Tohoku University Research Center for Policy Design Discussion Paper

TUPD-2026-006

The Self-Fulfilling Default Model: Its Validity, Applicability, and Implications

Michinao Okachi

Graduate School of Arts and Sciences, the University of Tokyo

Graduate School of Economics and Management, Tohoku University

April 2026

TUPD Discussion Papers can be downloaded from:

https://www2.econ.tohoku.ac.jp/~PDesign/dp.html

Discussion Papers are a series of manuscripts in their draft form and are circulated for discussion and comment purposes. Therefore, Discussion Papers cannot be reproduced or distributed without the written consent of the authors.

The Self-Fulfilling Default Model: Its Validity, Applicability, and Implications

Michinao Okachi

April 24, 2026

Abstract

This paper investigates expectation-driven sovereign debt crises, focusing on the Greek experience, through a self-fulfilling default framework. We first analytically characterize the model's micro-foundations, proving the existence of distinct debt thresholds that partition the state space into Safe, Crisis, and Default zones, thereby endogenously ruling out opportunistic default deviations. Quantitatively, we challenge the standard reliance on the Simulated Method of Moments, which forces unrealistic parameterizations and yields artificially low debt levels. By calibrating core parameters directly to empirical data and extending the model to feature long-term bonds, a partial default mechanism, and persistent market exclusion, we successfully replicate key empirical moments, notably the high debt-to-GDP ratios and realistic default frequencies of advanced economies. Finally, counterfactual simulations reveal that while forced austerity inflicts substantial short-term pain, it is strictly welfare-improving over the long run compared to a baseline of serial defaults. Because gradual macroeconomic improvements cannot restore debt sustainability, we conclude that external interventions are essential; their strict conditionality acts as a vital commitment device to enforce deleveraging and eliminate self-fulfilling risks. (JEL Classification: F34, H63)

Key words: Self-fulfilling Sovereign Default, Advanced Economies

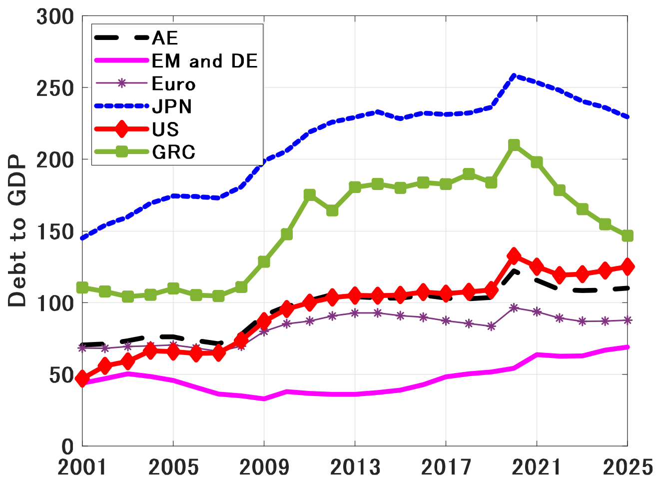
---

An associate professor at Graduate School of Arts and Sciences, the University of Tokyo (E-mail: michinao.okachi@waka.c.u-tokyo.ac.jp), and a visiting associate professor at Policy Design Lab, Tohoku University.

1 Introduction

Global public debt levels have been on a persistent upward trajectory over the past few decades. Figure 1 illustrates the evolution of the general government gross debt-to-GDP ratio for selected countries and economic groups. Although the pace of debt accumulation briefly moderated in 2023 and 2024 due to the denominator effect of high global inflation, the underlying long-term trend remains decidedly upward. While the average debt-to-GDP ratio in emerging market and developing economies has historically hovered around 50 percent, the average for advanced economies has grown steadily, now exceeding 100 percent of GDP.

Figure 1: Transition of the Debt-to-GDP Ratio for Some Selected Countries and Groups



Source: International Monetary Fund (IMF) World Economic Outlook (WEO) database.

Crucially, this high-debt environment has left even advanced economies increasingly vulnerable to shifts in market sentiment and fiscal policy missteps. For instance, the United Kingdom experienced a sudden and severe spike in sovereign bond spreads in 2022 following the announcement of unfunded tax cuts under Prime Minister Truss. Similarly, France's sovereign spread over German bunds has remained elevated since 2024, reflecting growing market scrutiny regarding its fiscal sustainability.

To explain the dynamics of sovereign bond yield spreads and the sovereign's default decision, economists have developed various quantitative frameworks. The traditional workhorse in this literature is the strategic default model, pioneered by Eaton and Gersovitz (1981) and later extended for quantitative applications by Aguiar and Gopinath (2006) and Arellano (2008). This framework assumes that a sovereign will willingly default if the continuation value of defaulting strictly exceeds the value of repayment. Since then, numerous researchers have extended this framework, incorporating features such as long term bonds (Chatterjee and Eyigungor, 2012), debt renegotiation (Yue, 2010), and production economies (Mendoza and Yue, 2012), to improve the models' empirical fit and align them more closely with real world cases.

However, a key limitation of standard strategic models is that they do not incorporate lenders' subjective anticipations and sentiment shifts into the probability of default. As demonstrated by the European Central Bank's Outright Monetary Transactions (OMT) announcement in 2012, mere policy signaling can drastically alter lender beliefs and sovereign default probabilities, even in the absence of immediate changes to economic fundamentals. To address this limitation, recent literature has actively developed self-fulfilling debt crisis models, building on the foundational work of Cole and Kehoe (2000) and further advanced by studies such as Bocola and Dovis (2019) and Aguiar et al. (2022). In these models, the sovereign's default decision is typically made after the bond auction, allowing lenders' beliefs to directly influence the borrowing cost and, consequently, the actual default decision. By endogenizing this expectation-driven mechanism, self-fulfilling models provide a theoretical framework to explain the sudden spikes and high volatility in sovereign spreads, without relying on implausibly massive fundamental shocks.

Despite these theoretical advancements, existing self-fulfilling models inherit several unresolved quantitative discrepancies from the strategic default literature. To match targeted empirical moments, researchers are frequently forced to adopt parameterizations that contradict empirical evidence or established literature. Specifically, to generate default events within the simulated environment, standard calibrations must provide a sufficient incentive to default by assuming an unrealistically myopic government, excessively mild economic costs of default, or both. Consequently, canonical studies routinely calibrate the government's subjective discount factor to values between 0.80 and 0.95, whereas the broader macroeconomic literature typically assumes values much closer to unity, such as 0.98 or 0.99. Such an extreme degree of impatience implies an annualized discount rate that is fundamentally at odds with the institutional reality of advanced democracies. While severe institutional instability in developing nations might occasionally justify a myopic policy horizon, governments in advanced economies are constrained by established institutional frameworks, long-term electoral accountability, and a commitment to intergenerational welfare.

Furthermore, default penalties, which are typically modeled as an exogenous decline in output and a temporary exclusion from financial markets, remain highly modest. They are often calibrated to a mere 2 to 4 percent drop in GDP[1] and a one to four year exclusion period. While assuming a mild output decline aligns with empirical estimates for emerging economies like Argentina or Mexico (Sturzenegger, 2004; Levy Yeyati and Panizza, 2011), applying this magnitude to advanced economies is structurally flawed. The Greek crisis, for instance, precipitated a catastrophic real GDP contraction of more than 25 percent from its pre-crisis peak. Critics might argue that such a severe contraction is merely an ex post realization, and that the government did not anticipate such devastating costs ex ante when contemplating default. However, we contend that for an advanced economy deeply integrated into the global financial system, profound economic dislocation is a widely foreseeable consequence. As Brunnermeier et al. (2016) point out, sovereign risk and banking risk deteriorate in tandem, triggering a severe feedback loop within the economy. Moreover, the relative size of the public sector in advanced economies is substantially larger than in emerging and developing nations. Therefore, treating the Greek crisis as analogous to emerging market defaults ignores critical institutional heterogeneity.

Beyond output costs, standard models critically misspecify the duration and nature of market exclusion. While empirical studies such as Dias, Richmond, and Westfahl (2024) find that defaulted countries can issue new bonds almost immediately after default, labeling this as partial market access, they also document that achieving full market access, defined as borrowing exceeding 1 percent of GDP, takes a median of 11 years. Furthermore, most canonical sovereign models assume a fresh start upon market reentry, meaning the outstanding debt of the sovereign is completely wiped out without any residual credit stigma. Thus, calibrating the model to the timeline of full market access, rather than a fleeting period of

[1] Following Chatterjee and Eyigungor (2012), it has become standard practice to employ nonlinear asymmetric cost functions, which assume that the punishment for default is disproportionately mild during severe recessions. This specification mechanically induces a strong incentive to default during adverse economic states.

exclusion, provides a much sounder empirical foundation.

Ultimately, the combination of these standard assumptions, including an excessively low discount factor, mild output costs, rapid full market reentry, and zero residual debt upon recovery, artificially inflates the sovereign incentive to default. By relying on these extreme parameterizations solely to match empirical default frequencies, canonical models systematically fail to explain how real world governments can sustain high debt-to-GDP ratios, a significant quantitative discrepancy also emphasized by Bolton et al. (2023). Appendix A summarizes these controversial parameter choices, specifically regarding the discount factor, GDP penalty, and exclusion duration, alongside the simulated debt-to-GDP ratios and default frequencies reported in prior canonical research.

The fundamental cause of these quantitative failures and unrealistic parameterizations largely stems from the standard application of the Simulated Method of Moments (SMM). This methodology relies on the premise of structural accuracy within the model and calibrates unobservable parameters by minimizing the distance between simulated moments and actual data. Because the literature predominantly targets empirical default frequencies, this calibration strategy mechanically forces the modeled government to be excessively myopic and the default penalties to be disproportionately mild. Consequently, while the model successfully generates frequent defaults, the government is strictly disincentivized from accumulating substantial debt to avoid the corresponding interest burden, which results in artificially low equilibrium debt levels.

One of the primary objectives of this paper is to examine whether sovereign default events, specifically in the context of the Greek debt crisis, can be attributed to a self-fulfilling default mechanism. To this end, prior to conducting numerical simulations, we provide a rigorous analytical characterization of the model's micro-foundations. Unlike many applied studies that rely heavily on computational algorithms to identify equilibrium regions, we mathematically prove the theoretical conditions under which a self-fulfilling crisis can occur. Crucially, we incorporate long-term government bonds into this framework. The introduction of long-term debt instruments is essential because the government's burden during a no-rollover event is limited strictly to the repayment of maturing debt, rather than the entire outstanding stock. This creates a distinct trade-off between the short-term liquidity burden and long-term solvency, providing a more realistic assessment of the government's incentives under liquidity pressure.

Building on this maturity structure, we formally prove the existence and uniqueness of two distinct debt thresholds corresponding to lenders' rollover and no-rollover expectations. By establishing the monotonic ordering of these thresholds, we mathematically partition the state space into three mutually exclusive regions: a Safe Zone, a Crisis Zone characterized by equilibrium multiplicity, and a Default Zone. Furthermore, we demonstrate how the equilibrium pricing mechanism endogenously enforces the government's ex post incentive compatibility constraint. This effectively rules out opportunistic deviations in which the sovereign issues massive debt only to default immediately, without relying on ad hoc timing restrictions. This rigorous theoretical foundation ensures that our subsequent quantitative results are driven by robust, mathematically proven economic mechanisms rather than being mere artifacts of numerical computation.

For parameter calibration, we deliberately avoid applying the Simulated Method of Moments with a full model simulation. Instead, we calibrate as many parameters as possible using estimates from existing literature or direct empirical data. In terms of key parameters, the steady state government discount factor is set to 0.966. This value is derived by multiplying the pure rate of time preference by a political discount factor, which reflects the probability of political survival based on the historical frequency of administration changes. The output penalty and the duration of market exclusion are calibrated to 0.07 and 27 quarters, respectively, based on ex post empirical data. The Simulated Method of Moments is applied only for the partial calibration of parameters outside the core model. We then evaluate the model validity by assessing whether the simulated moments, such as the default frequency, debt-to-GDP ratios, and sovereign spreads, align with historical data from Greek sovereign default episodes using these disciplined parameters.

However, the baseline self-fulfilling model with long term bonds relies on rigid assumptions, such as total debt repudiation and fleeting liquidity shocks, making it fundamentally inadequate to capture the protracted and complex nature of actual sovereign crises. Therefore, to improve model performance and capture the realities of the Greek experience, we extend the baseline framework in two critical dimensions. First, we introduce a partial default mechanism with a positive recovery rate. Unlike the standard assumption where default eliminates all debt, we model a scenario where the government retains a portion of its obligations. Under this assumption, the continuation value of default declines because the government still owes debt even after regaining a non default state. Furthermore, the sharp decline in sovereign spreads is mitigated because the bonds retain residual value even during a default episode. Second, we introduce persistence to the no rollover state. While standard models assume a liquidity crisis is a transient one period shock, historical evidence, such as the prolonged market exclusion of Greece, suggests these crises are durable. We impose a restriction where the economy may remain in a state of financial autarky for an extended period. This extension significantly widens the gap between the value of normal market access and the value of a liquidity crisis, thereby expanding the Crisis Zone to realistic levels.

Our findings indicate that the baseline specification of the conventional self-fulfilling default model, even with plausible parameters, fails to reproduce key empirical moments of the Greek crisis, such as the default frequency, the debt-to-GDP ratio, and sovereign spreads. These three simulated results are substantially lower than the actual data. Regarding the low debt-to-GDP ratio, the value of defaulting is artificially high due to the assumption of zero debt upon regaining a non default state. Meanwhile, the government is reluctant to accumulate debt to the point where the value of default exceeds the value of repayment. Consequently, debt accumulation stagnates well before the threshold unless an abnormally large exogenous shock occurs. In addition, the welfare cost of a temporary exclusion from the bond market is relatively small compared to the cost of outright default. As a result, the difference between the value of repayment and the value of a no rollover event becomes negligible, shrinking the Crisis Zone to a point where self-fulfilling crises rarely occur under realistic debt levels. Thus, both the default frequency and sovereign spreads fall short of empirical observations.

However, under the extended model, we successfully capture the empirical moments for default frequency and the debt-to-GDP ratio. The simulated result for default frequency is 0.77 percent per quarter, which closely aligns with the actual data of 0.5 percent. The average debt-to-GDP ratio reaches 118.0 percent in the simulation, precisely matching the empirical average of 117.1 percent. Regarding the period immediately prior to default, a gap remains, with the data showing 160 percent compared to 121.6 percent in the simulation. While the simulated sovereign spread improves significantly from the baseline model, it still remains lower than the actual data. This improvement occurs because the extended Crisis Zone prolongs the duration of positive default risk. Nevertheless, accurately capturing the exceptionally high spreads observed in reality remains a structural challenge for economic models. This quantitative discrepancy is also recognized by Bolton et al. (2023) as a persistent bond pricing puzzle, which similarly remains in our framework.

Our comparative analysis demonstrates that the extended self-fulfilling default model aligns with the empirical data significantly better than alternative specifications. By accurately reproducing the targeted moments of the Greek crisis, this framework proves highly applicable for sovereign debt analysis. Having established its empirical credibility, we utilize the model as a quantitative laboratory. Because it captures the structural vulnerabilities characteristic of advanced economies, it provides a robust foundation for evaluating policy interventions, thereby allowing us to transition from empirical validation to the exploration of practical crisis resolution strategies.

For policy implications, we quantitatively evaluate the dynamics and tradeoffs of sovereign debt crises using Generalized Impulse Response Functions and forward-looking counterfactual simulations. Our quantitative analysis reveals that once an economy enters a crisis zone, relying on gradual improvements in macroeconomic fundamentals is insufficient to restore debt sustainability. Positive endowment or favorable risk-free interest rate shocks reduce spreads and debt, but their impacts are strictly marginal. In stark contrast, an increase in the subjective discount factor of the government exhibits an overwhelmingly powerful and nonlinear impact. A more patient, forward-looking government triggers a virtuous cycle of rapid deleveraging that drastically compresses sovereign spreads. Furthermore, our counterfactual simulations highlight the chronic nature of debt crises. Under baseline policies, the economy faces severe serial default risks. While a forced austerity regime mechanically eliminates these risks and lowers the long-term debt burden, it inflicts substantial short-term pain by requiring primary surpluses that severely depress expected consumption. However, by evaluating the expected cumulative welfare over a long-term horizon, we demonstrate that the eventual benefits of structural deleveraging ultimately outweigh these profound short-term utility losses. Despite the immediate economic toll, strict adherence to fiscal consolidation proves to be a strictly welfare-improving strategy compared to the baseline cycle of recurring defaults.

These findings offer a strong theoretical justification for external interventions in expectation-driven crises. Because abrupt intrinsic changes in sovereign patience are practically unfeasible, external backstops are essential. Programs by a lender of last resort succeed by functioning as external commitment devices. Their strict conditionality effectively mimics a positive preference shock, binding the sovereign to a stringent, welfare-improving deleveraging path and successfully eliminating self-fulfilling default risk.

The remainder of the paper is organized as follows. Section 2 presents the theoretical model, detailing the environment and equilibrium definitions. Section 3 describes the functional forms, calibration strategy, and solution methods. Section 4 reports the results of the baseline model and discusses its limitations. Section 5 introduces the extended model, incorporating partial default and persistent liquidity crises, and presents the main quantitative findings. Finally, Section 6 concludes the paper and discusses the limitations of our framework along with avenues for future research.

2 Model

2.1 Environment, Timing and Recursive Equilibrium

2.1.1 Environment

Time is discrete, indexed by $t \in \{0, 1, 2, \dots\}$. The economy is small and open and is populated by three types of agents: households, a government, and foreign lenders. Households and foreign lenders each consist of a continuum of identical agents. A representative household derives utility from consumption c_t with expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{s=0}^{t-1} \beta_s \right) u(c_t)$$

where $u : \mathbb{R}_+ \to \mathbb{R}$ is continuously differentiable, strictly increasing and concave, and $\beta_s \in (0, 1)$ denotes a stochastic discount factor. In quantitative macroeconomics, discount factors are typically calibrated to high values (for instance, around 0.99 at a quarterly frequency). In contrast, the sovereign default literature frequently adopts unrealistically low discount factors to generate defaults in equilibrium, a practice that severely misrepresents the institutional stability and long horizon planning typical of advanced economies. To reconcile this tension, we depart from a constant parameter and introduce preference shocks that render intertemporal discounting time varying. Motivated by the empirical observation that political regime changes and expansionary fiscal episodes frequently shift the government effective rate of time preference, this stochastic specification allows the model to capture realistic long run savings behavior while preserving the possibility of endogenous sovereign default. Ultimately, capturing this political uncertainty enables the framework to reflect how sudden fluctuations in patience alter the government vulnerability to expectation driven crises, without relying exclusively on exogenous output shocks.

The government receives an exogenous, stochastic endowment y_t each period and transfers resources to households, possibly adjusting the timing of resources by issuing unsecured long-term bonds to risk-neutral foreign lenders. Government bonds are non-contingent, carry no commitment to repay, pay a common coupon each period, and have a long duration. At the beginning of period t, the government owes outstanding face value debt B_t, which is a bundle of claims with different remaining maturities $\{b_{t,t+n}\}_{n\geq 0}$. Following Chatterjee and Eyigungor (2012) and Bocola and Dovis (2019), we assume the cross-section of remaining maturities obeys an exponential law,

$$b_{t,t+n} = (1-\lambda)^n \lambda B_t \tag{1}$$

where $\lambda \in (0,1)$ is the maturity rate. Claims with $n = 0$ mature within period t, i.e., $b_{t,t} = \lambda B_t$.

For tractability and to match the reduced-form evidence on the approximately geometric run-off of sovereign debt portfolios, we treat λ as exogenous. This captures institutional features such as benchmark issuance programs and rollover norms that evolve slowly relative to business-cycle risk, and it permits the aggregation of the maturity distribution into a single endogenous state variable, B_t. Our approach aligns with recent literature, such as Aguiar et al. (2022), which similarly models the maturity rate as an exogenous parameter. All outstanding bonds pay a common coupon rate r_t, which is driven by a latent state variable ω_t. This variable captures persistent macro-financial conditions that simultaneously affect the risk-free rate and the pricing kernel. For convenience, we refer to ω_t simply as the interest rate shock. Consequently, the total coupon obligations at period t amount to $r_t B_t$.

In line with Arellano (2008) and much of the subsequent literature, the government's feasibility condition is given by the aggregate resource constraint of the small open economy.[2] When the government chooses to repay its debt, the aggregate feasibility condition is

[2] A growing strand of literature imposes an explicit government budget constraint with taxes and expenditures. We adopt the resource-constraint formulation to maintain comparability with the canonical sovereign default literature.

expressed as

$$c_t = y_t - \lambda B_t - r_t B_t + \Lambda_t$$

where Λ_t denotes net revenues from primary issuance, excluding the repayment of current debt obligation $b_{t,t}$. The term Λ_t can be explicitly formulated as

$$\begin{aligned} \Lambda_t &= \sum_{n=1}^{\infty} q_{t,t+n}[b_{t+1,t+1+n} - b_{t,t+n}] \\ &= \sum_{n=1}^{\infty} q_{t,t+n}\left[(1-\lambda)^{n-1}\lambda B_{t+1} - (1-\lambda)^n \lambda B_t\right] \end{aligned} \tag{2}$$

with $q_{t,t+n}$ the price at time t of a claim maturing at $t+n$. Equation (2) makes explicit that under the exponential maturity structure, net issuance depends solely on the aggregate debt states (B_{t+1}, B_t) and the maturity parameter λ.

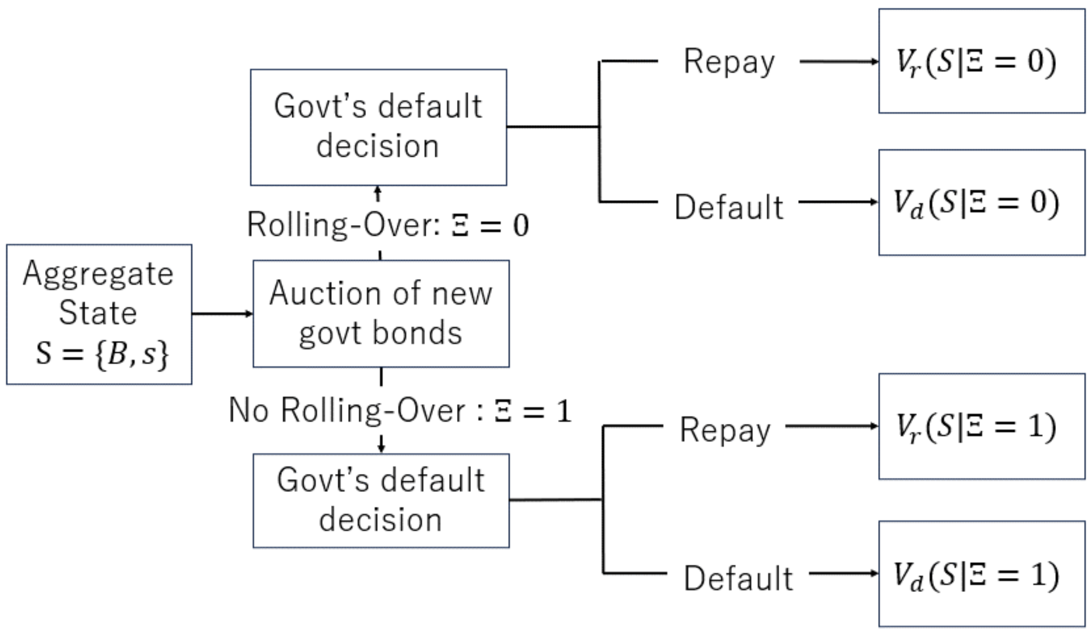
The term $q_{t,t+n}[b_{t+1,t+1+n} - b_{t,t+n}]$ represents the period-t cash flow from the primary market, namely the proceeds from newly issued claims net of buybacks of previously outstanding claims with remaining maturity n. A positive bracketed difference implies that the government raises funds through net issuance, whereas a negative difference indicates an outflow due to debt buybacks. This auction-related financing flow is distinct from debt service $(\lambda + r_t)B_t$, which appears separately in the aggregate resource constraint under repayment.

2.1.2 Timing and Equilibria

We characterize a recursive competitive equilibrium with a Markov structure. The timing of events in non-default periods is illustrated in Figure 2. At the beginning of period t, the government enters with outstanding face value debt B, and the exogenous state variables are realized. Let $s = (y, \omega, \beta, \zeta)$ collect the endowment y, the interest rate shock ω, the stochastic discount factor β and a sunspot variable ζ that coordinates lenders' rollover decisions. The aggregate state is summarized as $S = (B, s)$.

A key departure from the conventional strategic default setup is that the government bond auction takes place before the government's default decision. In the auction, the government announces its issuance such that next period's face value debt is B'. Risk-neutral foreign lenders then decide whether to purchase at the offered schedule. If they purchase, the government's obligations are rolled over; if not, rollover fails.

Figure 2: Timing of the Model



In standard strategic default models, the auction is assumed to occur after the repayment decision. Consequently, bond prices in those models reflect only future default risk, as they are determined conditional on the government having already honored its current obligations. By contrast, in the self-fulfilling default environment, bond prices incorporate the immediate risk that default is realized immediately after the auction within the same period. This timing ensures that lenders' expectations about rollover directly discipline the government subsequent default choice.

Following Cole and Kehoe (2000), we assume that all exogenous variables, including the sunspot ζ, are observed by both the government and lenders prior to the auction. This implies that lenders' expectations regarding rollover are a realized outcome driven by the sunspot, rather than a latent probability.[3]

After the auction outcome is realized, the government chooses whether to repay coupons and the maturing principal λB. If the government repays, households consume the endow-

[3] Some recent contributions, such as Aguiar et al. (2022), render expectations probabilistic rather than binary by introducing an idiosyncratic shock to the default payoff. Because this political or institutional shock is realized after the auction but prior to the default decision, lenders form non-degenerate beliefs over the rollover outcome based on partial information.

ment net of debt service plus net issuance proceeds. If it defaults, coupons and maturing principal are not honored, and output is reduced by an exogenous fraction. The government's value functions fundamentally depend on both its repayment decision and the realized rollover outcome at the auction. If the economy is in default, it re-enters financial markets in a subsequent period with an exogenous probability. The period ends with next period's outstanding debt B', which becomes the state variable carried into period $t+1$.

The Government Problem

The benevolent government that maximizes the representative household's welfare. The sequence of choices within the period is central for what follows: first the government chooses next period's face value debt B' at a primary-market auction; after the auction outcome is realized, it decides whether to repay or to default.

If the government honors both the coupon and the maturing principal in period t, households consume the endowment net of debt service, plus net issuance. Thus, the continuation value under repayment V_r is therefore

$$V_r(S;B') = u(y - \lambda B - r(\omega)B + \Lambda(S;B')) + \beta\mathbb{E}[V(S')|(S)],$$

where V is the value for the government prior to the decision of default or repayment. Two features are worth highlighting. First, the auction choice B' affects resources contemporaneously through Λ, not just through tomorrow's state. Second, because the auction precedes the default decision, bond prices $q_n(S)$ incorporate the endogenous default risk within the period rather than only at the period's end.

If the government chooses to default, it services neither the coupons nor the maturing share of principal in period t. Because the primary-market auction is held before the default decision, the net proceeds from issuance or buybacks at the auction, denoted Λ, are nevertheless realized within the period, even if the government subsequently defaults.[4] Output then falls by an exogenous fraction $\chi \in (0,1)$, capturing the output costs of default.[5] With

[4]As in Chatterjee and Eyigungor (2012), $\Lambda_t = \sum_{n\geq 1} q_{t,t+n}[b_{t+1,t+1+n} - b_{t,t+n}]$ represents the net cash flow from the primary market at the auction, namely the proceeds from new issuance net of buybacks. This is a financing flow realized at the auction rather than a debt service obligation. Total debt service, consisting of coupons plus the maturing share, is paid only on the repayment branch.

[5]This constant proportional decline in output during default follows Aguiar and Gopinath (2006). In contrast, other canonical studies, such as Arellano (2008) and Chatterjee and Eyigungor (2012), employ state-dependent functional forms for the endowment loss that vary with the realization of output. We discuss this modeling choice in greater detail in the following section.

probability $\psi \in (0,1)$ the economy regains market access next period and re-enters with zero debt; otherwise it remains in financial autarky. The value from default V_d can be written as

$$V_d(S; B') = u(y[1-\chi] + \Lambda(S; B')) + \beta\{(1-\psi)\mathbb{E}[\underline{V}(s')|(s)] + \psi\mathbb{E}[V(0, s')|(s)]\}, \tag{3}$$

where $\underline{V}$ is the default value that this state continues to the next period. When the default state continue to the next period, the issuance of new bonds and repayment are both zero, so

$$\underline{V}(s) = u(y[1-\chi]) + \beta\{(1-\psi)\mathbb{E}[\underline{V}(s')|(s)] + \psi\mathbb{E}[V(0, s')|(s)]\}.$$

The optimal problem for the government expressed as:

$$V(S) = \max_{B'} \max_{d\in\{0,1\}} \{(1-d)V_r(S; B') + dV_d(S; B')\}, \tag{4}$$

where $d \in \{0,1\}$ represents the government's default decision, taking the value 0 for repayment and 1 for default.

The Foreign Lender

Risk-neutral foreign lenders purchase government bonds in international markets to maximize expected profits. Let $b_{t,t+n}$ denote face value outstanding at t that matures in $t+n$. Under a long-term bond with geometric amortization λ, the change in notional across maturity buckets implied by the auction choice is $(b'_{n-1} - b_n) = (1-\lambda)^{n-1}\lambda B' - (1-\lambda)^n \lambda B$ for $\forall n \in \mathbb{N}$. New bonds trade at the auction if lenders expect rollover to succeed in period t.

Let $\Xi(S) \in \{0,1\}$ denote the realized rollover outcome at the auction (0 = rollover, 1 = no rollover). While the sunspot ζ is an exogenous trigger, the realized rollover outcome $\Xi(S)$ is an endogenous equilibrium object. Lenders form rational expectations: they will coordinate on a successful rollover ($\Xi(S) = 0$) only if they anticipate that the government will optimally choose repayment subsequent to the auction. In regions of the state space where the government's optimal decision strictly depends on the lenders' rollover coordination, the exogenous sunspot ζ acts as the coordinating device to resolve strategic uncertainty.

Let $m(s, s')$ be the lenders' pricing kernel, which is stochastically and exogenously determined. The zero-profit pricing condition for an n-period claim is

$$
\begin{aligned}
q_n(S;B') &= (1-\Xi(S))\mathbb{E}\left[m(s,s')(1-\Xi(S'))\right. \\
&\quad \left.((r(\omega')+q_{n-1}(S'))|(S)\right] \text{ for } n \geq 1
\end{aligned} \tag{5}
$$

with the convention $q_0(S) = 1$. Thus, if lenders coordinate on no rollover at the auction ($\Xi(S) = 1$), the price of all newly issued bonds is zero, $q_n(S) = 0$ for all $n \geq 1$. To economize on notation, we write $q_n(S)$, leaving the dependence on the auction choice B' implicit.

Definition: A *Markov recursive equilibrium* consists of a value function V, government policies $\{B'(S), d(S)\}$, a bond price schedule $\{q_n\}_{n\geq 0}$ and a rollover outcome map $\Xi(S)$ such that:

1. Given the bond price schedule and the rollover outcome map, the value function $V(S)$ and policy function $\{B'(S), d(S)\}$ solve the government's problem (4).

2. Given the government policies and the rollover outcome map, the bond prices $\{q_n\}_{n\geq 0}$ satisfies the condition (5) for all $n \geq 1$, with $q_0(S) = 1$.

3. The rollover outcome $\Xi(S)$ is self-fulfilling, consistent with the optimal default policy $d(S|\Xi)$. Under equilibrium multiplicity, the sunspot coordinates beliefs such that $\Xi(S) = \zeta$.

As in self-fulfilling rollover models, the environment can exhibit equilibrium multiplicity; we delineate the precise conditions for this multiplicity below.

Static Multiplicity

The self-fulfilling default environment admits multiple equilibria. By contrast, the standard strategic-default model typically features a unique equilibrium (Auclert and Rognlie, 2016). In the present framework, multiplicity arises because the primary-market auction precedes the government's default decision. Under this timing, sunspot-driven rollover beliefs regarding whether lenders coordinate on rollover or on refusal can directly shape the government's repayment choice. Intuitively, if lenders expect rollover, bond prices are positive, refinancing is feasible, and repayment can be optimal; if they expect no rollover, the price of newly issued bonds collapses to zero, refinancing fails, and default can become optimal. Hence, lenders' rollover beliefs and the government's subsequent decision can be mutually reinforcing.

We now delineate the conditions that distinguish environments with multiple equilibria from those with a unique equilibrium.

(a) Case: No Rollover at the Auction ($\Xi = 1$)

If lenders coordinate on no rollover, the price of all newly issued bonds is zero and the auction delivers no proceeds, i.e. $\Lambda = 0$. If the government nevertheless repays in period t, it must do so without new financing; next period's face value is $B' = (1 - \lambda)B$. The continuation value under repayment is

$$V_r(S|\Xi = 1) = u(y - \lambda B - r(\omega)B) + \beta\mathbb{E}[(V(B', s')|(S)]. \tag{6}$$

Meanwhile, if the government chooses to default, coupons and maturing principal are not honored, so the default value under lenders' no-rollover expectation equals the autarky value, meaning net revenues from primary issuance $\Lambda = 0$:

$$\begin{aligned} V_d(S|\Xi &= 1) = \underline{V}(s) \\ &= u(y[1 - \chi]) + \beta\{(1 - \psi)\mathbb{E}[\underline{V}(s')|(s)] + \psi\mathbb{E}[V(0, s')|(s)]\}. \end{aligned}$$

The post-auction problem, defined in Equation (4) is expressed as:

$$V(S|\Xi = 1) = \max_{d\in\{0,1\}} \{(1 - d)V_r(S|\Xi = 1) + d\underline{V}(s)\}.$$

Before proceeding, it is crucial to establish the existence and uniqueness of a debt threshold at which the government is indifferent between repayment and autarky. This threshold delineates the precise point where, under no-rollover beliefs, the government's incentive shifts from repayment to default. Demonstrating its existence ensures a well-defined partition of the state space into repayment and default regions, thereby providing a rigorous foundation for identifying the "Safe Zone," which represents the region where the government optimally chooses to repay even when lenders coordinate on a rollover failure.

For a given state s, define repayment set under the lenders' anticipation of no rolling-over as:[6]

$$\underline{\mathcal{R}}(s) \equiv \{B \in [0, \infty) : V_r(S|\Xi = 1) \geq \underline{V}(s)\}.$$

The set $\underline{\mathcal{R}}(s)$ collects debt levels for which the repayment value is at least as attractive as the default value. We can then define the debt threshold that separates the repayment and default regions under the foreign lenders' expectation of no rollover.

Proposition 1. *For a given state s, there exists a unique threshold debt level $\underline{B} \in [0, \infty)$ such that*

$$\underline{B}(s) \equiv \sup \underline{\mathcal{R}}(s)$$

Proof: See Appendix.

Because $V_r(S|\Xi = 1) \geq \underline{V}(s)$ at $B = 0$ and $V_r(S|\Xi = 1)$ is strictly decreasing in B, it follows that for all $B \in [0, \underline{B}(s)]$, we have $V_r(S|\Xi = 1) \geq \underline{V}(s)$. Thus, the government optimally chooses repayment even when lenders initially anticipate no rollover in state s.

Under these circumstances, the expected return from newly issued government bonds is strictly positive, making the strategy of withholding funds strictly suboptimal. Consequently, rational lenders have no incentive to abstain from the primary market, and their optimal behavior is to supply funds. Anticipating this, the government expects successful rollover (i.e., $\Xi = 0$) at the time of issuance for all $B \in [0, \underline{B}(s)]$. Following Cole and Kehoe, this region is therefore referred to as the Safe Zone, where the no-rollover belief cannot be self-fulfilling and repayment is the unique equilibrium outcome. Meanwhile, if $B \in (\underline{B}(s), \infty)$, we have $V_r(S|\Xi = 1) < \underline{V}(s)$,implying that the government will optimally choose to default. In this region, the realization of a no-rollover expectation by lenders directly leads to the government's default, thereby rendering the no-rollover belief self-fulfilling.

(b) Case: Rollover at the Auction ($\Xi = 0$)

When lenders coordinate on rollover, newly issued bonds sell at positive prices and the primary-market cash inflow $\Lambda(S|\Xi = 0)$ is realized within the period t. The repayment value is

$$V_r(S|\Xi = 0) = u(y - \lambda B - r(\omega)B + \Lambda(S|\Xi = 0)) + \beta \mathbb{E}[(V(B', s'))|(S)],$$

[6] The upper bound of the admissible debt set can, without loss of generality, be taken as $+\infty$; if $y < (\lambda + r(\omega))B$ then consumption would be negative and utility diverges to $-\infty$. In such cases, the government strictly prefers default, ensuring that the economically relevant repayment set $\underline{\mathcal{R}}(s)$ is endogenously bounded from above.

where the issuance choice B' is determined at the auction. Prices satisfy the no-arbitrage condition

$$q_n(S|\Xi = 0) = \mathbb{E}[m(s,s')(1 - \Xi(S'))((r(\omega') + q_{n-1}(S'))||(S)] \tag{7}$$

for $n = 1, 2, \cdots$, and $q_0(S) = 1$.

If the government defaults after a rollover auction, the default value is

$$V_d(S|\Xi = 0) = u(y[1-\chi] + \Lambda(S|\Xi = 0)) + \beta\{(1-\psi)\mathbb{E}[\underline{V}(s')|(s)] + \psi\mathbb{E}[V(0,s')|(s)]\}.$$

The term $\Lambda(S|\Xi = 0)$ enters current utility even when default is chosen after the auction, reflecting that funds are raised at the auction before the default decision is finalized.

Consequently, under rollover expectations ($\Xi = 0$), the government solves

$$V(S) = \max_{B'} \max_{d \in \{0,1\}} \{(1-d)V_r(S|\Xi = 0) + dV_d(S|\Xi = 0)\}.$$

Given this nested maximization structure, the government sequentially chooses the issuance level B' prior to making its final default decision d. This specific intra-period timing theoretically creates a severe moral hazard problem: the government might be tempted to act opportunistically by issuing a massive volume of new debt, securing the primary-market proceeds Λ, and then immediately choosing $d = 1$ to evade the ensuing repayment obligations. To understand how the model rules out deviations in which the government issues large amounts of debt and immediately defaults, consider the government's ex post incentive compatibility (IC) constraint for repayment:

$$V_r(S|\Xi = 0; B') \geq V_d(S|\Xi = 0; B').$$

Let $\widehat{B}'$ denote the upper bound of the issuance choices that satisfy this condition, provided such a bound exists. If the current debt B is sufficiently low such that a valid $\widehat{B}'$ exists and repayment is preferred to pure autarky (i.e., $\max_{B' \leq \widehat{B}'} V_r(S|\Xi = 0; B') > \underline{V}(s)$), the government might be tempted to issue $B' > \widehat{B}'$. However, rational lenders perfectly anticipate that such an issuance inevitably leads to a subsequent default. Consequently, lenders price this debt at zero ($q = 0$), yielding no primary-market proceeds ($\Lambda = 0$), and the government's payoff collapses to the pure autarky value $\underline{V}(s)$. Thus, the government strictly prefers to choose $B' \leq \widehat{B}'$ and repay. Conversely, if the current debt B is already so high that no feasible B' satisfies the IC constraint (i.e., $V_r < V_d$ for all B'), lenders anticipate default regardless of the issuance choice. The bond price is universally zero ($q = 0$ and $\Lambda = 0$), and the government simply defaults, obtaining $\underline{V}(s)$. In both cases, the equilibrium pricing mechanism ensures that the government can never realize positive proceeds ($\Lambda > 0$) followed by an immediate default. This formally eliminates the possibility of issuing debt and immediately defaulting across the entire state space.[7]

Building on this logic, we establish the existence and uniqueness of a debt threshold at which the government is indifferent between repayment and autarky under lenders' expectation of a rollover. First, we define the rollover-feasible set for a given s as:

$$\overline{\mathcal{R}}(s) \equiv \{B \in [0, \infty) : \sup_{B' \in \Gamma(B,s)} [V_r(S|\Xi = 0) - V_d(S|\Xi = 0)] \geq 0\},$$

where $\Gamma(B, s)$ is the feasible set of issuance choices at (B, s).

The set $\overline{\mathcal{R}}(s)$ collects debt levels for which some feasible issuance plan B' makes repayment under rollover beliefs at least as attractive as default. Then, we can define the threshold that separates the government's choice to default or repay under foreign lenders' expectation of a rollover.

Proposition 2. *For given s, there exists a unique debt threshold $\overline{B}(s) \in [0, \infty)$ such that*

$$\overline{B}(s) \equiv \sup \overline{\mathcal{R}}(s)$$

Proof: See Appendix.

Lemmas in the Appendix guarantee (i) non-emptiness for small B (default losses and continuity), (ii) boundedness from above for large B (current-period debt service dominates), and (iii) monotonicity of the repayment–default value gap in B. These properties imply that

[7] While our framework eliminates such opportunistic deviations endogenously through the equilibrium pricing mechanism under the timing convention of Aguiar et al. (2022), other studies adopt different approaches. For instance, Bocola and Dovis (2019) adopt a market microstructure in which, if the government chooses to default in period t, the period-t primary-market funding is effectively not realized, as debt issuance is incompatible with a same-period default. Their alternative timing assumption rules out outcomes where the government issues debt and subsequently defaults within the same period by assumption, rather than by equilibrium pricing.

$\overline{\mathcal{R}}(s)$ is an interval of the form $[0, B(s)]$ and pin down a unique threshold $B(s)$ at which $V_r = V_d$ under $\Xi = 0$.

Distinction of Three Zones

Building on the analysis in cases (a) and (b), we formalize how lenders' rollover beliefs partition the debt space into three regions. To do so, establishing the monotonic ordering $\underline{B}(s) \leq \overline{B}(s)$ is crucial. This ordering fundamentally allows us to rule out pathological outcomes, such as the government choosing to default when lenders expect a rollover, but repaying when lenders expect no rollover.

To compare the two thresholds directly, we must bridge their definitions. Recall that $\underline{B}(s)$ is defined relative to the pure autarky value $\underline{V}(s)$, whereas $\overline{B}(s)$ is defined relative to the default value under rollover expectations, $V_d(S|\Xi = 0)$.

Therefore, to facilitate a direct comparison between $\underline{B}(s)$ and $\overline{B}(s)$, we first demonstrate that at the exact boundary point $\overline{B}(s)$, the repayment value $V_r((\overline{B}(s), s)|\Xi = 0; B'_*)$ intersect with the autarky value $\underline{V}(s)$. This implies the following proposition:

Proposition 3. *For given s, there exists an optimal issuance plan B'_* such that at the unique rollover threshold $\overline{B}(s)$:*

$$V_r((\overline{B}(s), s)|\Xi = 0; B'_*) = V_d((\overline{B}(s), s)|\Xi = 0; B'_*) = \underline{V}(s)$$

Proof: See Appendix.

This normalization allows one to dispense with explicit references to $V_d(S|\Xi = 0)$ when comparing thresholds: at the rollover threshold, repayment equals default and both coincide with the autarky benchmark $\underline{V}(s)$.

As outstanding debt B increases, both $V_r(S|\Xi = 0)$ and $V_r(S|\Xi = 1)$ decrease monotonically due to high amortization and interest burdens, whereas $\underline{V}(s)$ is independent of B. Moreover, for any (B, s),

$$\sup_{B'} V_r(S|\Xi = 0; B') \geq V_r(S|\Xi = 1)$$

because the expectation of rollover ($\Xi = 0$) strictly relaxes the within-period budget constraint via Λ and permits the optimal choice of B' to improve the continuation value. Combining this weak dominance with the equality established in Proposition 3, we obtain:

Proposition 4. *For given s,* the no-rollover threshold is weakly below the rollover threshold:

$$\underline{B}(s) \leq \overline{B}(s)$$

Proof: See Appendix.

With this separation, we can clearly distinguish following three regions, depending on the debt outstanding B.

1. Safe Zone: $V_r(S|\Xi = 1) \geq \underline{V}(s)$ (equivalently $B \in [0, \underline{B}]$).

2. Crisis Zone: $V_r(S|\Xi = 0) \geq \underline{V}(s) \geq V_r(S|\Xi = 1)$ (equivalently $B \in [\underline{B}, \overline{B}]$)

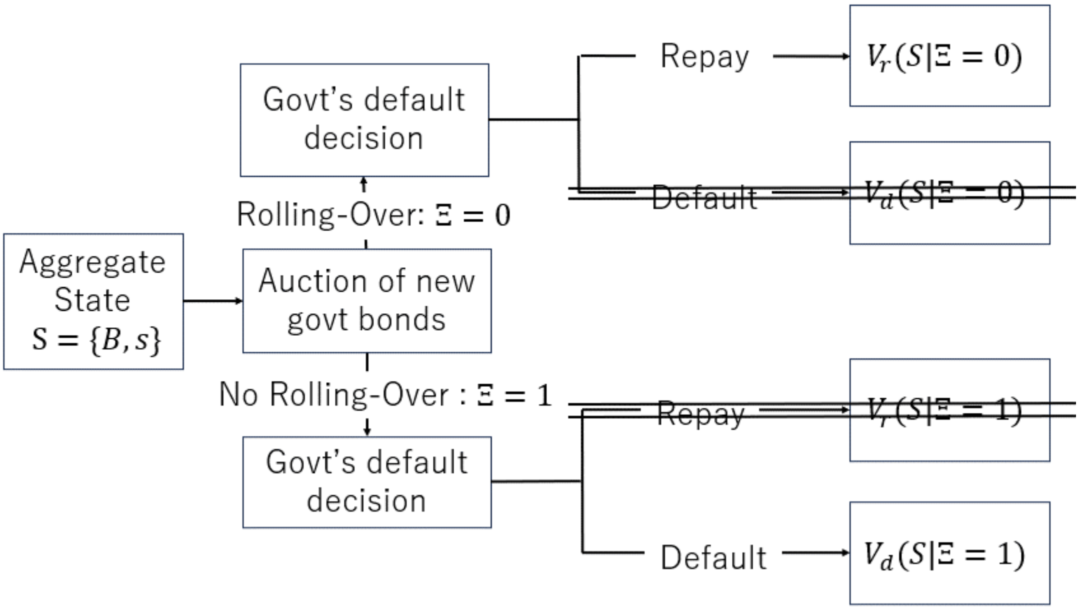
3. Default Zone: $V_r(S|\Xi = 0) > \underline{V}(s)$ (equivalently $B \in [\overline{B}, \infty)$).

In the Safe Zone, regardless of lenders' expectation of no rollover, the government would not choose to default. So, lenders understand its behavior, so they will not expect default in the first place. Default would never happen. In the Crisis Zone, if $\Xi = 1$ is realized, then $\underline{V}(s) \geq V_r(S|\Xi = 1)$ and the government defaults; if $\Xi = 0$ is realized, then $V_r(S|\Xi = 0) \geq \underline{V}(s)$ and the government repays. Thus, lenders' beliefs are pivotal: the realized coordination of sunspot ζ determines the government's action. In the Default Zone, regardless of lenders' expectation of rollover, the government would choose to default. So, lenders understand its behavior, so they will expect default in the first place. Default is inevitable. Consequently, the timing in Figure 2 is revised as shown in Figure 3. There is no possibility of lenders' rollover expectation leads default, and lenders' no rollover expectation leads repayment.

2.1.3 Strategic Default

To precisely isolate the quantitative and theoretical implications of self-fulfilling rollover crises, we introduce a canonical strategic default model as a comparative benchmark. In reality, observed sovereign spreads and default events are driven by a combination of fundamental insolvency and pure liquidity runs. By formulating an environment where default is driven exclusively by fundamentals, we can benchmark our main model against the standard framework in the literature (e.g., Eaton and Gersovitz, 1981; Arellano, 2008).

Figure 3: Modified Timing of the Model



The fundamental distinction between the self-fulfilling environment described above and this canonical strategic default framework lies in the intra-period timing of the government's decisions. In the strategic default model, the government makes its default decision before accessing the primary market. Consequently, if the government chooses to default, it is immediately excluded from financial markets for the current period, rendering any contemporaneous debt auction impossible.

Under this timing protocol, the value of default, denoted V_d^{SD}, depends solely on the exogenous aggregate state s, as the government neither services its existing debt nor raises new funds $\Lambda = 0$. Output contracts by the standard penalty parameter χ, and the economy enters financial autarky with probability $1 - \psi$. The default value is strictly independent of any auction choice B':

$$V_d^{SD}(s) = u(y[1-\chi]) + \beta\{(1-\psi)\mathbb{E}[V_d^{SD}(s')|(s)] + \psi\mathbb{E}[V^{SD}(0, s')|(s)]\}.$$

Alternatively, if the government commits to repayment, it honors its current debt service and subsequently enters the primary market to issue new debt B'. The repayment value V_r^{SD}

involves optimizing over the choice of next period's face value B' subject to the endogenously determined bond price schedule q^{SD}:

$$V_r^{SD}(S) = \max_{B'} u(y - \lambda B - r(\omega)B + \Lambda^{SD}(B', s)) + \beta\mathbb{E}[(V^{SD}(B', s')|(s)],$$

where $\Lambda^{SD}(B', s)$ represents the net cash flow from the primary market under strategic default pricing. The government's period t problem is therefore to choose the maximum between the repayment value and the default value:

$$V^{SD}(S) = \max_{d^{SD}\in\{0,1\}} \{(1 - d^{SD})V_r^{SD}(S) + d^{SD}V_d^{SD}(s)\}.$$

Because the default decision precedes the auction, foreign lenders form their prices based solely on the fundamentals (B', s) rather than relying on a sunspot-driven rollover variable Ξ. Risk-neutral lenders yield the following zero-profit pricing condition for newly issued bonds:

$$q_n^{SD}(B', s) = \mathbb{E}[m(s, s')(1 - d^{SD}(B', s'))((r(\omega') + q_{n-1}^{SD}(B', s'))|(s)]$$

for $n \geq 1$, with $q_0^{SD} = 1$.

This structural shift in timing fundamentally alters the equilibrium properties of the model. In the strategic default benchmark, the bond price q^{SD} is purely a function of the sovereign's fundamental incentives to repay tomorrow, completely insulating the current period's funding market from arbitrary shifts in lender sentiment. Consequently, the strategic default model generates a unique equilibrium. To parallel our previous analysis, we can define the repayment set for the strategic default model as:

$$\mathcal{R}^{SD}(s) \equiv \{B \in [0, \infty) : V_r^{SD}(S) \geq V_d^{SD}(s)\}.$$

Proposition 5. For a given state s in the strategic default model, there exists a unique debt threshold $B^{SD}(s) \equiv \sup \mathcal{R}^{SD}(s)$ such that the government weakly prefers repayment for $B \leq B^{SD}(s)$ and strictly prefers default for $B > B^{SD}(s)$.

Proof: See Appendix.

The uniqueness of this threshold implies that the state space is strictly partitioned into only two distinct regions:

1. Repayment Zone: $B \in [0, B^{SD}(s)]$, where $V_r^{SD}(S) \geq V_d^{SD}(s)$.

2. Default Zone: $B \in (B^{SD}(s), \infty)$, where $V_r^{SD}(S) < V_d^{SD}(s)$.

Crucially, the intermediate Crisis Zone, which characterizes the self-fulfilling model, vanishes entirely. Without the assumption that funds can be raised at an auction prior to the default decision, lenders cannot coordinate on a "no-rollover" equilibrium that forcibly pushes an otherwise solvent government into default.

3 Functional Form, Calibration and Solution Strategies

This section specifies the functional forms and calibrates the parameters. In the existing literature, most studies calibrate parameters using the Simulated Method of Moments (SMM) to match the moments of key macroeconomic data, such as the frequency of default or the risk premium on government bonds. However, this method assumes the accuracy of the underlying default mechanism. Consequently, even if the model's mechanism is misspecified, SMM forces the parameters to adjust to match empirical moments, often resulting in estimated parameter values that are inconsistent with the broader macroeconomic literature. For example, the time discount factor β is usually set to extremely low values, between 0.8 and 0.9, in the sovereign default literature, while most standard macroeconomic research sets this value closer to one (e.g., 0.98 or 0.99). By assuming a highly myopic government, prior studies structurally build in a strong incentive to consume today, which leads to more frequent default choices in the model.

Furthermore, the sovereign default literature typically sets low default costs and short exclusion durations. For instance, in their canonical analyses of the Argentinean default, Arellano (2008) and Aguiar and Gopinath (2006) set the endowment decline during the default state to 3.1% and 2.0%, respectively, with average exclusion periods of about one year and 2.5 years. However, empirical analyses suggest significant heterogeneity in default costs (Levy Yeyati and Panizza, 2011). To accurately analyze the Greek economy, we need to assume severe default penalties consistent with its actual historical experience. Otherwise, the model would generate an artificially high incentive for voluntary default, whereas Greece's actual default might not have been strictly a voluntary policy choice.

In addition, most of the literature assumes that outstanding debt is fully discharged upon regaining access to financial markets. This assumption also creates a strong incentive for the government to default. While we initially assume zero debt upon recovery in our baseline model to evaluate its comparability with conventional models, we subsequently modify this framework to incorporate residual debt that cannot be eliminated through restructuring.

In our baseline estimation, we calibrate the parameters without relying on a model-based SMM. The primary purpose of this study is to examine whether defaults can be triggered under a self-fulfilling mechanism. Instead of a model-based SMM, we calibrate the parameters using standard values from prior research, direct empirical data, and a data-based SMM approach. Specifically, we apply this independent SMM solely to parameters related to the pricing kernel of foreign lenders and the endowment process. This ensures that fundamental variables governing the default triggers, such as the discount factor, the endowment decline during default, and the duration of default, are not artificially adjusted.

3.1 Functional Form

The households' utility function is assumed to take a conventional constant relative risk aversion (CRRA) form:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

where σ is the coefficient of relative risk aversion. Following Aguiar and Gopinath (2006), we specify a fixed proportion of endowment decline due to default as $\chi \in (0, 1)$, regardless of the realization of the endowment y. In contrast, other canonical papers assume that the endowment decline depends on the realization of y to structurally provide a high incentive for the government to choose default. For example, Arellano (2008) sets the amount of endowment decline by default as $\max\{0, y - \overline{y}\}$, with $\overline{y}$ calibrated to 0.969. This implies that if the exogenous endowment falls by more than 3.1%, there is no further punishment for the government, effectively capping the upper bound of the endowment at 0.969 even when the actual realization is higher. Similarly, Chatterjee and Eyigungor (2012) define the endowment decline by default as $\max\{0, d_0 y + d_1 y^2\}$, calibrating $d_0 < 0$ and $d_1 > 0$. Under this specification, the cost is zero for $0 \leq y \leq -d_0/d_1$ and rises more than proportionally if

$y > -d_0/d_1$. Consequently, when the endowment is low, the default cost is assumed to be negligible, which again provides a strong incentive for the government to default. However, to the best of our knowledge, empirical evidence does not necessarily support the premise that default costs are exceptionally mild during severe recessions. Thus, these two mainstream functional forms appear to be arbitrarily specified merely to generate a sufficiently high default frequency within the model. Since one of the primary objectives of this paper is to examine whether moments, including the default probability, can be matched using a conventional parameterization without relying on artificial incentives, we avoid adopting such peculiar settings. Instead, we employ the simple proportional cost function described above.[8] Given the limited abundance of sovereign default data in advanced economies, adopting a simple proportional cost function with fewer parameters to estimate is methodologically preferable.

Following Ang and Piazzesi (2003) and subsequent affine term structure models such as Johri, Khan, and Sosa-Padilla (2022) and Bocola and Dovis (2019), we define the stochastic discount factor as:

$$m_{t+1} = \exp(-r_t)\frac{\xi_{t+1}}{\xi_t} \tag{8}$$

where r_t denotes the risk-free short rate, interpreted here as the continuously compounded yield on a one-quarter zero-coupon bond, and ξ_t is the Radon–Nikodym derivative that maps expectations under the physical measure to the risk-neutral measure. The risk-free interest rate r_t follows an affine process driven by a shock to a latent state variable ω_t:

$$r_t = r_0 + r_1\omega_t \tag{9}$$

where r_0 and r_1 are parameters. The state variable ω_t evolves according to a stationary AR(1) process:

$$\omega_{t+1} = \mu_\omega(1 - \rho_\omega) + \rho_\omega\omega_t + \sigma_\omega\varepsilon_{\omega,t+1} \tag{10}$$

with persistence parameter $\rho_\omega \in (0, 1)$, innovation volatility σ_ω, and i.i.d. Gaussian shocks $\varepsilon_{\omega,t+1} \sim N(0, 1)$. This specification captures the predictable yet stochastic movements of macro-financial fundamentals that drive both interest rates and risk premia.

[8] An additional reason for avoiding a quadratic or asymmetric form is that its parameters must typically be estimated via SMM to match targeted moments.

The Radon–Nikodym derivative is assumed to follow a log-normal process reflecting time-varying risk pricing:

$$\xi_{t+1} = \xi_t \exp(-\frac{1}{2}\nu_t^2\sigma_\omega^2 - \nu_t\sigma_\omega\varepsilon_{\omega,t+1}) \tag{11}$$

where ν_t represents the market price of risk. Following the affine tradition, we model this risk price as a linear function of the state variable:

$$\nu_t = \nu_0 + \nu_1\omega_t \tag{12}$$

where ν_0 and ν_1 are parameters. This specification allows risk premia to co-move with the same fundamental factor ω_t that drives the short rate, consistent with empirical evidence on bond yields and sovereign spreads. Substituting Equations (9) and (11) into the definition of m_{t+1} in equation (8), we obtain the following log-normal form of the pricing kernel:

$$m_{t+1} = \exp(-\frac{1}{2}\nu_t^2\sigma_\omega^2 - r_0 - r_1\omega_t - \nu_t\sigma_\omega\varepsilon_{\omega,t+1}) \tag{13}$$

This expression explicitly links the stochastic discount factor to observable and latent state variables, where $r_t(= r_0 + r_1\omega_t)$ determines the risk-free discounting, and the term $\nu_t\sigma_\omega\varepsilon_{\omega,t+1}$ captures the compensation for exposure to the underlying innovation $\varepsilon_{\omega,t+1}$.

The endowment process y_t is specified as a log-normal AR(1) process that is jointly affected by its own idiosyncratic shock and the innovation ω_t :

$$\ln y_{t+1} = \rho_y \ln y_t + \sigma_y\varepsilon_{y,t+1} + \sigma_{y\omega}\varepsilon_{\omega,t+1} \tag{14}$$

where ρ_y, σ_y, and $\sigma_{y\omega}$ are parameters. The inclusion of the cross-term $\sigma_{y\omega}\varepsilon_{\omega,t+1}$ allows endowment shocks and financial shocks to be correlated, reflecting the empirical co-movement between output fluctuations and sovereign yield spreads.

The time-varying discount factor is assumed to follow a standard log-normal AR(1) process:

$$\log(\beta_t) = (1 - \rho_\beta)\log(\overline{\beta}) + \rho_\beta \log(\beta_{t-1}) + \varepsilon_t^\beta \tag{15}$$

where $\rho_\beta \in (0, 1)$ is the persistence parameter, $\overline{\beta} \in (0, 1)$ is the long-run mean of the discount factor, and $\varepsilon_t^\beta \sim N(0, \sigma_\beta^2)$ is the exogenous preference shock.

While the probability of a self-fulfilling crisis is typically driven by an independent, purely exogenous sunspot variable in canonical frameworks (e.g., Bocola and Dovis, 2019), in our framework, the realization of the sunspot $\zeta \in \{0,1\}$, which dictates the lenders' beliefs regarding rollover ($\Xi = 0$) or no-rollover ($\Xi = 1$), is intrinsically linked to the value functions $V_r(S|\Xi = 0)$, $V_r(S|\Xi = 1)$, and $\underline{V}(s)$. Outside the crisis region, beliefs are deterministic: $\Xi = 0$ in the safe region (where $\underline{V}(s) \leq V_r(S|\Xi = 1)$) and $\Xi = 1$ in the default region (where $\underline{V}(s) \geq V_r(S|\Xi = 0)$). However, when the economy enters the crisis region, the realization of the lenders' belief becomes probabilistic. As $\underline{V}(s)$ moves away from $V_r(S|\Xi = 1)$ and closer to $V_r(S|\Xi = 0)$, the probability of a no-rollover coordination failure increases. We define an auxiliary variable δ_t as:

$$\delta_t = \frac{\underline{V}(s) - V_r(S|\Xi = 1)}{V_r(S|\Xi = 0) - V_r(S|\Xi = 1)}$$

The cutoff probability υ_t is then drawn from a Beta distribution:

$$\upsilon_t \sim Beta(\delta_t; \phi_0, \phi_1) \tag{16}$$

where ϕ_0 and ϕ_1 are parameters. Drawing a random variable $U_t \sim Uniform(0,1)$, the sunspot realization determining the rollover belief ($\zeta = 0 \Rightarrow \Xi = 0$) occurs if $U_t \leq \upsilon_t$, and the no-rollover belief ($\zeta = 1 \Rightarrow \Xi = 1$) is realized if $U_t > \upsilon_t$. Consequently, the government's default decision $d_t \in \{0,1\}$ can be summarized as follows:

$$d_t = \begin{cases} 0 & \text{if } \underline{V}(s) \leq V_r(S|\Xi = 1) \\ 0 & \text{if } V_r(S|\Xi = 1) < \underline{V}(s) < V_r(S|\Xi = 0) \text{ and } U_t \leq \upsilon_t \\ 1 & \text{if } V_r(S|\Xi = 1) < \underline{V}(s) < V_r(S|\Xi = 0) \text{ and } U_t > \upsilon_t \\ 1 & \text{if } \underline{V}(s) \geq V_r(S|\Xi = 0) \end{cases}$$

Conventional approaches often introduce the sunspot variable as an additional, independent Markov process to capture coordination failures (e.g., Cole and Kehoe, 2000; Lorenzoni and Werning, 2019). While conceptually appealing, this purely exogenous approach increases the number of state variables and substantially raises computational complexity. To maintain tractability in the high-dimensional value function iteration, we instead assume that the distribution of the sunspot variable is directly governed by the relative distances between the value functions. Unlike prior research, which treats the probability of panics uniformly across the crisis region, our formulation captures the nuanced severity of the government's proximity to default without artificially expanding the state space.

3.2 Calibration

We calibrate parameters using quarterly data. Table 1 summarizes the calibrated parameters. First, the risk aversion coefficient σ is set to 2, a value widely applied in the literature. The probability of reentry, ψ, is set to 0.036, calculated as the reciprocal of the duration of the default period. The duration of default is approximately 27 quarters, corresponding to the period from the onset of the Greek default in July 2011 (Asonuma and Trebesch, 2016) to August 2018, when the country officially ended its reliance on bailouts provided by external organizations.

The endowment decline during the default, χ, is set to 0.07. This value is obtained as the average GDP decline between 2011:Q3 and 2018:Q3 using detrended sampled GDP data. To extract the cyclical component of the endowment process for this analysis, we must detrend the raw data without introducing end-of-sample bias. While the Hodrick-Prescott (HP) filter is standard in the literature, its two-sided nature uses future data to smooth past trends, which artificially incorporates post-crisis collapses into the trend itself and understates the severity of the crisis (Cogley and Nason, 1995; Hamilton, 2018). To overcome this issue, we adopt a fixed-parameter local projection approach based on Hamilton (2018). Specifically, we estimate the trend parameters using a linear projection restricted strictly to the pre-crisis subsample (2000:Q1 to 2008:Q4). By projecting these pre-crisis structural coefficients onto the post-crisis period, the resulting cyclical component captures the magnitude of the prediction error, the deviation from the prevailing growth path that economic agents had expected prior to the default. A detailed explanation is provided in Appendix C.

The average bond maturity for Greece between 2003 and 2010 was 7.2 years, based on OECD data. Thus, the average maturity rate λ is set to $0.035(= 1/(7.2\times4))$. To calibrate the preference shock process defined in Equation (15), we must determine three parameters: the long-run mean of the discount factor $\overline{\beta}$, the persistence parameter ρ_β, and the volatility of the exogenous preference innovation σ_β. From the standard Euler equation, we have:

$$1 = \mathbb{E}_t \left[\beta_t \frac{U'(C_{t+1})}{U'(C_t)} R_t \right]$$

Assuming that consumption volatility is sufficiently small ($C_{t+1} \approx C_t$), this relationship can be approximated as $\ln(\beta_t) \approx -\ln(R_t)$, where R_t denotes the gross risk-free real interest rate. Thus, we can estimate the dynamics of β_t using empirical real interest rate data.

We measure the nominal interest rate using the yield on newly issued 3-month Treasury bills and calculate inflation via the GDP deflator. Our sample spans from 2001:Q1 (the adoption of the euro) to 2011:Q2 (the quarter immediately preceding the default in July 2011). Over this period, the average real interest rate was approximately zero. By fitting an AR(1) process to the real interest rate ($r_t = c + \rho_r r_{t-1} + u_t$), we estimate an autoregressive coefficient of $\rho_r = 0.67$ and a residual standard deviation of $\sigma_u = 0.003$. Consequently, we set the preference shock parameters to $\rho_\beta = 0.67$ and $\sigma_\beta = 0.003$.

Regarding the long-run mean $\overline{\beta}$, setting it close to unity (implied by a near-zero real interest rate) would make the sovereign unrealistically forward-looking. To address this, we define the effective discount factor as the product of the government's pure time preference parameter, $\widetilde{\beta}$, and the probability of the incumbent administration remaining in power, ϱ (i.e., $\overline{\beta} = \widetilde{\beta}\varrho$). This formulation is well-established in the political economy literature. For instance, Alesina and Tabellini (1990) argue that the possibility of losing an upcoming election induces myopic behavior in incumbent governments. Similarly, Cuadra and Sapriza (2008) explicitly discount the sovereign's future utility by the probability of political survival. In the case of Greece, the near-zero average real interest rate during the 2001:Q1 to 2011:Q2 period implies a pure time preference parameter of $\widetilde{\beta} \approx 1$. To calibrate ϱ, we note that Greece experienced five changes in administration between its democratization in 1974:Q3 and 2011:Q2, a period spanning 148 quarters. Thus, the quarterly probability of political survival is calculated as $\varrho = 1 - 5/148 \approx 0.966$. Consequently, the long-run mean of the effective discount factor is calibrated to $\overline{\beta} = 0.966$.

To analyze the shape of the Beta distribution and establish the mapping between the theoretical debt-to-GDP ratio and the implied default probabilities extracted from CDS spreads, we specify a nonlinear relationship using the cumulative distribution function (CDF) of the Beta distribution. Sovereign default risks typically exhibit highly nonlinear dynamics, where market concerns spike abruptly as debt approaches critical levels. After normalizing the debt levels between a safe bound and a default bound, we estimate the shape parameters $\{\phi_0, \phi_1\}$ via Non-Linear Least Squares (NLS). Crucially, the absolute level of implied default probabilities backed out from CDS spreads varies significantly depending on the haircut rate assumed by lenders. Under the baseline scenario, the haircut rate is assumed to be 100%. However, as discussed in a subsequent section, we also examine a scenario with a haircut rate of 53.5%. In this alternative case, the estimated values of the shape parameters $\{\phi_0, \phi_1\}$ differ from the baseline calibration.

The parameters related to the pricing kernel of foreign lenders (from r_0 to σ_ω in Table 1) and endowment process (from ρ_y to $\sigma_{y\omega}$ in Table 1) are calibrated using the SMM in two steps. First, the parameters formulated to Equation (9), Equation (10), Equation (12) and Equation (13), $(r_0, r_1, \nu_0, \nu_1, \mu_\omega, \rho_\omega, \sigma_\omega)$ are calibrated to match the moments of yield curve of government bonds. Next, we estimate the parameters related to the endowment process $(\rho_y, \sigma_y, \sigma_{y\omega})$ in Equation (14) alongside the shock $\varepsilon_{\omega,t+1}$, derived from the pricing kernel with the calibrated parameters. A detailed explanation of the calibration of these parameters is provided in Appendix C.

4 Results

In this section, we examine the validity of both self-fulfilling and strategic models default that we develop in the previous section. We shows the comparative statistics first, then simulate the economy, whether they are compatible to the actual default or not.

4.1 Comparative Statistics

Figure 4 shows the comparative statistics of important variables. Figure 4 (a) plots the value functions against the debt-to-GDP ratio, evaluated at the average value of the exogenous state variables. It compares the strategic default model (repayment and default) with the self-fulfilling default model (repayment with rollover, repayment without rollover, and default). In the self-fulfilling model, for debt-to-GDP ratios up to slightly above 60%, the value of repayment without rollover remains higher than the value of default; consequently,

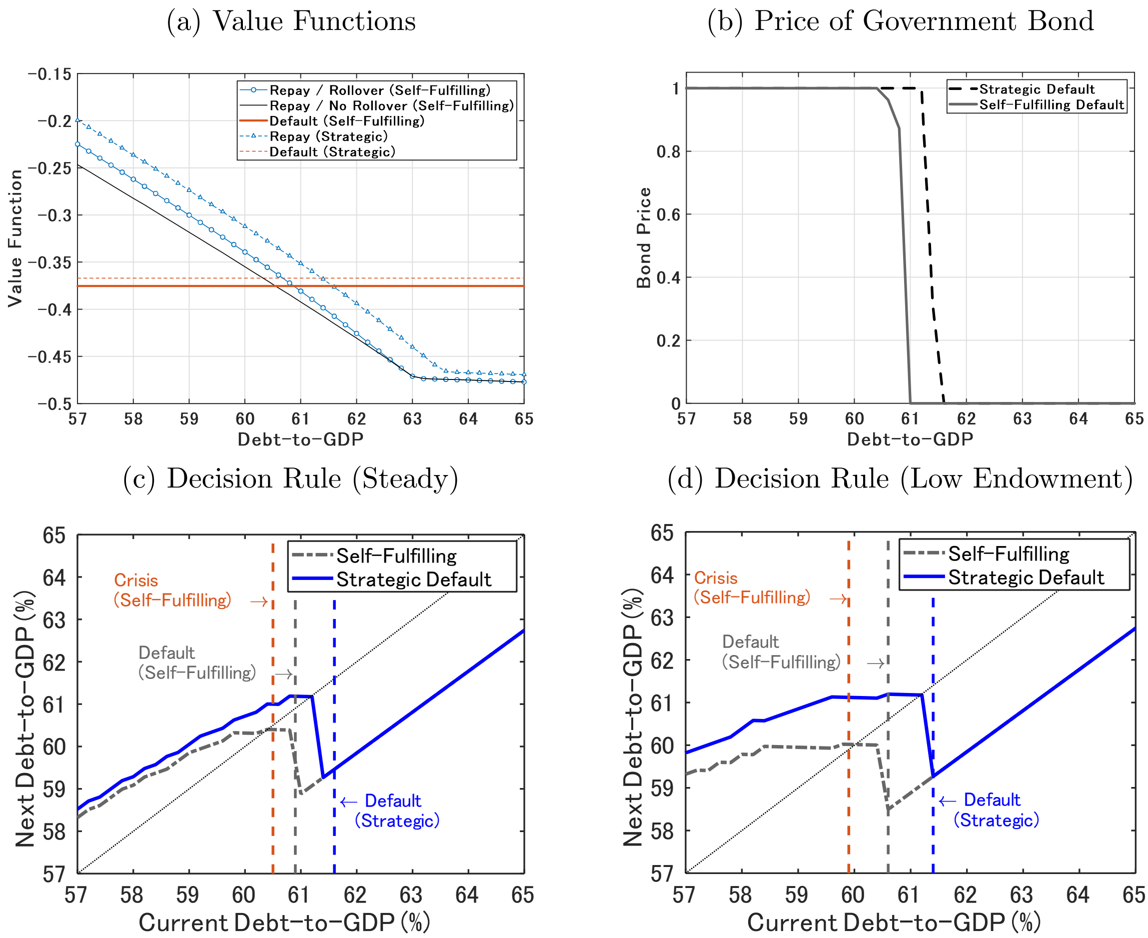
Table 1: Parameter Values

	Value	Target/Source
σ:	2	Standard value
ψ:	0.036	Data
χ:	0.07	Data
$\overline{\lambda}$:	0.035	Data
$\overline{\beta}$:	0.966	Data and Estimation
ρ_β:	0.67	Estimation
σ_β:	0.003	Estimation
ϕ_0:	0.32	Estimation
ϕ_1:	0.01	Estimation
r_0:	0.0046	Data Based SMM
r_1:	0.0022	Data Based SMM
ν_0:	0.20	Data Based SMM
ν_1:	-0.045	Data Based SMM
μ_ω:	-0.36	Data Based SMM
ρ_ω:	0.91	Data Based SMM
σ_ω:	0.09	Data Based SMM
ρ_y:	0.92	Estimation
σ_y:	0.026	Estimation
$\sigma_{y\omega}$:	-0.0002	Estimation

the government does not choose to default. If the ratio exceeds this threshold, meaning the economy enters the crisis region. However, because the difference between the both repayment values are small, repayment value with rollover also falls below default value with only a slightly higher level of debt accumulation. This is because the lenders' expectation of default cost is merely the cessation of government bond purchases. Thus, even though the government cannot roll over its debt, resulting in a decline in current consumption, it can reduce its outstanding debt. This mechanism and the resulting narrow crisis region are consistent with the findings in Bianchi and Mondragon (2022). In terms of strategic default model, the threshold that repayment value falls below default value is higher than self-fulfilling default model at about one percent of debt-to-GDP ratio. This is because the price of government in the self-fulfilling model began to decline at lower debt since the repayment without rollover falls below default value, so the low price of government bond declines the threshold of the intersection between default and repayment values in the self-fulfilling default model.

Figure 4 (b) plots the government bond prices corresponding to the value functions. When the debt-to-GDP ratio is sufficiently low (below approximately 60%), the price is close to

Figure 4: Value Functions and Price of Bond, Normal Time



Note 1: In Panel (d), the low endowment scenario is defined as the lowest of five grid points (spanning ± 3 unconditional standard deviations around the steady state), representing a roughly 19.9% drop from the unconditional mean.

Note 2: The debt-to-GDP ratio is evaluated using the steady-state GDP, not the low endowment level.

one, reflecting a zero probability of default. As the ratio exceeds 60.5% in the self-fulfilling model and roughly 61% in the strategic default model, the price drops abruptly to zero. This dramatic price fluctuation is consistent with the prior literature. Even in the self-fulfilling default model, because the gap between the two repayment values is so narrow, the bond price schedule does not deviate significantly from the baseline strategic model.

Figures 4 (c) and (d) display the optimal decision rules for government bond issuance, mapping the current debt-to-GDP ratio to the chosen ratio for the next period. The former illustrates the policy under steady-state exogenous variables, while the latter depicts a low-endowment scenario with all other exogenous variables held at their steady states. Dotted lines represent the self-fulfilling model, and solid lines represent the strategic default baseline. In the self-fulfilling model, the government tends to accumulate debt until the debt-to-GDP ratio reaches approximately 60%, as indicated by the decision rule lying above the 45-degree line. However, as debt approaches the threshold where repayment without rollover becomes suboptimal, the policy rule dips below the 45-degree line under steady-state conditions. This indicates that the government preemptively implements austerity measures to avoid entering the Crisis Zone.

By contrast, under a low endowment realization, the point at which the debt issuance rule crosses below the 45-degree line nearly coincides with the threshold for repayment without rollover. Consequently, during a severe recession, the government is willing to issue bonds to smooth consumption, even if it pushes the economy slightly into the crisis zone. Since this specific scenario assumes the other two exogenous variables remain at their steady states, an accompanying high-risk premium shock or a low preference shock would make the government even more likely to enter the Crisis Zone. In the strategic default model, the debt threshold at which default becomes optimal is strictly higher than the point where the decision rule crosses below the 45-degree line. Thus, the government optimally reduces bond issuance well in advance to avoid falling into default. To generate equilibrium defaults, prior research typically relies on parameter calibrations that conflict with the broader macroeconomic consensus and empirical data, such as assuming an extremely low government discount factor or negligible economic costs of default. However, as our results demonstrate, when the model is calibrated with standard parameters consistent with the broader macroeconomic literature, a purely strategic sovereign default rarely occurs in equilibrium.

4.2 Simulated Result

To evaluate the quantitative properties of the models, we simulate a time series of 1,000,000 periods. We discard the first 100,000 periods as a burn-in phase to eliminate the influence of initial conditions. A detailed explanation is provided in Appendix D. The moments of key variables are reported in Table 2.

In terms of default frequency, our baseline simulations for the self-fulfilling default model and the strategic default model yield approximately 0.008% and 0.002% per quarter, respectively. This implies that a default occurs once every 3,000 years and 14,000 years,

Table 2: Simulated Result

	Data	Self-fulfilling	Strategic
Default Frequency	0.5%	0.008%	0.002%
Average between 2005Q1 to 2011Q2			
Debt-to-GDP ratio	117.1%	60.6%	61.4%
Corr Debt-to-GDP and Output	-0.91	-0.97	-0.96
Spread	0.95%	0.002%	0.005%
SD Spread	1.32%	0.012%	0.033%
Corr Spread and Output	-0.05	0.04	0.003
Just Before Default			
Debt-to-GDP Ratio	160%	61.3%	61.9%
Spread	3.40%	0.005%	4e-15%

respectively. According to Reinhart and Trebesch (2015), Greece has defaulted four times on its external creditors since 1829. Calculating this over the period between 1829 and 2025 yields an actual default frequency of approximately 0.5% per quarter. While sovereign defaults are historically rare events, particularly among advanced economies, these simulated frequencies are far lower than the actual Greek experience.[9] As discussed previously, the strategic model does not generate frequent defaults under conventional parameterization. Furthermore, in the self-fulfilling model, because the gap between the values of repayment with rollover and repayment without rollover is very narrow, the government has a strong incentive to avoid states that spike the spread or depress the price of government bonds. Consequently, the occurrence of default remains extremely rare in both models.

Next, we compare the simulated moments with actual Greek data from 2005:Q1 to 2011:Q2. The actual average debt-to-GDP ratio during this period was approximately 117%, whereas our simulation for both models yields an average of slightly higher than 60%. Under the baseline modeling framework, several factors contribute to this counterfactually low debt level. First, the assumption that the government regains market access with zero outstanding debt after recovering from a default state lowers the debt threshold at which the value of default exceeds the value of repayment. Thus, the government tends to default before accumulating a higher amount of debt. A second explanation for the higher empirical debt levels lies in political economy. As Bolton et al. (2023) point out, actual overborrowing often

[9] Meanwhile, emerging economies default relatively more frequently than advanced economies. For reference, Argentina, which is frequently examined in the sovereign default literature, has defaulted seven times since World War II, including technical defaults.

stems from political agency problems, where self-interested politicians exhibit an inherent bias toward accumulating debt.[10] We address the first issue by incorporating a partial debt discharge mechanism to generate a higher debt-to-GDP ratio in the subsequent extension section.

Furthermore, the baseline models struggle to replicate the pricing of sovereign risk. As shown in the table, the simulated average spreads are virtually zero (0.002% and 0.005% for the self-fulfilling and strategic models, respectively), which drastically underestimates the actual Greek average spread of 0.95%. Consequently, the standard deviation of spreads generated by the models is also negligible (0.012% and 0.033%) compared to the empirical volatility (1.32%). This discrepancy naturally arises from the extremely low default frequencies discussed above. Since defaults are so rare in the baseline calibrations, lenders demand almost no default risk premium in normal times.

Despite these shortcomings in asset pricing, the models perform reasonably well in capturing the cyclicality of debt. Both models generate a strong negative correlation between the debt-to-GDP ratio and output (-0.97 and -0.96), which aligns closely with the empirical moment (-0.91). This confirms that the baseline framework successfully preserves the countercyclical borrowing behavior standard in the literature, where the government issues more debt during recessions to smooth consumption. The correlation between spread and output, while slightly negative in the data (-0.05), is close to zero in the models, reflecting the lack of meaningful spread variations.

However, the most striking failure of the baseline models appears in the crisis dynamics, specifically the moments labeled "Just Before Default." In the actual Greek data, the run-up to the default was characterized by a massive accumulation of debt (reaching 160% of GDP) and a severe spike in borrowing costs, with spreads jumping to 3.40%. In stark contrast, the simulated moments just prior to default show almost no amplification. The debt-to-GDP ratios remain stagnant at around 61%, and the spreads barely react (0.005% in the self-fulfilling model and virtually zero in the strategic model). This implies that under standard parameters, defaults occur abruptly without generating the nonlinear spread spikes or the deep crisis zone distress observed during the actual European debt crisis.

Taken together, these results underscore the quantitative limitations of the standard

[10] Another factor could be the lack of uncertainty regarding the default cost. In the sovereign default literature, the cost of default, comprising the proportion of endowment decline and the probability of regaining market access, is exogenously given. In reality, however, it is highly difficult to predict these costs prior to default.

framework. To resolve these puzzles, namely, the low debt capacity, the absence of spread volatility, and the lack of spread spikes prior to default, we introduce key extensions, including a partial default mechanism with a haircut rate and a persistent state of liquidity dry-up, in the following section.

5 Extension

In this section, we extend the baseline framework to better align its predictions with the stylized facts of the Greek sovereign debt crisis. As discussed in the previous section, the baseline model struggles to replicate key empirical moments, severely underestimating the debt-to-GDP ratio, the default frequency, and the mean and volatility of the spread. We argue that these quantitative failures stem from two specific simplifications: the assumption of a full default and the transient nature of liquidity crises.

Regarding the first point, a critical limitation of the baseline framework is the assumption of full default, wherein all outstanding debt is completely eliminated. This assumption artificially inflates the value of default relative to repayment, providing the government with an unrealistically strong incentive to default at low debt levels. Empirical evidence, however, demonstrates that sovereign debt obligations typically persist post-default. For instance, Sturzenegger and Zettelmeyer (2006) estimate the haircut for the 2001 Argentine default at 42%, leaving the government with substantial residual debt. Indeed, Argentina's debt-to-GDP ratio remained around 50% when it regained market access in 2016. Similarly, during the Greek debt restructuring in the early 2010s, a massive stock of debt exceeding 150% of GDP was rolled over into new instruments or transferred to official creditors rather than being wiped out. Furthermore, assuming a zero recovery rate implies that bond prices fall to zero as default becomes certain, which can lead to an unrealistic explosion of spreads, known as interest rate overshooting. Incorporating a nonzero recovery rate mitigates this pricing anomaly by bounding the bond price away from zero.

Regarding the second point, the baseline model assumes that the cost associated with lenders' refusal to roll over debt is limited to a single period of market exclusion. However, the state in which lenders refuse to purchase medium- and long-term government bonds often continues for an extended duration. In the actual Greek case, the government last issued medium- and long-term bonds in March 2010 but did not officially default until the third quarter of 2011.[11] Consequently, as shown in Figure 4 (a), this temporary loss of access in the baseline model generates only a limited welfare cost, resulting in a narrow margin between the value of repayment and the value of default.

To address these limitations, we introduce two realistic modifications: (i) a partial default mechanism where defaulted debt retains a nonzero recovery value, and (ii) a persistent "no-rollover" state that captures the prolonged duration of market exclusion.

5.1 Model

We first describe the modeling of residual debt upon regaining market access. While the sovereign debt literature often employs Nash bargaining frameworks to endogenize haircuts (e.g., Yue, 2010; Benjamin and Wright, 2013), we maintain tractability by adopting a parsimonious approach that treats the recovery rate as an exogenous parameter.[12] This simplification is justified for several reasons. First, standard bargaining models typically rely on unobservable parameters, such as the government's bargaining power, to match the average recovery rate observed in the data. Consequently, these endogenous mechanisms often amount to a complex way of calibrating a free parameter, offering limited additional insight into the specific dynamics we aim to study. Second, real-world debt restructuring involves a multitude of actors beyond private bondholders, including official sector creditors and international organizations like the IMF, whose lending capacity and political mandates significantly influence the final haircut. Modeling this complex multi-party negotiation with micro-foundations is beyond the scope of this paper.

To capture the complexity of real-world debt restructuring, we distinguish between the haircut incurred by private creditors and the actual reduction in the government's overall debt burden achieved through a partial default. In the Greek case, although the government imposed a remarkably high haircut rate on private bondholders, its total outstanding debt did not decline proportionately. This is mainly because a substantial portion of the debt had been transferred to official sector creditors, such as the ECB and the IMF, whose claims were exempt from the restructuring. Additionally, the government had to assume new official

[11] The government did, however, continue to issue short-term Treasury bills during this interim period.

[12] For alternative approaches, such as partial default without renegotiation or models incorporating IMF interventions to dampen spreads, see Arellano et al. (2023) and Gennaioli, Martin, and Rossi (2018), respectively.

debt to finance the cash incentives used in the bond exchange and to recapitalize domestic banks that were severely hit by the partial default. Thus, we assume a common exogenous haircut rate $h \in [0,1]$ applied to all outstanding bonds, representing the loss incurred by private lenders. Concurrently, we define $\alpha \in [0,1]$ as the fraction of the outstanding debt that the government retains upon regaining the non-default state.

First, we formalize the pricing of defaulted claims. Even after a default, legacy bonds retain value because holders expect a recovery upon market re-entry. This residual value mitigates the pre-default surge in spreads. To price these defaulted claims in a secondary market, we assume risk-neutral buyers purchase one unit of face value and receive no coupons until market re-entry. The zero-profit condition implies that the price per unit of face value of a defaulted claim, denoted by $q^h(s)$, is given by:

$$q^h(s) = \mathbb{E}\left[\sum_{j=1}^{\infty}(1-h)\psi(1-\psi)^{j-1}\left(\prod_{k=1}^{j} m(s_{k-1}, s_k)\right)|s_0 = s\right]. \tag{17}$$

We define the linear pricing operator $(Af)(s)$ as:

$$(Af)(s) := \mathbb{E}[m(s,s')f(s')|s],$$

where $f(s\prime)$ represents an arbitrary state-contingent payoff in the subsequent period. This operator compactly evaluates the present value of future cash flows. Then, Equation (17) can be simplified as:

$$\begin{aligned} q^h(s) &= (1-h)\psi\sum_{j=1}^{\infty}(1-\psi)^{j-1}(A^j 1)(s) \\ &= (1-h)\psi\left[A(I-(1-\psi)A)^{-1}\right](s). \end{aligned} \tag{18}$$

Since coupon payments are suspended during default and holders receive only the recovery value upon market re-entry, the price of defaulted debt is independent of its original maturity n. Accordingly, the market value of a specific defaulted bond with face value b_n is $q^h(s)b_n$. Consequently, the primary market price of government bonds at the time of auction, $q_n(B', s)$, is modified from Equation (7) to incorporate the recovery value:

$$q_n(B', s) = \mathbb{E}\left[m(s, s')\left\{(1 - \Xi(S^{'}))(r(\omega') + q_{n-1}(S^{'})) + \Xi(S^{'})q^h(s')\right\}\right]. \tag{19}$$

The distinction between this extended pricing function and the baseline version lies in the treatment of future default scenarios. Within the expectation term, $\Xi(S^{'})$ captures the case where the government survives the current period but defaults in the subsequent period. Even if such a crisis occurs, bondholders can sell their defaulted claims valued at $q^h(s')$ to buyers in the secondary market.

Turning to the actual debt reduction achieved by the government, let $\alpha \in [0, 1]$ denote the fraction of the initial debt burden that remains upon regaining market access. Owing to the transfer of debt to the official sector and the additional costs of recapitalizing domestic banks as discussed above, the residual debt ratio α is systematically larger than the recovery rate $(1 - h)$ experienced by private bondholders (i.e., $\alpha > 1 - h$). Under this specification, the government's remaining liability upon resolving the default becomes $B^r = \alpha B$, and the post-restructuring position is $b_n^h = \alpha b_n$ for each maturity bucket n. Accordingly, we modify the continuation value of default as follows:

$$\underline{V}(B, s) = u(y[1 - \chi]) + \beta\{(1 - \psi)\mathbb{E}[\underline{V}(B, s')|(s)] + \psi\mathbb{E}[V(B^r, s')|(s)]\}.$$

This dual-parameter approach, separating the private creditors' haircut h from the government's overall debt retention α, improves the quantitative performance and realism of our model. If we were to naively assume that the government's debt was reduced by the exact same magnitude as the private haircut (i.e., $\alpha = 1 - h$), the model would severely overstate the fiscal relief provided by the default. By explicitly recognizing that α remains relatively high despite a large h, our framework successfully prevents the artificial inflation of the default continuation value. Consequently, it ensures that the government's incentive to default aligns with the empirical reality of advanced economies: sovereign defaults are extremely costly, involve prolonged market exclusion, and ultimately do not offer a clean slate.

To address the second limitation, we incorporate a mechanism that allows for prolonged episodes of rollover failures. In the baseline model, the cost associated with lenders' refusal to roll over debt is restricted to a single period of market exclusion. Consequently, as illustrated in Figure 4 (a), this temporary loss of access generates only a modest welfare cost, resulting in a narrow margin between the value of repayment and the value of default. In reality, however, sudden stops in capital flows often persist for multiple periods. The Greek sovereign debt crisis exemplifies this persistence. Following the revelation of misreported fiscal statistics in late 2009, yields on Greek government bonds began to surge. Although the government managed to issue bonds twice in March 2010, foreign demand for these instruments dropped off appreciably. By April, secondary market spreads had widened further, and with large amortizations falling due in May, Greece effectively lost market access. This culminated in an agreement with the Troika (the IMF, the European Commission, and the ECB) in early May 2010 for a financial assistance program (IMF, 2013, pp. 8, 9). This timeline illustrates that the government faced severe, prolonged rollover difficulties long before the actual default materialized in 2011:Q3. To capture this persistence, we extend the baseline model by introducing an exogenous probability $\psi_{nr} \in (0,1)$ that governs the transition dynamics during a liquidity crisis. Specifically, in a no-rollover state, the government is forced into default with probability ψ_{nr}, while with probability $1-\psi_{nr}$, the no-rollover state persists into the subsequent period. Accordingly, the value function for the no-rollover state is modified from Equation (6) as follows:

$$V_r(S|\Xi = 1) = u(y - \lambda B - r(\omega)B) + \beta\mathbb{E}[(1-\psi_{nr})V_r(B', s'|\Xi = 1) + \psi_{nr}V(B', s')|(S)],$$

where $B' = (1-\lambda)B$ represents the reduced debt level after the current period's amortization. This modification significantly amplifies the expected welfare cost of losing market access, thereby strengthening the government's incentive to repay its debt.

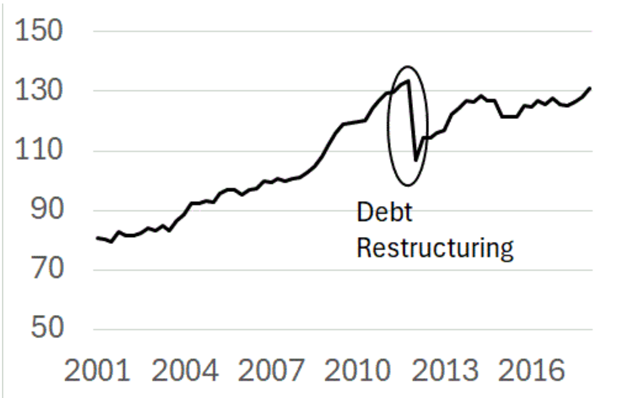
5.2 Calibration and Results

Calibration

To calibrate the extended model, we must determine three additional parameters: the haircut rate for lenders h, the proportion of debt retained at the time of recovery α, and the exogenous probability of a continued liquidity crisis ψ_{nr}. First, we set the haircut rate for lenders, h. The private sector involvement (PSI) debt restructuring in Greece in 2012 resulted in a reduction of approximately €107 billion on a total of €197 billion in privately held bonds. This implies a haircut of roughly 53.5%. Consequently, we set the recovery rate to 0.465, which aligns with Zettelmeyer, Trebesch, and Gulati (2013).[13]

However, as Figure 5 shows, the actual magnitude of debt reduction was not as large. This is mainly because, although existing debt was reduced, the government acquired new loans from the Troika. The actual overall debt reduction amounted to approximately 19.9%. Therefore, we also examine an alternative scenario where the debt retention rate α is set to 0.801.

Figure 5: Actual Debt Transition



Note:	The actual debt transition is adjusted using the GDP deflator. Debt is normalized by the average GDP between 2005 and 2011.
Source:	Quarterly government debt data is from Eurostat, and the GDP deflator is from FRED.

Finally, we calibrate the transition probability of the liquidity crisis, ψ_{nr}. As discussed in the previous section, the duration of market exclusion during the no rollover period was approximately one year in the Greek case, starting from 2010:Q2 to 2011:Q2. Assuming a quarterly model, we set $\psi_{nr} = 0.2$, which implies an average duration of five quarters.

Comparative Statistics

The resulting value functions and government bond prices are depicted in Figure 6 (a). Compared to the baseline results, the gap between the value of repayment with rollover ($V_r(\Xi = 0)$ and the value of repayment without rollover ($V_r(\Xi = 1)$) widens. This is because,

[13] They calculate the Greek haircut rate at approximately 64.6% under a baseline discount rate of 15.3%.

in this extension, the no rollover state persists for multiple periods, entailing a larger cumulative cost of market exclusion compared to the one period shock in the baseline model. Consequently, the crisis region expands. Furthermore, the debt-to-GDP threshold at which the government optimally chooses to default is higher than in the baseline case. This shift occurs because default no longer eliminates the government's entire debt obligation; instead, the government retains a portion of the debt determined by the recovery rate. This residual burden reduces the value of default ($\underline{V}$), thereby incentivizing the government to sustain higher levels of debt before exercising the default option. Specifically, the thresholds where the repayment values with and without rollover intersect the default value occur at approximately 128% and 120% of GDP, respectively. However, these two thresholds are lower than those in the extended strategic default model, as shown in Figure 6 (b). This is because the price of government bonds in the self-fulfilling default model begins to decline as soon as the value of repayment without rollover falls below the default value. Consequently, the value of repayment with rollover also declines alongside the drop in bond prices.

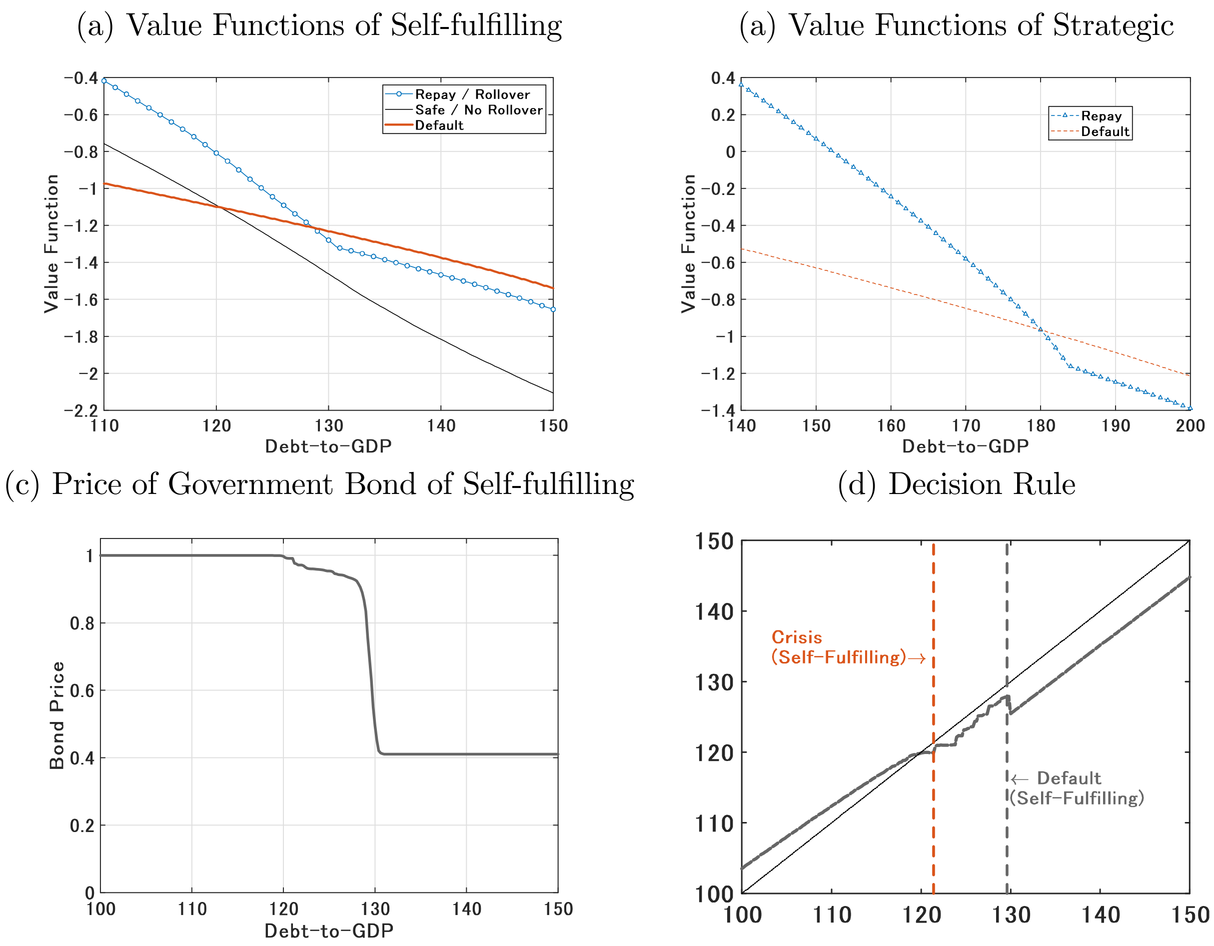
The bond pricing function in Figure 6 (c) is significantly altered in two ways. First, even in the default region where the default probability reaches 100% (e.g., at debt-to-GDP ratios exceeding 130%), the bond price does not collapse to zero. This reflects the fact that defaulted bonds retain positive value in the secondary market. Lenders anticipate that, upon future market re-entry, these bonds will be exchanged for new claims at the recovery rate α, preventing the price from falling to zero. Furthermore, the price fluctuation is milder in the extended model because the crisis region expands compared to the baseline model, leading to a more gradual price decline.

Finally, Figure 6 (d) displays the decision rule for government bond issuance. The government tends to accumulate debt within the safe region. However, when the debt-to-GDP ratio enters the crisis region, the decision rule dips slightly below the 45 degree line, indicating the government's willingness to reduce debt accumulation to avoid default.

Simulated Results

Next, we turn to the simulation results of the extended model, with the moments of key variables reported in Table 3. In terms of default frequency, the extended model generates a probability of 0.77% per quarter. This implies a default occurrence of once every 32.5 years, which is much closer to the actual historical frequency compared to the baseline result of 0.008%. This increase occurs because the wider crisis region increases the likelihood of the

Figure 6: Value Functions of Various Extended Assumptions



(a) Value Functions of Self-fulfilling

(a) Value Functions of Strategic

(c) Price of Government Bond of Self-fulfilling

(d) Decision Rule

government entering this zone, thereby triggering a no rollover event by lenders. In addition, even when the government regains market access, the outstanding debt is reduced by only about 20% from the level at the time of default, increasing the long term debt burden.

Next, we compare the simulated averages of the extended model with the actual Greek data. The most striking improvement is the model's ability to replicate high levels of sovereign indebtedness. The simulated average debt-to-GDP ratio reaches 118.0%, which is remarkably close to the actual data's 117.1% and a massive improvement over the baseline model's 60.6%. This indicates that the extended framework successfully allows the government to sustain the high debt levels characteristic of advanced economies like Greece. The mean and standard deviation of the spread in the simulation (0.16% and 0.22%, respectively) are also closer to the empirical moments than those in the baseline model, though they remain somewhat distant from the actual Greek case.

The model's ability to match the correlation between output and the debt-to-GDP ratio, as well as between the spread and output, declines compared to the baseline. Regarding the debt-to-GDP correlation, the baseline model features a safe region that extends very close to the default threshold. In this safe region, a low endowment leads to high bond issuance to smooth consumption, generating a strong negative correlation. In the extended model, however, the government faces mixed incentives within the newly expanded crisis region. It still desires to issue debt to smooth consumption, but simultaneously attempts to reduce debt to avoid default. As a result, the simulated correlation drops to merely -0.17. Similarly, the correlation between the spread and output is also weakened.

Just prior to default, the model's predictions for the debt-to-GDP ratio and spreads deviate somewhat from the data, but the improvements are notable. The simulated pre default debt-to-GDP ratio surges to approximately 122%. This marks a vast improvement over the baseline model's 61.3% and moves significantly closer to the empirical 160% observed just before the actual Greek default. However, the pre default spread in the simulation (0.36%) does not yet capture the dramatic, nonlinear spikes seen in the actual data (3.40%).

Table 3: Simulated Result

	Data	Self-fulfilling	Strategic
Default Frequency	0.5%	0.77%	0.16%
Average between 2005Q1 to 2011Q2			
Debt-to-GDP ratio	117.1%	118.0%	173.7%
Corr Debt-to-GDP and Output	-0.91	-0.17	-0.98
Spread	0.95%	0.16%	0.02%
SD Spread	1.32%	0.22%	0.09%
Corr Spread and Output	-0.05	0.25	-0.003
Just Before Default			
Debt-to-GDP Ratio	160%	121.6%	172.5%
Spread	3.4%	0.36%	0.011%

Extraction of Unobservable States and Simulation Results

In this section, we evaluate the quantitative performance of our model by assessing its ability to reproduce the empirical data, focusing on the debt-to-GDP ratio and the spread from 2005:Q1 to 2011:Q2. We also extract the unobservable state of the government's time preference. We employ a particle filter and a particle smoother to extract the historical trajectory of this latent preference state from the data.

Because our model is characterized by strong nonlinearity arising from self-fulfilling default crises and borrowing constraints, standard linear approximations are inadequate for accurately evaluating the posterior distribution of the state variables. Instead, we utilize a sequential importance sampling particle filter to nonparametrically approximate the posterior distribution of the preference shock, conditional on the observable data of sovereign spreads, the debt-to-GDP ratio, the endowment, and the risk shock. To utilize information from the entire sample period, we subsequently apply a particle smoother. The detailed theoretical framework of these algorithms is provided in Appendix D.

Figure 7: Comparison of Simulated Results and Actual Data Prior to Default: Extended Model

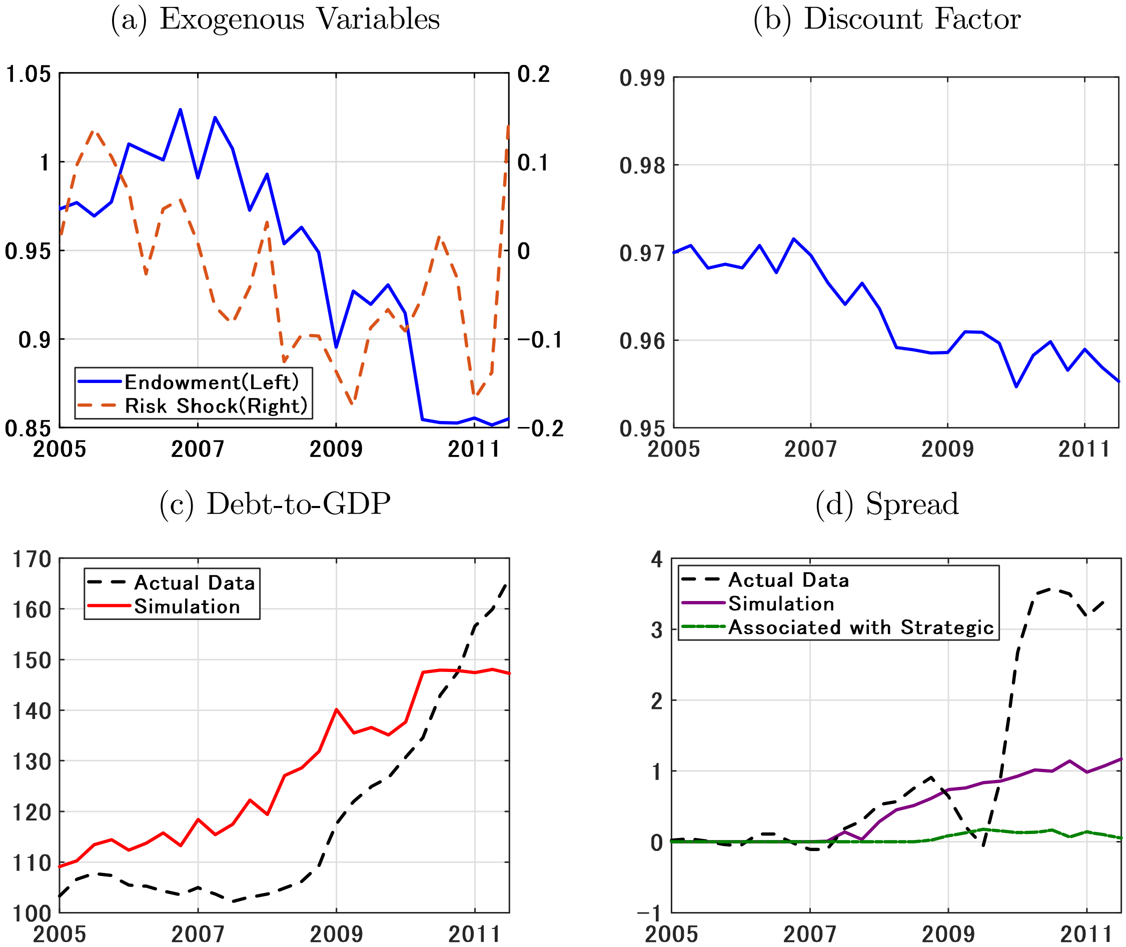


Figure 7 (a) presents the evolution of the directly observable exogenous state variables:

the endowment on the left axis and the risk shock on the right axis. As the figure illustrates, following the 2008 global financial crisis, the endowment exhibits a persistent decline, while the risk shock experiences sharp spikes and elevated volatility, capturing the deteriorating macroeconomic environment and heightened uncertainty.

Figure 7 (b) displays the trajectory of the unobservable preference parameter extracted via the particle smoother. The steady-state value of the discount factor is 0.966, and the estimated parameter remains slightly above this baseline until the period leading up to the 2008 Global Financial Crisis (GFC). Around the onset of the crisis, this value declines notably, falling below 0.96. This drop indicates that the government became increasingly myopic during this turbulent period.

Figure 7 (c) and (d) show the debt-to-GDP ratio and the spread, generated by feeding the three exogenous shocks back into the model to simulate the endogenous variables and compare them with the empirical data. Figure 7 (c) plots the actual debt-to-GDP ratio against the simulated path. The model's predictions track the empirical trend relatively well, demonstrating that the framework effectively captures the fiscal dynamics over the sample period. However, the simulated debt-to-GDP ratio is higher than the actual data until the crisis becomes more apparent around 2010. The primary reason for this discrepancy is that during the economic boom leading up to the global financial crisis, high tax revenues sustained the increase in actual debt, a dynamic not explicitly covered in our model. Furthermore, while the government in the model adopts austerity measures to avoid increasing debt once inside the crisis zone, the actual Greek debt-to-GDP ratio continued to rise until the default. Figure 7 (d) presents the evolution and decomposition of the sovereign spread. The figure plots the actual spread, the overall simulated spread, and the spread associated solely with strategic default risk. The overall simulated spread successfully replicates the empirical trend, beginning to increase around 2009. However, while the actual spread spiked dramatically, the simulated spread remains much milder starting about six quarters prior to the default. We decompose the drivers of the spread increase to isolate the strategic default risk, which represents the government's fundamental willingness to default. This strategic component remains quite low throughout the period, hovering near zero percent. This implies that the sharp increase in sovereign spreads during the actual crisis was heavily driven by self-fulfilling panics, rather than the government's intrinsic willingness to default.

5.3 Quantitative Measurements of Model Fitness

To objectively evaluate the empirical validity of the self-fulfilling and strategic default models under both baseline and extended specifications, we quantify their goodness-of-fit by computing the distance between the model-simulated moments and the empirical moments in the data. Specifically, we employ a Simulated Method of Moments (SMM) approach, where the distance measure, D, is defined as follows:

$$D = (m_{data} - m_{sim})' \cdot W \cdot (m_{data} - m_{sim})$$

where m_{data} and m_{sim} are vectors of empirical and model-generated moments, respectively, and W is a weighting matrix. The matrix W is constructed using a bootstrap method to account for the sampling variability of the empirical moments.

We assess the model fit using three progressively comprehensive sets of moments. The first set, (A) Pre-default, targets the period leading up to the crisis, covering 2005:Q1 to 2011:Q2. The matching moments in this set include: (i) the default frequency, (ii) the average debt-to-GDP ratio, (iii) the mean and standard deviation of sovereign bond spreads, and (iv) the correlation of the debt-to-GDP ratio with both the endowment and the spread. The second set, (B) Pre-default + Just before default, appends the moments strictly from one period before the default event (i.e., the average debt-to-GDP ratio and the spread exactly at the brink of default) to the first set. The final set, (C) Full set, covers the second set plus the post-default period, which spans 28 quarters following the government's reentry into the financial markets from the default state. Note that for the moments specific to the exact period before default and the post-default recovery, the default frequency is naturally excluded by construction, focusing instead on the levels and volatilities of debt and spreads.

Table 4: Simulated Result

	(A) Pre-default	(B) Pre-default + Just Before Default	(C) Full Set
(1) Self-fulfilling, Baseline	3.188	4.566	7.438
(2) Strategic, Baseline	3.160	4.536	7.487
(3) Self-fulfilling, Extension	2.303	3.158	5.560
(4) Strategic, Extension	3.002	4.002	6.595

Table 4 reports the minimized distance D for each of the four model specifications across the three sets of targeted moments. A lower value of D indicates a tighter fit to the empirical data. As expected in SMM estimations, the distance inherently increases as more moment restrictions are added from column (A) to (C). Several key findings emerge from this comparison. First, the extended versions of both the self-fulfilling and strategic models consistently and significantly outperform their baseline counterparts across all three criteria. This substantial reduction in the distance measure highlights that the additional mechanisms introduced in our extended framework are crucial for capturing the true dynamics of sovereign debt.

Second, while the baseline self-fulfilling and strategic models exhibit very similar goodness-of-fit (e.g., $D = 3.188$ and 3.160 under the pre-default criteria, respectively), a clear divergence appears when comparing the extended models. The extended self-fulfilling model yields the lowest distance across all configurations ($D = 2.303$ for Set A, 3.158 for Set B, and 5.560 for Set C), strictly dominating the extended strategic model. Importantly, the extended self-fulfilling model maintains its superior fit even when required to match the sharp deterioration exactly one period before the default and the subsequent trajectory upon market reentry. These quantitative results strongly suggest that self-fulfilling expectations driven by non-fundamental risk are not merely a theoretical possibility, but an essential component for accurately reproducing the rapid surge in spreads and the broader fiscal dynamics observed in the data.

5.4 Policy Implication

This section explores the policy implications for resolving a severe sovereign debt crisis through two quantitative exercises. First, we use a Generalized Impulse Response Function (GIRF) analysis to evaluate the economy's response to shocks within a "Crisis Zone," highlighting the necessity of external commitment devices. Second, we conduct a counterfactual simulation initialized at the 2010:Q2 crisis peak to quantify the macroeconomic and welfare trade-offs between endogenous default dynamics and a forced austerity regime.

Generalized Impulse Response Function

To evaluate the state-dependent dynamics of our non-linear model, we employ the Generalized Impulse Response Function (GIRF) approach (Koop, Pesaran, and Potter, 1996).

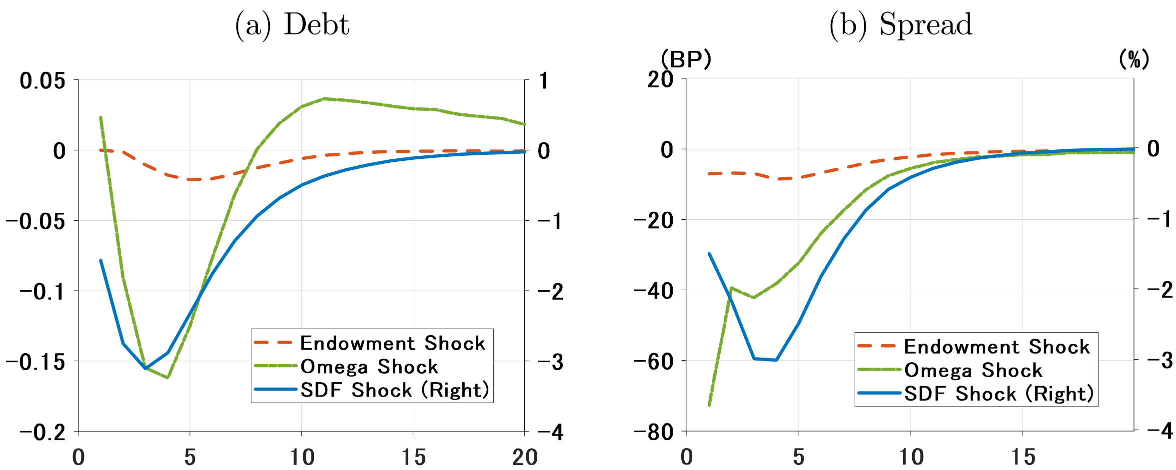
Formally, the GIRF for a given variable x_{t+h} at horizon h, conditional on an initial state Ω_t and an initial shock of size δ, is defined as:

$$GIRF_t(h, \delta, \Omega_t) = \mathbb{E}[x_{t+h}|\epsilon_t = \delta, \Omega_t] - \mathbb{E}[x_{t+h}|\epsilon_t = 0, \Omega_t]$$

To compute these conditional expectations, we conduct Monte Carlo simulations using 5,000 independent paths over a horizon of 20 quarters (5 years). We specifically focus on the economy's response when the initial state Ω_t is situated in a "Crisis Zone," which we identify and extract from the smoothed historical data. Under this high-stress initial condition, we simulate the responses of the debt level and sovereign spreads to three distinct scenarios: (1) a positive endowment shock ($+3\sigma$), (2) a negative interest rate shock (-3σ), and (3) a positive SDF shock ($+3\sigma$).

Figures 8 (a) and (b) plot the dynamic responses, measured as deviations from the baseline path, of the debt level and sovereign spreads following each shock. Note the dual-axis setup in both figures: the responses to the endowment and interest rate shocks are plotted on the left axis, while the response to the SDF shock is plotted on the right axis.

Figure 8: Generalized Impulse Responses of Debt and Spreads to Exogenous Shocks



First, shocks that improve macroeconomic fundamentals, namely an increase in the endowment or a decrease in interest rate risk, push spreads and debt in the desired downward direction, but their quantitative impacts are notably limited. Figure 8 (a) reveals that the deleveraging effect of these fundamental shocks is modest, hovering between -0.05% and -0.15% on the left axis, and their effects dissipate to near zero after approximately 10 quarters. As shown on the left axis of Figure 8 (b), a positive endowment shock reduces the spread by approximately 40 basis points, and a negative interest rate shock reduces it by roughly 60 basis points. Consequently, the relief in debt servicing costs is marginal. Second, the SDF shock, which captures an increase in the government's degree of forward-lookingness, exhibits a highly non-linear and overwhelmingly powerful impact in the crisis zone. As illustrated on the right axes of both figures, a more forward-looking government generates a powerful deleveraging effect that persistently reduces the debt level by nearly 3 percentage points within one year. This strong commitment to curb debt accumulation drastically compresses the sovereign spread by approximately 350 basis points.

These findings carry profound policy implications. When an economy enters a "Crisis Zone," relying exclusively on gradual improvements in fundamentals, such as positive endowment or favorable interest rate shocks, is insufficient to restore debt sustainability. While solid fundamentals are a necessary condition for long-term stability, they lack the quantitative power to immediately reverse a severe, panic-driven spike in borrowing costs. Conversely, a positive shift in the government's forward-lookingness yields an immediate, massive reduction in spreads and triggers a virtuous cycle of rapid debt reduction.

This quantitative result strongly suggests that breaking a "bad equilibrium" driven by self-fulfilling expectations requires a profound shift in the government's fiscal commitment. In reality, however, abrupt changes in a sovereign's intrinsic patience are difficult to achieve. Therefore, external interventions would be essential to resolve the crisis, provided they enforce this forward-looking behavior. Policies such as the provision of ample liquidity by a lender of last resort, most notably the European Central Bank's Outright Monetary Transactions (OMT) program, serve as a powerful backstop. Crucially, the strict conditionality attached to such programs acts as an external commitment device that compels the government to behave with a high degree of forward-lookingness. By binding the government to a strict deleveraging path, these institutional frameworks successfully eliminate self-fulfilling default risk and pull the sovereign out of the crisis.

Forward Counterfactual Analysis Conditioned on the 2010 Crisis

To evaluate the dynamic evolution of the economy and assess the efficacy of fiscal consolidation policies during a severe sovereign debt crisis, we conduct a forward-looking counterfactual analysis. Rather than starting the simulations from an arbitrary steady state, we initialize the economy using the exact latent states of endowment, risk-free interest rate, preference, and debt level extracted by the particle smoother for the second quarter of 2010.

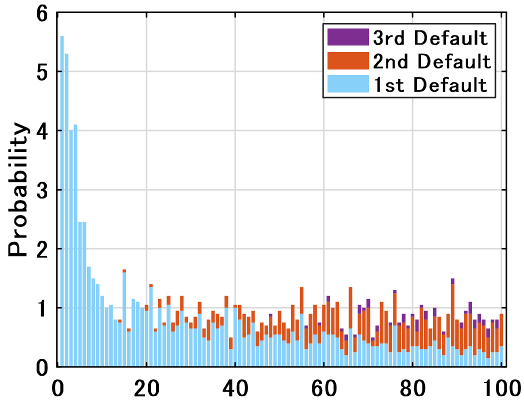
From this crisis state, we perform a Monte Carlo simulation generating 2,000 independent future paths over a horizon of 100 quarters. The exogenous shocks are drawn from their respective estimated distributions. We then compare the expected paths of the economy under two primary scenarios. The first is the Baseline scenario, where the government follows the optimal Markov-perfect equilibrium policy, choosing debt issuance and default strategies endogenously based on the realized state variables. The second is the Austerity scenario, in which the government is forced into a strict deleveraging regime. In this counterfactual, the government commits to a sustained reduction of its debt stock by setting next period's debt issuance strictly below the maturing debt until the total debt level is reduced by 20% from its 2010:Q2 level. We assume the duration to achieve this 20% reduction is 15 years.

The simulation results highlight the severe vulnerability of the economy during a crisis and the stark macroeconomic trade-offs associated with forced fiscal consolidation. Figure 9 illustrates these dynamics, with panels (a), (b), and (c) depicting the probability distribution of default events, expected consumption, and expected debt, respectively.

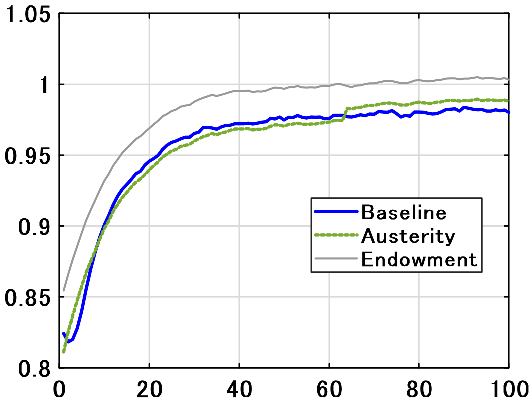
We begin by analyzing the baseline default dynamics. Figure 9 (a) presents the probability distribution of the timing of default events under the baseline policy, conditioned on the 2010:Q2 crisis state. The histogram breaks down the occurrences into the 1st, 2nd, and 3rd defaults over the 100-quarter horizon. Two critical insights emerge from this distribution. First, the probability of a first default is heavily concentrated in the quarters immediately following 2010:Q2. This reflects the severity of the initial crisis state: burdened by high debt and characterized by a myopic fiscal stance (a low SDF), the sovereign is highly vulnerable to even minor adverse shocks in the near term. Second, the visible presence of "2nd Default" and "3rd Default" events illustrates the phenomenon of serial default. When the government defaults, it experiences a temporary exclusion from financial markets and a haircut on its debt, but it eventually reenters the market. Because the structural vulnerabilities remain, the sovereign often accumulates debt again and falls back into default. The model successfully captures this chronic nature of sovereign debt crises, demonstrating that resolving a crisis via a single default event does not guarantee long-term fiscal sustainability if the underlying macroeconomic and non-fundamental risks persist.

Figure 9: Counterfactual Dynamics Following the 2010:Q2 Crisis State

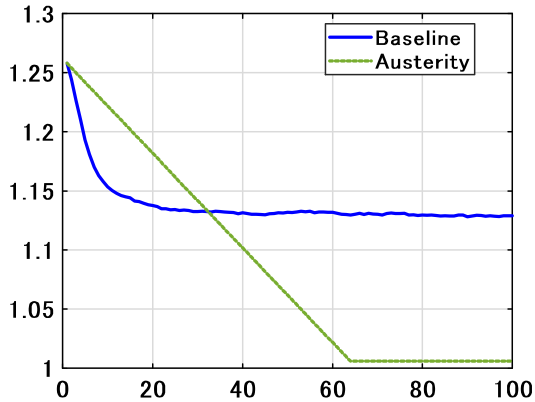
(a) Distribution of Default



(a) Expected Consumption



(b) Expected Debt



Given this high baseline default risk, we next examine the real economic costs of navigating the crisis under different policy regimes. As depicted in Figure 9 (b), in the Baseline scenario, consumption remains consistently below the expected endowment path due to the occurrence of defaults, which leads to an endowment reduction by a fraction χ. Moreover, even when the government does not default, the high debt burden restrains it from issuing large amounts of bonds in order to avoid default, further depressing consumption. Under the Austerity scenario, however, the government must run substantial primary surpluses to pay down the principal. Consequently, consumption in the short-to-medium term experiences a severe drop, falling to a level comparable to that of the baseline path.

This severe real economic cost is driven by the forced deleveraging process illustrated in

Figure 9 (c). Under the Baseline scenario, the expected debt level remains persistently high, slowly drifting downward only as a consequence of endogenous defaults or gradual economic recovery. In contrast, the Austerity scenario mechanically forces the debt level down to the targeted lower threshold. Together, these dynamics illustrate the classic "short-term pain for long-term gain" dilemma in sovereign debt management. Although austerity eventually mitigates the serial default risk and leads to a lower debt burden, the immediate welfare cost inflicted upon households during the consolidation phase is substantial. Over the very long run, however, this reduction in debt and interest payments can allow consumption to recover and even surpass the baseline. To formally evaluate this intertemporal trade-off, we compute the expected cumulative welfare over the 100-quarter simulation horizon, calculated as the expected present value of the utility function in our model, discounted by the realized path of the time-varying subjective discount factor. Our quantitative analysis reveals that the long-term benefits of structural deleveraging ultimately outweigh the severe short-term utility losses; the average post-event welfare under the austerity regime (-31.13) strictly dominates that of the baseline scenario (-31.17). This indicates that a credible commitment to fiscal consolidation, despite its profound initial economic toll, constitutes a welfare-improving strategy.

6 Conclusion

In this paper, we quantitatively investigate the mechanisms of sovereign debt crises, with a specific focus on the Greek experience. To address the limitations of canonical models, we extend the standard self-fulfilling default framework by incorporating two critical features: a partial default mechanism with a positive recovery rate and a persistent state of market exclusion. These extensions allow our model to realistically widen the crisis zone and accurately capture key empirical moments, such as the default frequency and the high debt-to-GDP ratio observed in advanced economies.

Having validated our extended framework, we conduct policy simulations to evaluate crisis resolution strategies. Our analysis reveals that once an economy enters a severe crisis zone, relying solely on gradual improvements in macroeconomic fundamentals is insufficient to restore debt sustainability. Instead, breaking the vicious cycle of self-fulfilling expectations requires a profound shift in government commitment. In this context, programs provided by a lender of last resort serve as powerful external commitment devices. By imposing strict conditionality, these interventions enforce a forward-looking deleveraging path, effectively eliminating default risks and stabilizing the economy.

While our study provides valuable insights into the dynamics of expectation-driven crises, it is subject to several limitations regarding empirical fit and structural assumptions, which point toward important avenues for future research. First, the actual movement of sovereign spreads cannot be fully captured by our simulated model, a quantitative discrepancy often referred to as the spread puzzle (Bolton et al., 2023). Several real-world mechanisms omitted from our framework could explain this gap. Standard models assume foreign lenders have unlimited lending capacity, whereas in severe crises, financial intermediaries face severe liquidity constraints and strict capital requirements that force them to demand substantial liquidity premiums regardless of the fundamental default probability. Furthermore, our driving process does not incorporate severe tail risks, and in the specific context of Greece, a significant portion of the observed spread was driven by redenomination risk (the fear of a Eurozone exit). Addressing these frictions remains an important task for future studies.

Second, our framework assumes that the penalties for default, consisting of an endowment decline and a specific duration of market exclusion, are fully known to all agents beforehand. This assumed predictability significantly affects the core mechanics of the model, specifically the government's decision to default and the subsequent spreads priced in by lenders. In reality, particularly for advanced economies, these costs are highly uncertain prior to the actual event, aligning more closely with Knightian uncertainty rather than measurable risk. A default might ignite broader financial and currency crises requiring massive bailout funds, making the assumption that agents can perfectly anticipate default costs a simplification of a deeply uncertain environment.

Finally, while our framework emphasizes the strategic, willingness-to-pay aspect of sovereign crises, actual defaults may also stem from a fundamental inability to repay. We assume the government theoretically possesses sufficient resources but chooses default based on cost-benefit calculations. However, under catastrophic conditions, such as the unprecedented contraction of output in Greece, structural collapse and strict limits on tax capacity can push a sovereign into sheer insolvency. This vulnerability is particularly acute for countries lacking monetary sovereignty, as they cannot rely on a domestic central bank to monetize debt. Abstracting from this pure inability to repay is a standard quantitative simplification, but exploring the intersection of expectation-driven liquidity crises and fundamental insolvency offers a critical direction for future research.

References

[1] Aguiar, M., Amador, M., Hopenhayn, H., and Werning, I. (2016). "The Economics of Sovereign Debt and Default." Handbook of Macroeconomics, Vol. 2, 1479-1519. Elsevier.

[2] Aguiar, M., Chatterjee, S., Cole, H., and Stange, Z. (2022). "Self-Fulfilling Debt Crises, Revisited." Journal of Political Economy, 130(5), 1147–1183.

[3] Aguiar, M., and Gopinath, G. (2006). "Defaultable Debt, Interest Rates and the Current Account." Journal of International Economics, 69(1), 64-83.

[4] Alesina, A., and Tabellini, G. (1990). "A Positive Theory of Fiscal Deficits and Government Debt." The Review of Economic Studies, 57(3), 403-414.

[5] Ang, A., and Piazzesi, M. (2003). "A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables." Journal of Monetary Economics, 50(4), 745-787.

[6] Arellano, C. (2008). "Default Risk and Income Fluctuations in Emerging Economies." American Economic Review, 98(3), 690-712.

[7] Arellano, C., Mateos-Planas, X., and Rios-Rull, J.-V. (2023). "Partial Default." Journal of Political Economy, 131(6), 1435–1484.

[8] Asonuma, T., and Trebesch, C. (2016). "Sovereign Debt Restructurings: Preemptive or Post-Default." Journal of the European Economic Association, 14(1), 175-214.

[9] Auclert, A., and Rognlie, M. (2016). "Unique Equilibrium..." Journal of Monetary Economics, 84, 134-146.

[10] Baxter, M., and King, R. G. (1999). "Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series." Review of Economics and Statistics, 81(4), 575-593.

[11] Benjamin, D., and Wright, M. L. J. (2013). "Recovery before Redemption: A Theory of Delays in Sovereign Debt Renegotiations." Working paper, Federal Reserve Bank of Chicago and State University of New York Buffalo.

[12] Bi, H. (2012). “Sovereign Default Risk Premia, Fiscal Limits, and Fiscal Policy.” *European Economic Review*, 56(3), 389-410.

[13] Bianchi, J., and Mondragon, J. (2022). "Monetary Independence and Rollover Crises." The Quarterly Journal of Economics, 137(1), 435–491.

[14] Bocola, L., and Dovis, A. (2019). "Self-Fulfilling Debt Crises: A Quantitative Analysis." American Economic Review, 109(12), 4343-4377.

[15] Bolton, P., Gulati, M., and Panizza, U. (2023). "Sovereign Debt Puzzles." Annual Review of Financial Economics, 15, 239–263.

[16] Borensztein, E., and Panizza, U. (2009). "The Costs of Sovereign Default." IMF Staff Papers, 56(4), 683-741.

[17] Brunnermeier, M. K., Garicano, L., Lane, P. R., Pagano, M., Reis, R., Santos, T., Thesmar, D., Van Nieuwerburgh, S., and Vayanos, D. (2016). "The Sovereign-Bank Diabolic Loop and ESBies." American Economic Review, 106(5), 508-512.

[18] Burns, A. F., and Mitchell, W. C. (1946). Measuring Business Cycles. National Bureau of Economic Research.

[19] Calvo, G. A. (1998). "Capital Flows and Capital-Market Crises: The Simple Economics of Sudden Stops." Journal of Applied Economics, 1(1), 35-54.

[20] Calvo, G. A., Izquierdo, A., and Mejia, L.-F. (2004). "On the Empirics of Sudden Stops: The Relevance of Balance-Sheet Effects." NBER Working Paper No. 10520.

[21] Chatterjee, S., and Eyigungor, B. (2012). "Maturity, Indebtedness, and Default Risk." American Economic Review, 102(6), 2674-2699.

[22] Cochrane, J. H., and Piazzesi, M. (2005). "Bond Risk Premia." American Economic Review, 95(1), 138–160.

[23] Cogley, T., and Nason, J. M. (1995). "Effects of the Hodrick-Prescott Filter on Trend and Difference Stationary Time Series: Implications for Business Cycle Research." Journal of Economic Dynamics and Control, 19(1-2), 253-278.

[24] Cole, H. L., and Kehoe, T. J. (2000). "Self-Fulfilling Debt Crises." The Review of Economic Studies, 67(1), 91-116.

[25] Cuadra, G., and Sapriza, H. (2008). "Sovereign Default, Interest Rates and Political Uncertainty in Emerging Markets." Journal of International Economics, 76(1), 78-88.

[26] Daniel, B. C., and Nam, J. (2022). "The Greek debt crisis: Excusable vs. strategic default." Journal of International Economics, 138, 103632.

[27] Dias, D. A., Richmond, C., and Westfahl, G. (2024). "Duration of Capital Market Exclusion: An Empirical Investigation." Finance and Economics Discussion Series 2024-093, Board of Governors of the Federal Reserve System (U.S.).

[28] Eaton, J., and Gersovitz, M. (1981). "Debt with Potential Repudiation: Theoretical and Empirical Analysis." The Review of Economic Studies, 48(2), 289-309.

[29] Gennaioli, N., Martin, A., and Rossi, S. (2018). "Banks and Sovereign Debt." Annual Review of Economics, 10, 483-503.

[30] Grossman, H. I., and Van Huyck, J. B. (1988). "Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation, and Reputation." *American Economic Review,* 78(5), 1088–1097.

[31] Hamilton, J. D. (2018). "Why You Should Never Use the Hodrick-Prescott Filter." The Review of Economics and Statistics, 100(5), 831-843.

[32] Hatchondo, J. C., Martinez, L., and Sapriza, H. (2009). "Heterogeneous Borrowers in Quantitative Models of Sovereign Default." International Economic Review, 50(4), 1129-1151.

[33] Hatchondo, J. C., Martinez, L., and Sapriza, H. (2010). "Quantitative Properties of Sovereign Default Models: Solution Methods." Review of Economic Dynamics, 13(4), 919–933.

[34] International Monetary Fund. (2013). Greece: Ex post evaluation of exceptional access under the 2010 stand-by arrangement (IMF Country Report No. 13/156). Washington, DC.

[35] Johri, A., Khan, S. U., and Sosa-Padilla, C. (2022). "Interest Rate Uncertainty and Sovereign Default Risk." Journal of International Economics, 139, 103681.

[36] Kitagawa, G. (1996). "Monte Carlo filter and smoother for non-Gaussian nonlinear state space models." Journal of Computational and Graphical Statistics, 5(1), 1-25.

[37] Koop, G., Pesaran, M. H., and Potter, S. M. (1996). "Impulse response analysis in nonlinear multivariate models." Journal of Econometrics, 74(1), 119-147.

[38] Kuvshinov, D., and Zimmermann, K. (2019). "Sovereigns Going Bust: Estimating the Cost of Default." European Economic Review, 119, 1–21.

[39] Levy Yeyati, E., and Panizza, U. (2011). "The Elusive Costs of Sovereign Defaults." Journal of Development Economics, 94(1), 95-105.

[40] Lorenzoni, G., and Werning, I. (2019). "Slow Moving Debt Crises." American Economic Review, 109(9), 3229-3263.

[41] Mendoza, E. G. (2010). "Sudden Stops, Financial Crises, and Leverage." American Economic Review, 100(5), 1941-1966.

[42] Mendoza, E. G., and Yue, V. Z. (2012). "A General Equilibrium Model of Sovereign Default and Business Cycles." The Quarterly Journal of Economics, 127(2), 889-946.

[43] Na, S., Schmitt-Grohe, S., Uribe, M., and Yue, V. Z. (2018). "The Twin Ds: Optimal Default and Devaluation." American Economic Review, 108(7), 1773-1819.

[44] Reinhart, C. M., and Trebesch, C. (2015). "The Pitfalls of External Dependence: Greece, 1829–2015." NBER Working Paper, 21664.

[45] Schuler, Y. S. (2018). "On the cyclical properties of Hamilton's regression filter." Deutsche Bundesbank Discussion Paper No 03/2018.

[46] Sturzenegger, F. (2004). "Toolkit for the Analysis of Debt Problems." Journal of Restructuring Finance, 1(1), 201-203.

[47] Sturzenegger, F., and Zettelmeyer, J. (2006). Debt Defaults and Lessons from a Decade of Crises. MIT Press.

[48] Tomz, M., and Wright, M. L. J. (2007). "Do Countries Default in 'Bad Times'?" Journal of the European Economic Association, 5(2-3), 352-360.

[49] Yue, V. Z. (2010). "Sovereign Default and Debt Renegotiation." Journal of International Economics, 82(2), 176-187.

[50] Zettelmeyer, J., Trebesch, C., and Gulati, M. (2013). "The Greek Debt Restructuring: An Autopsy." Economic Policy, 28(75), 513-563.

Appendix A: Controversial Parameters and Simulated Results of Prior Research

In this appendix, Table 5 summarizes the controversial parameter choices regarding the discount factor, GDP decline, and exclusion duration, alongside the simulated debt-to-GDP ratios and default frequencies from prior canonical research. As discussed in the main text, researchers frequently rely on parameterizations that contradict empirical evidence and established literature. Consequently, the simulated debt-to-GDP ratios consistently fall short of actual observed levels, a quantitative discrepancy also highlighted by Bolton et al. (2023). Regarding default frequency, because most prior studies explicitly target this empirical moment during their calibration process, the simulated frequencies generally align well with the data. Notable exceptions include early studies such as Aguiar and Gopinath (2006), which did not apply the Simulated Method of Moments. Furthermore, research focusing on economies that avoided an outright default, such as the analysis of Italy by Bocola and Dovis (2019), naturally omits default frequency from its reported results.

Table 5: Controversial Parameters and Simulated Results of Prior Research

	Discount Factor	GDP Decline	Exclusion	Debt-to-GDP	Default Frequency
Aguiar and Gopinath (2006)	0.8	0.02	2.5 years	18-27%	27- years
Arellano (2008)	0.953	Asymmetric	0.89 years	5.95%	8.3 years
Yue (2010)	0.72	0.02	permanent	10.13%	-
Chatterjee and Eyigungor (2012)	0.954	Asymmetric	6.49 years	70%	3.7 years
Mendoza and Yue (2012)	0.88	Endogenous	3 years	22.9%	36.0 years
Aguiar et al. (2016)	0.842	Asymmetric	2 years	66%	12.5 years
Na et al. (2018)	0.85	Asymmetric	6.5 years	60%	38.5 years
Bocola and Dovis (2019)	0.98	Asymmetric	5.1 years	82.8%	-
Bianchi and Mondragon (2022)	0.983	Asymmetric	4.2 years	28.6%-31.8%	-
Arellano et al. (2023)	0.954	Asymmetric	None	32%	37%

Note 1: All discount factors are standardized to a quarterly frequency.

Note 2: In Aguiar et al. (2016), the reported value of 0.842 corresponds to the stochastic growth specification, whereas the deterministic growth specification utilizes a value of 0.892. In Bianchi and Mondragon (2022), the value of 0.983 corresponds to the flexible exchange rate regime, while the fixed exchange rate regime uses 0.961. The default frequency reported for Arellano et al. (2023) represents the fraction of time the economy spends in a state of partial default, rather than the probability of entering a new default episode.

To replicate the empirical regularity of countercyclical sovereign defaults, the quantitative literature widely adopts non-linear, asymmetric output cost functions. Early work by Arellano (2008) introduces a kinked cost function where the endowment at the time of default is restricted to $y^{def} = \min(y, 0.969\mathbb{E}[y])$. This implies that there is no direct output penalty during severe recessions (when $y \leq 0.969\mathbb{E}[y]$), but any excess income above this threshold is entirely confiscated during economic expansions.

To overcome the computational instabilities associated with kinked functions, Chatterjee and Eyigungor (2012) introduce a smooth, convex penalty function: $\max\{0, d_0 y + d_1 y^2\}$. Using SMM, they calibrate the parameters to $d_0 = -0.188$ and $d_1 = 0.246$. Under this baseline calibration, the penalty is highly sensitive to the economic state and accelerates during booms. Specifically, when the endowment is 10 percent below its steady state, the output loss is roughly 3.0 percent. The penalty fraction gradually increases to 4.3 percent when output is 5 percent below the steady state, 5.8 percent at the steady state, and 7.4 percent when output is 5 percent above the steady state. Subsequent studies frequently adapt this quadratic specification. For instance, Na et al. (2018) set $d_0 = -0.35$ and $d_1 = 0.44$ to target an average output loss of 7 percent while in bad financial standing, which yields an equilibrium average output loss of 5 percent per period. Similarly, Bocola and Dovis (2019) incorporate a scaling factor, defining the cost as $\max\{0, d_0 \tau y + d_1 \tau y^2\}$, calibrated via SMM to $d_0 = 0.058$ and $d_1 = 0.092$.

Alternative functional forms have been developed to address specific computational or dynamic requirements. Bianchi and Mondragon (2022) propose a logarithmic specification defined as $\max\{0, d_0 + d_1 \ln(y)\}$. Because the income process is typically modeled as a log-normal AR(1) process, incorporating $ln(y)$ ensures that the marginal penalty changes linearly with respect to the state variable grid. This functional form guarantees higher computational stability, which is essential for complex frameworks involving self-fulfilling crises. Furthermore, unlike the quadratic acceleration in Chatterjee and Eyigungor (2012), the logarithmic form implies that while the absolute penalty increases during economic expansions, its growth rate gradually decelerates. They calibrate $d_0 = 0.140$ and $d_1 = 1.116$ for flexible exchange rate regimes, and $d_0 = 0.131$ and $d_1 = 0.395$ for fixed regimes. Another functional variant is seen in Aguiar et al. (2016), who utilize an exponential penalty form $d_0 \exp(y)^{d_1}$.

Finally, the literature has also expanded to endogenize these costs or accommodate partial defaults. Mendoza and Yue (2012) endogenize the output loss by modeling a production economy where default limits access to working capital, triggering an endogenous decline in labor and imported inputs to match a targeted average GDP decline of 13 percent. In the context of partial defaults, Arellano et al. (2023) introduce a multiplicative penalty structure characterized by $(1 - d_0 x)(1 - d_1(y' - y^*))$, where x denotes the chosen degree of default. This specification uniquely captures the interaction between the size of the haircut and the underlying productivity shock, penalizing larger partial defaults more severely during economic expansions.

In terms of default frequency, Yue (2010) assumes permanent market exclusion following a default event, which naturally precludes the computation of a recurring default frequency. Bocola and Dovis (2019) and Bianchi and Mondragon (2022) calibrate their frameworks to the Italian and Spanish economies, respectively. Because these advanced economies lack sovereign default events in modern history, these studies do not target an empirical default frequency, relying instead on the dynamics of sovereign spreads to discipline default risk. Arellano et al. (2023) depart from the standard full-default framework by introducing a partial default mechanism where market exclusion is absent and the sovereign continues to borrow while in arrears. Consequently, their reported default frequency of 37 percent does not measure the probability of entering a new default episode, but rather represents the unconditional probability, or the fraction of time, that the economy spends in a state of partial default.

While some seminal papers such as Lorenzoni and Werning (2019) and Aguiar et al. (2022) provide crucial theoretical foundations, their numerical exercises are designed primarily to illustrate qualitative mechanisms rather than to quantitatively match empirical moments. Consequently, we exclude them from this quantitative comparison.

Appendix B: Proofs of Propositions

Proposition 1. *For a given state s, there exists a unique threshold debt level* $\underline{B} \in [0, \infty)$ *such that*

$$\underline{B}(s) \equiv \sup \underline{\mathcal{R}}(s)$$

To prove Proposition 1, we establish the following two lemmas.

Lemma 1. *For a given* s, $V_r(S|\Xi = 1) \geq \underline{V}(s)$ *if* $B = 0$.

Proof. Under $\Xi = 1$, the primary market delivers zero proceeds ($\Lambda = 0$). If $B = 0$, the government faces no debt service obligations, meaning contemporaneous consumption exactly equals the endowment y. Furthermore, since the demand for newly issuance of bond is zero, $B' = 0$. Consequently, the repayment value simplifies to:

$$V_r(S|\Xi = 1) = u(y) + \beta\mathbb{E}[V(0, s')|(S)]$$

The autarky value satisfies

$$\underline{V}(s) = u(y[1 - \chi]) + \beta\{(1 - \psi)\mathbb{E}[\underline{V}(s')|(s)] + \psi\mathbb{E}[V(0, s')|(s)]\},$$

Taking the difference yields:

$$\begin{aligned} \Delta &= V_r(S|\Xi = 1) - \underline{V}(s) \\ &= \underbrace{[u(y) - u(y_t[1 - \chi])]}_{\text{(i)}} + \underbrace{\beta(1 - \psi)\,[\mathbb{E}(V(0, s')|(S)] - \mathbb{E}[\underline{V}(s')|(s)]]}_{\text{(ii)}} \end{aligned}$$

The first term (i) is strictly positive because u is strictly increasing and $\chi \in (0, 1)$. For the second term (ii), by the government's optimal decision rule Equation (4), re-entering the market with zero debt strictly dominates or weakly equals remaining in autarky, meaning $V(0, s') \geq \underline{V}(s')$ state by state; hence the expectation difference is non-negative. Since $\beta > 0$ and $1 - \varphi > 0$, (ii) is non-negative. Therefore $\Delta \geq 0$, proving the claim. Q.E.D.

Lemma 2. *The repayment value under no-rollover beliefs,* $V_r(S|\Xi = 1)$, *is strictly decreasing function in* B.

Proof.

Step 1: Finite horizon

Fix a terminal horizon T. Define the sequence of finite-horizon repayment values $\{V_{r,t}\}_{t=1}^{T}$ recursively by

$$V_{r,t}(S|\Xi = 1) = u(y - \lambda B - r(\omega)B) + \beta\mathbb{E}[(V_{r,t-1}((1-\lambda)B, s')|(S)]$$

with $V_{r,0}(\cdot)$ arbitrary but bounded and independent of B. Differentiating w.r.t.B and applying the chain rule yields,

$$\frac{\partial V_{r,t}(S|\Xi = 1)}{\partial B} = u'(\cdot)(-\lambda - r(\omega)) + \beta(1-\lambda)\mathbb{E}\left[\frac{\partial V_{r,t-1}((1-\lambda)B, s')}{\partial B}|S\right],$$

At the initial iteration step $t = 1$, the continuation term drops out of the value function because no future states remain to be considered, hence

$$\frac{\partial V_{r,1}(S|\Xi = 1)}{\partial B} = u'(\cdot)(-\lambda - r(\omega)) < 0,$$

because $u' > 0$, and both λ and $r(\omega)$ are positive.

Proceed by induction on the number of horizons. Suppose $\partial V_{r,t-1}/\partial B < 0$ for all states. Then the conditional expectation $\mathbb{E}[\partial V_{r,t-1}/\partial B|S] < 0$ is negative, and multiplying by $\beta(1-\lambda) > 0$ preserves the negative sign. Adding the strictly negative current-period term yields

$$\frac{\partial V_{r,t}(S|\Xi = 1)}{\partial B} < 0 \text{ for all } t = 1, \cdots, T$$

Hence $V_{r,t}$ is strictly decreasing in B for each finite horizon t.

Step 2: Infinite horizon

Let $\mathcal{T}$ denote the Bellman operator under $\Xi = 1$:

$$(\mathcal{T}W)(S) = u(y - \lambda B - r(\omega)B) + \beta\mathbb{E}[(W((1-\lambda)B, s')|(S)]$$

On the space of bounded continuous functions with the sup norm, $\mathcal{T}$ is a contraction with modulus β. Therefore V_r is the unique fixed point of $\mathcal{T}$, and the Picard iterates $V_{r,t}$ converge uniformly to V_r as $t \to \infty$. Moreover, from the derivative recursion above and the boundedness of u' on relevant compact sets of consumption, there exists a uniform bound $M < \infty$

such that $|\partial V_{r,t}/\partial B| \leq M$ for all t. Hence by dominated convergence and the contraction mapping argument for derivatives, we may pass limits through the conditional expectation and the linear operator, obtaining

$$\frac{\partial V_r(S|\Xi = 1)}{\partial B} = u'(\cdot)(-\lambda - r(\omega)) + \beta(1-\lambda)\mathbb{E}\left[\frac{\partial V_r((1-\lambda)B, s')}{\partial B}|S\right] < 0$$

By Step 1 the sequence $\{\partial V_{r,t}/\partial B\}_t$ is strictly negative pointwise; the limit therefore satisfies $\partial V_r/\partial B \leq 0$. Since the first term on the right-hand side is strictly negative (because $u' > 0$, and λ and r are both positive), $\partial V_r/\partial B < 0$ everywhere. Q.E.D.

Proof of Proposition 1.

First, by lamma 1, $V_r(S|\Xi = 1) \geq \underline{V}(s)$ holds at $B = 0$. Hence the repayment set, defined as

$$\underline{\mathcal{R}}(s) \equiv \{B \in [0, \infty) : V_r(S|\Xi = 1) \geq \underline{V}(s)\}$$

is nonempty. Moreover, by Lemma 2, the mapping $B \to V_r(S|\Xi = 1)$ is strictly decreasing, while $\underline{V}(s)$ is independent of B. This monotonicity ensures that if $B \in \underline{\mathcal{R}}(s)$ and $0 \leq \widetilde{B} \leq B$, then $\widetilde{B} \in \underline{\mathcal{R}}(s)$. Thus $\underline{\mathcal{R}}(s)$ must be an interval of the form $[0, \widetilde{B}]$ or $[0, \infty)$.

To exclude the case $[0, \infty)$, note that under $\Xi = 1$ the government cannot issue new debt in period t. Repayment consumption is given by

$$c = y - \lambda B - r(\omega)B$$

For fixed y, with $\lambda > 0$ and $r(\omega) \geq 0$, c is strictly decreasing in B. If B becomes sufficiently large, c becomes negative, violating the non-negativity constraint on consumption (or equivalently, causing $u(c) \to -\infty$). Because u is continuous and strictly increasing, the repayment value $V_r(S|\Xi = 1)$ eventually falls strictly below the finite autarky value $\underline{V}(s)$. Therefore, $\underline{\mathcal{R}}(s)$ cannot equal $[0, \infty)$; it must be of the form $[0, \widetilde{B}]$ with $\widetilde{B} < \infty$.

Next, define the threshold $\underline{B}(s) \equiv \sup \mathcal{B}$. By standard continuity of the value function in B, implied by the contraction mapping argument for V_r and continuity of u, the mapping $B \mapsto V_r(S|\Xi = 1)$ is continuous. Since $V(s)$ is constant in B, taking limits from below yields

$$\lim_{B \uparrow \underline{B}(s)} (V_r(S|\Xi = 1) - \underline{V}(s)) = 0$$

Thus equality holds at the boundary, i.e.

$$V_r(S|\Xi = 1) = \underline{V}(s) \text{ at } B = \underline{B}(s)$$

Finally, because $V_r(S|\Xi = 1)$ is strictly decreasing in B (Lemma 2) and $\underline{V}(s)$ is independent of B, their graphs can intersect at most once. Since an intersection occurs at $\underline{B}(s)$, this threshold is unique. Q.E.D.

Proposition 2. *For given s, there exists a unique debt threshold $\overline{B} \in [0, \infty)$ such that*

$$\overline{B}(s) \equiv \sup \overline{\mathcal{R}}(s)$$

To prove Proposition 2, we establish the following lemmas 3 to 5.

Lemma 3. For any sufficiently small debt level $B \geq 0$, the government can choose an issuance plan B' that satisfies

$$V_r(S|\Xi = 0; B') \geq V_d(S|\Xi = 0; B').$$

Consequently, the repayment set $\overline{\mathcal{R}}(s)$ is non-empty.

Proof. Choosing $B' = 0$ eliminates inherited debt in the next period. By monotonicity and the autarky comparison, the continuation value under repayment weakly dominates that under default. Formally, this implies:

$$\mathbb{E}[(V(0, s'))|(S)] > (1 - \psi)\mathbb{E}[\underline{V}(s')|(s)] + \psi\mathbb{E}[V(0, s')|(s)]$$

Furthermore, in the current period, consider the case where outstanding debt is exactly zero ($B = 0$). The default penalty $\chi > 0$ ensures that the contemporaneous utility strictly favors repayment:

$$u(y + \Lambda) - u(y(1 - \chi) + \Lambda) > 0.$$

Because the utility function u is continuous in B, this strict preference for repayment does not disappear immediately as B increases. By continuity, the inequality persists for any sufficiently small $B > 0$. Q.E.D.

Lemma 4. Suppose that the amortization rate is strictly positive ($\lambda > 0$) and the interest rate is non-negative ($r(\omega) \geq 0$). Then, for any given state s and any issuance plan B', there exists a sufficiently large debt level $\widetilde{B} > 0$ such that for all $B > \widetilde{B}$,

$$V_r(S|\Xi = 0; B') < V_d(S|\Xi = 0; B').$$

Proof. Consider the government's consumption under the repayment branch:

$$c = y - \lambda B - r(\omega)B + \Lambda(S|\Xi = 0; B')$$

As outstanding debt $B \to \infty$, the total debt service $(\lambda + r(\omega))B$ increases linearly toward infinity because $\lambda > 0$. Crucially, the revenue from new debt issuance λ is naturally bounded from above by the endogenous debt Laffer curve since bond prices fall to zero as default risk rises.

Because the endowment y is finite and the new borrowing Λ is bounded, the debt service burden eventually dominates all available resources. Consequently, as $B \to \infty$, the implied consumption c becomes strictly negative, which violates the non-negativity constraint on consumption (or equivalently, drives the contemporaneous utility $u(c) \to -\infty$). This implies that the value of repayment approaches negative infinity:

$$\lim_{B \to \infty} V_r(S|\Xi = 0; B') = -\infty$$

By contrast, consider the value of default $V_d(S|\Xi = 0; B')$. Since default eliminates all existing debt obligations, the continuation value is entirely independent of B and depends solely on the exogenous state s. Furthermore, while the contemporaneous consumption $y(1-\chi)+\Lambda(S|\Xi = 0)$ may depend on B, the boundedness of Λ ensures that this consumption remains strictly positive and bounded from below. Thus, V_d is bounded from below by some finite value.

Because V_d is bounded while V_r diverges to $-\infty$, there must exist some threshold $\widetilde{B}$ beyond which repayment becomes strictly worse than default. Q.E.D.

Lemma 5. Define the value gap for issuance plan B':

$$\Delta(B; B') \equiv V_r(S|\Xi = 0; B') - V_d(S|\Xi = 0; B').$$

For each fixed B', $\Delta(B; B')$ is at least weakly decreasing in B. Consequently, the upper envelope

$$\Delta^*(B) \equiv \sup_{B'} \Delta(B; B')$$

is also weakly decreasing in B. Under standard regularity (lower bound on marginal utility, boundedness of $\partial\Lambda/\partial B$), the upper envelope Δ^* is strictly decreasing.

Proof. For a fixed issuance plan B', the continuation value is completely determined by B' and is therefore independent of the current debt level B. Thus, the derivative of the value gap $\Delta(B; B')$ with respect to B depends exclusively on the current-period utilities. Differentiating the current-period utility difference with respect to B yields:

$$\frac{\partial\Delta(B; B')}{\partial B} \equiv u'(c_r)\left[-(\lambda + r(\omega)) + \frac{\partial\Lambda_r}{\partial B}\right] - u'(c_d)\left[\frac{\partial\Lambda_d}{\partial B}\right]$$

where subscription of r and d represents of under the government's state of repayment and default, respectively.

Because the debt service term $-(\lambda+r(\omega))$ strictly reduces contemporaneous consumption, the gap falls with B as long as the marginal impact of B on new debt issuance ($\partial\Lambda/\partial B$) is sufficiently bounded. Under the stated standard regularity conditions, the strict negativity of this derivative is guaranteed:

$$\frac{\partial\Delta(B; B')}{\partial B} < 0$$

Thus, for each fixed B', $\Delta(B; B')$ is strictly decreasing in B. Since the supremum of strictly decreasing functions (that satisfy the regularity conditions uniformly) preserves this property, the upper envelope $\Delta^*(B)$ is also strictly decreasing. Q.E.D.

Then, we can prove the proposition 2.

Proof of Proposition 2.

By Lemma 3, the repayment set $\overline{\mathcal{R}}(s) \neq \emptyset$ is non-empty, meaning $\Delta^*(B) \geq 0$ for sufficiently small $B \geq 0$. By Lemma 4, for sufficiently large B, repayment becomes strictly worse than default for any admissible B'. This implies that $\Delta^*(B) < 0$ for large B, and hence such B cannot belong to $\overline{\mathcal{R}}(s)$. It follows that $\overline{\mathcal{R}}(s)$ is bounded from above and thus admits a finite supremum. Under standard regularity conditions, Berge's Maximum Theorem guarantees that the upper envelope function $\Delta^*(B)$ is continuous in B. Furthermore, by Lemma 5, $\Delta^*(B)$ is strictly decreasing in B.

Since $\Delta^*(B)$ is continuous, strictly decreasing, initially positive or zero, and eventually negative, the intermediate value theorem implies that there exists exactly one root where $\Delta^*(B) = 0$. Consequently, the repayment set $\overline{\mathcal{R}}(s) = \{B \geq 0 : \Delta^*(B) \geq 0\}$ must be a closed interval of the form $[0, B(s)]$, where the unique threshold $B(s) \equiv \sup \overline{\mathcal{R}}(s)$ is precisely the root that satisfies $\Delta^*(B) = 0$. Equivalently, this threshold is defined by:

$$\sup_{B' \in \Gamma(B,s)} \{V_r(S|\Xi = 0; B') - V_d(S|\Xi = 0; B')\} = 0.$$

Because the strict monotonicity of $\Delta^*(B)$ allows for at most one intersection with zero, and the intermediate value property guarantees its existence, the debt threshold $B(s)$ exists and is unique. Q.E.D.

Proposition 3. *For given s, there exist an issuance plan B'_* and a unique threshold $\overline{B}(s)$ such that*

$$V_r((\overline{B}(s), s)|\Xi = 0; B'_*) = V_d((\overline{B}(s), s)|\Xi = 0; B'_*) = \underline{V}(s)$$

Proof.

By Proposition 2 and the properties of the value gap function established above, the threshold $\overline{B}(s) \equiv \sup \overline{\mathcal{R}}(s)$ is the exact boundary where the government is indifferent between repayment and default under rollover expectations. Thus, there exists an optimal issuance plan B'_* such that the repayment and default values are equalized:

$$V_r((\overline{B}(s), s)|\Xi = 0; B'_*) = V_d((\overline{B}(s), s)|\Xi = 0; B'_*) \tag{20}$$

To ascertain the exact magnitude of these values at the boundary, we evaluate the equilibrium pricing condition that newly issued debt is priced at zero ($q = 0$, yielding $\Lambda = 0$) if default is inevitable. By analyzing the market pricing in this region and subsequently applying the continuity of the value functions, we can deduce the boundary values. For any debt level strictly above the threshold, $B > \overline{B}(s)$, no issuance choice can satisfy the ex post incentive compatibility constraint. Because lenders perfectly anticipate default in this region, this zero-pricing condition applies. As shown in the main text, without these proceeds, the default value function mathematically collapses exactly to the pure autarky value:

$$V_d((B(s), s)|\Xi = 0; B') = \underline{V}(s) \text{ for all } B > \overline{B}(s)$$

Since the value functions are continuous in the debt level B, taking the limit as the debt approaches the threshold from above ($B \downarrow \overline{B}(s)$) implies that the default value at the exact boundary must also continuously extend to the pure autarky value:

$$V_d((\overline{B}(s), s)|\Xi = 0; B'_*) = \underline{V}(s) \tag{21}$$

Combining equations (20) and (21) establishes that at the rollover threshold, the repayment value, the default value, and the pure autarky value perfectly intersect:

$$V_r((\overline{B}(s), s)|\Xi = 0; B'_*) = V_d((\overline{B}(s), s)|\Xi = 0; B'_*) = \underline{V}(s)$$

Q.E.D.

Proposition 4. *For given s,* the no-rollover threshold is weakly below the rollover threshold:

$$\underline{B}(s) \leq \overline{B}(s).$$

Proof:

Define the value gaps relative to autarky under each lenders' expectation as:

$$f_0(B) \equiv V_r(S|\Xi = 1) - \underline{V}(s)$$

$$f_1(B) \equiv \sup_{B' \in \Gamma(B,s)} \{V_r(S|\Xi = 0; B') - \underline{V}(s)\}$$

By construction, $f_0(B)$ compares repayment to autarky when lenders coordinate on no rollover, whereas $f_1(B)$ performs the same comparison when lenders coordinate on rollover, maximizing over feasible issuance $B' \in \Gamma(B, s)$.

First, note that the strategy set underlying $f_1(B)$ weakly contains the one implicit in f_0:

choosing $B' = 0$ in the $\Xi = 0$ problem replicates the within-period financing constraint of $\Xi = 1$. Hence, for every B,

$$f_1(B) \geq V_r(S|\Xi = 0; B' = 0) - \underline{V}(s) \geq V_r(S|\Xi = 1) - \underline{V}(s) = f_0(B) \tag{22}$$

Second, by Lemma 1, $f_0(0) \geq 0$. By (22), $f_1(0) \geq f_0(0) \geq 0$. By Lemma 2, f_0 is decreasing in B. By the regularity conditions used in Propositions 2 and 3, f_1 is continuous and decreasing in B as well. Because both functions are initially non-negative and strictly decreasing, each crosses zero exactly once. Let $\underline{B}(s)$ be the unique root for f_0, such that $f_0(\underline{B}(s)) = 0$. Crucially, by Proposition 3, the unique root of f_1 is precisely the rollover threshold $\overline{B}(s)$, meaning $f_1(\overline{B}(s)) = 0$.

Finally, inequality (22) implies pointwise dominance $f_1(B) \geq f_0(B)$ for all B. For two continuous, strictly decreasing functions with $f_1 \geq f_0$ everywhere, the zero of f_1 cannot lie to the left of the zero of f_0. Therefore, we must have $\underline{B}(s) \leq \overline{B}(s)$. Q.E.D.

Proposition 5.

For a given state s in the strategic default model, there exists a unique debt threshold $B^{SD}(s) \equiv \sup \mathcal{R}^{SD}(s)$ such that the government weakly prefers repayment for $B \leq B^{SD}(s)$ and strictly prefers default for $B > B^{SD}(s)$.

Proof:

We first establish the properties of the repayment value function in the strategic default model, $V_r^{SD}(S)$, relative to the default value $V_d^{SD}(s)$.

Step 1: Repayment is preferred at $B = 0$.

If the government enters the period with no debt $B = 0$ and chooses to repay, it faces no debt service ($\lambda B = 0$, $r(\omega)B = 0$). The government can choose to issue new debt $B' = 0$, which yields $\Lambda^{SD}(0, s) = 0$. The repayment value is at least as high as the value of choosing $B' = 0$:

$$V_r^{SD}(0, s) \geq u(y) + \beta \mathbb{E}[V^{SD}(0, s')|s]$$

The default value under the strategic default timing is purely autarkic:

$$V_d^{SD}(s) = u(y[1 - \chi]) + \beta\{(1 - \psi)\mathbb{E}[V_d^{SD}(s')|s] + \psi \mathbb{E}[V^{SD}(0, s')|s]\}$$

Taking the difference yields:

$$\Delta^{SD} = V_r^{SD}(0,s) - V_d^{SD}(s) \geq [u(y) - u(y[1-\chi])] + \beta\{(1-\psi)\mathbb{E}[V^{SD}(0,s') - V_d^{SD}(s')|s]]$$

Because output penalty $\chi \in (0,1)$ and u is strictly increasing, $u(y) - u(y[1-\chi]) > 0$. By the optimal decision rule, entering the next period with zero debt is weakly better than entering with the autarky penalty, so $V^{SD}(0,s') \geq V_d^{SD}(s')$ state-by-state, making the second term non-negative. Therefore, $\Delta^{SD} > 0$, implying $V_r^{SD}(0,s) > V_d^{SD}(s)$. Hence, the set $R^{SD}(s) \equiv \{B \in [0,\infty) : V_r^{SD}(S) > V_d^{SD}(s)\}$ is non-empty.

Step 2: $V_r^{SD}(S)$ is strictly decreasing in B.

By the envelope theorem, the derivative of the repayment value with respect to existing debt B is driven by the current-period debt service burden:

$$\frac{\partial V_r^{SD}(S)}{\partial B} = u'(c)[-\lambda - r(\omega)] < 0$$

Since marginal utility $u'(c) > 0$ and the amortization and interest terms are positive, higher inherited debt strictly reduces the repayment value. Conversely, the strategic default value $V_d^{SD}(s)$ depends only on the exogenous state s and is entirely independent of B.

Step 3: Repayment becomes infeasible for sufficiently large B.

For a fixed state s, as outstanding debt $B \to \infty$, the required debt service $(\lambda + r(\omega))B \to \infty$. Even if the government issues the maximum feasible new debt B' (which is bounded by the finite willingness of lenders to provide funds Λ^{SD}), current consumption $c = y - (\lambda + r(\omega))B + \Lambda^{SD}$ becomes negative, driving $u(c) \to -\infty$. Because $V_d^{SD}(s)$ is finite and bounded below, there exists some large $\overline{B} < \infty$ such that for all $B > \overline{B}$, $V_r^{SD}(S) < V_d^{SD}(s)$. Thus, the set $R^{SD}(s)$ is bounded from above.

Step 4: Existence and uniqueness of the threshold $B^{SD}(s)$.

Because $V_r^{SD}(S)$ is strictly decreasing in B (Step 2) and $V_d^{SD}(s)$ is constant in B, the difference $V_r^{SD}(S) - V_d^{SD}(s)$ is strictly decreasing in B. By Step 1, this difference is strictly positive at $B = 0$. By Step 3, this difference is strictly negative for sufficiently large B. By the continuity of the value functions in B, the intermediate value theorem guarantees the existence of a root where the repayment value equals the default value, while the strict monotonicity guarantees its uniqueness. Thus, there exists exactly one point such that:

$$B^{SD}(s) \equiv \sup R^{SD}(s)$$

At this threshold, $V_r^{SD}(B^{SD}(s), s) = V_d^{SD}(s)$. Because the value gap is strictly monotonically decreasing in B, the government weakly prefers repayment for all $B \leq B^{SD}(s)$ and strictly prefers default for all $B > B^{SD}(s)$. Q.E.D.

Appendix C: Detailed Explanation of Calibration Method

Endowment

As discussed in the main text, we extract the cyclical component of the endowment process using a modified version of the linear projection method proposed by Hamilton (2018). This appendix provides the formal econometric specification of our detrending procedure. Let y_t denote the natural logarithm of the raw endowment data (e.g., real GDP). While Hamilton (2018) originally recommends a look-ahead horizon of two years ($h = 8$) for quarterly data, subsequent econometric literature has pointed out that this baseline setting may fail to properly extract typical, shorter-duration recessions (lasting 1 to 2 years) by overemphasizing medium-to-long-term cycles (Schuler, 2018). To address this, we set the look-ahead horizon to one year ($h = 4$ quarters) and the lag order to four quarters ($p = 4$). Our choice of $h = 4$ is highly consistent with classical macroeconomic theory, explicitly matching the conventional lower bound of business cycle frequencies (6 to 32 quarters) established by Burns and Mitchell (1946) and widely utilized in standard filtering methods (Baxter and King, 1999). This ensures that we accurately capture the relevant cyclical fluctuations without being contaminated by high-frequency noise. The baseline regression specification is given by:

$$y_{t+h} = \beta_0 + \beta_1 y_t + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \beta_4 y_{t-3} + \upsilon_{t+h}$$

To construct a trend that is uncontaminated by the post-crisis collapse, we restrict the estimation window of the parameter vector β exclusively to the pre-crisis subsample. Let T_{break} denote the final quarter of the pre-crisis period (2008:Q4). We estimate the coefficients via Ordinary Least Squares (OLS) over the restricted window $t = p, \cdots, T_{break} - h$:

$$\widehat{\boldsymbol{\beta}}_{pre} = \underset{\boldsymbol{\beta}}{\operatorname{argmin}} \sum_{t=p}^{T_{break}-h} (\widetilde{y}_{t+h} - \mathbf{x}_t' \boldsymbol{\beta})^2$$

where $\mathbf{x}_t = [1, y_t, y_{t-1}, y_{t-2}, y_{t-3}]'$ and $\boldsymbol{\beta} = [\beta_0, \beta_1, \beta_2, \beta_3, \beta_4]'$.

Using the estimated fixed coefficients $\widehat{\boldsymbol{\beta}}_{pre}$, we extrapolate the underlying trend component, denoted as y_{t+h}^{trend}, for the entire sample period (both pre- and post-crisis). This trend is defined as the conditional expectation based strictly on the pre-crisis structural relationship:

$$y_{t+h}^{trend} = \widehat{\boldsymbol{\beta}}'_{pre}\mathbf{x}_t$$

Finally, the cyclical component y_{t+h}^{cycle}, which theoretically corresponds to the prediction error or the deviation from the pre-crisis expected growth path, is obtained by subtracting the extrapolated trend from the realized data:

$$y_{t+h}^{cycle} = y_{t+h} - y_{t+h}^{trend}.$$

This resulting sequence $\{y_t^{cycle}\}$ is then utilized as the exogenous endowment process in our structural sovereign default model.

Calibration of the Beta Distribution Parameters

We explain the technical details of the calibration procedure used to map the observed debt-to-GDP ratio b_t to the market-implied default probabilities P_t^{obs}. First, we normalize the empirical debt sequence b_t into a variable x_t within the $[0, 1]$ interval. Based on the empirical distribution, we define a safe debt lower bound $b_{safe} = 5.228$ where default risk is assumed to be negligible, and a default upper bound $b_{default} = 6.648$ where default is practically certain:

$$x_t = \frac{b_t - b_{safe}}{b_{default} - b_{safe}}$$

For the normalized debt x_t, we model the theoretical default probability $P(b_t)$ using the cumulative distribution function (CDF) of the Beta distribution (the regularized incomplete beta function I_x):

$$P(b_t) = I_x(\phi_0, \phi_1) = \frac{1}{B(\phi_0, \phi_1)} \int_0^{x_t} u^{\phi_0 - 1}(1-u)^{\phi_1 - 1} du$$

where $B(\cdot, \cdot)$ is the Beta function, and $\{\phi_0, \phi_1\}$ are the shape parameters governing the curvature of the risk sensitivity. We estimate the parameter set $\Theta = \{\phi_0, \phi_1\}$ by minimizing the sum of squared residuals between the observed implied probabilities and the model predictions via Non-Linear Least Squares (NLS):

$$\widehat{\Theta} = \underset{\phi_0,\phi_1}{\operatorname{argmin}} \sum_{t=1}^{T} (P_t^{obs} - I_x(\phi_0, \phi_1))^2$$

To ensure the probability distribution is mathematically well-defined without overly restricting the curve's shape, we impose a standard strict positivity constraint on the parameters $\phi_0 > 0$, $\phi_1 > 0$ during the optimization process. Crucially, as the absolute magnitude of the implied default probabilities P_t^{obs} backed out from CDS spreads varies significantly depending on the haircut rate assumed by lenders, we perform this NLS estimation separately for two distinct scenarios: a baseline haircut rate of 100% and an alternative haircut rate of 53.5%. This procedure yields a specific set of $\{\phi_0, \phi_1\}$ for each scenario, allowing us to evaluate the theoretical model's quantitative implications under different empirical market assumptions.

Pricing Kernel and Term-Structure Block with the SMM

We derive the one-period stochastic discount factor from an exponential-affine kernel with a time-varying market price of risk, as in Equations (13) and (12) as

$$m_{t+1} = \exp(-\frac{1}{2}\nu_t^2\sigma_\omega^2 - r_0 - r_1\omega_t - \nu_t\sigma_\omega\varepsilon_{\omega,t+1}),\ \nu_t = \nu_0 + \nu_1\omega_t$$

where the state driver ω_t follows a mean-reverting AR(1):

$$\omega_{t+1} = \mu_\omega(1 - \rho_\omega) + \rho_\omega\omega_t + \sigma_\omega\varepsilon_{\omega,t+1},\ \varepsilon_{\omega,t+1} \sim N(0, 1)$$

Because the bond data are nominal, we model inflation separately and use it for both discounting and mapping nominal into real quantities. Quarterly inflation follows

$$\pi_{t+1} = \mu_\pi(1 - \rho_\pi) + \rho_\pi\pi_t + \sigma_\pi\varepsilon_{\pi,t+1},\ \varepsilon_{\pi,t+1} \sim N(0, 1) \tag{23}$$

We discount nominal payoffs by $\exp(-\pi_{t+1}) \approx (1 + \pi_{t+1})^{-1}$, which is accurate at quarterly horizons.

Let $q_{t,n}^f$ be the time t price of an n period risk-free zero coupon bond. No-arbitrage implies

$$q_{t,n+1}^f = \mathbb{E}_t\left[m_{t+1}\exp(-\pi_{t+1})q_{t+1,n}^f\right].$$

Following Ang and Piazzesi (2003), we postulate exponential-affine bond prices

$$q^f_{t,n} = \exp(A_n + B_n\omega_t + C_n\pi_t),\ (A_0, B_0, C_0) = (0, 0, 0),$$

and define continuously compounded spot yields $x_{t,n}$ and forward rates $f_{t,n}$ in the usual way:

$$x_{t,n} \equiv -\frac{1}{n}\log q^f_{t,n}, \quad f_{t,n} \equiv n x_{t,n} - (n-1)x_{t,n-1}.$$

One-period excess returns from holding an n-period bond for one period $rx_{t+1,n}$ are

$$\begin{aligned} rx_{t+1,n} &\equiv f_{t,n} - x_{t,1} \\ &= n x_{t,n} - (n-1)x_{t,n-1} - x_{t,1}. \end{aligned} \tag{24}$$

Plugging the conjecture $q^f_{t,n} = \exp(A_n + B_n\omega_t + C_n\pi_t)$ into the pricing recursion and using the joint normality of $(\varepsilon_{\omega,t+1}, \varepsilon_{\pi,t+1})$ yields closed-form affine recursions:

$$\begin{aligned} A_{n+1} &= A_n - r_0 + B_n\mu_\omega(1-\rho_\omega) + (C_n - 1)\mu_\pi(1-\rho_\pi) \\ &\quad + \frac{1}{2}B_n^2\sigma_\omega^2 - \sigma_\omega^2 B_n\nu_0 + \frac{1}{2}\sigma_\pi^2(C_n-1)^2 + \rho_{\omega,\pi}\sigma_\omega\sigma_\pi[B_n(C_n-1) - \nu_0(C_n-1)] \end{aligned}$$

$$B_{n+1} = -r_1 + B_n\rho_\omega - \sigma_\omega^2 B_n\nu_1 - \rho_{\omega,\pi}\sigma_\omega\sigma_\pi(C_n-1)\nu_1,$$

$$C_{n+1} = \rho_\pi(C_n - 1)$$

with stationarity requiring $|\rho_\omega| < 1$ and $|\rho_\pi| < 1$. These recursions ensure bond prices remain exponential-affine even when the market price of risk ν_t depends on the state ω_t.

To discipline the term-structure block, we match moments that summarize both the level/slope of the yield curve and the predictability of bond excess returns. We first fit a Nelson–Siegel–Svensson (NSS) curve to Euro-area AAA issuer data. From the fitted NSS coefficients $(\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2)$ we construct, for each maturity m in years, an annual percentage yield and convert it into a quarterly continuously-compounded spot rate

$$z_{t,m} = \log\left((1 + NSS_t(m)/100)^{1/4}\right).$$

Monthly observations are averaged by calendar quarter to obtain quarterly series $z_{t,n}$ for maturities $n \in \{1, 3, 4, 7, 8, 11, 12, 15, 16, 19, 20, 31.2\}$ quarters. The last corresponds to the average maturity of Greek government bonds used in calibration.

From these spot rates we form one-quarter holding-period excess returns on n-quarter bonds as in Cochrane–Piazzesi (2005). In continuously-compounded units, Equation (24) becomes

$$rx_{t+1,n} = nz_{t,n} - (n-1)z_{t+1,n-1} - z_{t,1}$$

We also construct the forward rates $f_{t,n}$ from spot rates

$$f_{t,4} = 4z_{t,4} - 3z_{t,3},\ f_{t,8} = 8z_{t,8} - 7z_{t,7},\ \cdots$$

Following Cochrane and Piazzesi (2005), we estimate a single forecasting factor by regressing the cross-maturity average of excess returns on the short rate and the forward curve. Let

$$\overline{rx}_{t+1} = \frac{1}{5} \sum_{n \in \{4,8,12,16,20\}} rx_{t+1,n}.$$

We run the first-stage regression

$$\overline{rx}_{t+1} = \gamma_0 + \gamma_{spot} y_{t,1} + \sum_{j=1}^{5} \gamma_{4j} f_{t,4j} + \varepsilon_{t+1},$$

and define the CP factor as the fitted value

$$CP_t \equiv \widehat{\gamma}_0 + \widehat{\gamma}_{spot} y_{t,1} + \sum_{j=1}^{5} \widehat{\gamma}_{4j} f_{t,4j}.$$

In the second stage for each maturity n we estimate

$$rx_{t+1,n} = a_n + b_n CP_t + u_{t+1,n},\ u_{t+1,n} \sim N(0, \sigma_n^2),$$

and retain the slopes b_n and residual standard deviations σ_n. We also summarize the dynamics of the CP factor with an AR(1)

$$CP_{t+1} = a_{cp} + b_{cp} CP_t + \varepsilon_{cp,t+1},$$

Finally, to connect nominal term-structure moments to real pricing in the model block, inflation measured from the CPI is used to form real short-rate moments and correlations. The empirical moment vector stacks $(\mu_{cp}, \rho_{cp}, \sigma_{cp})$ from the CP AR(1), the predictive intercept/slope and residual s.d. for the 5-year excess return, $corr(z_{t,1} - \pi_t, \pi_t)$, the mean and standard deviation of $z_{t,1} - \pi_t$, and the AR(1) parameters $(\mu_\pi, \rho_\pi, \sigma_\pi)$. These are exactly the targets saved by the data routines and matched in the SDF estimation.

We collect SDF parameters in $\theta = (r_0,\ r_1,\ \nu_0,\ \nu_1,\ \mu_\omega,\ \rho_\omega,\ \sigma_\omega)$, treat $(\mu_\pi, \rho_\pi, \sigma_\pi)$ as data-driven from the CPI block, stack the empirical targets in m_{data}, and compute model-simulated counterparts $m_{sim}(\theta)$ from the affine recursions and simulated paths (ω_t, π_t). The parameters are estimated by minimizing the Simulated Method of Moments criterion,

$$J(\theta) = (m_{sim}(\theta) - m_{data})^\intercal W (m_{sim}(\theta) - m_{data}),$$

with a positive-definite weighting matrix W. We use diagonal weights based on absolute moments; inverse-variance weights can be adopted iteratively. Identification requires $|\rho_\omega| < 1$ and $|\rho_\pi| < 1$. Because risk adjustments enter via $\nu_t \sigma_\omega$, we restrict ν_1 and σ_ω away from zero and leverage the slope/variance targets (b_n, σ_n) to avoid weak identification.

Using the estimated θ, we simulate large samples (e.g., $N = 20,000$ quarters), compute yields $x_{t,n}$, forward rates $f_{t,n}$, and excess returns $rx_{t+1,n}$, and form the model moments listed above. The recursive formulas guarantee that the entire yield curve up to $n = 20$ quarters can be generated consistently with the exponential-affine SDF and the inflation block.

Appendix D: Numerical Simulation Algorithm

We use value function iteration to numerically solve the model and approximate the three value functions and the price of government bonds. Although the theoretical state vector includes the sunspot variable ζ, our endogenous formulation of the coordination failure probability allows us to integrate out the realization of lenders' beliefs without adding it to the computational grid. Therefore, the numerical state space strictly consists of four state variables: one endogenous state variable B and three exogenous fundamental state variables $\{y, \omega, \beta\}$.

The Price of Government Bond

In terms of the price of government bonds, rather than computing prices for each individual maturity $q_n(S; B')$ one by one, we aggregate across maturities with geometric weights and solve directly for

$$Q(S; B') = \sum_{n=1}^{\infty} \lambda(1-\lambda)^{n-1} q_n(S; B') \tag{25}$$

where $q_n(S; B')$ is defined in Equation (5). Under a Calvo-style maturity structure, a fraction λ of principal is amortized each period while the remainder $(1-\lambda)$ is carried over. With default indicator $\Xi(S) \in \{0, 1\}$ and stochastic discount factor $m(s, s')$, this aggregation yields the single fixed-point equation

$$Q(S; B') = (1 - \Xi(S))\mathbb{E}[m(s, s')(1 - \Xi(S')) \cdot (r(\omega') + \lambda + (1-\lambda)Q(S^{'}; B''))]$$

This representation is both economically transparent and numerically advantageous. Economically, it preserves the interpretation that coupons $r(\omega')$ are paid while a λ share of face value is redeemed each period and the surviving $(1-\lambda)$ share continues trading. Numerically, it collapses an infinite family of pricing recursions into a single contraction-like operator in Q. The factor $(1-\lambda)$ acts like additional discounting inside the expectation, which strengthens the contraction property and typically improves convergence. In operator form, letting A denote the linear price operator induced by transition and discounting and $D = diag(1-\Xi)$, we have

$$Q = DAD[(r + \lambda) + (1-\lambda)Q]$$

Then, we numerically obtain the price as:

$$Q_{new} = DAD[(r + \lambda) + (1 - \lambda)Q_{old}]$$

so Q can be obtained by a resolvent solve or by a rapidly convergent fixed-point iteration. From a computational standpoint, replacing $\{q_n\}_{n\geq 1}$ with Q significantly reduces both arithmetic and memory requirements. Computing q_n for $n = 1, \cdots, N$ individual maturities entails N state-contingent pricing equations, meaning matrix–vector multiplications and storage scale as $O(N)$ per iteration. In contrast, the aggregated formulation requires solving for only a single unknown function Q, bringing the per-iteration cost down to $O(1)$ relative to N.

Once the aggregate price Q is determined, the net revenues from primary issuance, Λ, can be derived compactly. The net revenues are given by:

$$\begin{aligned} \Lambda &= \sum_{n=1}^{\infty} q_n \left[\lambda(1-\lambda)^{n-1}B' - \lambda(1-\lambda)^n B\right] \\ &= Q(S;B')B' - (1-\lambda)Q(S;B')B \end{aligned}$$

Because the new issuance of government bond B_{new} can be defined as $B_{new} = B' - (1-\lambda)B$, this simplifies nicely to:

$$\Lambda = Q(S;B')B_{new}$$

We now extend this aggregated pricing framework to the case of partial default. As previously discussed, the price of each individual bond incorporates a recovery value in the event of default, as shown in Equation (19).

$$q_n(B', s) = \mathbb{E}\left[m(s, s')\left\{(1 - \Xi(S'))(r(\omega') + q_{n-1}(S')) + \Xi(S')q^h(s')\right\}\right]$$

Applying the same geometric aggregation to these individual prices yields the aggregate price under partial default:

$$
\begin{aligned}
Q(S;B') &= \sum_{n=1}^{\infty} \lambda(1-\lambda)^{n-1} q_n(S;B') \qquad (26)\\
&= \mathbb{E}\left[m(s,s') \left\{ (1-\Xi(S^{'}))(r(\omega') + \lambda + (1-\lambda)Q(S^{'};B'')) + \Xi(S^{'})Q^h(s') \right\} \right],
\end{aligned}
$$

where $Q^h(s') \equiv \sum_{n=1}^{\infty} \lambda(1-\lambda)^{n-1} q^h(s)$, and $q^h(s)$ is defined in (18).

Crucially, to maintain the numerical advantages established in the baseline model, this extended equation can also be cast into operator form. Letting $\widetilde{D} = I - D = diag(\Xi)$ denote the default indicator matrix, the fixed-point equation becomes

$$
Q = DA \left[D((r+\lambda) + (1-\lambda)Q) + \widetilde{D} Q^r_d \right]
$$

Consequently, even in the presence of partial default, the aggregate price Q can still be efficiently solved using the same $O(1)$ fixed-point iteration approach.

Solution of Discrete State Space

We solve a sovereign-default model in which the government chooses next-period debt B' taking bond prices Q as endogenous, and the payoff of bonds depends on whether the economy repays or defaults in the future. Numerical difficulty arises from the functional dependence of the pricing operator on the value functions, which necessitates solving for a joint fixed point of prices and values.

Our solution strategy is a projected value-function iteration with Chebyshev collocation over the exogenous states and a uniform grid for debt. To compute the coupled fixed point in values and prices, we employ a simultaneous one-loop iteration scheme rather than traditional nested inner-outer loops because this method is faster than the two-loop algorithm (Hatchondo et al. (2010)). At each step, we stack the bond prices and the Chebyshev coefficients of the value functions into a single comprehensive vector. We then apply Type-I Anderson acceleration with a small memory parameter and L_2 regularization directly to this joint system. This simultaneous updating approach elegantly captures the deep cross-dependencies between prices and continuation values, substantially accelerating global convergence while entirely bypassing the computational burden of inner-loop price iterations.

We approximate three value objects: default value with no issuance in period t, $\underline{V}(s)$, repayment value with no rollover, $V_r(S|\Xi = 1)$, and repayment value with rollover, $V_r(S|\Xi = 0)$. For notational convenience, we hereafter denote these as V_d, V_{nr} and V_{rr}, respectively.

We solve the model on a discretized state space that combines a uniform grid for the endogenous state of outstanding government debt B with tensor Chebyshev collocation for the exogenous block $s = (y, \omega, \beta)$. The debt grid spans $[0, 8.0]$ and contains $n_B = 1001$ equally spaced nodes. The exogenous processes for the endowment and spread shocks are truncated at ± 3.0 standard deviations about their respective means, while the preference shock is truncated at ± 3.0 standard deviations (with an explicit upper bound to ensure $\beta < 1$). We place 5 Chebyshev nodes per dimension, yielding $n_s = 5 \times 5 \times 5 = 125$ tensor nodes. For speed and robustness, we use a coarse-to-fine continuation: a preliminary solve on a $3 \times 3 \times 3$ grid ($n_s = 27$) provides a high-quality initial guess that is then lifted to the $5 \times 5 \times 5$ grid. At the fine level, this implies $n_B \times n_s = 1001 \times 125 = 125{,}125$ collocation nodes per value object.

In the rollover branch (i.e., $V_{rr}(S)$), the next-period debt choice B' is chosen within a local trust region centered on the amortized carry-over $(1-\lambda)B$. Define issuance by $B_{new} \equiv B' - (1-\lambda)B$. Instead of restricting choices to the global debt grid, we construct a local continuous interval $B_{new} \equiv B' - (1-\lambda)B$. The lower bound is fixed at $B_{new} \equiv B' - (1-\lambda)B$. To ensure the government can always feasibly roll over its maturing debt, the upper bound is state-dependent: $B_{new}^{\max} = \max(0.40, \lambda B + 0.05)$. We evaluate $B_{new}^{\max} = \max(0.40, \lambda B + 0.05)$ evenly spaced B_{new} candidates within this local interval. For each candidate \$B'\$, the expected continuation value is evaluated via linear interpolation over the global debt grid. This trust-region restriction curbs implausibly large jumps in B' and stabilizes the coupled price and value fixed point while leaving the economically relevant portion of the choice set intact.

Approximation proceeds by projection on the exogenous states and direct tabulation on the debt grid. For each of the three value objects $x \in \{d, rn, rr\}$, we write

$$V_x(S) \approx \Phi_x(B)\mathbb{T}(\boldsymbol{s}) \tag{27}$$

where $\mathbb{T}(\boldsymbol{s})$ is the tensor-product Chebyshev basis evaluated at the n_s exogenous nodes and $\Phi_x(B)$ is the corresponding $1 \times n_s$ coefficient row indexed by the debt node B. Given level evaluations $V_x(S)$ on the collocation set, coefficients are recovered by the row-wise projection $\Phi_x(B) = V_x(B, \cdot)\mathbb{T}^{-1}$, with $\mathbb{T}^{-1}$ precomputed once and reused throughout the iterations to avoid repeated inversions.

The Bellman structure compares three branches at any state (B, s). In the default branch, the economy incurs an output loss χ, consumes $\exp(y)$ net of that loss, and faces re-entry with probability ψ, so the continuation is a convex combination of next-period default and repayment-with-rollover values. Under repayment without rollover, the cash flow is $\exp(y)/4 - \lambda B - r(\omega)B$ with next-period debt $(1-\lambda)B$, and the continuation value is the maximum of default and no-rollover repayment next period. Under repayment with rollover, the government chooses B' subject to a region $[(1-\lambda)B + B_{new}^{\min}, (1-\lambda)B + B_{new}^{\max}]$ to stabilize the policy search; within that set, current consumption adds the issuance proceeds $Q(S; B')[B' - (1-\lambda)B]$ and expectations are taken over the next-period maximum of default versus no-rollover repayment at (B', s'). Expectations are computed by tensor quadrature on the Chebyshev abscissae with precomputed weights.

Bond prices satisfy a nonlinear fixed point with partial rollover. It is convenient to collect discounting and transition into the quadrature operator

$$(\mathcal{A}f)(s) \equiv \mathbb{E}_{s \to s'}\left[m(s, s')f(s')\right]$$

so that the pricing equation for the baseline case can be written as:

$$Q(S; B') = \mathcal{A}_{s \to s'}\left[(1 - \mathbb{E}[\Xi(S^{'})|s])(r(\omega') + \lambda + (1-\lambda)Q(B''(S^{'}), s'))\right]$$

And for the extended case with partial default:

$$Q(S; B') = \mathcal{A}_{s \to s'}\left[\left\{(1 - \mathbb{E}[\Xi(S^{'})|s])(r(\omega') + \lambda + (1-\lambda)Q(B''(S^{'}), s')) + \mathbb{E}[\Xi(S^{'})|s]Q^h(s')\right\}\right]$$

Let

$$\Delta_n(S') = V_{rn}(S') - V_d(s'),\ \Delta_r(S') = V_{rr}(S') - V_d(s')$$

In "safe" regions where $\Delta_n \geq 0$, rollover is certain so $\mathbb{E}[\Xi(S^{'})|s] = 0$. In "crisis" regions where $\Delta_n \leq 0 \leq \Delta_r$, repayment occurs unless a crisis arrives with reduced-form probability $\pi_{cri}(s') \in [0, 1]$, hence $\mathbb{E}[\Xi(S^{'})|s] = \pi_{cri}(s')$. In "default" regions where $\Delta_r < 0$, default is certain so $\mathbb{E}[\Xi(S^{'})|s] = 1$. Equivalently,

$$\mathbb{E}[\Xi(S^{'})|s] = \mathbf{1\{\Delta_r(S') < 0\}} + \mathbf{1\{\Delta_n \leq 0 \leq \Delta_r\}}\pi_{cri}(s')$$

The default-branch payoff Q^h itself is a linear fixed point. This is because default triggers an exogenous exclusion and probabilistic re-entry process, eliminating the government's debt-issuance choice during default and thereby removing any non-linear policy dependence. With re-entry probability ψ and an exogenous recovery parameter $\alpha(S') \in [0,1]$, the price of a defaulted bond only depends on the exogenous state s:

$$\begin{aligned} Q^h(s) &= \psi \mathcal{A}_{s\to s'}\alpha + (1-\psi)\mathcal{A}_{s\to s'}Q^h(s') \\ &= \psi \mathcal{A}(\boldsymbol{I} - (1-\psi)\mathcal{A})^{-1}\alpha \end{aligned}$$

In computation we evaluate this matrix-free via the Picard iteration

$$q^{(k+1)} = \psi \mathcal{A}\alpha + (1-\psi)\mathcal{A}q^{(k)}$$

with light damping. Finally, note that the "rollover choice" $\Xi(B,s) \in [0,1]$ used in the main text (0 = rollover, 1 = no-rollover) is absorbed here into the policy mapping $B''(B', s')$: conditional on repayment at s', the policy determines the newly issued amount, and the surviving principal $(1-\lambda)$ is repriced at the endogenously chosen B''.

Let $Q(B', s)$ be the ex-dividend price of a claim on next period's debt B'. If the economy repays at (S'), the lender receives the amortization λ and the surviving principal $(1-\lambda)$ is immediately repriced at $Q(B''(S'), s')$, where $B''(S')$ is the next issuance choice implied by the rollover policy in state (S'). If the economy defaults, the lender receives a default payoff $Q^h(s')$ (zero in the baseline without recovery). Writing $\pi_{rep}(S') = 1 -$ $\mathbb{E}[\Xi(S^{'})|s]$ for the repayment probability and taking discounting/transition via the Chebyshev quadrature operator $\mathcal{A}$, the price solves the nonlinear fixed point discussed above.

The procedure of code is as follows:

0. Bounds, grids, and precomputations

Set upper and lower bounds for every state and discretize. For the endogenous debt state B, build a uniform grid $\{B_k\}_{k=1}^{n_B} \subset [0, B_{\max}]$. For the exogenous block $s = (y, \omega, \beta)$, construct tensor Chebyshev collocation nodes from the chosen quadrature abscissae and weights. Evaluate the tensor-product Chebyshev basis $\mathbb{T}(s_j)$ at all $n_s = n_y \times n_\omega \times n_\beta$ nodes and precompute the inverse $\mathbb{T}^{-1}$ once. Also precompute the weights needed to apply the expectation/pricing operator $\mathcal{A}_{s \to s'}$. In the rollover branch, fix a trust region for issuance via

$$B' \in [(1-\lambda)B + B_{new}^{\min}, (1-\lambda)B + B_{new}^{\max}]$$

and map that interval to the nearest indices on the global B-grid for each current B.

1. Initialization of coefficients and prices

Provide initial guesses for the coefficient rows of the three value objects, $\{\Phi_d^{(0)}, \Phi_{rn}^{(0)}, \Phi_{rr}^{(0)}\}$, and for the bond-pricing surface $Q^{(0)}$ on the collocation set.

2. Using the projection identity,

$$V_x^{(n-1)}(B_k, s_j) \approx \Phi_x^{(n-1)}(B_k)\mathbb{T}(s_j),\; x \in \{d, rn, rr\}$$

recover level values $V_x^{(n-1)}$ at all collocation nodes by a row-wise multiplication with the precomputed basis $\mathbb{T}$.

3. Bellman updates at each exogenous node

For each exogenous node s and each debt grid point B:

- Default branch V_d: Current consumption applies the output loss χ. The default-branch price Q_d solves the linear fixed point $Q_d = \psi\mathcal{A}_{s\to s'}\alpha + (1-\psi)\mathcal{A}_{s\to s'}Q_d$, which is computed matrix-free by Picard iteration on $q = \psi\mathcal{A}\alpha + (1-\psi)\mathcal{A}Q_d$ with light damping. Expectations use the precomputed quadrature rule.
- Repay without rollover V_{rn}: Next debt is deterministically $B' = (1-\lambda)B$. Current consumption is $\exp(y)/4 - \lambda B - r(\omega)B$. The continuation value integrates $\max\{V_d, V_{rn}\}$ at (B', s') under the operator $\mathcal{A}_{s\to s'}$, reflecting the next-period discrete choice.

- Repay with rollover V_{rr} and issuance policy: Restrict B' to the trust region $[(1-\lambda)B + B_{new}^{\min}, (1-\lambda)B + B_{new}^{\max}]$ and evaluate that interval on the nearest B-grid subgrid. For each candidate B':

- Add issuance proceeds to current consumption: $Q(B,s) \cdot (B' - (1-\lambda)B)$.
  - The continuation again integrates $\max\{V_d, V_{rn}\}$ at (B', s') under $\mathcal{A}_{s \to s'}$.

  Select the maximizer to obtain $V_{rr}(B,s)$ and the optimal issuance $B'^{*}(B,s)$. This policy defines the "instant repricing" argument $B^{''}(B,s) = B'^{*}(B,s)$ that recurs in the price equation.

4. Assemble the RHS of the price fixed point at the current s.

With default determined by value comparisons, write the pricing right-hand side as:

$$RHS(B,s) = \mathcal{A}_{s \to s'} \begin{bmatrix} (1 - \mathbb{E}[\Xi(B^{'}, s')|s])(r'(\omega') + \lambda + (1-\lambda)Q(B''(B', s'), s')) \\ +\mathbb{E}[\Xi(B^{'}, s')|s]Q_d(B^{'}, s') \end{bmatrix}$$

where $B' = (1-\lambda)B + issuance(B,s)$ is given by the optimal rollover branch. Enforce any theoretical shape restrictions in B (e.g., non-increasingness) by a light monotonicity clip on this right-hand side. This per-s block is the workhorse update and can be threaded/parallelized across exogenous nodes.

5. Project levels back to coefficients

Obtain new coefficient rows by multiplying levels with $\mathbb{T}^{-1}$, row by row:

$$\Phi_x^{(n)}(B_k) = V_x^{(n)}(B_k, \cdot)\mathbb{T}^{-1}, x \in \{d, rn, rr\}$$

Stabilize with damping,

$$\Phi_x \leftarrow (1 - \alpha\nu)\Phi_x + \alpha\nu\Phi_x^{(n)},\ \alpha\nu \in (0,1)$$

with $\alpha\nu$ adapted to progress.

6. Light inner price relaxation

Treat the assembled right-hand side as the current target and update the price array by a few steps of successive over-relaxation (SOR) plus mild Anderson acceleration, vectorized over all (B, s):

$$q^{(k+1)} \leftarrow (1-\alpha_q)q^{(k)} + \alpha_q RHS,\; q^{(k+1)} \leftarrow AA(q^{(k+1)}, q^{(k)}),$$

where $\alpha_q \in (0,1)$ is the SOR weight, and the acceleration uses small memory and light regularization. Define the price residual $d_q = ||RHS - q||_\infty$.

7. Convergence checks and adaptive controls

Compute the maximum coefficient move:

$$d\nu = \max\left\{||\Phi_d^{(n)} - \Phi_d||_\infty, ||\Phi_{rn}^{(n)} - \Phi_{rn}||_\infty, ||\Phi_{rr}^{(n)} - \Phi_{rr}||_\infty\right\}$$

If $d_V \leq \varepsilon_V$ and $d_q \leq \varepsilon_Q$, accept and stop. Otherwise, return to Step (2).

Simulation and Moment Computation

To evaluate the empirical and quantitative properties of the solved model, we simulate the economy using the converged value objects and policy functions. The simulation generates artificial time series from which we extract unconditional macroeconomic moments, crisis-specific statistics, and the typical dynamics preceding a default episode. Because the numerical solution relies on a discrete debt grid and Chebyshev collocation over the exogenous states, we construct continuous approximations of the required functions using a hybrid approach: combining exact Chebyshev polynomial evaluations for the exogenous states with linear interpolation over the endogenous debt grid. We then simulate a long sample of $T = 1,000,000$ periods, discarding the first 10% periods as a burn-in to eliminate the influence of initial conditions.

The simulation and data extraction procedure is structured as follows:

0. Continuous approximation of value and policy objects

For any realized exogenous state $s = (y, \omega, \beta)$, we first evaluate the tensor-product Chebyshev basis polynomials $\mathbb{T}(\boldsymbol{s})$. By multiplying these evaluated bases with the converged coefficient vectors $\Phi(B_k)$, we obtain the precise evaluations of the three value functions (V_d, V_{nr}, V_{rr}) at each discrete debt node B_k. We then use one-dimensional linear interpolation over the debt grid to evaluate the functions at any continuous debt level B. We apply the same hybrid approach (Chebyshev evaluation for s, linear interpolation for B) to approximate the issuance policy $B_{new}(S)$, the bond price schedule $q(S)$, and the default probability $\pi_{cri}(s)$.

1. Initialization of the simulation

We set the initial exogenous state to the unconditional mean of the respective processes, (y_0, ω_0, β_0), and set the initial debt to zero. We draw a sequence of pseudo-random normal innovations $\{\epsilon_{y,t}, \epsilon_{\omega,t}, \epsilon_{\beta,t}\}_{t=1}^{T}$ for the exogenous processes, and sequences of uniform random shocks $\{u_t\}_{t=1}^{T} \in [0,1]$ and $\{u_t^{re}\}_{t=1}^{T} \in [0,1]$ to determine the realization of crisis events and market re-entry, respectively.

2. State transitions and Bellman evaluations

For each period t, if the economy is currently in good standing (i.e., not in default), the exogenous state $s_t = (y_t, \omega_t, \beta_t)$ evolves according to its AR(1) laws of motion, truncated at the boundaries of the state space.

Given the current state (B_{t-1}, s_t), we evaluate the interpolated value functions V_d, V_{nr} and V_{rr}, along with the crisis probability π_{cri}. Following the theoretical setup, the economy finds itself in the "crisis vulnerability region" if repayment with rollover is preferred to default, but default is preferred to repayment without rollover. A crisis materializes if the economy is in this region and the uniform shock u_t falls below π_{cri}. The crisis indicator is thus given by:

$$I_t^{crisis} = 1_{\{V_{nr} \leq V_d \leq V_{rr}\}} \cdot 1_{\{u_t < \pi_{cri}\}}$$

3. Default decision and updating rules

The government defaults if either it is strictly optimal to do so as strategic default or if an illiquidity crisis materializes. The default decision $d_t \in \{0, 1\}$ is defined as: $d_t = 1_{\{V_d > V_{rr}\}} + I_t^{crisis}$

- Default state ($d_t = 1$): Output is penalized by the fraction χ, consumption equals the penalized output, and the sovereign debt is completely erased ($B_t = 0$). The economy is excluded from financial markets. In the subsequent periods, the economy randomly re-enters good standing with probability ψ, evaluated using the uniform shock u_t^{re}.

- Repayment state ($d_t = 0$): The government issues new debt $B_{new,t}$ according to the interpolated policy function. The outstanding debt evolves deterministically via the amortized carry-over rule: $B_t = (1-\lambda)B_{t-1}+B_{new,t}$. Current consumption is calculated via the government budget constraint, adding the issuance proceeds $q \cdot B_{new,t}$ and subtracting the coupon and principal payments. We also record the annualized bond spread relative to the risk-free rate.

4. Moment computation and event extraction

After executing the main loop and discarding the burn-in period, we partition the remaining sample into specific event windows to compute relevant statistics:

- Unconditional non-default moments: We isolate periods where $d_t = 0$. Using this sub-sample, we compute standard macroeconomic moments: the mean and standard deviation of the debt-to-GDP ratio, the correlation between debt and output, and the mean, standard deviation, and auto-correlation of both issuance B_{new} and total debt changes. We also calculate the first and second moments of the sovereign spread.

- Crisis statistics: Conditioning strictly on periods where $I_t^{crisis} = 1$, we compute the distributional properties of the model-implied crisis probability π_{cri}. We evaluate its mean, standard deviation, skewness, and kurtosis, as well as its correlation with output during crisis episodes.

- Event study prior to default: To investigate the macroeconomic dynamics leading up to a default, we extract an event window of 27 quarters (corresponding empirically to the period from 2004Q1 to 2011Q2) immediately preceding each default event. By averaging across all default occurrences in the simulation, we construct the typical run-up paths for output, consumption, debt, and spreads. We additionally extract a short 1-quarter window to capture the exact debt levels and interest rates at the brink of default.

Particle Filter and Smoother

To align the model with the empirical data, we utilize the particle filtering and smoothing algorithms discussed in the main text to extract the unobservable latent state variable. The economic model analyzed in this paper can be formulated as a non-linear state-space model consisting of the following two equations:

$$x_t = f(x_{t-1}, \epsilon_t)$$

$$y_t = h(x_t, Z_t) + \eta_t$$

representing the state equation and observation equation, respectively. Here, x_t is the unobservable latent state variablem which is the preference shock, at time t, while Z_t is the vector of observable exogenous driving forces, the endowment and risk shocks. The vector y_t represents the observable endogenous data, which are sovereign spreads and the debt-to-GDP ratio. The terms ε_t and η_t represent the innovations for the state and observation equations, respectively, which are drawn from known probability distributions. The non-linear functions $f(\cdot)$ and $h(\cdot)$ are derived from the equilibrium conditions of the dynamic model.

The objective of filtering is to compute the posterior probability density of the unobservable state $p(x_t|y_{1:t}, Z_{1:t})$ conditional on the history of observations up to period t. Due to the strong non-linearities, obtaining an analytical solution is infeasible. Therefore, we use the Sequential Importance Resampling (SIR) algorithm. We approximate the posterior distribution using a set of N particles $\{x_t^{(i)}, w_t^{(i)}\}_{i=1}^N$:

$$p(x_t|y_{1:t}, Z_{1:t}) \approx \sum_{i=1}^{N} w_t^{(i)} \delta(x_t - x_t^{(i)})$$

where $\delta(\cdot)$ is the Dirac delta function, and $w_t^{(i)}$ is the normalized weight of particle i such that $\sum_{i=1}^N w_t^{(i)} = 1$.

In each period t, the algorithm proceeds in three steps:

0. Prediction: Propagate the previous particles $x_{t-1}^{(i)}$ forward through the state equation $f(\cdot)$ to generate a sample from the prior distribution, $x_t^{(i)}$.

1. Update: Upon observing y_t and Z_t, update the weights using the likelihood from the observation equation: $\widetilde{w}_t^{(i)} = w_{t-1}^{(i)} p(y_t|x_t^{(i)}, Z_t)$. The weights are then normalized to

obtain $w_t^{(i)}$.

2. Resampling: To prevent the degeneracy problem, we resample the particles with replacement. Particles are drawn with probabilities proportional to $w_t^{(i)}$ to generate a new set of equally weighted particles ($\{x_t^{(i)}\}_{i=1}^N$ with $w_t^{(i)} = 1/N$)

While the filter provides the density conditional on concurrent and past data, accurately reconstructing the historical trajectory of the preference shock requires evaluating the smoothed distribution $p(x_t|y_{1:T}, Z_{1:T})$, which is conditional on the full sample of information. Following the literature on Monte Carlo smoothers (e.g., Kitagawa, 1996), we perform smoothing by tracing the genealogy of the particles generated during the filtering stage. Starting from the final posterior at time T, the smoothed weights $w_{t|T}^{(i)}$ are computed recursively backward in time. Finally, the expected value of the smoothed distribution,

$$\mathbb{E}[x_t|y_{1:T}, Z_{1:T}] \approx \sum_{i=1}^{N} w_{t|T}^{(i)} x_t^{(i)}$$

is extracted and used as the realized path of the latent preference shock for the simulations presented in the main text.

Appendix E: Data

Macroeconomic Aggregates:

- **Endowment (real GDP)**: Real gross domestic product, millions of chained 2010 euros, seasonally adjusted, quarterly. Source: Eurostat (retrieved via FRED, Federal Reserve Bank of St. Louis).
- **Consumption:** Real private final consumption expenditure for Greece (GDP by expenditure), constant prices (euros), seasonally adjusted, quarterly. Source: Organization for Economic Co-operation and Development (retrieved via FRED, Federal Reserve Bank of St. Louis).
- **Inflation**: GDP implicit price deflator, seasonally adjusted, quarterly. Source: OECD.

Fiscal and Debt Indicators:

- **Debt (Level)**: General government consolidated gross debt, millions of national currency, quarterly. Source: Eurostat.
- **Debt-to-GDP ratio**: General government consolidated gross debt as a percentage of GDP.

  - Greece: Quarterly data. Source: Eurostat.

  - World: Source: International Monetary Fund (IMF) World Economic Outlook (WEO) database.

- **Public deficit**: General government deficit (net borrowing/lending) from quarterly non-financial accounts, seasonally adjusted. Source: Eurostat.
- **Debt maturity**: Average residual maturity for total government debt securities (non-consolidated outstanding amounts, in years). Source: European Central Bank (ECB) Data Portal.

Interest Rates and Spreads:

- **3-Month spot rates:**

  - Greece: 13-week Treasury bill yields. Source: Greek Ministry of Economy and Finance.

  - Benchmark: 3-month yield curve spot rate for AAA-rated euro area central government bonds (changing composition). Source: ECB Data Portal.

- **Yield curve parameters**: All estimated parameters of the Nelson-Siegel-Svensson yield curve model for AAA-rated euro area sovereign bonds (changing composition). Source: ECB Data Portal.

- **Sovereign CDS spreads**: 5-year sovereign credit default swap (CDS) spreads for Greece. Source: Bloomberg L.P.

Historical Default Data:

- Default frequency: Historical frequency of sovereign defaults by the Greek government on external creditors since independence. Source: Reinhart and Trebesch (2015), "The Pitfalls of External Dependence: Greece, 1829–2015".