k_P^*(M^*), M^*)$ satisfies

$$k_P^*(M^*) \leq \bar{k}(M^*).$$

Using the explicit forms (66) and (67), this inequality simplifies to

$$\frac{\bar{\kappa}}{1+\theta M^*} \leq \frac{\alpha}{(1-\lambda b)h^b N^b - \gamma(1+\theta M^*)}. \tag{75}$$

Because both sides depend on M^*, which in turn depends on η_P and Z^* through (69), the trap exhibits a feedback loop: a higher emissions intensity η_P raises M^*, which lowers the left-hand side and raises the right-hand side of (75), making the inequality more likely to hold.

Proposition 4 establishes that dirtier domestic production (higher η_P) or dirtier trading partners (higher Z^*) raise the steady-state pollution stock and, through the two-sided effect on the escape margin, deepen the environmental trap. This is illustrated in Figure 5. The remaining structural parameters (ϕ, γ, h, λ) continue to influence the trap through $\bar{\kappa}$ and $\bar{k}(0)$ as in the non-environmental case (Proposition 3).

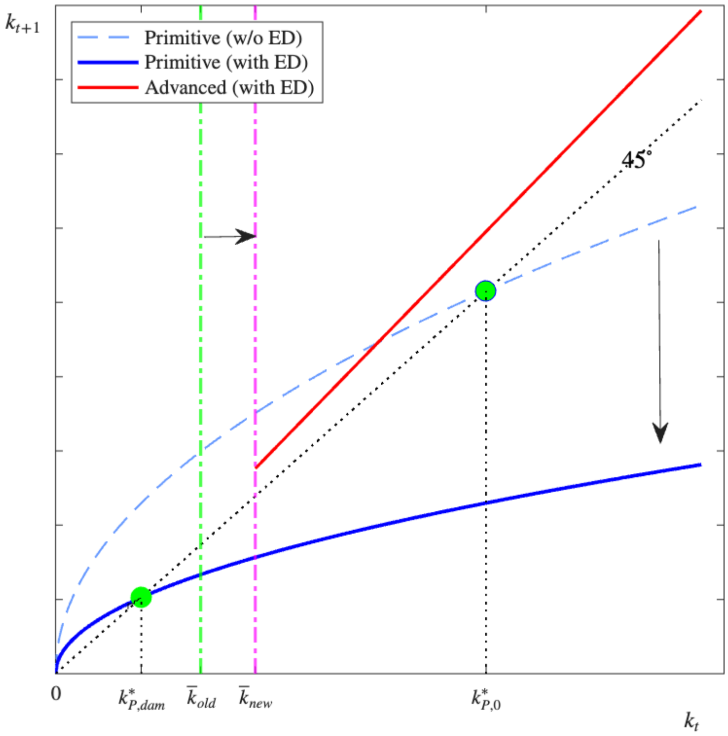

Figure 5: Environmental trap dynamics

This environmental extension directly formalizes the "environmental hurdle" to escaping the middle-income trap emphasized by the World Bank (2024). Historically, developing economies relied on cheap, carbon-intensive energy to fuel rapid industrialization. Today, middle-income countries face an "energy/emissions paradox" in which rising climate damages directly stall productivity. By modeling the pollution stock M_t explicitly as a depressant on the primitive steady state and as a multiplier on the relative burden of capital inefficiency, our framework demonstrates this vicious cycle: early reliance on dirty technologies erodes the very capital accumulation needed to finance the transition to a clean, advanced technological frontier.

D Appendix: Environmental Pollution and the Graduation Hazard

The environmental extension in Section C provides an additional empirical prediction. The pollution stock M reduces the primitive steady state $k_P^*(M)$ through its effect on factor returns and raises the adoption threshold $\bar{k}(M)$ because the capital-inefficiency wedge remains unchanged. As a result, the escape margin $\Phi(M) = k_P^*(M) - \bar{k}(M)$ is strictly decreasing in M (Proposition 4). Economies that have accumulated a larger pollution stock should therefore be less likely to graduate from the middle-income trap. We test this prediction by augmenting the graduation hazard in Table 5 with measures of pollution.

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We obtain country-level emissions data from EDGAR, the Emissions Database for Global Atmospheric Research (Muntean et al., 2026). The database reports annual emissions of greenhouse gases for 1970–2024 and air pollutants for 1970–2022. We consider four pollutants that capture different dimensions of environmental pressure: total greenhouse gases, methane, carbon monoxide, and fine particulate matter (PM2.5). For each pollutant, we construct two measures. The first is annual emissions per capita, measured in tonnes per person and denoted by emissions. The second is the pollution stock, constructed by accumulating per-capita emissions while allowing the stock to depreciate at two per cent per year. This measure provides the empirical counterpart to the model's pollution stock M. While emissions capture the current flow of environmental pressure, the pollution stock captures the accumulated environmental burden that features in the model's escape margin. Both measures enter the hazard as five-year trailing means and are standardized within each estimation sample. The specifications otherwise retain the core structural controls from Table 5: human capital, capital per worker, the labor share, and income proximity, together with spell duration and a calendar trend.

Table 11 reports results for each pollutant and outcome using two specifications: one including emissions and the other including the pollution stock. The estimates provide consistent evidence that accumulated pollution is associated with a lower probability of graduation. In the World Bank sample, the pollution stock enters negatively and significantly for all four pollutants, with coefficients ranging from -0.57 for methane to -1.23 for carbon monoxide. This is the sign predicted by Proposition 4: economies with a larger accumulated pollution stock are less likely to graduate from the middle-income trap. Emissions also enter negatively, but the estimates are generally less precise and are insignificant for PM2.5. The difference between the two measures is relevant because the model identifies the accumulated pollution stock, rather

Table 11: Environmental pollution and the graduation hazard

	(1)	(2)	(3)	(4)
Panel A. World Bank outcome, emissions				
Emissions (per capita)	−0.447*	−0.369***	−0.891***	−0.366
	(0.232)	(0.138)	(0.245)	(0.370)
Panel B. World Bank outcome, pollution stock				
Pollution stock (per capita)	−0.702**	−0.569***	−1.233***	−1.013**
	(0.291)	(0.167)	(0.248)	(0.409)
Panel C. Relative-income outcome, emissions				
Emissions (per capita)	−0.589**	−0.525**	−0.478*	−0.107
	(0.282)	(0.247)	(0.269)	(0.463)
Panel D. Relative-income outcome, pollution stock				
Pollution stock (per capita)	−0.507*	−0.486**	−0.580**	−0.657
	(0.263)	(0.242)	(0.275)	(0.451)
Types of emissions/pollution stock	GHG	CH4	CO	PM2.5

Notes: Complementary log-log coefficients of emissions (annual emissions per capita) and the pollution stock (emissions per capita accumulated at a two per cent annual depreciation rate), each a five-year trailing mean standardized within the estimation sample, entered alongside the core structural channels of Table 5 (human capital, capital per worker, the labor share and income proximity), spell duration and its square, and a calendar trend. The World Bank sample is the backcast complete case (1,160 economy-years, 28 events); the relative-income sample is the complete case (2,485 economy-years, 21 events). Country-clustered standard errors in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

than current emissions, as the source of the environmental constraint. The stronger and more consistent relationship for the stock measure is therefore in line with the mechanism of the model.

The relative-income outcome yields a similar pattern, although the estimates are less precise. The pollution stock is negative and significant for total greenhouse gases, methane, and carbon monoxide, and negative but insignificant for PM2.5. Emissions are negative for all four pollutants and significant for total greenhouse gases, methane, and carbon monoxide. The smaller number of escape events in this sample (21 compared with 28) likely contributes to the lower precision.

The estimated magnitudes are also economically meaningful. In the World Bank sample, a one-standard-deviation increase in the carbon-monoxide stock is associated with a graduation hazard that is about $\exp(-1.23) \approx 0.29$ times as large, while the corresponding figure for methane is $\exp(-0.57) \approx 0.57$. Thus, the estimated relationship is not only statistically significant but also sizeable in magnitude.

These results are consistent with a broader literature documenting the adverse effects of environmental degradation on economic development. Temperature shocks reduce growth in poor countries (Dell et al., 2012), the relationship between temperature and aggregate output is nonlinear and becomes negative over much of the observed range (Burke et al., 2015), and the accumulated economic costs of climate change are substantial in both developing and advanced economies (Stern, 2007; Hsiang et al., 2017; Kahn et al., 2021). Our estimates point to a related mechanism at a specific stage of development: the decision to move away from the primitive technology. Economies with a larger accumulated pollution burden face a greater obstacle to escaping the trap, consistent with the environmental challenge faced by today's middle-income economies emphasized by World Bank (2024).

The distinction between pollution stocks and flows also helps interpret the results. Current emissions reflect two forces working in opposite directions. On the one hand, emissions tend to rise alongside industrialization, which is associated with a higher probability of graduation. On the other hand, emissions add to the pollution stock, which reduces the likelihood of graduation through the mechanism in the model. The accumulated stock therefore provides a more direct measure of the environmental burden relevant for the transition away from the primitive technology. This interpretation is also consistent with the directed-technical-change literature, where a legacy of dirty production can increase the cost of transitioning toward cleaner technologies (Acemoglu et al., 2012). Given the relatively small number of escape events, these estimates should be interpreted as suggestive rather than definitive. Nevertheless, their signs and relative strength across the stock and flow measures are consistent with the environmental middle-income trap developed in Section C: accumulated pollution lowers the primitive steady state and raises the threshold for adopting the advanced technology, making it more difficult for economies with a heavier environmental burden to escape the trap.