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Free College and Economic Growth: A Paradox of Innovation Productivity

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Abstract

This paper investigates the macroeconomic effects of tuition subsidies in an overlapping-generations model with endogenous growth and innovation. Calibrated to the Japanese economy, the model explores the "growth puzzle" where expanded educational attainment often yields modest aggregate productivity gains. We identify a "paradox of innovation productivity": while subsidies can achieve a Pareto improvement in low-innovation environments, highly productive innovation may cause technological progress to outpace capital accumulation. This dynamic destabilizes the tax base by eroding the capital-effective labor ratio, rendering aggressive subsidies fiscally infeasible in equilibrium. Our findings suggest that the feasibility of free college policies depends critically on the economy's innovation productivity and its resulting impact on the dynamic interaction between technological progress and the fiscal foundation.

JEL classification: E13, O41, I22, H52
Keywords: higher education, tuition subsidies, innovation, economic growth, fiscal feasibility, overlapping generations

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1 Introduction

Policies that expand access to higher education are often justified by their potential to promote innovation and long-run economic growth. In many countries, governments have introduced or considered policies that reduce or eliminate college tuition in order to increase the supply of highly educated workers and stimulate research activity. The underlying idea is straightforward: a larger higher education sector should generate more innovation and thereby enhance economic performance. However, the macroeconomic consequences of such policies are not necessarily straightforward. Education policies interact with capital accumulation, factor prices, and fiscal capacity, potentially generating unintended general equilibrium effects.

This paper studies the macroeconomic effects of tuition subsidies in a dynamic general equilibrium model with higher education and endogenous technological progress. The model is an overlapping-generations economy in which higher education institutions play a dual role: they produce specialized labor used in final goods production and generate technological progress through research activities. Tuition subsidies initially expand the higher education sector and increase research inputs, thereby accelerating technological progress. Over time, however, these changes affect factor prices and the distribution of income across generations through general equilibrium adjustments, with important implications for capital accumulation and fiscal capacity.

The central mechanism of the model is that technological progress may outpace capital accumulation. As innovation increases labor productivity and reduces the relative scarcity of labor and specialized skills, the income growth of the young generation—the tax base used to finance tuition subsidies—may become insufficient to sustain large education subsidies. As a result, policies intended to promote higher education and innovation may weaken their own fiscal foundations and become infeasible in equilibrium. This mechanism highlights a fundamental trade-off between innovation-driven growth and fiscal sustainability.

The analysis yields three main results. First, in the absence of government intervention, the competitive equilibrium features inefficiently low investment in higher education due to uninternalized technological externalities. Second, when the productivity of innovation is sufficiently low, tuition subsidies accelerate technological progress and improve the welfare of all generations. The welfare gains are larger for later-born generations, reflecting the cumulative effects of faster technological progress. For generations born after the policy is introduced, the magnitude of the welfare gains increases with the subsidy rate, although this monotonic relationship does not necessarily hold for the generation that is already old at the time of implementation. Third, when the productivity of innovation is sufficiently high, technological progress may outpace capital accumulation, weakening the tax base and rendering tuition subsidies fiscally infeasible.

The model is calibrated to match key features of the Japanese economy. This calibration is particularly relevant given Japan's demographic trends and ongoing policy discussions on higher education financing, making it a natural setting to evaluate the interaction between education policy, economic growth, and fiscal sustainability.

This paper is related to the literature that studies the macroeconomic effects of education policies in dynamic general equilibrium models. An early contribution is Glomm and Ravikumar (1992), who analyze public investment in education in an overlapping-generations framework, focusing on how these institutions affect long-run human capital accumulation and income inequality. More closely related to the present study is Gerson (1994), who analyzes tuition subsidies in a growth model based on the Ramsey-Cass-Koopmans framework with endogenous technological change. He shows that policies aimed at expanding higher education actually curb capital accumulation and, paradoxically, lead to its contraction, because higher equilibrium tuition discourages household saving. A similar contraction of the higher education sector can also arise in the present model under certain parameter configurations. In both Gerson's model and the present framework, tuition subsidies may reduce the capital-labor ratio in the broad sense, but the mechanism differs. In Gerson's model, the decline arises from higher tuition and lower saving, whereas in the present model it emerges because technological progress accelerates relative to capital accumulation.

These theoretical possibilities of policy-induced contraction or stagnation align with broader empirical and structural concerns. For instance, Pritchett (2001) documents a "growth puzzle" where massive expansions in educational attainment often fail to translate into higher aggregate productivity, suggesting that the link between schooling and growth is mediated by complex general equilibrium interactions. Furthermore, if subsidies draw talent away from direct production into less productive academic activities, the net effect on innovation can be negative, as discussed in the literature on talent misallocation (Murphy et al., 1991; Hsieh et al., 2019).

Recent studies highlight additional mechanisms through which education policies interact with long-run growth and welfare. Morimoto and Tabata (2020) show that while higher education subsidies can enhance the entry of new firms, they may ultimately reduce long-run growth by diminishing each firm's incentives for process innovation. Okada (2023) further investigates the role of public education expenditure in an OLG model of R&D-based growth, demonstrating that such expenditure has an inverted U-shaped effect on both the steady-state growth rate and social welfare. This suggests that while a certain level of government intervention is growth-enhancing, excessive spending can eventually suppress growth and welfare by reducing households' savings and R&D investment.

While these studies identify various structural and fiscal channels through which education policies may fail to promote growth, the present paper highlights a different and more fundamental constraint: the race between technological progress and capital accumulation. In our model, even when tuition subsidies accelerate technological progress, the resulting decline in the capital-effective labor ratio can destabilize the tax base, rendering the policy fiscally infeasible in equilibrium.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes the equilibrium. Section 4 analyzes the effects of tuition subsidies on economic growth and fiscal feasibility. Section 5 concludes.

2 Model

Time is discrete and the economy starts operation in period 1.

2.1 Overlapping Generations

At the beginning of a generic period t (≥ 1), a continuum of young agents of measure N_t are born. They are two-period-lived and have the following lifetime utility function:

$$U_t = \log c_t^y + \beta \log c_{t+1}^o, \tag{1}$$

where c^y and c_{t+1}^o, respectively, denote the amounts of consumption in their young and old ages, and β is a constant satisfying $\beta \in (0, 1)$.

When young, the agents choose their academic background between a high-school graduate or a college graduate. To become a college graduate, the agents must pay p_t units of the final good as tuition, but they do not have to pay anything to become a high-school graduate. If they choose to become a high-school graduate, they supply A_t units of general labor in that period and earn $w_t^0 A_t$ units of income in return, where w_t^0 is the price of one unit of general labor. If they choose to become a college graduate, they supply A_t units of specialized labor in that period and earn $w_t^1 A_t$ units of income in return, where w_t^1 is the price of one unit of specialized labor. Since they paid tuition, their net income for this period is $w_t^1 A_t - p_t$. Naturally, young agents try to choose an academic background that offers them more income. As will be assumed in the next subsection, the general labor services supplied by high-school graduates and the specialized labor supplied by college graduates are both essential to the production of the final good, and thus their arbitrage behavior equalizes the incomes of high-school graduates and college graduates as follows:

$$w_t^0 A_t = w_t^1 A_t - p_t,$$

or

$$w_t^0 = w_t^1 - \tilde{p}_t, \tag{2}$$

where $\tilde{p}_t = p_t/A_t$.

Eq.(2) implies that, regardless of their choice of academic background, the budget constraint of the agents born in period t can be expressed as

$$c_t^y + s_t = w_t^0 A_t, \qquad c_{t+1}^o = r_{t+1} s_t, \tag{3}$$

where s_t and r_{t+1}, respectively, denote the amount of savings in the young age and the rental rate of capital in period $t+1$. Savings take the form of capital formation, and capital is completely depleted in one period.

Maximizing (1) subject to (3) yields

$$c_t^y = w_t^0 A_t/(1+\beta) \tag{4}$$
$$c_t^o = \beta r_{t+1} w_t^0 A_t/(1+\beta) \tag{5}$$
$$s_t = \beta w_t^0 A_t/(1+\beta). \tag{6}$$

Let K_{t+1} be the total supply of capital in period $t+1$. Under the above assumptions about capital formation, the following equation holds:

$$K_{t+1} = s_t N_t = \beta w_t^0 A_t N_t/(1+\beta). \tag{7}$$

In period 1, there are initial old of measure N_0, each of whom is endowed with K_1/N_0 units of capital. They supply this capital inelastically and receive $r_1 K_1/N_0$ units of final goods as compensation. They consume this good and die at the end of that period. The population of each generation changes over time according to the following rules:

$$\forall t \geq 0, \qquad N_{t+1} = bN_t, \tag{8}$$

where b is a positive constant.

2.2 Production Sector

In every period, the final good is competitively produced using a Cobb-Douglas production technology from capital, general labor, and specialized labor. Specifically, the production function is formulated as follows:

$$Y_t = \Phi^P (K_t^P)^{1-\alpha_0-\alpha_1} (L_t)^{\alpha_0} (E_t^P)^{\alpha_1}, \tag{9}$$

where Y_t, K_t^P, L_t and E_t^P, respectively, denote the total output of the final good in period t, and the total amounts of capital, general labor, and specialized labor that are put in the production of the final good, with Φ^P, α_0 and α_1 being positive constants satisfying $\alpha_0 + \alpha_1 < 1$.

The factor demand of this sector in period t must satisfy

$$\begin{aligned} r_t &= \Phi^P(1-\alpha_0-\alpha_1)(K_t^P)^{-\alpha_0-\alpha_1}(L_t)^{\alpha_0}(E_t^P)^{\alpha_1} \\ w_t^0 &= \Phi^P\alpha_0(K_t^P)^{1-\alpha_0-\alpha_1}(L_t)^{\alpha_0-1}(E_t^P)^{\alpha_1} \\ w_t^1 &= \Phi^P\alpha_1(K_t^P)^{1-\alpha_0-\alpha_1}(L_t)^{\alpha_0}(E_t^P)^{\alpha_1-1}, \end{aligned}$$

which can be rewritten as

$$\frac{w_t^0}{r_t} = \frac{\alpha_0}{1-\alpha_0-\alpha_1}\cdot\frac{K_t^P}{L_t} \tag{10}$$

$$\frac{w_t^1}{r_t} = \frac{\alpha_1}{1-\alpha_0-\alpha_1}\cdot\frac{K_t^P}{E_t^P} \tag{11}$$

$$\Phi^P = \left(\frac{r_t}{1-\alpha_0-\alpha_1}\right)^{1-\alpha_0-\alpha_1}\left(\frac{w_t^0}{\alpha_0}\right)^{\alpha_0}\left(\frac{w_t^1}{\alpha_1}\right)^{\alpha_1}. \tag{12}$$

2.3 Higher Education Sector

In the model, higher education institutions play two distinct roles: they produce specialized labor and generate technological progress through research activities.

Specialized labor is competitively produced in the higher education sector through the following technology:

$$E_t = \Phi^E(K_t^E)^{1-\gamma}(E_t^E)^{\gamma}, \tag{13}$$

where E_t, K_t^E and E_t^E, respectively, denote the total supply of specialized labor in period t, and the total amounts of capital and specialized labor that are put in the production of specialized labor, with Φ^E and γ being positive constants satisfying $\gamma < 1$.

Notations K_t^P, K_t^E, E_t, E_t^P, and E_t^E satisfy the following relations:

$$K_t = K_t^P + K_t^E \tag{14}$$

$$E_t = E_t^P + E_t^E. \tag{15}$$

Because a high-school graduate supplies A_t units of general labor, and because a college graduate supplies A_t units of specialized labor, L_t, E_t^P, and E_t^E satisfy

$$L_t = A_t N_t^0, \qquad E_t^P = A_t N_t^{1P}, \qquad E_t^E = A_t N_t^{1E}, \tag{16}$$

where N_t^0, N_t^{1P}, and N_t^{1E}, respectively, represent the populations of the agents born in period t who are high-school graduates, college graduates working in the production sector, and college graduates working in the higher education sector. Hence, if we define N_t^1 as $N_t^1 \equiv N_t^{1P} + N_t^{1E}$, then it represents the population of college-educates in this generation. Since $N_t^0 + N_t^{1P} + N_t^{1E} = N_t$, eq.(16) implies that

$$L_t + E_t^P + E_t^E = A_t N_t. \tag{17}$$

For analytical convenience, define n_t^0, n_t^1, n_t^{1P} and n_t^{1E} as

$$\begin{aligned} n_t^0 \equiv N_t^0/N_t, \qquad n_t^1 \equiv N_t^1/N_t \\ n_t^{1P} \equiv N_t^{1P}/N_t, \quad n_t^{1E} \equiv N_t^{1E}/N_t. \end{aligned} \tag{18}$$

Obviously, n_t^0, n_t^1, n_t^{1P} and n_t^{1E}, respectively, represent the percentages of the agents born in period t who are high-school graduates, college graduates, college graduates working in the production sector, and college graduates working in the higher education sector. They satisfy

$$n_t^0 + n_t^1 = n_t^0 + n_t^{1P} + n_t^{1E} = 1. \tag{19}$$

Moreover, eqs.(15)(16) and (18) jointly imply that

$$\begin{aligned} n_t^0 = L_t/A_tN_t, \qquad n_t^1 = E_t/A_tN_t \\ n_t^{1P} = E_t^P/A_tN_t, \quad n_t^{1E} = E_t^E/A_tN_t. \end{aligned} \tag{20}$$

Competitive production of specialized labor establishes the following equations:

$$\begin{aligned} r_t &= \tilde{p}_t \Phi^E (1-\gamma)(K_t^E)^{-\gamma}(E_t^E)^{\gamma} \\ w_t^1 &= \tilde{p}_t \Phi^E \gamma (K_t^E)^{1-\gamma}(E_t^E)^{\gamma-1}, \end{aligned}$$

which can be rewritten as

$$\frac{w_t^1}{r_t} = \frac{\gamma}{1-\gamma} \cdot \frac{K_t^E}{E_t^E} \tag{21}$$

$$\Phi^E \tilde{p}_t = \left(\frac{r_t}{1-\gamma}\right)^{1-\gamma} \left(\frac{w_t^1}{\gamma}\right)^{\gamma}. \tag{22}$$

This sector also improves the productivity of young agents. Specifically, it increases the value of A_t as follows:

$$A_{t+1} - (1-\delta)A_t = \Phi^I (K_t^E/N_t)^{1-\phi_1-\phi_2}(E_t^E/N_t)^{\phi_1}(A_t)^{\phi_2}, \tag{23}$$

where Φ^I, ϕ_1, ϕ_2 and δ are positive constants satisfying $\phi_1 + \phi_2 < 1$ and $\delta \in (0,1)$. This improvement is assumed to be a byproduct of the activities of the higher education institution. In this economy, innovation is the result of off-the-job activities performed by members of the higher education institutions using their facilities, and these institutions do not consider this activity when determining the amount of capital and specialized labor employed. It is also assumed that the results of this improvement are available to everyone for free. In other words, productivity improvements obtained through innovation are pure public goods. Let us also note two characteristics of (23). First, the inputs to innovation are capital and specialized labor normalized by the total population. This assumption is adopted so that larger economies do not necessarily grow faster than smaller economies. Second, a δ fraction of production knowledge is lost after one period. This assumption is made based on the observations that there is usually a competitive relationship between production technologies, and that technologies that lose the competition are no longer used and will eventually be forgotten.

3 Equilibrium

3.1 Temporary Equilibrium

In this subsection, we will see that in period t, the values of all endogenous variables are uniquely determined by k_t, through establishing the following facts: First, given a value of q_t ($\equiv w_t^1/r_t$), the values of some endogenous variables are determined by it; Second, given the values of k_t and q_t, the values of the remaining endogenous variables are also determined by them; Third, the value of q_t is uniquely determined by k_t.

Using (2) to eliminate $\tilde{p}_t$ from (22), we can derive

$$\frac{w_t^0}{r_t} = q_t \left(1 - \frac{1}{\Gamma \Phi^E q_t^{1-\gamma}}\right), \tag{24}$$

where $\Gamma \equiv (1-\gamma)^{1-\gamma}\gamma^\gamma$. Substituting (24) into (12) produces

$$r_t = \Lambda \Phi^P \left(1 - \frac{1}{\Gamma \Phi^E q_t^{1-\gamma}}\right)^{-\alpha_0} q_t^{-\alpha_0-\alpha_1}, \tag{25}$$

where $\Lambda \equiv (\alpha_0)^{\alpha_0}(\alpha_1)^{\alpha_1}(1-\alpha_0-\alpha_1)^{1-\alpha_0-\alpha_1}$. Eqs.(24)(25) and $q_t = w_t^1/r_t$ jointly yield

$$w_t^0 = \Lambda \Phi^P \left(1 - \frac{1}{\Gamma \Phi^E q_t^{1-\gamma}}\right)^{1-\alpha_0} q_t^{1-\alpha_0-\alpha_1} \tag{26}$$

$$w_t^1 = \Lambda \Phi^P \left(1 - \frac{1}{\Gamma \Phi^E q_t^{1-\gamma}}\right)^{-\alpha_0} q_t^{1-\alpha_0-\alpha_1}, \tag{27}$$

which, combined with (2), produce

$$\tilde{p}_t = \frac{\Lambda \Phi^P}{\Gamma \Phi^E} \left(1 - \frac{1}{\Gamma \Phi^E q_t^{1-\gamma}}\right)^{-\alpha_0} q_t^{\gamma-\alpha_0-\alpha_1}. \tag{28}$$

Substituting (26) into (7), we can also obtain

$$a_{t+1}k_{t+1} = \frac{\beta \Lambda \Phi^P}{b(1+\beta)} \left(1 - \frac{1}{\Gamma \Phi^E q_t^{1-\gamma}}\right)^{1-\alpha_0} q_t^{1-\alpha_0-\alpha_1}, \tag{29}$$

where $a_{t+1} \equiv A_{t+1}/A_t$ and $k_{t+1} \equiv K_{t+1}/A_{t+1}N_{t+1}$. Eqs.(25)-(29) establish the fact that the values of r_t, w_t^0, w_t^1, $\tilde{p}_t$ and $a_{t+1}k_{t+1}$ are uniquely determined by q_t, although these equations make sense only when q_t satisfies

$$\Gamma \Phi^E q_t^{1-\gamma} - 1 > 0. \tag{30}$$

Our next task is to show that the remaining endogenous variables, specifically n_t^1, n_t^{1P}, n_t^{1E}, k_t^P ($\equiv K_t^P/A_tN_t$), k_t^E ($\equiv K_t^E/A_tN_t$), a_{t+1}, and k_{t+1}, are uniquely determined by k_t and q_t. To this end, let us first consider how E_t^P, E_t^E, K_t^P, and K_t^E are determined given K_t and E_t. Using (11)(15)(21) and $q_t = w_t^1/r_t$, we can rewrite (14) as

$$\begin{aligned}
K_t &= K_t^P + K_t^E \\
&= \frac{1-\alpha_0-\alpha_1}{\alpha_1} q_t E_t^P + \frac{1-\gamma}{\gamma} q_t E_t^E \\
&= q_t \left[\frac{1-\alpha_0-\alpha_1}{\alpha_1} E_t^P + \frac{1-\gamma}{\gamma}(E_t - E_t^P)\right] \\
&= q_t \left[\frac{\gamma(1-\alpha_0)-\alpha_1}{\gamma\alpha_1} E_t^P + \frac{1-\gamma}{\gamma} E_t\right].
\end{aligned}$$

By solving this equation with respect to E_t^P, we obtain

$$E_t^P = \frac{\gamma\alpha_1}{\gamma(1-\alpha_0)-\alpha_1}\left(\frac{K_t}{q_t} - \frac{1-\gamma}{\gamma}E_t\right). \tag{31}$$

Combined with (11)(15) and (21), eq.(31) implies that

$$E_t^E = E_t - E_t^P = \frac{\gamma(1-\alpha_0-\alpha_1)}{\gamma(1-\alpha_0)-\alpha_1}\left(E_t - \frac{\alpha_1}{1-\alpha_0-\alpha_1}\frac{K_t}{q_t}\right) \tag{32}$$

$$K_t^P = \frac{1-\alpha_0-\alpha_1}{\alpha_1} q_t E_t^P = \frac{\gamma(1-\alpha_0-\alpha_1)}{\gamma(1-\alpha_0)-\alpha_1} q_t \left(\frac{K_t}{q_t} - \frac{1-\gamma}{\gamma}E_t\right) \tag{33}$$

$$K_t^E = \frac{1-\gamma}{\gamma} q_t E_t^E = \frac{(1-\gamma)(1-\alpha_0-\alpha_1)}{\gamma(1-\alpha_0)-\alpha_1} q_t \left(E_t - \frac{\alpha_1}{1-\alpha_0-\alpha_1}\frac{K_t}{q_t}\right). \tag{34}$$

So far, we have taken E_t as a given, but in fact, it is an endogenous variable determined by (13). Then, substituting (32) and (34) into (13) produces

$$E_t = \frac{(1-\alpha_0-\alpha_1)\Gamma\Phi^E q_t^{1-\gamma}}{\gamma(1-\alpha_0)-\alpha_1}\left(E_t - \frac{\alpha_1}{1-\alpha_0-\alpha_1}\frac{K_t}{q_t}\right).$$

By solving this equation with respect to E_t, we obtain

$$E_t = \frac{\alpha_1 \Gamma \Phi^E q_t^{-\gamma} K_t}{(1-\alpha_0-\alpha_1)(\Gamma \Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1}. \tag{35}$$

Substituting (35) into (31)-(34) yields

$$E_t^P = \frac{\alpha_1(\Gamma \Phi^E q_t^{1-\gamma} - \gamma)}{(1-\alpha_0-\alpha_1)(\Gamma \Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} \cdot \frac{K_t}{q_t} \tag{36}$$

$$E_t^E = \frac{\gamma \alpha_1}{(1-\alpha_0-\alpha_1)(\Gamma \Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} \cdot \frac{K_t}{q_t} \tag{37}$$

$$K_t^P = \frac{(1-\alpha_0-\alpha_1)(\Gamma \Phi^E q_t^{1-\gamma} - \gamma)}{(1-\alpha_0-\alpha_1)(\Gamma \Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} K_t \tag{38}$$

$$K_t^E = \frac{(1-\gamma)\alpha_1}{(1-\alpha_0-\alpha_1)(\Gamma \Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} K_t. \tag{39}$$

Then, dividing both hands of (35)-(39) by $A_t N_t$, we obtain

$$n_t^1 = \frac{\alpha_1 \Gamma \Phi^E q_t^{1-\gamma}}{(1-\alpha_0-\alpha_1)(\Gamma \Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} \cdot \frac{k_t}{q_t} \tag{40}$$

$$n_t^{1P} = \frac{\alpha_1(\Gamma \Phi^E q_t^{1-\gamma} - \gamma)}{(1-\alpha_0-\alpha_1)(\Gamma \Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} \cdot \frac{k_t}{q_t} \tag{41}$$

$$n_t^{1E} = \frac{\gamma \alpha_1}{(1-\alpha_0-\alpha_1)(\Gamma \Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} \cdot \frac{k_t}{q_t} \tag{42}$$

$$k_t^P = \frac{(1-\alpha_0-\alpha_1)(\Gamma \Phi^E q_t^{1-\gamma} - \gamma)}{(1-\alpha_0-\alpha_1)(\Gamma \Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} k_t \tag{43}$$

$$k_t^E = \frac{(1-\gamma)\alpha_1}{(1-\alpha_0-\alpha_1)(\Gamma \Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} k_t. \tag{44}$$

Eqs.(40)-(44) establish the fact that the values of n_t^1, n_t^{1P}, n_t^{1E}, k_t^P and k_t^E are uniquely determined by q_t and k_t, although these equations make sense only when q_t satisfies

$$\Gamma \Phi^E q_t^{1-\gamma} - \gamma > 0. \tag{45}$$

Since k_t^E and n_t^{1E} are uniquely determined by q_t and k_t, the value of a_{t+1} is also uniquely determined by them, since eq.(23) can be rewritten as

$$a_{t+1} = 1 - \delta + \Phi^I (k_t^E)^{1-\phi_1-\phi_2} (n_t^{1E})^{\phi_1}. \tag{46}$$

This result further implies that the value of k_{t+1} is also uniquely determined by q_t and k_t, since the value of $a_{t+1} k_{t+1}$ is uniquely determined by q_t as shown in (29).

Our final task is to establish the fact that the value of q_t is uniquely determined by k_t. From (10)(24) and (43), we can derive

$$\begin{aligned} n_t^0 &= \frac{\alpha_0}{1-\alpha_0-\alpha_1} \cdot \frac{k_t^P}{w_t^0/r_t} \\ &= \frac{\alpha_0 \Gamma \Phi^E q_t^{-\gamma} k_t}{(1-\alpha_0-\alpha_1)(\Gamma \Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} \\ &\quad \times \left(1 + \frac{1-\gamma}{\Gamma \Phi^E q_t^{1-\gamma} - 1}\right). \end{aligned} \tag{47}$$

Plugging (40) and (47) into $n_t^0 + n_t^1 = 1$, we obtain

$$\begin{aligned} &\frac{\alpha_1 \Gamma \Phi^E q_t^{-\gamma} k_t}{(1-\alpha_0-\alpha_1)(\Gamma \Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} \\ &\times \left[1 + \frac{\alpha_0}{\alpha_1}\left(1 + \frac{1-\gamma}{\Gamma \Phi^E q_t^{1-\gamma} - 1}\right)\right] = 1. \end{aligned} \tag{48}$$

This equation can be interpreted as the equality of specialized labor demand and supply in period t. Specifically, the LHS of (48) is the sum of job openings for high school and college graduates in period t, which is obtained by summing up the RHSs of (40) and (47). On the other hand, the RHS of (48) is the total number of job seekers in period t, which equals the population of the generation born in that period, i.e., 1. Our interest is whether there exists a value of q_t that would equalize the specialized labor demand and supply. The next lemma answers this question in the affirmative.

Lemma 1. *For $\forall k_t > 0$, there is a unique value of q_t that satisfies (30)(45) and (48).*

Proof. Whenever condition (30) is true, condition (45) is also true, which can be verified directly from their definitions. Thus in the following, we only need to consider whether condition (30) is true or not. Eq.(48) can be rewritten as

$$\begin{aligned} \frac{1}{k_t} &= \frac{\alpha_1 \Gamma \Phi^E q_t^{-\gamma}}{(1-\alpha_0-\alpha_1)(\Gamma \Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} \\ &\quad \times \left[1 + \frac{\alpha_0}{\alpha_1}\left(1 + \frac{1-\gamma}{\Gamma \Phi^E q_t^{1-\gamma} - 1}\right)\right]. \end{aligned} \tag{49}$$

Let the RHS of (49) be $f(q_t)$. Also define $\underline{q}_1$ as $\underline{q}_1 \equiv (\Gamma \Phi^E)^{-1/(1-\gamma)}$. It is easy to verify that $q_t > \underline{q}_1$ is equivalent to (30). On $(\underline{q}_1, +\infty)$, $f(q_t)$ is a continuously decreasing function of q_t, which can be verified by differentiation, with

$$\lim_{q_t \to \underline{q}_1 + 0} f(q_t) = +\infty, \qquad \lim_{q_t \to +\infty} f(q_t) = 0.$$

These properties jointly mean that the graphs of LHS and RHS of (49) intersect only once on $(\underline{q}_1, +\infty)$, implying that the value of q_t is uniquely determined by k_t, and that it satisfies (30) (i.e., $q_t > \underline{q}_1$). □

Then we can state

Proposition 1. *In a generic period t (≥ 1), the values of q_t, r_t, w_t^0, w_t^1, $\tilde{p}_t$, a_{t+1}, k_{t+1}, k_t^P, k_t^E, n_t^0, n_t^1, n_t^{1P} and n_t^{1E} are uniquely determined by k_t.*

Proof. As Lemma 1 shows, the value of q_t is uniquely determined by k_t. Also, as we saw above, once q_t and k_t are given, the values of the remaining endogenous variables are uniquely determined. □

3.2 Intertemporal Equilibrium

Now we are in a position to define the equilibrium path of this economy.

Definition 1. *An equilibrium path of this economy is a sequence*

$$\{(q_t, r_t, w_t^0, w_t^1, \tilde{p}_t, a_{t+1}, k_{t+1}, k_t^P, k_t^E, n_t^0, n_t^1, n_t^{1P}, n_t^{1E})\}_{t=1}^{\infty}$$

generated by (25)-(30) and (40)-(48), given $k_1 > 0$.

Then we can state

Proposition 2. *Given $k_1 > 0$, there is a unique equilibrium path in this economy.*

Proof. We have already seen that, in a generic period t, k_t uniquely determines the value of q_t through (48), and then that k_t and q_t uniquely determine the values of r_t, w_t^0, w_t^1, $\tilde{p}_t$, a_{t+1}, k_{t+1}, k_t^P, k_t^E, n_t^0, n_t^1, n_t^{1P} and n_t^{1E} through (25)-(29) and (40)-(46). These results imply that the equilibrium path of this economy is uniquely determined, once the value of k_1 is given. □

This proposition establishes the existence and uniqueness of the equilibrium path, but does not fully characterize its dynamics. In fact, a full analytical characterization is difficult, mainly because equation (48) cannot be explicitly solved for q_t. We therefore study the dynamic behavior of the model using numerical simulations.

Table 1 summarizes the parameter values used in the numerical analysis, all of which are determined by calibrating the model to the Japanese economy.[1] Setting the parameters to these values ensures that the equilibrium converges to a unique BGP, regardless of the initial conditions.

Figure 1 illustrates the equlibrium paths of k_t for several values of k_1, specifically, $10^{-3}\bar{k}$, $10^{-2}\bar{k}$, $10^{-1}\bar{k}$, $\bar{k}$, $10\bar{k}$, $10^2\bar{k}$ and $10^3\bar{k}$, where $\bar{k} \approx 97.5289$,

[1]For details on how to determine parameter values, see Appendices A and B.

Table 1: Baseline Parameter Values for the Numerical Analysis

symbol	value	description
β	0.8521	discount factor (40 years)
b	0.75	gross rate of population growth (40 years)
Φ^P	138.8168	productivity of the production sector
α_0	0.3103	share of general labor in production
α_1	0.3564	share of specialty in production
Φ^E	0.8196	productivity of the higher education sector
γ	0.54	share of specialty in the production of specialty
Φ^I	1.6458	productivity of innovation
ϕ_1	1/3	share of specialty in innovation
ϕ_2	1/3	share of existing knowledge in innovation
δ	0.9	depreciation rate of knowledge (40 years)

which is the value of k_t on BGP. As that figure shows, k_t converges to $\bar{k}$ in the long run, regardless of the initial value.[2] According to Lemma 1, q_t is uniquely determined when k_t is determined, so when k_t converges to a certain value, q_t also converges to a certain value. Furthermore, given the values of k_t and q_t, the values of the other endogenous variables (i.e., a_t, r_t, w_t^0, w_t^1, $\tilde{p}_t$, k_t^P, k_t^E, n_t^0, n_t^1, n_t^{1P} and n_t^{1E}) are also uniquely determined, so they also converge to a certain value. Therefore, as long as the values of the parameters are given as in Table 1, it is safe to say that the model economy will converge to the same BGP in the long run.

4 Growth Effects of College Tuition Reductions

In the model considered here, technological progress is realized as a byproduct of higher education. The outcome of technological progress is a public good available to all free of charge, and thus higher education institutions do not take into account the effect of their input choices on technological progress. This creates an externality, which tends to result in a lower level of research activity than is socially optimal. A natural policy response to this inefficiency is to reduce or eliminate college tuition in order to promote the expansion of higher education. However, as we show below, the effects of such a policy are not straightforward. While tuition subsidies may initially expand the higher education sector and accelerate technological progress, their impact on equilibrium dynamics can generate unintended consequences. In particular, the general equilibrium effects of faster technological progress may alter income distribution and savings behavior, thereby affecting the sustainability of such policies.

[2] Note that $\log_{10} \bar{k} \approx 1.9891$, and that when $k_1 = \bar{k}$, $\log_{10} k_t = \log_{10} \bar{k}$ for $\forall t \geq 2$.

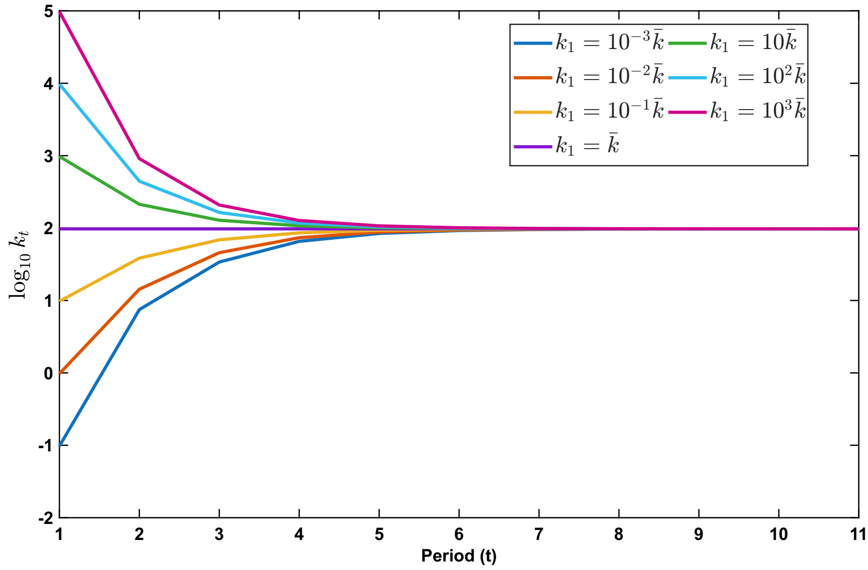


Figure 1: Convergence to the same BGP

4.1 Equilibrium Conditions

In this subsection, we derive the equilibrium conditions of the economy under a tuition reduction policy.

Consider a policy such that, in each period, the government subsidizes θ ($\in (0,1]$) percentage of college tuition, and that the funds are raised through a lump-sum tax on the young agents in that period.[3] When this policy is implemented, the government's budget constraints for period t would be

$$\tau_t N_t = \theta p_t N_t^1, \tag{50}$$

where τ_t is the lump-sum tax imposed on each agent born in that period. For analytical convenience, we divide both hands of (50) by $A_t N_t$ to obtain

$$\tilde{\tau}_t = \theta \tilde{p}_t n_t^1, \tag{51}$$

where $\tilde{\tau}_t \equiv \tau_t / A_t$.

This policy changes the levels of disposal incomes of high-school graduates and college graduates as $w_t^0 A_t - \tau_t$ and $w_t^1 A_t - \tau_t - (1-\theta) p_t$, respectively. In equilibrium, being a high-school graduate and being a college graduate must be indifferent for young agents in period t, so the following equation

[3]This policy is functionally equivalent to a policy such that, in each period, the government subsidizes θ^o ($\in (0,1)$) percentage of the operating costs of institutions of higher education, and that the funds are raised through a lump-sum tax on the young agents in that period. For details, see Appendix C.

holds:

$$w_t^0 A_t = w_t^1 A_t - (1-\theta) p_t,$$

or equivalently

$$w_t^0 = w_t^1 - (1-\theta)\tilde{p}_t. \tag{52}$$

Under this policy, the agents' optimization problem is modified as

$$\max_{c_t^y, c_{t+1}^o, s_t} \log c_t^y + \beta \log c_{t+1}^o$$
$$\text{s.t. } c_t^y + s_t = w_t^0 A_t - \tau_t, \quad c_{t+1}^o = r_{t+1} s_t,$$

the solutions to which are

$$\begin{aligned} c_t^y &= (w_t^0 A_t - \tau_t)/(1+\beta) && (53)\\ c_{t+1}^o &= \beta r_{t+1}(w_t^0 A_t - \tau_t)/(1+\beta) && (54)\\ s_t &= \beta(w_t^0 A_t - \tau_t)/(1+\beta). && (55) \end{aligned}$$

Using (55), we obtain the capital for period $t+1$ as follows:

$$K_{t+1} = s_t N_t = \frac{\beta(w_t^0 A_t - \tau_t) N_t}{1+\beta}. \tag{56}$$

Eqs.(22) and (52) jointly produce

$$\frac{w_t^0}{r_t} = q_t \left(1 - \frac{1-\theta}{\Gamma \Phi^E q_t^{1-\gamma}}\right). \tag{57}$$

By substituting (57) into (12), we obtain

$$r_t = \Lambda \Phi^P \left(1 - \frac{1-\theta}{\Gamma \Phi^E q_t^{1-\gamma}}\right)^{-\alpha_0} q_t^{-\alpha_0-\alpha_1}. \tag{58}$$

Combined with the definition formula for q_t, eqs.(57) and (58) imply that

$$\begin{aligned} w_t^0 &= \Lambda \Phi^P \left(1 - \frac{1-\theta}{\Gamma \Phi^E q_t^{1-\gamma}}\right)^{1-\alpha_0} q_t^{1-\alpha_0-\alpha_1} && (59)\\ w_t^1 &= \Lambda \Phi^P \left(1 - \frac{1-\theta}{\Gamma \Phi^E q_t^{1-\gamma}}\right)^{-\alpha_0} q_t^{1-\alpha_0-\alpha_1}. && (60) \end{aligned}$$

Eqs.(52)(59) and (60) jointly produce

$$\tilde{p}_t = \frac{\Lambda \Phi^P}{\Gamma \Phi^E} \left(1 - \frac{1-\theta}{\Gamma \Phi^E q_t^{1-\gamma}}\right)^{-\alpha_0} q_t^{\gamma-\alpha_0-\alpha_1}. \tag{61}$$

Since the values of r_t, w_t^0, w_t^1, and $\tilde{p}_t$ are all positive, eqs.(58)-(61) make sense only when q_t satisfies

$$\Gamma\Phi^E q_t^{1-\gamma} - 1 + \theta > 0. \tag{62}$$

It is easy to verify that equilibrium conditions (40)-(46) remain unchanged by this policy. Using (42) and (61), we can rewrite (51) as

$$\tilde{\tau}_t = \frac{\theta\alpha_1\Lambda\Phi^P[1-(1-\theta)/\Gamma\Phi^E q_t^{1-\gamma}]^{-\alpha_0} q_t^{-\alpha_0-\alpha_1} k_t}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-\gamma)+(1-\gamma)\alpha_1}. \tag{63}$$

Furthermore, eqs.(56)(59) and (63) jointly produce

$$\begin{aligned} a_{t+1}k_{t+1} &= \frac{\beta(w_t^0 - \tilde{\tau}_t)}{b(1+\beta)} \\ &= \frac{\beta\Lambda\Phi^P}{b(1+\beta)}\left(1-\frac{1-\theta}{\Gamma\Phi^E q_t^{1-\gamma}}\right)^{1-\alpha_0} q_t^{1-\alpha_0-\alpha_1} \\ &\quad \times \left\{1-\frac{\theta\alpha_1[1-(1-\theta)/\Gamma\Phi^E q_t^{1-\gamma}]^{-1}(k_t/q_t)}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-\gamma)+(1-\gamma)\alpha_1}\right\}. \end{aligned} \tag{64}$$

In equilibrium, k_{t+1} takes a positive value for $\forall t \geq 1$, and so the following must hold:

$$1-\frac{\theta\alpha_1[1-(1-\theta)/\Gamma\Phi^E q_t^{1-\gamma}]^{-1}(k_t/q_t)}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-\gamma)+(1-\gamma)\alpha_1} > 0. \tag{65}$$

From (10)(43) and (57), we can derive

$$\begin{aligned} n_t^0 &= \frac{\alpha_0}{1-\alpha_0-\alpha_1}\cdot\frac{k_t^P}{w_t^0/r_t} \\ &= \frac{\alpha_0\Gamma\Phi^E q_t^{-\gamma}k_t}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-\gamma)+(1-\gamma)\alpha_1} \\ &\quad \times\left(1+\frac{1-\gamma-\theta}{\Gamma\Phi^E q_t^{1-\gamma}-1+\theta}\right). \end{aligned} \tag{66}$$

Plugging (40) and (66) into $n_t^0 + n_t^1 = 1$, we obtain

$$\begin{aligned} &\frac{\alpha_1\Gamma\Phi^E q_t^{-\gamma}k_t}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-\gamma)+(1-\gamma)\alpha_1} \\ &\times\left[1+\frac{\alpha_0}{\alpha_1}\left(1+\frac{1-\gamma-\theta}{\Gamma\Phi^E q_t^{1-\gamma}-1+\theta}\right)\right] = 1, \end{aligned} \tag{67}$$

which simplifies (65) as

$$\Gamma\Phi^E q_t^{1-\gamma} - 1 + (\alpha_0/\alpha_1)(1-\gamma-\theta) > 0. \tag{68}$$

The following lemma is useful in that it reduces the number of conditions that need to be checked when examining equilibrium.

Lemma 2. *(a) When* $\theta \in (0, (1-\gamma)\alpha_0/(\alpha_0+\alpha_1)]$*, condition (62) implies conditions (45) and (68). (b) When* $\theta \in ((1-\gamma)\alpha_0/(\alpha_0+\alpha_1), 1]$*, condition (68) implies conditions (45) and (62).*

Proof. To establish part (a), it suffices to compare the RHSs of conditions (45), (62), and (68), and to show that condition (62) is the most restrictive. This is equivalent to showing that

$$1 - \theta \geq \gamma \tag{69}$$

$$1 - \theta \geq 1 - (\alpha_0/\alpha_1)(1 - \gamma - \theta). \tag{70}$$

Of these two conditions, condition (69) can be rewritten as $1 - \gamma \geq \theta$, which is satisfied since in this case the following hold:

$$\theta \leq (1-\gamma)\alpha_0/(\alpha_0+\alpha_1) < 1 - \gamma.$$

Condition (70) can be rewritten as $\theta \leq (1-\gamma)\alpha_0/(\alpha_0+\alpha_1)$, which is satisfied by assumption. To establish part (b), it suffices to show that condition (68) is the most restrictive among the three conditions, which is equivalent to showing that

$$1 - (\alpha_0/\alpha_1)(1-\gamma-\theta) \geq 1 - \theta \tag{71}$$

$$1 - (\alpha_0/\alpha_1)(1-\gamma-\theta) \geq \gamma. \tag{72}$$

Of these two conditions, condition (71) can be rewritten as $\theta \geq (1 - \gamma)\alpha_0/(\alpha_0 + \alpha_1)$, which is satisfied since we are currently considering the case in which $\theta > (1-\gamma)\alpha_0/(\alpha_0+\alpha_1)$. Condition (72) can be rewritten as

$$(1-\gamma)(1-\alpha_0/\alpha_1) \geq -(\alpha_0/\alpha_1)\theta.$$

If $\alpha_0 \leq \alpha_1$, this inequality is true since its LHS is positive while its RHS is negative. If $\alpha_0 > \alpha_1$, the above inequality can be further rewritten as

$$(1-\gamma)(1-\alpha_1/\alpha_0) \leq \theta.$$

This inequality is true since in this case the following hold:

$$(1-\gamma)(1-\alpha_1/\alpha_0) < (1-\gamma)\alpha_0/(\alpha_0+\alpha_1) < \theta.$$

□

This lemma allows us to rephrase conditions (45)(62) and (68) as follows.

Proposition 3. *(a) When* $\theta \in (0, (1-\gamma)\alpha_0/(\alpha_0+\alpha_1)]$*, the equilibrium value of* q_t *must satisfy*

$$q_t > [(1-\theta)/\Gamma\Phi^E]^{1/(1-\gamma)} (\equiv \underline{q}_2). \tag{73}$$

(b) When $\theta \in ((1-\gamma)\alpha_0/(\alpha_0+\alpha_1), 1]$*, the equilibrium value of* q_t *must satisfy*

$$q_t > \left\{[1 - (\alpha_0/\alpha_1)(1-\gamma-\theta)] / \Gamma\Phi^E\right\}^{1/(1-\gamma)} (\equiv \underline{q}_3). \tag{74}$$

Proof. Note that conditions (62) and (68) can be rewritten as (73) and (74), respectively. In equilibrium, q_t must satisfy (45)(62) and (68) simultaneously. These conditions characterize the admissible range of q_t in equilibrium. As shown in Lemma 2, condition (62) implies conditions (45) and (68) when $\theta \in (0, (1-\gamma)\alpha_0/(\alpha_0+\alpha_1)]$, and condition (68) implies conditions (45) and (62) when $\theta \in ((1-\gamma)\alpha_0/(\alpha_0+\alpha_1), 1]$. Thus, in each case, the set of three conditions reduces to a single binding condition, which is given by (73) in the first case and (74) in the second case. □

These conditions provide the basis for studying the feasibility and dynamic effects of tuition reductions in the following subsections.

4.2 Upper Limits of Tuition Reduction Rate

In this subsection, we examine whether eq.(67) admits a well-defined equilibrium value of q_t for a given k_t. As we show below, the existence of such an equilibrium depends critically on the tuition reduction rate.

The next proposition asserts that this is the case when θ is not too large.

Proposition 4. *Suppose that $\theta \in (0, (1-\gamma)\alpha_0/(\alpha_0+\alpha_1)]$, and let $\underline{q}_2$ be as defined in (73). Then, for $\forall k_t > 0$, eq.(67) has a unique solution satisfying $q_t > \underline{q}_2$.*

Proof. By Proposition 3, any equilibrium value of q_t must satisfy $q_t > \underline{q}_2$. Eq.(67) can be rewritten as

$$\begin{aligned} \frac{1}{k_t} = & \frac{\alpha_1 \Gamma \Phi^E q_t^{-\gamma}}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} \\ & \times \left[1 + \frac{\alpha_0}{\alpha_1} \left(1 + \frac{1-\gamma-\theta}{\Gamma\Phi^E q_t^{1-\gamma} - 1 + \theta} \right) \right]. \end{aligned} \tag{75}$$

Let the RHS of (75) be $f(q_t)$. On $(\underline{q}_2, +\infty)$, $f(q_t)$ is a continuously decreasing function of q_t with

$$\lim_{q_t \to \underline{q}_2 + 0} f(q_t) = +\infty, \qquad \lim_{q_t \to +\infty} f(q_t) = 0.$$

These properties jointly mean that the graphs of LHS and RHS of (75) intersect only once on $(\underline{q}_2, +\infty)$, implying that the value of q_t is uniquely determined by k_t, and that it satisfies $q_t > \underline{q}_2$. □

This result shows that when the tuition reduction rate is sufficiently small, a temporary equilibrium exists and is uniquely determined for any level of capital. However, once the tuition reduction rate exceeds a critical threshold, the existence of equilibrium is no longer guaranteed. In this case, the solution to (67) exists only when k_t is sufficiently large.

Proposition 5. *Suppose that $\theta \in ((1-\gamma)\alpha_0/(\alpha_0+\alpha_1), 1-\gamma]$, and let $\underline{q}_3$ be as defined in (74). In addition, define Ω as*

$$\Omega \equiv \frac{(1-\alpha_0-\alpha_1)[1-\gamma-(\alpha_0/\alpha_1)(1-\gamma-\theta)]+(1-\gamma)\alpha_1}{\alpha_0\{\Gamma\Phi^E[1-(\alpha_0/\alpha_1)(1-\gamma-\theta)]^{-\gamma}\}^{1/(1-\gamma)}} \times \frac{\theta(1+\alpha_1/\alpha_0)-1+\gamma}{\theta[1+\alpha_1/\alpha_0+(\alpha_1/\alpha_0)^2]-1+\gamma}. \tag{76}$$

Then, eq.(67) has a unique solution satisfying $q_t > \underline{q}_3$ if $k_t > \Omega$, and no such solution, otherwise.

Proof. Under the maintained assumption that $\theta \in ((1-\gamma)\alpha_0/(\alpha_0+\alpha_1), 1-\gamma]$, the equilibrium value of q_t must satisfy $q_t > \underline{q}_3$, as shown in Proposition 3. Eq.(67) can be rewritten as (75).

$$\frac{1}{k_t} = \frac{\alpha_1\Gamma\Phi^E q_t^{-\gamma}}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-\gamma)+(1-\gamma)\alpha_1} \times \left[1+\frac{\alpha_0}{\alpha_1}\left(1+\frac{1-\gamma-\theta}{\Gamma\Phi^E q_t^{1-\gamma}-1+\theta}\right)\right]. \tag{75}$$

As in the proof of Proposition 4, let the RHS of (75) be $f(q_t)$. Then, $f(q_t)$ is continuously decreasing on $(\underline{q}_3, +\infty)$ with

$$\lim_{q_t \to \underline{q}_3+0} f(q_t) = 1/\Omega, \qquad \lim_{q_t \to +\infty} f(q_t) = 0.$$

When $\theta > (1-\gamma)\alpha_0/(\alpha_0+\alpha_1)$, the following inequalities are true:

$$(\alpha_0/\alpha_1)(1-\gamma-\theta) < (1-\gamma)\alpha_0/(\alpha_0+\alpha_1) < 1-\gamma < 1$$
$$\theta[1+\alpha_1/\alpha_0+(\alpha_1/\alpha_0)^2] > \theta(1+\alpha_1/\alpha_0) > 1-\gamma,$$

which ensure that Ω takes a positive finite value. Thus, the graphs of LHS and RHS of (75) intersect only once on $(\underline{q}_3, +\infty)$ if $1/k_t < 1/\Omega$, or $k_t > \Omega$. If $k_t \leq \Omega$, then $1/k_t \geq 1/\Omega$, while $f(q_t) < 1/\Omega$ for all $q_t > \underline{q}_3$, so no such solution exists. □

This result implies that when the tuition reduction rate is sufficiently large, an equilibrium exists only if k_t is sufficiently large. In other words, aggressive tuition reduction policies may render the equilibrium infeasible unless the economy has accumulated a sufficiently large amount of capital relative to effective labor. If such a high rate of tuition reduction were to be implemented in an economy with $k_t \leq \Omega$, even taxing all of the income of the young generation would not be enough to cover the necessary government expenditures.[4]

[4]Even when $k_t \leq \Omega$, eq.(67) has a "solution" satisfying $q_t \in (\underline{q}_2, \underline{q}_3]$, although it does not satisfy (74), failing to make the disposable income of the young generation positive.

When $\theta > 1-\gamma$, the RHS of (75) is no longer a decreasing function of q_t. Because of this, the analysis in this case becomes very complicated but the conclusion remains unchanged: a sufficiently high tuition reduction rate can only be achieved when the value of k_t is sufficiently large.

Proposition 6. *Suppose that $\theta \in (1-\gamma, 1]$, and let $\underline{q}_3$ and Ω be as defined in (74) and (76), respectively. Then, the following statements hold: (i) if $k_t > \Omega$, then eq.(67) has a unique solution satisfying $q_t > \underline{q}_3$; (ii) if*

$$k_t \leq \frac{\alpha_0}{\alpha_0+\alpha_1} \cdot \frac{\theta[1+\alpha_1/\alpha_0+(\alpha_1/\alpha_0)^2]-1+\gamma}{\theta(1+\alpha_1/\alpha_0)-1+\gamma}\Omega, \tag{77}$$

then eq.(67) has no solution satisfying $q_t > \underline{q}_3$; and (iii) for intermediate values of k_t, that is,

$$\frac{\alpha_0}{\alpha_0+\alpha_1} \cdot \frac{\theta[1+\alpha_1/\alpha_0+(\alpha_1/\alpha_0)^2]-1+\gamma}{\theta(1+\alpha_1/\alpha_0)-1+\gamma}\Omega < k_t \leq \Omega, \tag{78}$$

the number of solutions to eq.(67) satisfying $q_t > \underline{q}_3$ may be zero, one, or two, depending on parameter values.

The proof of this proposition is rather lengthy and is therefore relegated to Appendix D. The proposition highlights that, when $\theta > 1-\gamma$, the equilibrium structure becomes more intricate: for intermediate values of k_t, eq.(67) may admit zero, one, or multiple solutions satisfying $q_t > \underline{q}_3$.

Taken together, these results show that the existence of equilibrium imposes an upper bound on the tuition reduction rate, which depends on the level of k_t. In particular, when the tuition reduction rate is sufficiently high, equilibrium can be sustained only if k_t is large enough. This characterization provides a basis for evaluating the feasible range of tuition policies in the quantitative analysis that follows.

4.3 Effects of College Tuition Reductions

The results in the previous subsection show that a temporary equilibrium may fail to exist when the tuition reduction rate is sufficiently large. Therefore, the existence of an intertemporal equilibrium is not guaranteed a priori and depends on whether the equilibrium conditions are satisfied in every period. We now define an intertemporal equilibrium for the economy under a tuition reduction policy, provided that a temporary equilibrium exists in each period.

Definition 2. *An equilibrium of the economy with the tuition reduction policy is a sequence*

$$\{(q_t, r_t, w_t^0, w_t^1, \tilde{p}_t, \tilde{\tau}_t, a_{t+1}, k_{t+1}, k_t^P, k_t^E, n_t^0, n_t^1, n_t^{1P}, n_t^{1E})\}_{t=1}^{\infty}$$

generated by (40)-(46)(58)-(64) and (66)-(68), given $k_1 > 0$ and $\theta \in (0,1)$.

As suggested by Propositions 4-6, these conditions restrict the set of feasible equilibrium paths. In particular, when the tuition reduction rate is sufficiently large, only economies with sufficiently high levels of k_t can sustain an equilibrium.

4.3.1 Simulation Results

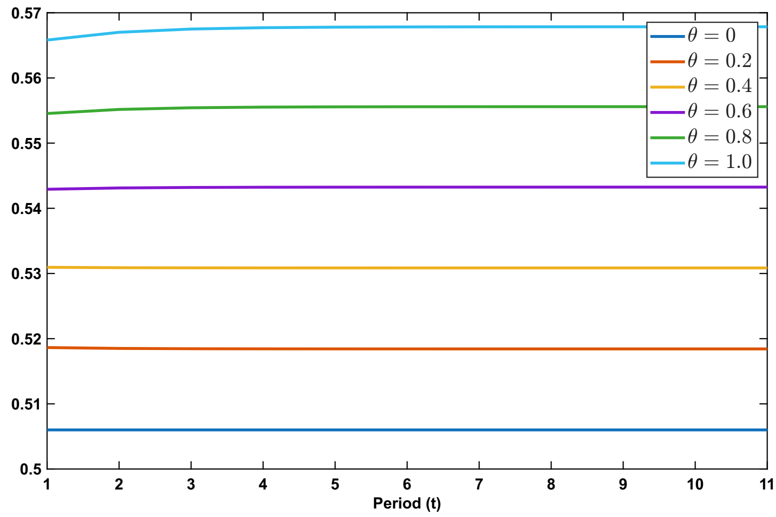
In conducting the simulation, we assume that the policy is implemented in period 1, and that the economy is on the BGP before it is implemented. The parameter values shall continue to be those specified in Table 1.

Figure 2 shows that this policy increases the number of college graduates and accelerates technological progress in the economy. Specifically, Figure 2a depicts the change in the percentage of college graduates (i.e., n_t^1) after the policy was implemented for $\theta = 0, 0.2, 0.4, 0.6, 0.8$, and 1, where $\theta = 0$ represents the case without the policy. As depicted in that figure, the percentage of college graduates jumps to a new level corresponding to the subsidy rate during the first period of policy implementation and remains at that level thereafter. The magnitude of the jump increases with the subsidy rate. Notably, the case where $\theta = 1$ is reported. This implies that, at least in the Japanese economy, tuition-free college education is feasible. Figure 2b depicts the change in the rate of technological progress (i.e., a_t) after the implementation for the same values of θ. As depicted in that figure, the rate of technological progress increases on impact and eventually settles at a new level, both of which are higher than the corresponding levels without the policy. The magnitude of the increase in the rate of technological progress also increases with the subsidy rate, but the increase is very small, even when $\theta = 1$. In that case, the policy raises the rate of technological progress from 1.47 to 1.57 in the long run, which amounts to an increase of only about 0.16% on an annualized basis.

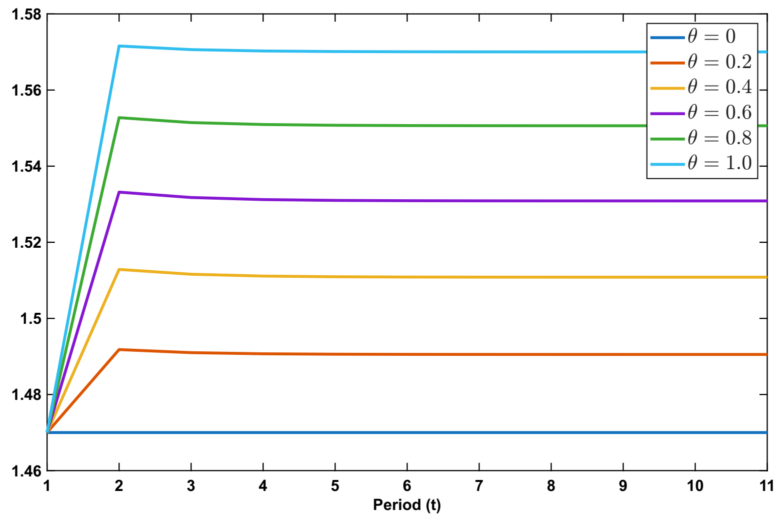
Table 2 summarizes the increase in lifetime utility resulting from this policy for the generations present at the time of implementation or born thereafter, where gen.t (≥ 1) in the table represents the generation born in period t, and gen.0 represents the old generation at the time of implementation. As shown in that table, for all generations, lifetime utility is higher when this policy is implemented, and the degree of improvement is greater for later generations. These results indicate that, at least in the Japanese economy, this policy is Pareto-improving.

4.3.2 How Did the Policy Bring About These Changes?

To understand the responses of n_t^1 and a_t to the tuition reduction policy, we must first understand the responses of q_t and k_t. As we have seen, this model operates such that, in any given period t, the value of q_t is determined first, given the value of k_t, and then q_t and k_t determine the values of other



(a) Responses of n_t^1



(b) Responses of a_t

Figure 2: Effects of the Tuition Reduction Policy (1)

Table 2: Welfare Gain by Generation

gen.	θ 0.2	0.4	0.6	0.8	1
0	0.0010	0.0015	0.0018	0.0017	0.0013
1	0.0082	0.0152	0.0211	0.0258	0.0295
2	0.0271	0.0520	0.0746	0.0950	0.1135
3	0.0503	0.0974	0.1413	0.1821	0.2199
4	0.0750	0.1461	0.2131	0.2762	0.3354
5	0.1004	0.1960	0.2870	0.3732	0.4547
6	0.1259	0.2465	0.3616	0.4713	0.5756
7	0.1515	0.2971	0.4366	0.5699	0.6970
8	0.1772	0.3478	0.5116	0.6687	0.8188
9	0.2029	0.3985	0.5868	0.7675	0.9406
10	0.2285	0.4492	0.6619	0.8663	1.0625
⋮	⋮	⋮	⋮	⋮	⋮

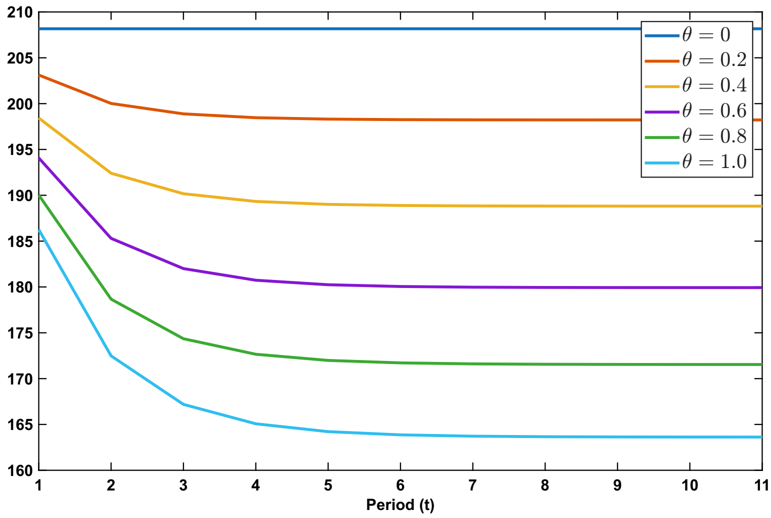
endogenous variables, including k_{t+1}.

Figure 3 depicts the responses of q_t and k_t to the policy for the same values of θ as in Figure 2. Since the value of k_1 is already given at the beginning of period 1, the value of q_1 is determined by (75) given the values of k_1 and θ. As can be easily verified, the RHS of (75) is a decreasing function of θ; thus, the larger θ is, the smaller the value of q_1 becomes. This explains why, in Figure 3a, the value of q_1 jumps downward at the start of the policy implementation, and the jump amplitude increases with θ.

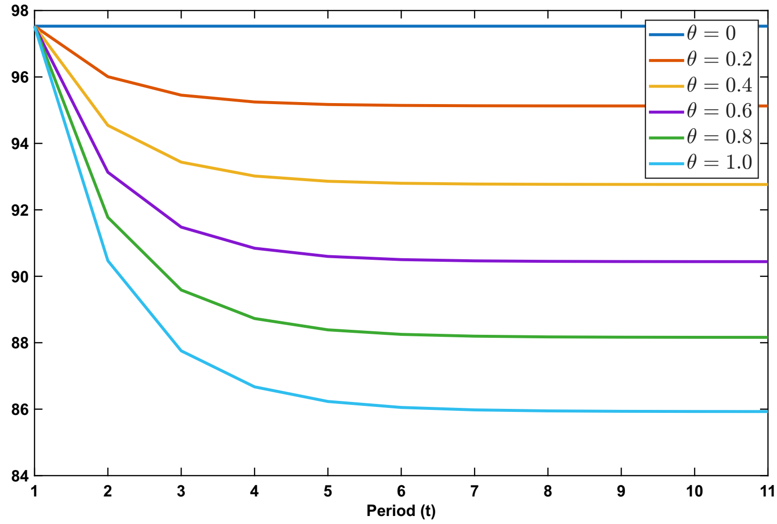
This downward jump in q_1 causes upward jumps in the values of n_1^1, n_1^{1E}, and k_1^E. This is because the RHS of eqs.(40)(42) and (44), which determine these values from q_1 and k_1, are all decreasing functions of q_1. Because n_1^{1E} and k_1^E have undergone upward jumps, eq.(46) concludes that $a_2 > a_1$. This explains why the upward jump of n_1^1 occurs in Figure 2a and why the increase in a_t occurs from period 1 to period 2 in Figure 2b.

These results are prediction generated by equilibrium conditions, independent of parameter values. In other words, implementing a policy like the one considered here will inevitably increase the number of college graduates and raise the rate of technological progress immediately after implementation. However, the subsequent course of the economy can only be investigated through simulation, because the determination of variables thereafter is so complex that observing equilibrium conditions alone cannot pinpoint their movement.

The simulation clearly shows that after period 1, the values of q_t and k_t continuously decline and eventually converge to new steady-state values (see Figure 3). These movements affect the evolution of n_t^1, n_t^{1E}, and k_t^E through (40)(42) and (44). Specifically, they stabilize n_t^1, increase n_t^{1E}, and decrease



(a) Responses of q_t



(b) Responses of k_t

Figure 3: Effects of the Tuition Reduction Policy (2)

k_t^E from period 2 onward. Importantly, the movements of n_t^{1E} and k_t^E exert opposing effects on the value of a_{t+1} through (46). While the increase in n_t^{1E} raises a_{t+1}, the decline in k_t^E works in the opposite direction. These two effects counteract each other, causing the value of a_t to stabilize from period 2 onward (see Figure 2b).

Let us also see how this policy improved the welfare of all generations. The policy causes an upward jump in r_1 at the start of its implementation, and the value of r_t continues to rise thereafter. This contributes to increasing the income of each generation during their old age. This is why the utility of the old generation in period 1 increased. The policy also causes $w_t^0 - \tilde{\tau}_t$ —the after-tax income earned by the younger generation for supplying one unit of general or specialized labor— to jump upward in period 1 before gradually declining. This change undoubtedly improves the welfare of the generation born in period 1, while simultaneously working against the utility of generations born in period 2 and beyond. Nevertheless, their economic welfare is remarkably improved by the policy, as shown in Table 2. This is because the policy increases the amount of general or specialized labor supplied by each of them, more than compensating for the decline in $w_t^0 - \tilde{\tau}_t$.

Finally, it cannot be overemphasized how strongly these results depend on the parameter values given in Table 1. As we will see shortly, under parameter values different from those in Table 1, the policy could have entirely different effects.

4.3.3 Innovation Productivity and the Feasibility of Free College Education

As we have seen, the tuition reduction policy under consideration here accelerates technological progress and economic growth, but the acceleration effect is very small. Even with full tuition waivers (i.e., $\theta = 1$), it amounts to only about 0.16% per year. The primary reason for this minimal acceleration lies in the low productivity of innovation, or the small value of $\Phi^I (= 1.6458)$. However, it is precisely this low productivity that makes permanent tuition-free higher education possible in this economy.

To examine how the feasibility of tuition reductions depends on innovation productivity, consider the following simulation. In period 1, the economy experiences a shock such that Φ^I becomes 2^i times its value in Table 1, where $i = 0, 1, 2, 3, 4, 5$. Fifty periods later, the government implements a tuition reduction policy with $\theta = 0.2, 0.4, 0.6, 0.8$, or 1.[5] We then compare the equilibrium responses across these economies.

Table 3 summarizes the responses of the number of college graduates and

[5] Here, $i = 0$ represents the absence of a shock. The 50-period lag between the shock and policy implementation is designed to allow the economy to converge to a new BGP consistent with post-shock innovation productivity. The values of parameters other than Φ^I shall remain unchanged.

Table 3: Effects of the Tuition Reduction Policy when $\Phi^I = 2^i \times 1.6458$

(a) Response Rates of n^1_{51} and n^1_{61}

i	θ = 0.2	0.4	0.6	0.8	1
0	1.0250	1.0493	1.0730	1.0959	1.1182
	1.0246	1.0491	1.0736	1.0980	1.1222
1	1.0472	1.0926	1.1360	1.1773	1.2164
	1.0453	1.0917	1.1391	1.1872	1.2358
2	1.1087	1.2137	1.3119	1.4012	1.4809
	1.0909	1.2011	NaN	NaN	NaN
3	1.3148	1.7001	NaN	NaN	NaN
	0.3145	NaN	NaN	NaN	NaN
4	1.5442	NaN	NaN	NaN	NaN
	0.1554	NaN	NaN	NaN	NaN
5	1.6145	NaN	NaN	NaN	NaN
	0.1347	NaN	NaN	NaN	NaN

(b) Response Rates of a_{52} and a_{62}

i	θ = 0.2	0.4	0.6	0.8	1
0	1.0148	1.0292	1.0430	1.0563	1.0691
	1.0140	1.0278	1.0414	1.0548	1.0680
1	1.0288	1.0562	1.0819	1.1061	1.1287
	1.0263	1.0526	1.0789	1.1051	1.1310
2	1.0669	1.1293	1.1858	1.2360	1.2799
	1.0512	1.1077	NaN	NaN	NaN
3	1.1889	1.3992	NaN	NaN	NaN
	0.4589	NaN	NaN	NaN	NaN
4	1.3174	NaN	NaN	NaN	NaN
	0.2922	NaN	NaN	NaN	NaN
5	1.3559	NaN	NaN	NaN	NaN
	0.2657	NaN	NaN	NaN	NaN

the rate of technological progress to the policy for different values of $\Phi^I = 2^i \times 1.6458$, where $i = 0, 1, 2, 3, 4, 5$. Tables 3a and 3b each contain two rows for a single value of i. The upper row shows the reaction immediately after policy implementation, while the lower row shows the reaction 10 periods after implementation. The figures shown in each row represent the ratio between the case where the policy was implemented and the case where it was not. Additionally, 'NaN' indicates that the corresponding value could not be obtained from the simulation.

This table shows that the feasibility of the policy deteriorates sharply as innovation productivity rises. In economies with high innovation productivity, 'NaN' values appear frequently, indicating that within 10 periods after policy implementation, k_t falls below the critical threshold required for the existence of a temporary equilibrium, as suggested by Propositions 5-6. Particularly in the economies with $i = 3, 4, 5$, simulations fail to obtain even the immediate reaction values following policy activation in nearly all cases where θ is 0.4 or higher. This occurs because, in these economies, the value of k_t is already low at the time of policy implementation. Consequently, the policies requiring a sufficiently large value of θ become impossible to implement from the outset. On the other hand, when $\theta = 0.2$, both short-term and long-term policy responses are reported for all economies. This is as asserted in Proposition 4. According to this proposition, the existence of equilibrium is guaranteed if θ satisfies the following condition:

$$\theta \leq (1 - \gamma)\alpha_0/(\alpha_0 + \alpha_1) \approx 0.2141,$$

and $\theta = 0.2$ is precisely the value that satisfies this condition. However, this modest tuition reduction policy does not always increase the number of college graduates or accelerate technological progress. Indeed, in the short run, it increases the number of college graduates and the rate of technological progress in all economies. However, in the long run, it significantly reduces both of them in the economies with high innovation productivity (i.e., $i = 3, 4, 5$). This outcome is quite ironic, as this policy was originally intended to accelerate technological progress by expanding the higher education sector. These results show that the tuition reduction policy can be implemented at various reduction rates and achieve their intended effects only in economies with relatively low innovation productivity.

Let us also see the impact of the policy on economic welfare. Table 3 summarizes the increase in the lifetime utility of generations born in periods 50 and 60 due to the policy for different values of $\Phi^I = 2^i \times 1.6458$, where $i = 0, 1, 2, 3, 4, 5$. As in Tables 3a and 3b, this table contains two rows for a single value of i. The upper row shows the increase in lifetime utility of the generation born in period 50, who are old at the time of policy implementation, while the lower row shows the increase in lifetime utility of the generation born in period 60. The figures shown in each row represent the difference between the case where the policy was implemented and

Table 4: Welfare Gain for Generations Born in Period 50 and Period 60

	θ				
i	0.2	0.4	0.6	0.8	1
0	0.0010	0.0015	0.0018	0.0017	0.0013
	0.2285	0.4492	0.6619	0.8663	1.0625
1	0.0031	0.0049	0.0056	0.0054	0.0042
	0.4218	0.8240	1.2044	1.5609	1.8917
2	0.0132	0.0201	0.0240	0.0231	0.0190
	0.7726	1.4231	NaN	NaN	NaN
3	0.0705	0.1239	NaN	NaN	NaN
	-8.2052	NaN	NaN	NaN	NaN
4	0.1641	NaN	NaN	NaN	NaN
	-20.2707	NaN	NaN	NaN	NaN
5	0.2066	NaN	NaN	NaN	NaN
	-23.6450	NaN	NaN	NaN	NaN

the case where it was not. Again, in this table, 'NaN' indicates that the corresponding value could not be obtained from the simulation.

The following can be inferred from this table. In economies with low innovation productivity (i.e., $i = 0, 1, 2$), the tuition reduction policy is likely to improve the economic welfare of all generations, provided it is sustainable in the long term. As the table shows, the policy increases the lifetime utility of the generations born in period 50 and period 60, but the increase is greater for the latter. This is because the policy raises the rate of technological progress, allowing later-born generations to reap greater benefits. Conversely, in economies with high innovation productivity (i.e., $i = 3, 4, 5$), even if the tuition reduction policy is sustainable in the long term, it does not contribute to improving the economic welfare of all generations. As the table shows, while the policy increases the lifetime utility of the generation born in period 50, the lifetime utility of the generation born in period 60 becomes significantly lower than if the policy had not been implemented. This is because the policy lowers the medium-to-long-term rate of technological progress to a level below what would have been achieved without it, thereby decreasing the amount of general and specialized labor supplied by the generation born in period 60. In other words, the policy improves the economic welfare of the current generation at the expense of the economic welfare of future generations.

To understand these differences, consider the underlying mechanism. The key point is that the tuition reduction policy induces an excessively rapid decline in the capital-effective labor ratio, k_t, in economies with high innovation productivity. When the policy is implemented, the price of specialized labor falls, leading the production sector to increase its demand for college graduates. This response temporarily expands the higher education sector and raises the rate of technological progress. In economies with high innovation productivity, however, this increase in technological progress becomes excessively rapid. As a result, the capital-effective labor ratio, k_t, declines sharply because effective labor grows quickly. In severe cases, this decline suppresses income growth for young cohorts and makes it difficult to generate sufficient tax revenue to sustain the policy. Even when such outcomes are avoided by lowering the reduction rate, the demand for college graduates weakens, forcing a contraction of the higher education sector and leading to a slowdown in economic growth over the medium to long run. By contrast, in economies with low innovation productivity, the increase in technological progress induced by the policy remains moderate. As a result, the decline in k_t is limited and does not threaten the feasibility of continued policy implementation. Instead, the policy steadily increases the demand for college graduates, supports the expansion of the higher education sector, and gradually raises the rate of technological progress. This process increases the disposable income of the young generation in each period, thereby improving the economic welfare of all generations.

5 Conclusion

This paper examined the impact of college tuition reductions on economic growth and generational welfare within an endogenous growth framework. The analysis revealed that when model parameters are calibrated to reflect the current Japanese economy, tuition subsidies—and even full tuition waivers—are likely to promote economic growth and improve economic welfare for all generations by accelerating technological progress. While this policy achieves a Pareto improvement, the resulting acceleration of technological progress remains modest, amounting to an annualized increase of approximately 0.16% even under a full waiver ($\theta = 1$).

The central contribution of this study is the finding that these positive outcomes depend critically on the relatively low productivity of innovation (Φ^I) inherent in the calibrated economy. In economies characterized by high innovation productivity, aggressive tuition reduction policies face a "paradox of innovation." In such cases, the rapid technological progress induced by the policy outpaces capital accumulation, leading to a sharp decline in the capital-effective labor ratio (k_t). This shift weakens the income growth of young cohorts—the primary tax base—thereby rendering high subsidy rates fiscally unsustainable within a few periods of implementation. Furthermore, even if low reduction rates are maintained to ensure sustainability, the general equilibrium effects in high-innovation economies can ironically lead to a long-term contraction of the higher education sector and sluggish growth.

A promising avenue for future research is to consider alternative financing mechanisms, such as proportional income taxation across generations. Such policies may help broaden the tax base and mitigate the fiscal pressure identified in this paper. However, the underlying mechanism emphasized here—where technological progress outpaces capital accumulation and weakens the effective tax base—suggests that fiscal sustainability concerns may not be fully resolved by changes in the tax structure alone. Ultimately, the decision to implement tuition reductions must be grounded in a precise evaluation of an economy's innovation productivity and its interaction with the fiscal foundation.

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A Existence of Balanced Growth Path

This appendix demonstrates that the model has at least one BGP. This task is necessary for the calibration performed in the next appendix. Calibration usually involves selecting parameter values that reproduce the real economy assuming that it is on a BGP. If there is no BGP in the model, such practice makes no sense.

Define a balanced growth path (BGP) of this economy as a time-invariant path of k_t, q_t, a_t, r_t, w_t^0, w_t^1, $\tilde{p}_t$, k_t^P, k_t^E, n_t^0, n_t^1, n_t^{1P} and n_t^{1E} that satisfies (25)-(30) and (40)-(48). Then we can state

Proposition 7. *There is at least one BGP in this model.*

Proof. The proof proceeds by reducing the BGP conditions to a single equation in q_t and showing that this equation admits at least one solution. Eq.(48) can be rewritten as

$$k_t = \frac{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-1)+(1-\gamma)(1-\alpha_0)}{(\alpha_0+\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-1)+(1-\gamma)\alpha_0} \times \left(1-\frac{1}{\Gamma\Phi^E q_t^{1-\gamma}}\right) q_t. \tag{79}$$

From (42)(44) and (46), we can derive

$$a_{t+1} = 1-\delta+(1-\gamma)^{1-\phi_1-\phi_2}\gamma^{\phi_1}\Phi^I \times \left[\frac{\alpha_1 k_t}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-1)+(1-\gamma)(1-\alpha_0)}\right]^{1-\phi_2} q_t^{-\phi_1}. \tag{80}$$

Using (79) to eliminate k_t from (80), we can obtain

$$a_{t+1} = 1-\delta+(1-\gamma)^{1-\phi_1-\phi_2}\gamma^{\phi_1}\Phi^I \times \left[\frac{\alpha_1}{(\alpha_0+\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-1)+(1-\gamma)\alpha_0}\right]^{1-\phi_2} \times \left(1-\frac{1}{\Gamma\Phi^E q_t^{1-\gamma}}\right)^{1-\phi_2} q_t^{1-\phi_1-\phi_2}. \tag{81}$$

Let $h(q_t)$ be the RHS of (81). Obviously, $h(q_t)$ is a continuous function of $q_t \in (\underline{q}_1, +\infty)$, where $\underline{q}_1$ is the lower bound ensuring that $\Gamma\Phi^E q_t^{1-\gamma} > 1$. Using $h(q_t)$, we can rewrite (29) as

$$k_{t+1} = \frac{\beta\Lambda\Phi^P}{b(1+\beta)}\left(1-\frac{1}{\Gamma\Phi^E q_t^{1-\gamma}}\right)^{1-\alpha_0}\frac{q_t^{1-\alpha_0-\alpha_1}}{h(q_t)}. \tag{82}$$

On a BGP, eqs.(79) and (82) are, respectively, simplified as

$$k = \frac{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q^{1-\gamma}-1)+(1-\gamma)(1-\alpha_0)}{(\alpha_0+\alpha_1)(\Gamma\Phi^E q^{1-\gamma}-1)+(1-\gamma)\alpha_0} \times \left(1-\frac{1}{\Gamma\Phi^E q^{1-\gamma}}\right) q.$$

and

$$k = \frac{\beta\Lambda\Phi^P}{b(1+\beta)}\left(1-\frac{1}{\Gamma\Phi^E q^{1-\gamma}}\right)^{1-\alpha_0} \frac{q^{1-\alpha_0-\alpha_1}}{h(q)},$$

where k and q are the values of k_t and q_t on the BGP, respectively. From these two equations, we obtain

$$\frac{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q^{1-\gamma}-1)+(1-\gamma)(1-\alpha_0)}{(\alpha_0+\alpha_1)(\Gamma\Phi^E q^{1-\gamma}-1)+(1-\gamma)\alpha_0} = \frac{\beta\Lambda\Phi^P}{b(1+\beta)}\left(1-\frac{1}{\Gamma\Phi^E q^{1-\gamma}}\right)^{-\alpha_0} \frac{q^{-\alpha_0-\alpha_1}}{h(q)}. \tag{83}$$

Eq.(83) implicitly defines the BGP value of q. To establish existence, we examine this equation by defining functions $f(q)$ and $g(q)$ as its LHS and RHS, respectively. Then

$$f(q) = \frac{1-\alpha_0-\alpha_1}{\alpha_0+\alpha_1} + \frac{(1-\gamma)\alpha_1/(\alpha_0+\alpha_1)}{(\alpha_0+\alpha_1)(\Gamma\Phi^E q^{1-\gamma}-1)+(1-\gamma)\alpha_0},$$

implying that $f(q)$ is a continuously decreasing function of $q \in (\underline{q}_1, +\infty)$ with

$$\lim_{q\to\underline{q}_1+0} f(q) = (1-\alpha_0)/\alpha_0$$
$$\lim_{q\to+\infty} f(q) = (1-\alpha_0-\alpha_1)/(\alpha_0+\alpha_1).$$

Thus, $f(q)$ decreases from $(1-\alpha_0)/\alpha_0$ to $(1-\alpha_0-\alpha_1)/(\alpha_0+\alpha_1)$ as q increases. Eq.(81) implies that $h(q) \to 1-\delta$ as $q \to \underline{q}_1 + 0$, and thus that $g(q)$ is a continuous function of $q \in (\underline{q}_1, +\infty)$ with

$$\lim_{q\to\underline{q}_1+0} g(q) = +\infty.$$

Since $h(q) > 1-\delta$, $g(q)$ satisfies the following inequalities:

$$0 < g(q) < \frac{\beta\Lambda\Phi^P}{b(1+\beta)}\left(1-\frac{1}{\Gamma\Phi^E q^{1-\gamma}}\right)^{-\alpha_0} \frac{q^{-\alpha_0-\alpha_1}}{1-\delta} \equiv \tilde{g}(q). \tag{84}$$

It is easy to verify that $\tilde{g}(q) \to 0$ as $q \to +\infty$, which, combined with (84), implies that

$$\lim_{q\to+\infty} g(q) = 0.$$

Hence, $g(q)$ decreases from infinity to zero as q increases. These properties jointly imply, by the intermediate value theorem, that the graphs of f and g intersect at least once on $(\underline{q}_1, +\infty)$, establishing the existence of a BGP. □

B Calibration

We calibrate the model by selecting parameter values so that key steady-state moments of the model match their empirical counterparts in the Japanese economy. Some parameters are set directly based on external evidence, while others are chosen to match specific target moments.

B.1 Parameter Configuration (1)

Among the parameters in Table 1, numerical values for α_0, α_1, β, and γ are set in the following manner. In setting these values, it is assumed that one period of the model is 40 years, and that the economy is on a balanced growth path.

The value of β is chosen to match the low level of real interest rates observed in Japan since 2000, which are typically below 1% per annum. We assume that the subjective discount factor is 0.999 at a quarterly frequency. Given that one year consists of four quarters and one model period corresponds to 40 years, the discount factor is given by

$$\beta = 0.999^{4\times 40} \approx 0.8521.$$

The parameter γ governs the share of specialized labor in the production of new knowledge. It is calibrated using the personnel cost ratio of private universities in Japan.[6] This is interpreted as the share of specialized labor in knowledge production, since personnel expenses primarily reflect payments to skilled workers engaged in education and research. Given $\gamma = 0.54$, the parameter Γ is determined from its definition in the model as

$$\Gamma = \gamma^{\gamma}(1-\gamma)^{1-\gamma} = 0.54^{0.54}(1-0.54)^{1-0.54} \approx 0.5016.$$

The parameters α_0 and α_1 are chosen to match the share of college-educated individuals and the wage premium of college graduates observed in the data.[7] In determining their values, it is assumed that the capital share

[6] See page 14 of the following document released by the Ministry of Education, Culture, Sports, Science and Technology (MEXT): https://www.mext.go.jp/content/20210317-mxt_sigsanji-000013293_9.pdf

[7] The percentage of college-bound students in the same generation is set at 0.506, based on MEXT's estimation. See page 4 of the following document: https://www.mext.go.jp/component/b_menu/shingi/giji/__icsFiles/afieldfile/2017/07/24/1386653_05.pdf. The wage ratio between college and high-school graduates is calculated as follows:

$$\frac{(1-0.451)\times 327.3 + 0.451\times 256.5}{(1-0.451)\times 266.1 + 0.451\times 194.3} \approx 1.2638,$$

where 0.451 is the percentage of women in the labor force in 2023, published by the Ministry of Health, Labor and Welfare (MHLW) (See page 1 of https://www.mhlw.go.jp/bunya/koyoukintou/josei-jitsujo/dl/23-01.pdf). The figures 327.3, 256.5, 266.1, and 194.3 are, respectively, the lifetime earnings of college-educated men, college-educated is 1/3, and thus that $\alpha_0 + \alpha_1 = 2/3$. Given this restriction, these two target moments jointly and uniquely pin down the values of α_0 and α_1. Specifically, the values of α_0 and α_1 are obtained as follows: From (26) and (27), one can derive

$$\frac{w_t^1}{w_t^0} = 1/\left(1 - \frac{1}{\Gamma\Phi^E q_t^{1-\gamma}}\right).$$

Since w_t^1/w_t^0 is the wage ratio between college and high-school graduates in this model and the data say that its long-run value is 1.2638, this equation implies that, on BGP, the following holds:

$$\Gamma\Phi^E q^{1-\gamma} = \frac{w_t^1/w_t^0}{w_t^1/w_t^0 - 1} \approx 4.791. \tag{85}$$

Next, from (40) and (48), one can derive

$$\alpha_1 = \frac{(\alpha_0 + \alpha_1)(\Gamma\Phi^E q_t^{1-\gamma} - \gamma)n_t^1}{\Gamma\Phi^E q_t^{1-\gamma} - 1 + (1-\gamma)n_t^1}.$$

Substituting the (long-run) values of $\alpha_0 + \alpha_1$, $\Gamma\Phi^E q_t^{1-\gamma}$, γ and n_t^1 into this equation yields

$$\begin{aligned} \alpha_1 &\approx 0.3564 \\ \alpha_0 &\approx 0.3103. \end{aligned}$$

Once the values of α_0 and α_1 are determined, the value of Λ is calculated as

$$\Lambda \approx 0.3339.$$

When the values of α_0, α_1, β and γ are set as above, the endogenous variables on BGP satisfy the following conditions:

$$\begin{aligned} k_t/q_t &= \frac{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-1)+(1-\gamma)(1-\alpha_0)}{(\alpha_0+\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-1)+(1-\gamma)\alpha_0} \times \left(1 - \frac{1}{\Gamma\Phi^E q_t^{1-\gamma}}\right) \\ &\approx 0.4685 \end{aligned} \tag{86}$$

$$n_t^0 = 1 - n_t^1 = 0.494 \tag{87}$$

$$n_t^{1P} = \frac{\alpha_1(\Gamma\Phi^E q_t^{1-\gamma} - \gamma)(k_t/q_t)}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} \approx 0.4490 \tag{88}$$

$$n_t^{1E} = \frac{\gamma\alpha_1(k_t/q_t)}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} \approx 0.0570 \tag{89}$$

$$k_t^P/k_t = \frac{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma} - \gamma)}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} \approx 0.8963 \tag{90}$$

$$k_t^E/k_t = \frac{(1-\gamma)\alpha_1}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma} - \gamma) + (1-\gamma)\alpha_1} \approx 0.1037. \tag{91}$$

women, high-school-educated men, and high-school-educated women in 2023, published by the Japan Institute for Labor Policy and Training (JILPT) (See page 5 of `https://www.jil.go.jp/kokunai/statistics/kako/2024/documents/useful2024_21_p301_331.pdf`).

Eqs.(88)-(91) mean that, in the long run, about 90% of college graduates and capital will be hired in the production sector and the rest in the higher education sector. Of these values, the value of n_t^{1E} is worth noting, since it is very close to the real-world percentage of their peers who go on to a master's degree. MEXT estimates that 5.5% of the generation born in 1992 entered a master's program and 0.7% of that generation entered a doctoral program.[8] Combined with the above prediction, this observation implies the possibility that, in Japan, master's programs contribute to some extent to the development of highly skilled personnel for research and development, while doctoral programs appear to play a more limited role.

B.2 Estimating Technological Progress Rates

In this subsection, we estimate the rate of technological progress implied by the model using observed macroeconomic data for Japan. We infer the model-implied growth rate of technology from the observable variables in the data.

In the model, technological progress is represented by the growth of a_t. Under the assumption that the economy is on the balanced growth path (BGP), k_t $(= K_t/A_tN_t)$ and y_t $(= Y_t/A_tN_t)$ are constant, and thus the following are true:

$$\begin{aligned} K_{t+1}/A_{t+1}N &= K_t/A_tN_t \\ Y_{t+1}/A_{t+1}N &= Y_t/A_tN_t. \end{aligned}$$

These relations imply that

$$\begin{aligned} A_{t+1}/A_t = (K_{t+1}/K_t)/(N_{t+1}/N_t) \\ A_{t+1}/A_t = (Y_{t+1}/Y_t)/(N_{t+1}/N_t), \end{aligned}$$

Thus, technological progress is not measured directly but inferred from observable long-run relationships implied by the model.

Table 5 summarizes the estimation results for alternative measures of labor supply. Estimates based on the capital-labor ratio were made for the period 2006-2024, and estimates based on GDP per capita were made for the period 2006-2023.[9] While these estimates have their respective advantages and disadvantages, we will adopt the estimate using the working-age population, following the proposal by Fernández-Villaverde et al. (2025). Since

[8] See page 4 of the following document: `https://www.mext.go.jp/component/b_menu/shingi/giji/__icsFiles/afieldfile/2017/07/24/1386653_05.pdf`.

[9] GDP and capital stock data were taken from National Account for 2023 and Quarterly Estimates of Net Capital Stocks of Fixed Assets published by the Cabinet Office. Data from the Labor Force Survey and population estimates published by the Statistics Bureau of the Ministry of Internal Affairs and Communications were used for the number of persons employed, labor force, working-age population (i.e., population aged 15 to 64), and population aged 15 and over, which are measures of labor supply.

Table 5: Estimation of Technological Progress Rates

Measure of labor supply	Estimation from capital-labor ratio			Estimation from GDP per capita		
	min	mean	max	min	mean	max
persons employed	0.988	0.999	1.011	0.958	0.999	1.027
labor force	0.990	1.000	1.011	0.953	1.000	1.027
population aged 15-64	0.983	1.010	1.021	0.961	1.010	1.041
population aged 15+	0.986	1.002	1.011	0.954	1.002	1.027

one period in the model corresponds to 40 years, we need to convert annual growth rates into 40-year equivalents. Specifically, the steady-state value of the technological progress rate a is set as the value obtained by raising the average progress rate (annual rate) for 2006-2023 to the 40th power:

$$a = 1.010^{40} \approx 1.47.$$

The population growth rate b is set as the value obtained by raising the average growth rate (annual rate) of the working-age population for the same period to the 40th power:

$$b = 0.993^{40} \approx 0.75.$$

The implied value of a is consistent with the slow productivity growth observed in Japan. When $a = 1.47$ and $b = 0.75$, the level of rental rate on that path is determined as follows. From (25) and (29), one can derive

$$r_t = \frac{b(1+\beta)a_{t+1}}{\beta} \cdot \frac{k_{t+1}}{q_{t+1}} \cdot \frac{q_{t+1}}{q_t} \left(1 - \frac{1}{\Gamma \Phi^E q_t^{1-\gamma}}\right)^{-1}.$$

Since $q_{t+1}/q_t = 1$ on BGP, substituting the values of a, b, β, k_{t+1}/q_{t+1} and $\Gamma \Phi^E q_t^{1-\gamma}$ into this equation yields

$$r_t \approx 1.4189, \tag{92}$$

implying that the annual net interest rate is 0.88%. This result is consistent with the observation that the level of real interest rates in Japan since 2000 has been lower than 1%.

The estimated rate of technological progress is then used to pin down the value of the innovation productivity parameter, Φ_I. Given the structure of the model, the estimated rate of technological progress uniquely determines the value of Φ_I. These estimates provide the basis for calibrating the innovation-related parameters in the next subsection.

B.3 Parameter Configuration (2)

We now complete the calibration by determining the remaining parameters.

Since there is limited empirical guidance for the values of ϕ_1, ϕ_2, and δ, we set them as in Table 1. The assumed values of ϕ_1 and ϕ_2 mean that the contribution of capital, specialized labor, and the accumulation of past knowledge to innovation is equal. The assumed value of δ means that, after a period of time (40 years), 90% of the current productive knowledge loses its usefulness. These choices are standard in the literature and allow for a balanced contribution of inputs to innovation.

The values of Φ^E, Φ^P and Φ^I are determined as follows. We have already computed the BGP value of w_t^0 as

$$w_t^0 = (1 - 0.451) \times 266.1 + 0.451 \times 194.3 \approx 233.7182,$$

where 0.451, 266.1 and 194.3 are, respectively, the percentage of women in the labor force and the lifetime earnings of high-school-educated men and high-school-educated women in 2023.[10] Moreover, the values of $\Gamma\Phi^E q_t^{1-\gamma}$ and r_t on BGP are given by (85) and (92), respectively. Substituting these values into (24) allows us to solve for the BGP value of q_t as

$$q_t = \frac{w_t^0}{r_t} \Big/ \left(1 - \frac{1}{\Gamma\Phi^E q_t^{1-\gamma}}\right) \approx 208.1675, \tag{93}$$

which, combined with (85), allows us to pin down the value of Φ^E as

$$\Phi^E \approx \frac{4.791}{\Gamma q_t^{1-\gamma}} \approx 0.8196.$$

By substituting the values of w_t^0, α_0, α_1, Λ, $\Gamma\Phi^E q_t^{1-\gamma}$ and q_t into (26), we can also determine the value of Φ^P as

$$\Phi^P = \frac{w_t^0}{\Lambda} \Big/ \left(1 - \frac{1}{\Gamma\Phi^E q_t^{1-\gamma}}\right)^{1-\alpha_0} q_t^{1-\alpha_0-\alpha_1} \approx 138.8168.$$

Eqs.(86) and (93) jointly determine the value of k_t on BGP as

$$k_t \approx 97.5289,$$

which, combined with (91), determines the value of k_t^E on BGP as

$$k_t^E \approx 10.1134.$$

Since $a = 1.47$ on the BGP, the innovation function implies a restriction that pins down the value of Φ^I.

$$\Phi^I = (a - 1 + \delta)/(k_t^E)^{1-\phi_1-\phi_2}(n_t^{1E})^{\phi_1}. \tag{94}$$

[10] See footnote 7.

We have already known the values of δ, ϕ_1, ϕ_2, k_t^E and n_t^{1E}. Substituting these values into (94), we can define the value of Φ^I as

$$\Phi^I \approx 1.6458. \tag{95}$$

C Functional Equivalence between College Tuition Reduction and Subsidizing Higher Education Institutions

Consider an alternative policy in which the government subsidizes a fraction $\theta^o \in (0,1)$ of the operating costs of higher education institutions, financed by a lump-sum tax on young agents. We show that this policy is functionally equivalent to the tuition-reduction policy considered in the main text, in the sense that it yields identical equilibrium conditions and thus identical equilibrium allocations.

When this policy is implemented, the government's budget constraints for period t would be

$$\tau_t^o N_t = \theta^o (w_t^1 A_t N_t^{1E} + r_t K_t^E),$$

where τ_t^o is the lump-sum tax imposed on each agent born in that period. By dividing both sides by $A_t N_t$, we can rewrite this equation as

$$\tilde{\tau}_t^o = \theta^o (w_t^1 n_t^{1E} + r_t k_t^E), \tag{96}$$

where $\tilde{\tau}_t^o \equiv \tau_t^o / A_t$.

This policy modifies (22) as follows:

$$\frac{\Phi^E \tilde{p}_t}{1-\theta^o} = \left(\frac{r_t}{1-\gamma}\right)^{1-\gamma} \left(\frac{w_t^1}{\gamma}\right)^{\gamma}. \tag{97}$$

In contrast, (10)-(12) and (21), which, like (22), are the result of optimization by firms and institutions of higher education, are not affected by this policy. This ensures that the equilibrium conditions (40)-(46) remain unchanged under this policy.

Combined with (2) and $q_t = w_t^1 / r_t$, eq.(97) produces

$$\frac{w_t^0}{r_t} = q_t \left(1 - \frac{1-\theta^o}{\Gamma \Phi^E q_t^{1-\gamma}}\right). \tag{98}$$

By substituting (98) into (97), we obtain

$$r_t = \Lambda \Phi^P \left(1 - \frac{1-\theta^o}{\Gamma \Phi^E q_t^{1-\gamma}}\right)^{-\alpha_0} q_t^{-\alpha_0 - \alpha_1}. \tag{99}$$

Combined with $q_t = w_t^1/r_t$, eqs.(98) and (99) imply that

$$w_t^0 = \Lambda\Phi^P \left(1 - \frac{1-\theta^o}{\Gamma\Phi^E q_t^{1-\gamma}}\right)^{1-\alpha_0} q_t^{1-\alpha_0-\alpha_1} \tag{100}$$

$$w_t^1 = \Lambda\Phi^P \left(1 - \frac{1-\theta^o}{\Gamma\Phi^E q_t^{1-\gamma}}\right)^{-\alpha_0} q_t^{1-\alpha_0-\alpha_1}. \tag{101}$$

Eqs.(2)(100) and (101) jointly produce

$$\tilde{p}_t = \frac{\Lambda\Phi^P}{\Gamma\Phi^E} \left(1 - \frac{1-\theta^o}{\Gamma\Phi^E q_t^{1-\gamma}}\right)^{-\alpha_0} q_t^{\gamma-\alpha_0-\alpha_1},$$

By substituting (42)(44)(99) and (101) into (96), we can obtain

$$\tilde{\tau}_t^o = \frac{\theta^o\alpha_1\Lambda\Phi^P[1-(1-\theta^o)/\Gamma\Phi^E q_t^{1-\gamma}]^{-\alpha_0} q_t^{-\alpha_0-\alpha_1}k_t}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-\gamma)+(1-\gamma)\alpha_1}. \tag{102}$$

The optimization problem for agents under this policy is exactly the same as in the tuition-reduction case, and the values of c_t^y, c_{t+1}^o, and s_t are determined as follows:

$$c_t^y = (w_t^0 A_t - \tau_t^o)/(1+\beta) \tag{103}$$

$$c_{t+1}^o = \beta r_{t+1}(w_t^0 A_t - \tau_t^o)/(1+\beta) \tag{104}$$

$$s_t = \beta(w_t^0 A_t - \tau_t^o)/(1+\beta). \tag{105}$$

Using (105), we can obtain the capital for period $t+1$ as follows:

$$K_{t+1} = s_t N_t = \frac{\beta(w_t^0 A_t - \tau_t^o)N_t}{1+\beta}. \tag{106}$$

Eqs.(100)(102) and (106) jointly produce

$$\begin{aligned} a_{t+1}k_{t+1} &= \frac{\beta(w_t^0 - \tilde{\tau}_t^o)}{b(1+\beta)} \\ &= \frac{\beta\Lambda\Phi^P}{b(1+\beta)} \left(1 - \frac{1-\theta^o}{\Gamma\Phi^E q_t^{1-\gamma}}\right)^{1-\alpha_0} q_t^{1-\alpha_0-\alpha_1} \\ &\quad \times \left\{1 - \frac{\theta^o\alpha_1[1-(1-\theta^o)/\Gamma\Phi^E q_t^{1-\gamma}]^{-1}(k_t/q_t)}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-\gamma)+(1-\gamma)\alpha_1}\right\}. \end{aligned} \tag{107}$$

In equilibrium, k_{t+1} takes a positive value for $\forall t \geq 1$, and so the following must hold:

$$1 - \frac{\theta^o\alpha_1[1-(1-\theta^o)/\Gamma\Phi^E q_t^{1-\gamma}]^{-1}(k_t/q_t)}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-\gamma)+(1-\gamma)\alpha_1} > 0. \tag{108}$$

From (10)(43) and (57), we can derive

$$
\begin{aligned}
n_t^0 &= \frac{\alpha_0}{1-\alpha_0-\alpha_1} \cdot \frac{k_t^P}{w_t^0/r_t} \\
&= \frac{\alpha_0 \Gamma \Phi^E q_t^{-\gamma} k_t}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-\gamma)+(1-\gamma)\alpha_1} \\
&\quad \times \left(1+\frac{1-\gamma-\theta^o}{\Gamma\Phi^E q_t^{1-\gamma}-1+\theta^o}\right).
\end{aligned}
\tag{109}
$$

Plugging (40) and (109) into $n_t^0 + n_t^1 = 1$, we obtain

$$
\begin{aligned}
&\frac{\alpha_1 \Gamma \Phi^E q_t^{-\gamma} k_t}{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-\gamma)+(1-\gamma)\alpha_1} \\
&\times \left[1+\frac{\alpha_0}{\alpha_1}\left(1+\frac{1-\gamma-\theta^o}{\Gamma\Phi^E q_t^{1-\gamma}-1+\theta^o}\right)\right] = 1,
\end{aligned}
\tag{110}
$$

which simplifies (108) as

$$
\Gamma\Phi^E q_t^{1-\gamma} - 1 + (\alpha_0/\alpha_1)(1-\gamma-\theta^o) > 0. \tag{111}
$$

We can now define the equilibrium under this policy as follows.

Definition 3. *An equilibrium of the economy with operation subsidy for higher education institutions is a sequence*

$$
\{(q_t, r_t, w_t^0, w_t^1, \tilde{p}_t, \tilde{\tau}_t^o, a_{t+1}, k_{t+1}, k_t^P, k_t^E, n_t^0, n_t^1, n_t^{1P}, n_t^{1E})\}_{t=1}^{\infty}
$$

generated by (40)-(46)(99)-(102)(107) and (109)-(111), given $k_1 > 0$ *and* $\theta^o \in (0,1)$.

Comparing the equilibrium conditions in this case with those in the tuition-reduction case, we find that they are exactly the same. In particular, all equilibrium prices, allocations, and laws of motion coincide under the two policies, except that the subsidy rate is denoted by θ^o in the former case and by θ in the latter, and the size of the lump-sum tax is denoted by $\tilde{\tau}_t^o$ in the former case and by τ_t in the latter case. Therefore, we can conclude that tuition-reduction policies and operating subsidies to institutions of higher education have exactly the same effect.

The reason why the two policies produce identical outcomes is that both policies effectively reduce the marginal cost of education faced by agents. As a result, they generate the same relative price distortions and therefore induce identical equilibrium behavior.

D Proof of Proposition 6

This appendix provides the proof of Proposition 6. We proceed by analyzing separately the cases corresponding to sufficiently large and small values of k_t, and the intermediate region.

D.1 Proof of Statements (i) and (ii)

The following lemma corresponds to statements (i) and (ii) of Proposition 6. As the lemma shows, if the value of k_t is sufficiently large, a unique solution exists. If the value of k_t is sufficiently small, no solution exists.

Lemma 3. *Suppose that $\theta \in (1-\gamma, 1]$, and let $\underline{q}_3$ and Ω be as defined in (74) and (76), respectively. Then, eq.(67) has a unique solution satisfying $q_t > \underline{q}_3$ if $k_t > \Omega$, and no such solution if*

$$k_t \leq \frac{\alpha_0}{\alpha_0+\alpha_1} \cdot \frac{\theta[1+\alpha_1/\alpha_0+(\alpha_1/\alpha_0)^2]-1+\gamma}{\theta(1+\alpha_1/\alpha_0)-1+\gamma}\Omega. \tag{77}$$

Proof. In this case, θ satisfies $\theta > (1-\gamma)\alpha_0/(\alpha_0+\alpha_1)$, and thus the equilibrium value of q_t must satisfy $q_t > \underline{q}_3$, as shown in Proposition 3. When $\theta > (1-\gamma)\alpha_0/(\alpha_0+\alpha_1)$, moreover, Ω takes a positive finite value, as shown in the proof of Proposition 5.

Before proceeding, we verify that condition (77) is well defined, i.e., that

$$0 < \frac{\alpha_0}{\alpha_0+\alpha_1} \cdot \frac{\theta[1+\alpha_1/\alpha_0+(\alpha_1/\alpha_0)^2]-1+\gamma}{\theta(1+\alpha_1/\alpha_0)-1+\gamma} < 1. \tag{112}$$

The first (left) inequality of (112) is obtained from the fact that, in this case, the following inequalities are true:

$$\theta[1+\alpha_1/\alpha_0+(\alpha_1/\alpha_0)^2]-1+\gamma > \theta(1+\alpha_1/\alpha_0)-1+\gamma > \theta-1+\gamma > 0.$$

To establish the second inequality in (112), rewrite its LHS as

$$\begin{aligned} LHS &= \frac{\theta[1+\alpha_1/\alpha_0+(\alpha_1/\alpha_0)^2]-1+\gamma}{\theta(1+\alpha_1/\alpha_0)^2-(1-\gamma)(1+\alpha_1/\alpha_0)} \\ &= 1 - \frac{(\alpha_1/\alpha_0)(\theta+\gamma-1)}{\theta(1+\alpha_1/\alpha_0)^2-(1-\gamma)(1+\alpha_1/\alpha_0)} < 1, \end{aligned}$$

the inequality of which is obtained from the fact that $\theta+\gamma-1>0$.

Define $g(q_t)$ and $h(q_t)$ as

$$g(q_t) \equiv \frac{(1-\alpha_0-\alpha_1)(\Gamma\Phi^E q_t^{1-\gamma}-\gamma)+(1-\gamma)\alpha_1}{\alpha_1\Gamma\Phi^E q_t^{-\gamma}} \tag{113}$$

$$h(q_t) \equiv 1+\frac{\alpha_0}{\alpha_1}\left(1+\frac{1-\gamma-\theta}{\Gamma\Phi^E q_t^{1-\gamma}-1+\theta}\right). \tag{114}$$

These functions have the following properties:

$$g'(q_t) = \frac{1-\alpha_0-\alpha_1}{\alpha_1 \Gamma \Phi^E q_t^{1-\gamma}} \left\{ \Gamma \Phi^E q_t^{1-\gamma} - \frac{\gamma[\gamma(1-\alpha_0)-\alpha_1]}{1-\alpha_0-\alpha_1} \right\} > 0 \tag{115}$$

$$g''(q_t) = \frac{(1-\gamma)[\gamma(1-\alpha_0)-\alpha_1] q_t^{\gamma-2}}{\alpha_1 \Gamma \Phi^E} \tag{116}$$

$$\lim_{q_t \to \underline{q}_3 + 0} g(q_t) = g(\underline{q}_3) \in (0, +\infty) \tag{117}$$

$$\lim_{q_t \to +\infty} g(q_t) = +\infty \tag{118}$$

$$h'(q_t) = \frac{-(\alpha_0/\alpha_1)(1-\gamma-\theta)(1-\gamma)\Gamma \Phi^E q_t^{-\gamma}}{(\Gamma \Phi^E q_t^{1-\gamma} - 1 + \theta)^2} > 0 \tag{119}$$

$$\begin{aligned} h''(q_t) = & \frac{(\alpha_0/\alpha_1)(1-\gamma-\theta)(1-\gamma)\Gamma \Phi^E q_t^{-\gamma}}{(\Gamma \Phi^E q_t^{1-\gamma} - 1 + \theta)^2} \\ & \times \left(\frac{\gamma}{q_t} + \frac{2\Gamma \Phi^E q_t^{1-\gamma}}{\Gamma \Phi^E q_t^{1-\gamma} - 1 + \theta} \right) < 0 \end{aligned} \tag{120}$$

$$\lim_{q_t \to \underline{q}_3 + 0} h(q_t) = h(\underline{q}_3) \in (0, +\infty) \tag{121}$$

$$\lim_{q_t \to +\infty} h(q_t) = 1 + \alpha_0/\alpha_1 \in (0 + \infty), \tag{122}$$

where the inequality of (115) is obtained from the fact that the following hold true for $\forall q_t \in (\underline{q}_3, +\infty)$:

$$\Gamma \Phi^E q_t^{1-\gamma} - \frac{\gamma[\gamma(1-\alpha_0)-\alpha_1]}{1-\alpha_0-\alpha_1} > \Gamma \Phi^E q_t^{1-\gamma} - \gamma > 0,$$

and the inequalities of (119) and (120) are obtained from the fact that $\theta > 1-\gamma$.

Using $g(q_t)$ and $h(q_t)$, we can rewrite (67) and $k_t > \Omega$ as

$$g(q_t)/k_t = h(q_t) \tag{123}$$

and

$$g(\underline{q}_3)/k_t < h(\underline{q}_3), \tag{124}$$

respectively. The strategy is to study the number of intersections between $g(q_t)/k_t$ and $h(q_t)$. Combined with (118) and (122), eq.(124) implies that the graphs of LHS and RHS of (123) intersect at least once on $(\underline{q}_3, +\infty)$, or that eq.(123) has at least one solution satisfying $q_t \in (\underline{q}_3, +\infty)$. Let q^s be the smallest one of such solutions. Then, q^s must satisfy

$$h'(q^s) < g'(q^s)/k_t. \tag{125}$$

To establish the fact that eq.(67) has a unique solution satisfying $q_t > \underline{q}_3$ when $k_t > \Omega$ is equivalent to showing that eq.(123) has no solution satisfying $q_t > q^s$, or that

$$h(q_t) < g(q_t)/k_t \quad \text{for } \forall q_t \in (q^s, +\infty). \tag{126}$$

To compare the slopes of $g(q_t)/k_t$ and $h(q_t)$, define

$$\begin{aligned}
G(q_t) &\equiv q_t^{\gamma}(\Gamma\Phi^E q_t^{1-\gamma} - 1 + \theta)^2 (g'(q_t)/k_t - h'(q_t)) \\
&= \frac{1-\alpha_0-\alpha_1}{\alpha_1 \Gamma\Phi^E k_t} q_t \left\{ \Gamma\Phi^E q_t^{1-\gamma} - \frac{\gamma[\gamma(1-\alpha_0)-\alpha_1]}{1-\alpha_0-\alpha_1} \right\} \\
&\quad \times \left[\Gamma\Phi^E - (1-\theta)/q_t^{1-\gamma} \right]^2 - (\alpha_0/\alpha_1)(\gamma+\theta-1)(1-\gamma)\Gamma\Phi^E.
\end{aligned}$$

One can verify that G is continuously increasing on $(\underline{q}_3, +\infty)$. Conditions (115) and (116) imply that $g'(q_t)/k_t$ is a continuous function of q_t with

$$\begin{aligned}
\lim_{q_t \to \underline{q}_3 + 0} g'(q_t)/k_t &= g'(\underline{q}_3)/k_t \in (0, +\infty) \\
\lim_{q_t \to +\infty} g'(q_t)/k_t &= (1-\alpha_0-\alpha_1)/\alpha_1 k_t \in (0, +\infty).
\end{aligned}$$

On the other hand, conditions (119) and (120) imply that $h'(q_t)$ is a continuously decreasing function of q_t with

$$\begin{aligned}
\lim_{q_t \to \underline{q}_3 + 0} h'(q_t) &= h'(\underline{q}_3) \in (0, +\infty) \\
\lim_{q_t \to +\infty} h'(q_t) &= 0.
\end{aligned}$$

These properties jointly mean that

$$\begin{aligned}
\lim_{q_t \to \underline{q}_3 + 0} G(q_t) &= G(\underline{q}_3) \in (-\infty, +\infty) \\
\lim_{q_t \to +\infty} G(q_t) &= +\infty.
\end{aligned}$$

When $G(\underline{q}_3) < 0$, equation $G(q_t) = 0$ has a unique solution q^* such that

$$\begin{aligned}
G(q_t) &< 0 \qquad \text{if } q_t \in (\underline{q}_3, q^*) \\
G(q_t) &= 0 \qquad \text{if } q_t = q^* \\
G(q_t) &> 0 \qquad \text{if } q_t \in (q^*, +\infty).
\end{aligned}$$

Since $q_t^{\gamma}(\Gamma\Phi^E q_t^{1-\gamma} - 1 + \theta)^2 > 0$, the above conditions can be reduced to

$$\begin{aligned}
g'(q_t)/k_t &< h'(q_t) \qquad \text{if } q_t \in (\underline{q}_3, q^*) \\
g'(q_t)/k_t &= h'(q_t) \qquad \text{if } q_t = q^* \\
g'(q_t)/k_t &> h'(q_t) \qquad \text{if } q_t \in (q^*, +\infty).
\end{aligned} \tag{127}$$

Combined with (125), condition (127) implies that $q^s \in (q^*, +\infty)$, and that function $g(q_t)/k_t - h(q_t)$ is continuously increasing on $(q^*, +\infty)$. These facts jointly establish (126) for this case. When $G(\underline{q}_3) \geq 0$, we have

$$G(q_t) > 0 \qquad \text{for } \forall q_t \in (\underline{q}_3, +\infty),$$

which can be reduced to

$$g'(q_t)/k_t > h'(q_t) \qquad \text{for } \forall q_t \in (\underline{q}_3, +\infty). \tag{128}$$

This inequality means that $q^s \in (\underline{q}_3, +\infty)$, and that function $g(q_t)/k_t - h(q_t)$ is continuously increasing on $(\underline{q}_3, +\infty)$. These facts jointly establish (126) for this case.

To establish the fact that eq.(67) has no solution satisfying $q_t > \underline{q}_3$ when condition (77) is true, we need to note that this condition is equivalent to $g(\underline{q}_3)/k_t \geq 1+\alpha_0/\alpha_1$. According to (115), $g(q_t)/k_t$ is continuously increasing on $[\underline{q}_3, +\infty)$, and thus the following hold:

$$1 + \alpha_0/\alpha_1 \leq g(\underline{q}_3)/k_t < g(q_t)/k_t \quad \text{for } \forall q_t \in (\underline{q}_3, +\infty). \tag{129}$$

On the other hand, conditions (119) and (122) jointly imply that

$$h(q_t) < 1 + \alpha_0/\alpha_1 \quad \text{for } \forall q_t \in (\underline{q}_3, +\infty). \tag{130}$$

From (129) and (130), we can conclude that

$$g(q_t)/k_t > h(q_t) \quad \text{for } \forall q_t \in (\underline{q}_3, +\infty),$$

or that equation $g(q_t)/k_t = h(q_t)$ has no solution on $(\underline{q}_3, +\infty)$. □

D.2 Proof of Statement (iii)

The next lemma corresponds to statement (iii) of Proposition 6, which argues that when the value of k_t is in the intermediate region, multiple solutions may arise.

Lemma 4. *Suppose that $\theta \in (1-\gamma, 1]$, and that k_t satisfies (78). In addition, let $\underline{q}_3$, $g(q_t)$ and $h(q_t)$ be as defined in (74)(113) and (114), respectively. If*

$$g'(\underline{q}_3)/h'(\underline{q}_3) \geq g(\underline{q}_3)/h(\underline{q}_3),$$

then eq.(67) has no solution satisfying $q_t > \underline{q}_3$. Otherwise, the number of solutions with $q_t > \underline{q}_3$ changes from 0 to 1 to 2 and back to 1 as k_t increases its value from the lower bound to the upper bound of (78).

Proof. Using $\underline{q}_3$, $g(q_t)$ and $h(q_t)$, we can rewrite (78) as

$$g(\underline{q}_3)/(1 + \alpha_0/\alpha_1) < k_t \leq g(\underline{q}_3)/h(\underline{q}_3), \tag{131}$$

which implies that $g(\underline{q}_3)/k_t \geq h(\underline{q}_3)$.

Note that, when $g'(\underline{q}_3)/k_t \geq h'(\underline{q}_3)$, eq.(67) has no solution satisfying $q_t \in (\underline{q}_3, +\infty)$. In this case, the sign of $g'(q_t)/k_t - h'(q_t)$ is positive for $\forall q_t \in (\underline{q}_3, +\infty)$, as shown in the proof of Lemma 3. In particular, from eq.(128) and the preceding argument, the sign of $g'(q_t)/k_t - h'(q_t)$ is positive for all

$q_t \in (\underline{q}_3, +\infty)$. Since, by construction, the sign of $G(q_t)$ coincides with that of $g'(q_t)/k_t - h'(q_t)$, this establishes the claim. Thus, function $g(q_t)/k_t - h(q_t)$ is continuously increasing on $(\underline{q}_3, +\infty)$. Since $g(\underline{q}_3)/k_t \geq h(\underline{q}_3)$, this result implies that

$$g(q_t)/k_t - h(q_t) > 0 \qquad \text{for } \forall q_t \in (\underline{q}_3, +\infty),$$

and thus that eq.(67), which is equivalent to $g(q_t)/k_t - h(q_t) = 0$, has no solution larger than $\underline{q}_3$.

Also note that, when $g'(\underline{q}_3)/k_t < h'(\underline{q}_3)$, the graph of $g(q_t)/k_t - h(q_t)$ is J-shaped, that is, it first decreases and then increases on $(\underline{q}_3, +\infty)$. In this case, the sign of $g'(q_t)/k_t - h'(q_t)$ is determined as (127). Combined with (118) and (122), condition (127) means that the value of $g(q_t)/k_t - h(q_t)$ decreases with $q_t \in (\underline{q}_3, q^*)$, reaches its minimum at $q_t = q^*$, and then increases with $q_t \in (q^*, +\infty)$, finally diverging to a positive infinity. Moreover, one can show that q^* is increasing with k_t, and that the minimum value of $g(q_t)/k_t - h(q_t)$ is decreasing with k_t. Since q^* is implicitly defined by $g'(q^*)/k_t = h'(q^*)$, taking a total derivative of this equation leads to

$$\frac{dq^*}{dk_t} = \frac{h'(q^*)/k_t}{g''(q^*)/k_t - h''(q^*)} > 0,$$

the inequality of which is obtained from condition (127), which implies that $g''(q^*)/k_t - h''(q^*) > 0$. Furthermore, differentiating $g(q^*)/k_t - h(q^*)$ yields

$$\frac{d}{dk_t}\left(\frac{g(q^*)}{k_t} - h(q^*)\right) = -\frac{g(q^*)}{k_t^2} + \left(\frac{g'(q^*)}{k_t} - h'(q^*)\right)\frac{dq^*}{dk_t} = -\frac{g(q^*)}{k_t^2} < 0,$$

the second equality of which is obtained from the fact that $g'(q^*)/k_t = h'(q^*)$.

When $g'(\underline{q}_3)/h'(\underline{q}_3) \geq g(\underline{q}_3)/h(\underline{q}_3)$, any k_t that satisfies (131) also satisfies $g'(\underline{q}_3)/k_t \geq h'(\underline{q}_3)$. Thus, in this case, eq.(67) has no solution satisfying $q_t > \underline{q}_3$.

When $g'(\underline{q}_3)/h'(\underline{q}_3) < g(\underline{q}_3)/h(\underline{q}_3)$, We divide the proof into two cases according to whether

$$1 + \alpha_0/\alpha_1 > h'(\underline{q}_3)$$

or not. When this inequality is true, the following hold:

$$g(\underline{q}_3)/(1 + \alpha_0/\alpha_1) < g'(\underline{q}_3)/h'(\underline{q}_3) < g(\underline{q}_3)/h(\underline{q}_3).$$

Thus, when $k_t \in (g(\underline{q}_3)/(1 + \alpha_0/\alpha_1), g'(\underline{q}_3)/h'(\underline{q}_3)]$, eq.(67) has no solution satisfying $q_t > \underline{q}_3$. When $k_t \in (g'(\underline{q}_3)/h'(\underline{q}_3), g(\underline{q}_3)/h(\underline{q}_3)]$, the graph of $g(q_t)/k_t - h(q_t)$ is J-shaped on $(\underline{q}_3, +\infty)$, as stated above. The question is whether this graph intersects the horizontal axis. Since q^* is an increasing function of k_t, it can be expressed as $q^*(k_t)$. Then, $q^*(k_t)$ satisfies

$$\lim_{k_t \to g'(\underline{q}_3)/h'(\underline{q}_3)+0} q^*(k_t) = \underline{q}_3$$
$$\lim_{k_t \to g(\underline{q}_3)/h(\underline{q}_3)-0} q^*(k_t) = q^*(g(\underline{q}_3)/h(\underline{q}_3)) > \underline{q}_3,$$

which imply that

$$\lim_{k_t \to g'(\underline{q}_3)/h'(\underline{q}_3)+0} \left\{ \frac{g[q^*(k_t)]}{k_t} - h[q^*(k_t)] \right\} = \frac{g(\underline{q}_3)}{g'(\underline{q}_3)/h'(\underline{q}_3)} - h(\underline{q}_3) > 0$$

$$\begin{aligned} \lim_{k_t \to g(\underline{q}_3)/h(\underline{q}_3)-0} \left\{ \frac{g[q^*(k_t)]}{k_t} - h[q^*(k_t)] \right\} &= \frac{g[q^*(g(\underline{q}_3)/h(\underline{q}_3))]}{g(\underline{q}_3)/h(\underline{q}_3)} \\ &\quad - h[q^*(g(\underline{q}_3)/h(\underline{q}_3))] \\ &< \frac{g(\underline{q}_3)}{g(\underline{q}_3)/h(\underline{q}_3)} - h(\underline{q}_3) = 0, \end{aligned}$$

the last inequality of which is obtained from the fact that, when $k_t = g(\underline{q}_3)/h(\underline{q}_3)$, the minimum of $g(q_t)/k_t - h(q_t)$ is attained at $q_t = q^*(g(\underline{q}_3)/h(\underline{q}_3))$. Combined with the fact that $g[q^*(k_t)]/k_t - h[q^*(k_t)]$ is continuously decreasing with $k_t \in (g'(\underline{q}_3)/h'(\underline{q}_3), g(\underline{q}_3)/h(\underline{q}_3)]$, these results jointly indicate the existence of k^* satisfying

$$\frac{g[q^*(k^*)]}{h[q^*(k^*)]} = \frac{g'[q^*(k^*)]}{h'[q^*(k^*)]} = k^* \in \left(\frac{g'(\underline{q}_3)}{h'(\underline{q}_3)}, \frac{g(\underline{q}_3)}{h(\underline{q}_3)} \right]. \tag{132}$$

Thus, on $(\underline{q}_3, +\infty)$, the graph of $g(q_t)/k_t - h(q_t)$ does not intersect the horizontal axis when $k_t \in (g'(\underline{q}_3)/h'(\underline{q}_3), k^*)$, touches it when $k_t = k^*$, intersects it twice when $k_t \in (k^*, g(\underline{q}_3)/h(\underline{q}_3))$, and intersects it once when $k_t = g(\underline{q}_3)/h(\underline{q}_3)$.[11] From the results obtained so far, we can conclude that, in this case, the number of such solutions to (67) that $q_t > \underline{q}_3$ is zero when $k_t \in (g(\underline{q}_3)/(1 + \alpha_0/\alpha_1), k^*)$, one when $k_t = k^*$, two when $k_t \in (k^*, g(\underline{q}_3)/h(\underline{q}_3))$, and one when $k_t = g(\underline{q}_3)/h(\underline{q}_3)$.

When $1 + \alpha_0/\alpha_1 \le h'(\underline{q}_3)$, the following hold:

$$g'(\underline{q}_3)/h'(\underline{q}_3) \le g(\underline{q}_3)/(1 + \alpha_0/\alpha_1) < g(\underline{q}_3)/h(\underline{q}_3),$$

implying that any k_t that satisfies (131) also satisfies $g'(\underline{q}_3)/k_t < h'(\underline{q}_3)$, and thus that the graph of $g(q_t)/k_t - h(q_t)$ is J-shaped on $(\underline{q}_3, +\infty)$. Let k^* be as defined in (132). Then, on $(\underline{q}_3, +\infty)$, the graph of $g(q_t)/k_t - h(q_t)$ does not intersect the horizontal axis when $k_t \in (g(\underline{q}_3)/(1 + \alpha_0/\alpha_1), k^*)$, touches it when $k_t = k^*$, intersects it twice when $k_t \in (k^*, g(\underline{q}_3)/h(\underline{q}_3))$, and intersects it once when $k_t = g(\underline{q}_3)/h(\underline{q}_3)$ if k^* satisfies the following inequalites:

$$g(\underline{q}_3)/(1 + \alpha_0/\alpha_1) < k^* < g(\underline{q}_3)/h(\underline{q}_3). \tag{133}$$

Since the second (right) inequality of (133) is already shown in (132), we only need to show that the first (left) inequality is true. To this end, suppose that

$$k^* \le g(\underline{q}_3)/(1 + \alpha_0/\alpha_1).$$

[11] Even when $k_t = g(\underline{q}_3)/h(\underline{q}_3)$, the graph of $g(q_t)/k_t - h(q_t)$ intersects the horizontal axis twice, but one of the intersection points is $q_t = \underline{q}_3$, which does not satisfy $q_t > \underline{q}_3$.

Then, for any $k_t \in (k^*, g(\underline{q}_3)/(1 + \alpha_0/\alpha_1)]$, the graph of $g(q_t)/k_t - h(q_t)$ intersects the horizontal axis twice, which contradicts Lemma 3, specifically the claim that eq.(67) has no solution satisfying $q_t > \underline{q}_3$ when $k_t \leq g(\underline{q}_3)/(1+\alpha_0/\alpha_1)$. Therefore, the assumption is false, and hence

$$g(\underline{q}_3)/(1 + \alpha_0/\alpha_1) < k^*.$$

This establishes (133) and completes the proof. □

D.3 Completion of the Proof

Lemmas 3 and 4 together establish Proposition 6.