Tohoku University Research Center for Policy Design Discussion Paper

TUPD-2026-001

Buyout Fund and Entrepreneurial Spawning in an Endogenous Growth Model

Zhang Peichang
School of Business Administration, South China University of Technology
Graduate School of Economics and Management, Tohoku University

Jan 2026

TUPD Discussion Papers can be downloaded from:

https://www2.econ.tohoku.ac.jp/~PDesign/dp.html

Buyout Fund and Entrepreneurial Spawning in an Endogenous Growth Model

Zhang Peichang[1,2*]

[1]School of Business Administration, South China University of Technology, Wushan Road, Guangzhou, 510641, Guangdong, China.
[2]Graduate School of Economics and Management, Tohoku University, Aobaku, Sendai, 9808576, Miyagi, Japan.

Corresponding author(s). E-mail(s): zhang.peichang.p1@alumni.tohoku.ac.jp;

Abstract

This paper develops an endogenous growth model featuring income-dependent risk preferences to explain the emergence and evolution of buyout funds. We propose a novel preference structure where high-income agents derive utility from the thrill of entrepreneurial risk-taking, leading them to acquire business ideas from capital-constrained innovators. The model demonstrates that buyout funds emerge as equilibrium contracts when income inequality exceeds a critical threshold, with wealthy investors paying premiums to participate in ventures. Conversely, in more equal economies, buyout funds serve as transitional institutions. Initial inequality enables the acquisition of ideas, but subsequent growth allows the original idea holders to become independent entrepreneurs, leading to the fund's eventual decline. Our framework provides microfoundations for understanding how income distribution shapes financial intermediation patterns and their growth consequences, offering new insights into the relationship between inequality, entrepreneurial spawning, and innovation-driven growth.

Keywords: Buyout Fund, Income Inequality, Entrepreneurial Spawning, Endogenous Growth, Risk Preferences

1 Introduction

Private equity funds account for a growing share of real investment in the modern economy, accompanied by increasing scholarly interest in their nature and effects. A well-established empirical literature demonstrates that private equity significantly contributes to innovation and economic growth (Kortum and Lerner 2000; Samila and Sorenson 2011; Audretsch et al. 2016). Notably, Jovanovic et al. (2022) distinguish between two fundamental components: venture capital, which provides financing to startups and early-stage growth firms, and buyout funds, which acquire existing firms, optimize their operations, and subsequently exit through sales to other investors or public markets. Their estimates indicate these private equity sectors collectively contribute between 14 and 21 percent of U.S. economic growth relative to a counterfactual without private equity.

Despite these significant contributions, theoretical research embedding private equity within macroeconomic frameworks remains underdeveloped, particularly regarding buyout funds. Existing studies have focused predominantly on venture capital, examining its procyclical nature (Opp 2019), the role of non-diversified agents' hedging motives (Petukhov 2019), and its relationship to growth through dynamic contracts (Greenwood et al. 2022a). This paper aims to fill the critical void in understanding buyout funds' origins and their relationship with economic growth.

A fundamental challenge in modeling private equity is the "Private Equity Premium Puzzle" (Moskowitz and Vissing-Jorgensen 2002). Empirical evidence consistently shows that private equity investments yield risk-adjusted returns no higher than public equities despite being highly concentrated and poorly diversified (Hamilton 2000; Cochrane 2005; Shane 2009; Hall and Woodward 2010; Harris et al. 2014; Gupta and Van Nieuwerburgh 2019). Traditional expected utility frameworks cannot explain why risk-averse investors would accept such inefficient portfolios. Crucially, survey evidence reveals that private equity suppliers are predominantly ultra-high-net-worth individuals (Rin et al. 2013).

Inspired by these findings, we resolve this puzzle by introducing agents whose preferences shift when income exceeds a threshold: they exhibit standard concave utility for safe investments but develop convex utility for risky entrepreneurial ventures above an income threshold. This preference structure generates several empirically observed behaviors. At low income levels, agents rely exclusively on safe savings methods. As income crosses the threshold, they develop concentrated positions in risky investments without diversification, since diversification would reduce the convexity-driven utility they derive from these investments. Most importantly, they begin seeking the "thrill" of entrepreneurial activities despite suboptimal financial returns, consistent with patterns documented by Campbell (2017). This approach aligns with explanations based on nonpecuniary benefits (Moskowitz and Vissing-Jorgensen 2002; Hurst and Pugsley 2015).[1]

Our preference specification relates to recent work on heterogeneous risk attitudes in macroeconomic settings. Notably, Araujo et al. (2025) demonstrates how risk-loving behavior at the wealth extremes can generate fat-tailed wealth distributions,

[1]Alternative explanations are surveyed in Astebro et al. (2014).

while Araujo et al. (2018) presents a general equilibrium framework with uncertainty-loving preferences. Unlike these studies that focus on exogenously given preference heterogeneity, our model endogenizes risk-loving behavior through income-dependent utility curvature. Specifically, we reconcile Araujo's observation of risk-loving wealthy investors with microfoundations: convex utility over risky assets emerges only above an income threshold, creating endogenous heterogeneity that varies with economic conditions. This mechanism complements their findings while providing new insights into how endogenous preference formation affects financial intermediation patterns.

The model features two distinct agent types differentiated by their endowments: wealthy agents who lack business ideas and poorer agents who possess them. These agents engage in Nash bargaining to establish cooperative ventures. Our modified risk preferences yield a crucial insight: the optimal contract naturally takes the form of a buyout fund. [2] Wealthy investors, motivated by the desire for entrepreneurial engagement, actively seek to acquire business opportunities, offering premiums proportional to their income. Entrepreneurs will only sell their ideas when acquisition prices adequately compensate for relinquished control. Consequently, buyout funds emerge exclusively when income inequality exceeds a certain threshold, and they disappear as incomes increase.

This dynamic mirrors real-world entrepreneurial spawning processes (Gompers et al. 2005). Initially, idea owners sell to funds due to capital constraints or attractive offers. However, growth-induced wage increases alter reservation utilities in bargaining. As entrepreneurs gain financial capacity and bargaining power, they retain ideas for independent implementation—the "Xerox effect" occurs precisely when inequality is moderate. Thus, buyout funds serve as transitional institutions intrinsically linked to income inequality.

The model provides valuable insights into the role of buyout funds in driving economic growth. By engaging wealthy investors in innovation processes, they increase the probability of R&D success. These successes expand the variety of intermediate goods in the economy, accelerating growth. Perhaps more significantly, the resulting wage increases enable previously constrained entrepreneurs to implement their ideas independently, creating a virtuous cycle of growth. This mechanism suggests that initial inequality can serve as a catalyst for sustained growth, with buyout funds acting as the transmission channel.

These findings make important contributions to the literature on inequality and growth. Earlier studies identified positive relationships through various channels, including physical capital investment (Galor and Moav 2004), and demand-driven innovation (Foellmi and Zweimüller 2006). Our model establishes a novel financial intermediation channel that operates differently from these established mechanisms. The buyout funds in our framework emerge as institutions whose existence depends on inequality levels. They directly accelerate variety expansion through increased innovation success rates, while their eventual decline through endogenous wage convergence

[2] In this paper, the term "buyout" is used to describe a setting in which a well-endowed agent acquires control rights over a *new entrepreneurial idea* (corresponding to an unlaunched variety) from an under-endowed idea holder. While the mechanism differs in emphasis from the conventional use of "buyout" in applied contexts, it shares important features with early-stage investment arrangements (such as angel financing or venture capital) in which better-resourced investors take over nascent projects and shape their development.

triggers widespread entrepreneurial spawning. This dual-phase process represents a distinct contribution to understanding how buyouts can temporarily stimulate R&D-driven growth while simultaneously creating conditions for their own reduction.

The remainder of this paper proceeds as follows. Section 2 presents the core model framework and details agent preferences. It derives the optimization problems for both well-endowed and under-endowed agents and characterizes the Nash bargaining solution that governs their interactions. Section 3 analyzes the resulting temporary equilibrium. Section 4 investigates the model's long-run dynamics, establishing conditions for the persistence or disappearance of the buyout fund. Section 5 discusses growth and policy implications. Section 6 provides numerical transition paths and sensitivity analysis. Section 7 concludes.

2 The Model

2.1 Agents and Preferences

We propose an overlapping generations economy populated by a continuum of agents who live for two periods. Each cohort has measure $L > 0$ and is born at the beginning of period t. Agents are of two types: a fraction $\theta \in (0, 1/2)$ are *well-endowed* and the remaining $1 - \theta$ are *under-endowed*. Preferences are discounted with factor $\beta \in (0, 1)$, entrepreneurial success is governed by a technological parameter $p \in (0, 1)$, and the curvature of the non-pecuniary payoff from entrepreneurial success is $\alpha > 1$.

In the first period of life, an under-endowed agent receives $e^u > 0$ units of labor and is endowed with a business idea, while a well-endowed agent receives $e^w \geq e^u$ units of labor but no business concept. These labor endowments satisfy

$$\theta e^w + (1 - \theta)e^u = \bar{e}, \tag{1}$$

where $\bar{e}$ denotes average labor supply per cohort. We impose

$$0 < e^u \leq \bar{e} \leq e^w < \bar{e}/\theta,$$

and assume labor is supplied inelastically. The restriction $\theta < 1/2$ reflects the empirical reality that ultra-high-net-worth individuals are fewer than idea-rich but capital-constrained entrepreneurs and simplifies the bargaining analysis.[3]

Young agents can transfer resources to old age through two channels. First, they can carry final goods to the next period as risk-free capital loans to incumbent firms. Second, they can invest in entrepreneurial startups that create new varieties: a project yields monopoly profits if successful and zero otherwise. Let I_t denote total entrepreneurial investment in period t. The success probability is

$$p(I_t) \;=\; \max\left\{0,\; p - \frac{1}{I_t}\right\},$$

[3] The analysis of the case $\theta \geq 1/2$ is essentially analogous and does not provide additional insight.

which implies a minimum viable scale $I_t > 1/p$ for entrepreneurship.[4]

A type-$i \in \{w, u\}$ agent born in period t chooses first-period consumption c_t^i, savings s_t^i, and entrepreneurial investment $I_t^i \in [0, s_t^i]$. Utility is

$$\ln c_t^i + \beta \left[\ln\left(r_{t+1}(s_t^i - I_t^i)\right) + p_t(R_{t+1}^E)^\alpha\right], \quad \alpha > 1, \tag{2}$$

where r_{t+1} is the rental rate of capital in period $t+1$ and R_{t+1}^E is the entrepreneurial return (monopoly profit, specified below).

Equation (2) separates the utility from safe intertemporal transfer, $\ln\bigl(r_{t+1}(s_t^i - I_t^i)\bigr)$, from a non-pecuniary component tied to entrepreneurial success, $p_t(R_{t+1}^E)^\alpha$. This reduced-form formulation is related to (but distinct from) the way non-pecuniary entrepreneurial motives are typically modeled in the empirical literature. In particular, Hurst and Pugsley (2015) and Jones and Pratap (2020) rationalize entrepreneurial choices by augmenting standard utility from consumption (or income) with a *direct* non-pecuniary payoff from operating one's own business.[5] Our approach differs in that we do not introduce an additive taste shifter; instead, we represent these motives in reduced form by allowing entrepreneurial success to enter utility with convex curvature, $p_t(R_{t+1}^E)^\alpha$. This delivers similar qualitative implications—entrepreneurship can be attractive even when expected pecuniary returns are modest—while remaining tractable for our general-equilibrium analysis of buyouts and portfolio concentration.

The key curvature implications of (2) are that the objective is *concave* in the safe component and *convex* in the entrepreneurial payoff. Specifically, the safe term $\ln\bigl(r_{t+1}(s_t^i - I_t^i)\bigr)$ is concave in I_t^i (reflecting increasing marginal opportunity costs of diverting resources away from safe saving), whereas the entrepreneurial-success term is convex in R_{t+1}^E when $\alpha > 1$.[6]

Separability between the safe and entrepreneurial components is important for our mechanism and comparative statics because it enables a clean comparison of marginal returns to risky versus safe investment across the wealth distribution. In particular, the marginal utility cost of increasing entrepreneurial investment through the safe term is

$$\frac{\partial}{\partial I_t^i} \ln\bigl(r_{t+1}(s_t^i - I_t^i)\bigr) \;=\; -\frac{1}{s_t^i - I_t^i},$$

which decreases in wealth s_t^i. Thus, for high-income the marginal value of allocating an additional unit to safe investment is relatively lower, making it more likely that the marginal gain from entrepreneurial exposure—operating through $p_t'(I_t^i)(R_{t+1}^E)^\alpha$—dominates. The combination of (i) separability between safe and entrepreneurial components and (ii) convex utility in the entrepreneurial payoff predicts *increasing*

[4] The introduced threshold and saturation point are not essential for our main purpose. Appendix C discusses alternative, smoother specifications that deliver the same main comparative statics.

[5] Specifically, $u(\cdot) + \gamma \cdot \mathbf{1}\{\text{operate own business}\}$, where $\gamma > 0$ captures autonomy, control, and other intrinsic benefits.While their interpretation emphasizes the "control motive," we term the utility from the act of risk-taking itself as the "thrill" motive. Both interpretation could be relevant though we emphasize the thrill motive.

[6] We discuss the precise domain over which convexity is operative when deriving the investment problem's first-order conditions and characterizing optimality in section 2.4.2.

portfolio concentration in risky private investments with wealth: as safe marginal utility declines with wealth, high-wealth agents should optimally tilt toward fewer, larger, and more concentrated positions in entrepreneurial assets. [7]

2.2 Production

The final good, serving as the numeraire in the economy, is produced through combining labor with a diverse range of intermediate goods. The production technology follows a constant returns to scale specification:

$$Y_t = \frac{1}{1-\zeta}\left(\int_0^{N_t} x_{it}^{1-\zeta} di\right) L_t^{\zeta}, \quad \zeta \in (0,1) \tag{3}$$

where x_{it} represents the quantity of intermediate good i utilized in production, while L_t denotes the aggregate labor input.

Under the labor market equilibrium condition where $L_t = L\bar{e}$, we can derive the inverse demand functions for intermediate goods along with the competitive wage rate. The price of intermediate good i responds to its quantity according to:

$$q_{it} = x_{it}^{-\zeta}(L\bar{e})^{\zeta} \tag{4}$$

while the wage rate determination incorporates all intermediate goods:

$$w_t = \frac{\zeta}{1-\zeta}\left(\int_0^{N_t} x_{it}^{1-\zeta} di\right)(L\bar{e})^{\zeta-1} \tag{5}$$

The economy features two distinct regimes for intermediate goods production. For existing varieties indexed $i \in [0, N_{t-1}]$, production occurs competitively with a one-to-one transformation from capital goods that fully depreciate each period. The supply function for these established varieties takes the form:

$$x_{it} = \left(\frac{1}{r_t}\right)^{1/\zeta} L\bar{e} \tag{6}$$

In contrast, newly invented varieties indexed $i \in (N_{t-1}, N_t]$ benefit from one-period patent protection, granting inventors temporary monopoly power. The optimal supply decision for patent holders yields:

$$x_{it} = \left(\frac{1-\zeta}{r_t}\right)^{1/\zeta} L\bar{e} \tag{7}$$

This monopolistic supply generates profits for each new variety inventor, with the profit flow given by:

$$R_t^E = \zeta\left(\frac{1-\zeta}{r_t}\right)^{(1-\zeta)/\zeta} L\bar{e} \tag{8}$$

[7] This can be tested empirically by, for example, relating the number of private deals held and position sizes to investors' wealth, which we leave for future empirical work. Appendix D discuss how strong the convexity need to be to prevent investors from diversifying across business ideas.

Combining these elements, the economy's factor price frontier emerges as:

$$w_t^{\zeta} r_t^{1-\zeta} = \left\{ \frac{\zeta}{1-\zeta} \left[N_{t-1} + (N_t - N_{t-1})(1-\zeta)^{(1-\zeta)/\zeta} \right] \right\}^{\zeta} \tag{9}$$

2.3 Initial Conditions

The economy commences with specific initial conditions. A continuum of old agents of measure L exists, each endowed with k_1 units of the capital good. These agents supply their entire capital endowment in the first period, after which the capital fully depreciates and the agents exit the economy. Notably, the model assumes no innovation occurs in the initial period, maintaining the variety count constant at $N_1 = N_0 = N > 0$.

2.4 Optimization under Self-Sufficiency

First, consider optimization when there is no match between well-endowed and under-endowed agents.

2.4.1 Well-Endowed Agents

Since well-endowed agents do not possess business ideas, they face a standard consumption-savings problem subject to the constraints $0 < s_t^w < w_t e^w$ and $I_t^w = 0$. Their optimization problem is:

$$\max_{s_t^w, I_t^w} \ln(w_t e^w - s_t^w) + \beta \ln r_{t+1}(s_t^w - I_t^w)$$

The solution yields optimal savings with no entrepreneurial investment:

$$(s_t^w, I_t^w) = \left[\frac{\beta w_t e^w}{1+\beta}, 0 \right]. \tag{10}$$

The corresponding indirect utility is:

$$u_t^{wr} = (1+\beta) \ln w_t e^w + \beta \ln r_{t+1} - \ln \frac{(1+\beta)^{1+\beta}}{\beta^{\beta}}. \tag{11}$$

2.4.2 Under-Endowed Agents

Under-endowed agents with business ideas solve a more complex problem subject to $0 < s_t^u < w_t e^u$ and $0 \leq I_t^u < s_t^u$. Their decision depends on wage levels as shown in Lemma 1.

Lemma 1 The under-endowed agents' optimal (s_t^u, I_t^u) choices follow two regimes: (a)When $w_t \leq 1/pe^u$:

$$(s_t^u, I_t^u) = \left[\frac{\beta w_t e^u}{1+\beta}, 0 \right] \tag{12}$$

(b)When $w_t > 1/pe^u$:

$$(s_t^u, I_t^u) = \begin{cases} \left[\dfrac{(I_t^u)^* + \beta w_t e^u}{1+\beta}, (I_t^u)^*\right] & \text{if } 0 \leq r_{t+1} \leq r^u(w_t) \\ \left[\dfrac{\beta w_t e^u}{1+\beta}, 0\right] & \text{if } r^u(w_t) \leq r_{t+1} \end{cases} \tag{13}$$

where the optimal investment is:

$$(I_t^u)^* = \frac{2 w_t e^u}{1 + \sqrt{1 + \frac{4 w_t e^u (1+\beta)}{\beta (R_{t+1}^E)^\alpha}}} \tag{14}$$

and $r^u(w_t)$ is a continuously increasing function of w_t ($\in [1/pe^u, +\infty)$) with $r^u(1/pe^u) = 0$ defined implicitly by:

$$\ln\left[1 - \frac{(I_t^u)^*}{w_t e^u}\right] + \frac{\beta}{1+\beta}\left[p - \frac{1}{(I_t^u)^*}\right](R_{t+1}^E)^\alpha = 0. \tag{15}$$

Proof See Appendix A. □

The indirect utility differs by case. When no entrepreneurship occurs ($w_t \leq 1/pe^u$ or $r^u(w_t) \leq r_{t+1}$):

$$u_t^{ur} = (1+\beta)\ln w_t e^u + \beta \ln r_{t+1} - \ln\frac{(1+\beta)^{1+\beta}}{\beta^\beta}; \tag{16}$$

With entrepreneurship ($w_t > 1/pe^u$ and $0 \leq r_{t+1} \leq r^u(w_t)$):

$$\begin{aligned} u_t^{ur} = (1+\beta)\ln[w_t e^u - (I_t^u)^*] + \beta\left[p - \frac{1}{(I_t^u)^*}\right](R_{t+1}^E)^\alpha \\ +\beta \ln r_{t+1} - \ln\frac{(1+\beta)^{1+\beta}}{\beta^\beta}. \end{aligned} \tag{17}$$

These indirect utilities serve as threat points in the Nash bargaining analysis. Moreover, note that the curvature of the reservation utility in R_{t+1}^E is regime-dependent. Only in the entrepreneurial regime—i.e., when $w_t > 1/(pe^u)$ and $0 \leq r_{t+1} < r^u(w_t)$—does the under-endowed agent choose a strictly positive entrepreneurial investment $(I_t^u)^* > 0$. [8]In that region, the indirect utility u_t^{ur} is strictly convex in the entrepreneurial payoff R_{t+1}^E. The reason is that, by the envelope theorem, changes in R_{t+1}^E affect the value function only through the direct term $\beta\big(p - 1/(I_t^u)^*\big)(R_{t+1}^E)^\alpha$, while the induced changes in the optimal choice $(I_t^u)^*$ do not enter the derivative.[9] Intuitively, this convexity becomes relevant only when income is high enough for entrepreneurship to be feasible and privately optimal.

[8]We view the implied threshold behavior as a tractable approximation to a steeper region of a smoother underlying transition in tastes or effective risk attitudes, rather than as a literal discontinuity.

[9]Formally, in the entrepreneurial regime the envelope theorem yields $\partial u_t^{ur}/\partial R_{t+1}^E = \beta\big(p - 1/(I_t^u)^*\big)\alpha(R_{t+1}^E)^{\alpha-1}$, and hence $\partial^2 u_t^{ur}/\partial(R_{t+1}^E)^2 = \beta\big(p - 1/(I_t^u)^*\big)\alpha(\alpha-1)(R_{t+1}^E)^{\alpha-2} > 0$ for $\alpha > 1$ and $R_{t+1}^E > 0$, since $(I_t^u)^* > 1/p$ implies $p - 1/(I_t^u)^* > 0$. At the threshold $r_{t+1} = r^u(w_t)$ (or when $w_t \leq 1/(pe^u)$), entrepreneurship is not chosen and the indirect utility is independent of R_{t+1}^E, so convexity is not operative.

2.5 Nash Bargaining

When a well-endowed agent matches with an under-endowed agent to implement a business idea, their agreement solves the following optimization problem:

$$\max_{s_t^w, s_t^u, I_t^w, I_t^u, R_{t+1}^w} \ln(w_t e^w - s_t^w) + \beta \left[\ln r_{t+1}(s_t^w - I_t^w) + \left(p - \frac{1}{I_t^w + I_t^u} \right) (R_{t+1}^w)^\alpha \right]$$

subject to the participation constraint for under-endowed agents:

$$\ln(w_t e^u - s_t^u) + \beta \left[\ln r_{t+1}(s_t^u - I_t^u) + \left(p - \frac{1}{I_t^w + I_t^u} \right) (R_{t+1}^E - R_{t+1}^w)^\alpha \right] = u_t^{ur} \quad (18)$$

and the limited liability condition:

$$0 \le R_{t+1}^w \le R_{t+1}^E \quad (19)$$

where R_{t+1}^w represents the well-endowed agent's profit share. Two crucial features characterize this bargaining solution. First, the participation constraint implies that under-endowed agents receive exactly their reservation utility u_t^{ur}, reflecting their lack of bargaining power. This outcome stems from competition among the measure $(1 - 2\theta)L$ of unmatched under-endowed agents who could potentially make better offers. [10] Second, the optimal profit allocation R_{t+1}^w equals R_{t+1}^E when solutions exist, as $\alpha > 1$ implies corner solutions and $R_{t+1}^w = 0$ would leave well-endowed agents no better off than autarky.

The problem can be reformulated using the variable transformations $\tilde{s}_t^w \equiv s_t^w + I_t^u$, $\tilde{s}_t^u \equiv s_t^u - I_t^u$, $\tilde{I}_t^w \equiv I_t^w + I_t^u$, and $a_t \equiv -I_t^u$:

$$\max_{\tilde{s}_t^w, \tilde{s}_t^u, \tilde{I}_t^w, a_t} \ln(w_t e^w - a_t - \tilde{s}_t^w) + \beta \left[\ln r_{t+1}(\tilde{s}_t^w - \tilde{I}_t^w) + \left(p - \frac{1}{\tilde{I}_t^w} \right) (R_{t+1}^E)^\alpha \right]$$

subject to:

$$\ln(w_t e^u + a_t - \tilde{s}_t^u) + \beta \ln r_{t+1} \tilde{s}_t^u = u_t^{ur} \quad (20)$$

The solution yields optimal policies:

$$\tilde{s}_t^u = \frac{\beta(w_t e^u + a_t)}{1+\beta} \quad (21)$$

$$\tilde{s}_t^w = \frac{\tilde{I}_t^w + \beta(w_t e^w - a_t)}{1+\beta} \quad (22)$$

$$\tilde{I}_t^w = \frac{2(w_t e^w - a_t)}{1 + \sqrt{1 + \frac{4(1+\beta)(w_t e^w - a_t)}{\beta (R_{t+1}^E)^\alpha}}} \quad (23)$$

---

[10] In Appendix B, we consider generalized Nash bargaining in which under-endowed agents are assigned bargaining weight $\phi \in (0, 1)$.

Lemma 2 For $\theta < 1/2$, the optimal transfer a_t follows:(a) When $w_t \leq 1/pe^u$ or when $w_t > 1/pe^u$ and $r^u(w_t) \leq r_{t+1}$:

$$a_t = 0 \tag{24}$$

(b) When $w_t > 1/pe^u$ and $0 \leq r_{t+1} \leq r^u(w_t)$:

$$a_t = \left[w_t e^u - (I_t^u)^*\right] \exp\left\{\frac{\beta}{1+\beta}\left[p - \frac{1}{(I_t^u)^*}\right](R_{t+1}^E)^\alpha\right\} - w_t e^u, \tag{25}$$

where $(I_t^u)^*$ and $r^u(\cdot)$ are defined in Lemma 1.

Proof See Appendix A. □

Substituting these results into the optimal policies reveals that well-endowed agents minimize idea acquisition costs, sometimes obtaining business ideas for free when condition (24) holds. This outcome reflects the competitive pressure from unmatched under-endowed agents in the market for ideas.

3 Temporary Equilibrium

The temporary equilibrium in period $t \geq 2$ depends on the entrepreneurial activities in period $t-1$. We characterize equilibria through three distinct regimes based on the composition of innovators in the previous period. [11]

3.1 Equilibrium Characterization

The capital market clears when aggregate demand K_t^D equals aggregate supply K_t^S. Capital demand comprises the capital required for producing both established and new intermediate goods:

$$K_t^D = N_{t-1}\left(\frac{1}{r_t}\right)^{1/\zeta} L\bar{e} + (N_t - N_{t-1})\left(\frac{1-\zeta}{r_t}\right)^{1/\zeta} L\bar{e}. \tag{26}$$

Capital supply originates from the net savings of the young cohort born in period $t-1$:

$$K_t^S = \theta L \cdot (s_{t-1}^w - I_{t-1}^w) + (1-\theta)L \cdot (s_{t-1}^u - I_{t-1}^u).$$

Using the optimal savings policies $s_{t-1}^i - I_{t-1}^i = \frac{\beta}{1+\beta}(w_{t-1}e^i - I_{t-1}^i)$ for $i = u, w$, we obtain the aggregate capital supply:

$$K_t^S = \frac{\beta}{1+\beta}\left[w_{t-1}\bar{e} - \mathcal{I}_{t-1}\right] L, \tag{27}$$

where $\mathcal{I}_{t-1} = \theta \cdot I_{t-1}^w + (1-\theta) \cdot I_{t-1}^u$, varies across three regimes:

[11] The two-period OLG structure keeps the state variable compact and makes the link from cohort decisions to next-period factor prices explicit, which allows us to characterize buyout regimes and inequality thresholds analytically. We view this as a tractable benchmark: the core mechanism—wealth-dependent willingness to pay for concentrated entrepreneurial implementation and the resulting reallocation and entry effects—does not rely on two-period lives, but would carry over to richer infinite-horizon heterogeneous-agent environments at the cost of additional state variables and numerical solution methods.

(a) **No Entrepreneurship**: $\mathcal{I}_{t-1} = 0$ and $N_t = N_{t-1}$.
(b) **Under-endowed Only**: $\mathcal{I}_{t-1} = (1-\theta)(I^u_{t-1})^*$ with new varieties:

$$N_t - N_{t-1} = (1-\theta)\left[p - \frac{1}{(I^u_{t-1})^*}\right] L. \tag{28}$$

(c) **Matched Pairs** ($\theta < 1/2$): The composition of entrepreneurial investment depends on whether unmatched under-endowed agents find independent innovation optimal under the equilibrium interest rate yield by this regime. When $r_t \le r^u(w_{t-1})$:

$$\mathcal{I}_{t-1} = \theta \tilde{I}^w_{t-1} + (1-2\theta)(I^u_{t-1})^*, \tag{29}$$

with new variety creation:

$$N_t - N_{t-1} = \left[\theta\left(p - \frac{1}{\tilde{I}^w_{t-1}}\right) + (1-2\theta)\left(p - \frac{1}{(I^u_{t-1})^*}\right)\right] L. \tag{30}$$

Otherwise, only matched pairs innovate: $\mathcal{I}_{t-1} = \theta \tilde{I}^w_{t-1}$ and $N_t - N_{t-1} = \theta\left(p - \frac{1}{\tilde{I}^w_{t-1}}\right) L$.

3.2 Market Clearing Conditions

Each regime yields a distinct market-clearing condition:
(a)**No Entrepreneurship**:

$$N_{t-1}\left(\frac{1}{r_t}\right)^{1/\zeta} = \frac{\beta}{1+\beta} w_{t-1}. \tag{31}$$

(b)**Under-endowed Only**:

$$N_{t-1}\left(\frac{1}{r_t}\right)^{1/\zeta} + (1-\theta)\left[p - \frac{1}{(I^u_{t-1})^*}\right]\left(\frac{1-\zeta}{r_t}\right)^{1/\zeta} L = \frac{\beta}{1+\beta}\left[w_{t-1} - (1-\theta)\frac{(I^u_{t-1})^*}{\bar{e}}\right]. \tag{32}$$

(c)**Matched Pairs**: When unmatched under-endowed agents innovate independently:

$$N_{t-1}\left(\frac{1}{r_t}\right)^{1/\zeta} + (N_t - N_{t-1})\left(\frac{1-\zeta}{r_t}\right)^{1/\zeta} L = \frac{\beta}{1+\beta}\left[w_{t-1} - \frac{\theta \tilde{I}^w_{t-1} + (1-2\theta)(I^u_{t-1})^*}{\bar{e}}\right]. \tag{33}$$

where $N_t - N_{t-1}$ is given in (30). Otherwise:

$$N_{t-1}\left(\frac{1}{r_t}\right)^{1/\zeta} + (N_t - N_{t-1})\left(\frac{1-\zeta}{r_t}\right)^{1/\zeta} L = \frac{\beta}{1+\beta}\left[w_{t-1} - \frac{\theta \tilde{I}^w_{t-1}}{\bar{e}}\right]. \tag{34}$$

and $N_t - N_{t-1} = \theta \left(p - \frac{1}{\tilde{I}^w_{t-1}} \right) L$.

In each regime $j \in \{(a), (b), (c)\}$, substituting the regime-specific investment rules into the corresponding capital market clearing condition yields an excess-demand function $E^j_{t-1}(r_t)$ in the single unknown r_t. For fixed (N_{t-1}, w_{t-1}) and parameters, individual investment rules are continuous in (r_{t+1}, R^E_{t+1}) and strictly decreasing in r_t, whereas incremental variety $N_t - N_{t-1}$ is either 0 or strictly decreasing in r_t. It follows that, within each regime, $E^j_{t-1}(r_t)$ is continuous and strictly decreasing in r_t and therefore admit at most one solution to $E^j_{t-1}(r_t) = 0$. In Section 6, we provide an algorithm to pin down the temporary equilibrium and simulate the transition path.

4 Long-Run Economic States

The economy can converge to three distinct long-run states, each characterized by different patterns of innovation and financial contracts. These states emerge endogenously from the model's dynamics and have markedly different implications for growth trajectories.

4.1 No-Innovation Steady State

In this stagnant equilibrium, the variety of intermediate goods stabilizes at some finite level $\overline{N}$. The economy reaches a stationary configuration where two key relationships govern factor prices. First, the factor price frontier simplifies to a static relationship between normalized wages and interest rates:

$$\left(\frac{\overline{w}}{\overline{N}} \right)^{\zeta} \overline{r}^{1-\zeta} = \left(\frac{\zeta}{1-\zeta} \right)^{\zeta} \tag{35}$$

Second, capital market clearing imposes a direct connection between these variables:

$$\left(\frac{1}{\overline{r}} \right)^{1/\zeta} = \frac{\beta}{1+\beta} \frac{\overline{w}}{\overline{N}} \tag{36}$$

These conditions jointly pin down the steady state values of the interest rate and the wage-to-variety ratio:

$$\overline{r} = \frac{1+\beta}{\beta} \cdot \frac{1-\zeta}{\zeta}, \quad \frac{\overline{w}}{\overline{N}} = \left(\frac{\beta}{1+\beta} \right)^{(1-\zeta)/\zeta} \left(\frac{\zeta}{1-\zeta} \right)^{1/\zeta} \tag{37}$$

Whether the economy reaches this state depends crucially on initial conditions. When the initial variety of goods N_0 exceeds a critical threshold $\overline{N}^*$, convergence to this stagnant equilibrium becomes impossible, as established in the next lemma:

Lemma 3 Let $\overline{w}$, $\overline{N}$ and A be, respectively, the values of w_t, N_t and w_t/N_t in the state of no-innovation. Then, there is a positive constant $\overline{N}^*$ such that

$$\overline{N} \leq \overline{N}^* \qquad \text{and} \qquad \overline{w} \leq A\overline{N}^*. \tag{38}$$

Proof See Appendix A. □

4.2 Sustained Innovation without Buyouts

When innovation persists exclusively through under-endowed entrepreneurs, the economy exhibits several characteristic growth patterns. The variety of intermediate goods grows without bound, with $N_t \to \infty$ as $t \to \infty$. Despite this growth, the share of income that entrepreneurs devote to innovation expenditures vanishes asymptotically, with $(I_t^u)^*/w_t \to 0$.

The economy settles into a balanced growth path where the long-run growth rate tends to zero, and factor prices maintain stable relationships. The interest rate converges to the same value found in the stagnant equilibrium:

$$\lim_{t\to\infty} r_t = \frac{1+\beta}{\beta} \cdot \frac{1-\zeta}{\zeta} \tag{39}$$

while the wage-to-variety ratio approaches[12]:

$$\lim_{t\to\infty} \frac{w_t}{N_t} = \left(\frac{\beta}{1+\beta}\right)^{(1-\zeta)/\zeta} \left(\frac{\zeta}{1-\zeta}\right)^{1/\zeta} \tag{40}$$

Entrepreneurial participation remains optimal throughout this growth path because the expected returns to innovation stay positive (consider the limit of the LHS of (15)).

4.3 Sustained Innovation with Buyouts

The economy with active buyouts exhibits similar asymptotic properties, including unbounded growth in matched entrepreneurial investment, declining relative investment shares despite absolute growth, and convergence of monopoly profits to a finite steady-state value:

$$\lim_{t\to+\infty} \tilde{I}_t^w = +\infty \tag{41}$$

$$\lim_{t\to+\infty} \tilde{I}_t^w / w_t = 0 \tag{42}$$

$$\lim_{t\to+\infty} R_{t+1}^E = \zeta^{1/\zeta} \left(\frac{\beta}{1+\beta}\right)^{(1-\zeta)/\zeta} L\bar{e} \equiv R^E \tag{43}$$

The limiting conditions (39), (40), and $(I_t^u)^*/w_t \to 0$ continue to hold in this regime. To analyze the capital market equilibrium, we transform equation (33) into:

$$\begin{aligned} &\left[\left(\frac{1}{r_t}\right)^{1/\zeta} - \frac{\beta}{1+\beta} \cdot \frac{w_{t-1}}{N_{t-1}}\right] \bar{e} \\ &+ \left\{\theta\left(p - \frac{1}{\tilde{I}_{t-1}^w}\right) + (1-2\theta)\left[p - \frac{1}{(I_{t-1}^u)^*}\right]\right\} \left(\frac{1-\zeta}{r_t}\right)^{1/\zeta} \frac{L\bar{e}}{N_{t-1}} \\ &+ \frac{\beta}{1+\beta}\left[\theta \frac{\tilde{I}_{t-1}^w}{N_{t-1}} + (1-2\theta)\frac{(I_{t-1}^u)^*}{N_{t-1}}\right] = 0. \end{aligned} \tag{44}$$

[12] See Appendix A for the derivation of limit conditions in this case.

As $t \to +\infty$, this equation simplifies considerably. The convergence results $(I_t^u)^*/w_t \to 0$, $(I_t^w)^*/w_t \to 0$, (40) collectively ensure that the second and third terms vanish asymptotically, leaving:

$$\lim_{t\to+\infty} \left[\left(\frac{1}{r_t}\right)^{1/\zeta} - \frac{\beta}{1+\beta} \cdot \frac{w_{t-1}}{N_{t-1}} \right] \bar{e} = 0. \tag{45}$$

This limiting condition reproduces the same interest rate and wage-to-variety ratio relationships established in the no-buyout case.

The optimal transfer amount a_t between agents exhibits important asymptotic behavior. Rewriting equation (25) as:

$$\frac{a_t}{w_t e^u} = \left[1 - \frac{(I_t^u)^*}{w_t e^u}\right] \exp\left\{\frac{\beta}{1+\beta}\left[p - \frac{1}{(\tilde{I}_t^u)^*}\right](R_{t+1}^E)^\alpha\right\} - 1. \tag{46}$$

and combining with the limits, we obtain:

$$\lim_{t\to+\infty} \frac{a_t}{w_t e^u} = \exp\left\{\frac{\beta}{1+\beta} p (R^E)^\alpha\right\} - 1 \tag{47}$$

which implies unbounded growth in absolute transfer amounts:

$$\lim_{t\to+\infty} a_t = +\infty. \tag{48}$$

The well-endowed agents' buyout decision depends on comparing the utility from purchasing ideas:

$$(1+\beta)\ln(w_t e^w - a_t - \tilde{I}_t^w) + \beta \ln r_{t+1} + \beta\left(p - \frac{1}{\tilde{I}_t^w}\right)(R_{t+1}^E)^\alpha - \frac{(1+\beta)^{1+\beta}}{\beta^\beta}$$

with their reservation utility from equation (11). The participation condition simplifies asymptotically to:

$$\ln\left(1 - \frac{a_t + \tilde{I}_t^w}{w_t e^w}\right) + \frac{\beta}{1+\beta}\left(p - \frac{1}{\tilde{I}_t^w}\right)(R_{t+1}^E)^\alpha \geq 0 \tag{49}$$

which converges to the key inequality:

$$\ln\left\{1 - \frac{e^u}{e^w}\left[\exp\left(\frac{\beta}{1+\beta} p (R^E)^\alpha\right) - 1\right]\right\} + \frac{\beta}{1+\beta} p (R^E)^\alpha \geq 0$$

This leads to our central existence result:

Proposition 1 In an economy with sustained innovation, buyout emerges if and only if

$$\frac{e^w}{e^u} > \exp\left[\frac{\beta}{1+\beta} p(R^E)^\alpha\right].$$

Proof The necessity follows from the asymptotic analysis above. For sufficiency, when the condition holds, equations (49) are satisfied for sufficiently large t, ensuring mutually beneficial idea transfers between agent types. □

Proposition 1 establishes that persistent buyouts emerge if and only if the endowment ratio e^w/e^u exceeds a critical threshold. The threshold is jointly determined by the success probability p, the intensity of non-pecuniary utility α, the discount factor β, and the steady-state entrepreneurial return R^E. These parameters influence the persistence of buyout activity through two distinct channels. First, higher values of p, α, β, or R^E increase the acquisition price required to compensate under-endowed agents for their forgone entrepreneurial utility, as shown in the asymptotic transfer condition (47). This reflects the fact that when entrepreneurial success is more likely, future profits are more valuable, or agents derive greater non-pecuniary benefits from entrepreneurship, idea holders demand higher compensation to relinquish control. Second, these same parameters simultaneously affect wealthy agents' willingness to pay for entrepreneurial participation. Higher values enhance the non-pecuniary utility wealthy agents derive from implementing business ideas, potentially increasing their reservation acquisition prices. Consequently, economies require greater initial inequality to sustain buyout activity when entrepreneurial prospects are more attractive.

5 Buyout Funds and Economic Growth

In the model, entrepreneurship provides intrinsic utility to idea holders, while affluent agents value the experience of implementing businesses enough to pay premiums that make transfers mutually beneficial. This mechanism captures empirical observations that private-equity participation is often driven by non-pecuniary motives as well as financial returns, and has important implications for inequality and economic growth.

5.1 Buyout and Growth

Buyouts affect innovation and variety expansion through two complementary channels. First, buyouts reallocate projects to agents with greater effective resources, increasing per-project success probabilities and accelerating the introduction of new intermediate goods. Without buyouts, only under-endowed idea owners implement projects and variety growth is

$$\Delta N_t^{\text{no-buyout}} = (1-\theta)\left(p - \frac{1}{(I_{t-1}^u)^*}\right) L. \tag{50}$$

where $(I_{t-1}^u)^*$ is the optimal investment of an under-endowed implementer. With active buyouts, transferred ideas are implemented by well-endowed agents who invest $\tilde{I}_{t-1}^w$, while the remaining unmatched under-endowed agents also begin businesses, so

$$\Delta N_t^{\text{buyout}} = \left[\theta\left(p - \frac{1}{\tilde{I}_{t-1}^w}\right) + (1-2\theta)\left(p - \frac{1}{(I_{t-1}^u)^*}\right)\right] L. \tag{51}$$

In our setup, both investment choices can be written as $I(x)$ evaluated at different effective resources,

$$(I_{t-1}^u)^* = I(w_{t-1}e^u), \qquad \tilde{I}_{t-1}^w = I(w_{t-1}e^w - a_{t-1}),$$

with

$$I(x) = \frac{2x}{1 + \sqrt{1 + \dfrac{4(1+\beta)x}{\beta(R_t^E)^\alpha}}}, \quad x > 0.$$

Since $I'(x) > 0$ for all $x > 0$, larger effective resources raise investment and hence the success probability. Under the buyout-persistence condition,

$$\frac{e^w}{e^u} > \exp\left\{\frac{\beta}{1+\beta} p(R^E)^\alpha\right\},$$

and the associated asymptotic transfer rule

$$\lim_{t\to\infty} \frac{a_t}{w_t e^u} = \exp\left\{\frac{\beta}{1+\beta} p(R^E)^\alpha\right\} - 1,$$

we have for large t, that

$$w_t e^w - a_t > w_t e^u.$$

hence $\tilde{I}_t^w > (I_t^u)^*$ and

$$p - \frac{1}{\tilde{I}_t^w} > p - \frac{1}{(I_t^u)^*}.$$

Thus, reallocating projects to wealthier implementers raises the per-project success probability and accelerates the short-run introduction of intermediate goods.[13]

Second, when $\theta < 1/2$ (the empirically relevant case where capital-abundant agents are fewer), a trickle-down effect emerges on the extensive margin. Initially, many under-endowed agents lack the resources to implement ideas, and growth is approximately θLp varieties per period. As matched (buyout-implemented) innovations expand the variety of intermediates and push up wages, previously constrained under-endowed agents acquire the means to launch their own projects. When the $(1-2\theta)L$ previously inactive under-endowed agents become active, variety growth increases to the expression given for $\Delta N_t^{\text{no-buyout}}$. Over time, wages and investments rise, and both success probabilities approach p, and long-run variety growth in either regime converges to $(1-\theta)Lp$.

[13] When the persistence condition fails, the inequality reverses. Therefore, buyout persists as long as it is socially beneficial.

These channels are consistent with the institutional view that buyouts are not merely a relabeling of ownership, but involve concentrated control rights and active governance and operational restructuring of portfolio companies (Kaplan and Stromberg 2009). Moreover, using industry-level evidence, Bernstein et al. (2017) find that private equity activity predicts subsequent improvements in industry performance, in line with our mechanism, which reallocates ownership to better-capitalized (and more actively involved) owners, thereby raising effective investment and innovation outcomes.

5.2 Policy Implications

The model yields a nuanced set of policy implications that depend on the economy's stage of development. In settings where under-endowed agents are initially unable to implement projects independently, some concentration of resources can play a constructive role: wealthy agents, motivated both by returns and non-pecuniary gains, provide up-front resources and acquisition premia that bring latent ideas to market. In such early-stage contexts, blunt policies that substantially raise the effective cost of private capital (for example, by sharply increasing taxation on active investment without compensating measures) risk suppressing a private channel that jump-starts innovation. As the economy develops, endogenous wage increases improve the outside options of idea holders and reduce the social necessity of buyouts. At this intermediate stage, policies that broaden credit access or offer targeted support to nascent entrepreneurs (subsidized lending, matching grants, or public seed equity) can accelerate the shift from buyout-mediated commercialization to broad-based independent entrepreneurship.

Tax and fiscal design have ambiguous effects: lower taxes on returns to active investment raise wealthy agents' willingness to bid for projects and may foster buyouts where they would otherwise be absent, whereas redistributive policies reduce wealthy agents' capacity to pay acquisition premia while simultaneously strengthening under-endowed agents' ability to implement ideas. Regulatory choices that affect contractual forms (earn-outs, equity retention) and market structure also matter: contracts that better share upside with originators can reduce required upfront premia and improve distributional outcomes without eliminating the matched-improvement gains.

Taken together, these considerations suggest a stage-dependent policy strategy. In nascent or credit-constrained environments, preserving incentives for active, control-oriented investors can be growth-enhancing; as financial access widens, the policy emphasis should shift toward expanding independent entrepreneurship and ensuring that buyout activity—if present—operates under contractual and regulatory arrangements that align incentives and limit value extraction. A full welfare assessment of these trade-offs requires quantitative extensions that embed taxes, subsidies, and alternative contracting into the calibrated model, which we leave for future work.

6 Numerical Illustrations

This section implements the model numerically in four steps. Section 6.1 introduces a light-touch calibration that maps the key preference and technology parameters

Table 1: Baseline parameter values for the numerical illustration

Symbol	Value	Description	Discipline / source
e^u	1	Endowment of under-endowed agents	Normalization
e^w	7.36	Endowment of well-endowed agents	Top 10% income share
θ	0.10	Population share of well-endowed agents	Top 10% of the wealth distribution
p	0.21	Success probability of a buyout-backed project	The fraction of "good" projects
α	1.5	Curvature of entrepreneurial "thrill" utility	Moderate convexity
β	0.985^{30}	Subjective discount factor	30 years per generation
ζ	2/3	Labor share in production	Standard
L	10	Population size	The scale of the economy

to simple empirical moments. Section 6.2 describes the algorithm used to compute temporary equilibria and transition paths under a given parameterization. Section 6.3 then uses this machinery to construct two benchmark transition paths that illustrate when buyouts disappear versus persist, and quantifies the contribution of the two growth channels. Finally, Section 6.4 conducts a sensitivity analysis, showing how the buyout regimes move with the parameters that enter the inequality threshold derived in Proposition 1.

6.1 Parameterization

The numerical illustration follows a light-touch calibration strategy. We anchor the key heterogeneity and innovation parameters to simple moments from the wealth distribution and from evidence on venture-backed project quality. Table 1 summarizes the main parameters used in the simulations. Throughout, we normalize the endowment of under-endowed agents to one, $e^u = 1$, so that all monetary variables can be interpreted in units of the under-endowed agent's period income $w_t e^u$.

We now briefly discuss how the key parameters linking the model to the data are chosen.

Inequality parameters θ and (e^w, e^u).

We interpret θ as the population share of "well-endowed" agents who are rich enough to finance buyouts. In the numerical exercises, we set $\theta = 0.10$, corresponding to the top 10% of the wealth distribution in advanced economies.

Given θ, we choose the ratio e^w/e^u to mimic a simple moment of the empirical wealth distribution. In our two-type model, with e^u normalized to one, the wealth share of well-endowed agents is

$$s^{\text{model}}(\theta, e^w) = \frac{\theta e^w}{\theta e^w + (1-\theta)e^u} = \frac{\theta e^w}{\theta e^w + (1-\theta)}.$$

The World Inequality Database reports that the top 10% hold roughly 42–47% of aggregate wealth in industrialized economies. We therefore target a central value $s^{\text{data}} = 0.45$ for the wealth share of the top decile and set e^w such that $s^{\text{model}}(\theta, e^w) = s^{\text{data}}$. With $\theta = 0.10$ and $e^u = 1$, this implies

$$e^w = \frac{s^{\text{data}}(1-\theta)}{\theta(1-s^{\text{data}})} = \frac{0.45 \times 0.9}{0.1 \times 0.55} \simeq 7.36,$$

so the ratio of endowments is $e^w/e^u \approx 7.36$. In other words, in the baseline parameterization the top 10% of agents hold about 45% of wealth, a level of inequality in line with the empirical range.

Success probability p.

In the model, p is the max probability that an implemented project yields a successful entrepreneurial payoff R^E. To discipline its magnitude, we draw on (Greenwood et al. 2022b), who sets the share of good ideas to $\rho = 0.21$, implying that roughly 21% of funded ventures are high-quality. We interpret this fraction as an empirical counterpart to the max success probability of a buyout-backed idea and therefore set $p = 0.21$ in our baseline simulations.

These choices provide a transparent benchmark for the dynamic experiments that follow. Given this baseline parameterization, we next describe how we compute equilibrium transition paths under different regimes.

6.2 Computation Algorithm

Given the parameterization above, we compute the transition path $\{N_t, r_t, w_t, I_t^u, I_t^w\}_{t=1}^T$ by iterating on the temporary equilibrium problem for each t, solving for r_{t+1} and classifying the regime. The algorithm proceeds as follows:

1. **Initialization.** Fix a horizon T, initial variety counts (N_0, N_1), and an initial interest rate r_1. Set $t = 1$.
2. **Factor prices.** Given (N_{t-1}, N_t, r_t), compute the wage w_t from (9). For a candidate r_{t+1}, compute $R^E_{t+1}(r_{t+1})$ from (8).
3. **Individual decisions.** For the current candidate regime $j \in \{(a), (b), (c)\}$ described in section 3, use $(w_t, r_{t+1}, R^E_{t+1})$ and Lemma 1–2 to compute the corresponding investment and transfer decisions $(I_t^u, I_t^w, \tilde{I}_t^w, a_t)$ and update N_{t+1} using equations (28)–(30).
4. **Capital market clearing.** Plug these decisions into the appropriate capital market clearing condition (31), (32), or (33) to construct $E_t^j(r_{t+1})$. Use a standard one-dimensional root-finding method to find the r_{t+1} such that $E_t^j(r_{t+1}) = 0$.
5. **Regime consistency and selection.** Check that the regime-defining inequalities are satisfied: in particular, verify that under-endowed agents' utility under the proposed contracts is at least as high as in autarky, and that well-endowed agents are willing to finance the stipulated buyouts. If regime j is not self-consistent, switch to another regime and repeat steps 3–4. In states where more than one regime is self-consistent, we select an equilibrium with active buyouts whenever

Table 2: Growth mechanism decomposition for disappearing buyout case ($e^w/e^u = 1.1$, $\theta = 0.1$)

Metric	Buyout Existence Periods ($n = 16$)	Long-Run (Last 200 Periods)
Total new varieties	1.03	1.56
Matched pairs	0.13 (13.05%)	0.00 (0.00%)
Extensive margin	0.90 (86.95%)	1.56 (100.00%)

Notes: Values in parentheses denote percentage contributions to $\Delta N_{\text{buyout},t}$. Matched pairs channel: $\theta \cdot (p - 1/\tilde{I}^w_{t-1}) \cdot L$; Extensive margin channel: $(1 - 2\theta) \cdot (p - 1/I^{u,s}_{t-1}) \cdot L$. Parameters: $p = 0.21$, $\alpha = 1.5$, $L = 10$. Total periods $T = 800$.

Table 3: Growth mechanism decomposition for persistent buyout case ($e^w/e^u = 7.64$, $\theta = 0.1$)

Metric	Buyout Existence Periods ($n = 799$)	Long-Run (Last 200 Periods)
Total new varieties	1.75	1.81
Matched pairs	0.21 (11.94%)	0.21 (11.58%)
Extensive margin	1.54 (88.06%)	1.60 (88.42%)

Notes: See Table 2 for variable definitions and parameters.

both types strictly prefer it to the no-buyout outcome, focusing on the privately and Pareto-superior equilibrium path.

6. **Update.** Record $\{w_t, r_{t+1}, I^u_t, I^w_t, \tilde{I}^w_t, a_t, N_{t+1}\}$, set $t \leftarrow t + 1$, and repeat until $t = T$.

The resulting simulated transition paths for wages, interest rates, investment, transfers, and varieties provide the basis for the numerical examples in the next subsection and for the sensitivity analysis in Section 6.4.

6.3 Numerical Examples

With the baseline parameterization and computation algorithm in place, we now illustrate the model's dynamics in two benchmark economies that differ in their degree of inequality. These examples are chosen to mirror the theoretical regimes characterized in the previous sections and to quantify the contributions of the matched-improvement and extensive-margin mechanisms to variety growth.

We first consider an economy in which buyouts initially emerge but eventually disappear. Figure 1 shows this scenario, which arises when Proposition 1's condition fails and well-endowed agents are scarce. With initial parameters $(e^w, e^u) = (1.1, 1)$ and $(N_1, r_1) = (30, 5)$, we observe three distinct phases. Early periods show no innovation due to low incomes ($I^w_t = a_t = 0$). As capital accumulation reduces interest rates and raises reservation utilities, under-endowed agents begin selling ideas at positive prices ($I^w_t > 0$, $a_t > 0$). The distance between the I^w_t and a_t lines represents total project commitment. Eventually, a kink emerges where well-endowed agents cease purchases

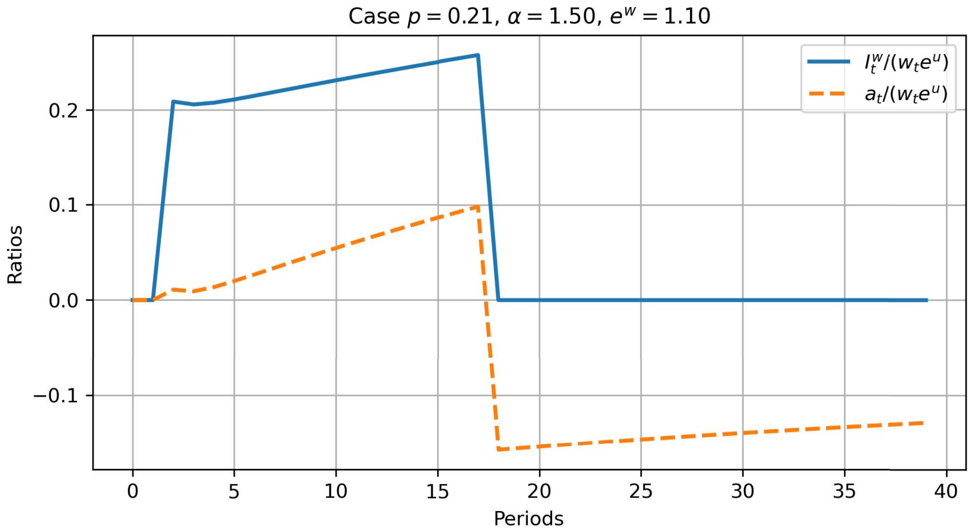

Fig. 1: Dynamics of investment and transfers in an economy without persistent buyouts in the first 40 periods. Investment of well-endowed agents: $I_t^w/(w_t e^u)$; transfer to under-endowed agents: $a_t/(w_t e^u)$.

($I_t^w = 0$) and under-endowed agents implement ideas independently ($I_t^u = -a_t > 0$), demonstrating how buyouts fade when endowment disparities are modest.

Table 2 reports the quantitative decomposition of variety growth in this disappearing-buyout case. During the 16-period buyout existence phase, the matched improvement channel (well-endowed agents raising project success probabilities) contributes 13.05% of new varieties, while the extensive margin channel (unmatched under-endowed agents entering entrepreneurship) accounts for 86.95%. In the long run, buyouts disappear entirely, leaving the extensive margin as the sole driver of growth ($\Delta N_{\text{buyout},t} = 1.56$, 100% from the extensive margin).

Figure 2 presents the contrasting case where Proposition 1's condition holds with $(e^w, e^u) = (5.54, 1)$. Here, equation (47) predicts that selling prices asymptotically become a constant fraction of under-endowed agents' income, leading to persistent buyout activity. Table 3 shows that in this regime the matched improvement channel contributes about 12% of new varieties, while the extensive margin accounts for roughly 88%, both during the long buyout phase and in the steady state. Together, these examples validate our key theoretical results about the conditions for buyout persistence and provide a quantitative sense of the relative strengths of the two growth mechanisms.

6.4 Sensitivity analysis

Building on the benchmark examples above, we next explore how the model's buyout regimes respond to changes in the key parameters that enter the inequality threshold

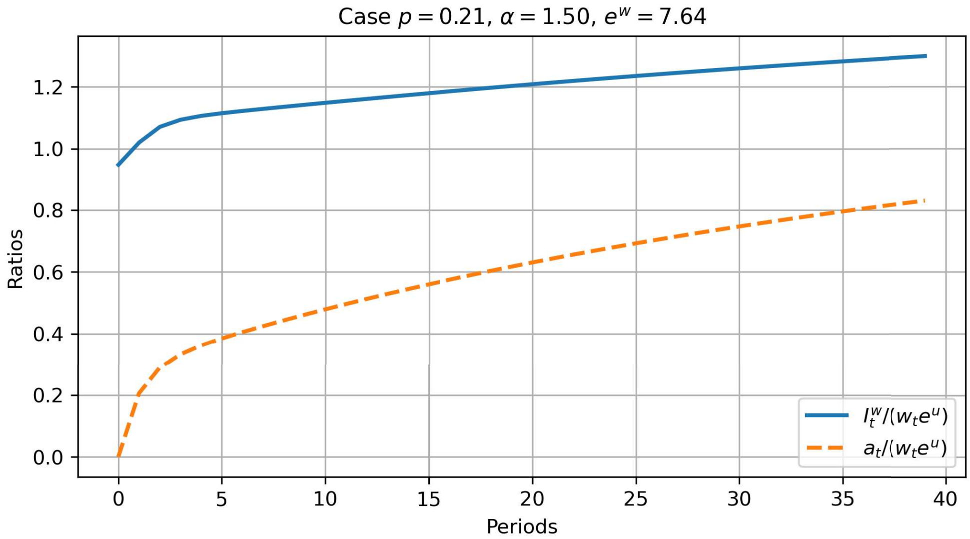

Fig. 2: Dynamics of investment and transfers in an economy with persistent buyouts in the first 40 periods.

in Proposition 1. In particular, we vary the maximum success probability p, the intensity of non-pecuniary entrepreneurial utility α, and the endowment ratio e^w/e^u, and examine how the simulated regime classification lines up with the theoretical threshold.

Table 4 summarizes the results. For each configuration, we compute the implied inequality threshold from Proposition 1, compare it to the chosen endowment ratio, and classify the simulated economy as exhibiting no buyouts, disappearing buyouts, or persistent buyouts.

The sensitivity results are consistent with the mechanism highlighted in Proposition 1 and with the benchmark examples in the previous subsection. For each value of the success probability p, increasing the non-pecuniary taste parameter α raises the required inequality threshold because under-endowed agents must be compensated more for giving up entrepreneurial control, and higher p similarly raises the threshold by increasing the value of future entrepreneurial payoffs. In the simulations, whenever the economy remains in the sustained innovation region where Proposition 1 applies, buyout activity is persistent precisely when the observed endowment ratio exceeds the theoretical threshold. For example, with $p = 0.18$ and $\alpha = 1.5$, the implied threshold is 1.789; when inequality is low ($e^w/e^u = 1.1 < 1.789$) buyouts eventually disappear, whereas with high inequality ($e^w/e^u = 3.1 > 1.789$) buyout activity persists. The same pattern holds across the higher values of p: for $p = 0.21$ and $p = 0.24$, economies with $e^w/e^u = 3.1$ always exhibit persistent buyouts once the threshold is exceeded, while economies with $e^w/e^u = 1.1$ lie below the threshold and display only transient buyouts that eventually vanish.

The three parameter configurations marked with a dagger in Table 4 are noteworthy. When both the success probability p and the entrepreneurial taste α are low, either

Table 4: Sensitivity Analysis

(a) $e^w/e^u = 1.1$

p	α	Threshold	Buyout regime
0.18	1.3	1.415	never†
0.18	1.5	1.559	disappeared†
0.18	1.7	1.764	disappeared
0.21	1.3	1.499	disappeared
0.21	1.5	1.678	disappeared
0.21	1.7	1.939	disappeared
0.24	1.3	1.588	disappeared
0.24	1.5	1.807	disappeared
0.24	1.7	2.132	disappeared

(b) $e^w/e^u = 3.1$

p	α	Threshold	Buyout regime
0.18	1.3	1.551	disappeared†
0.18	1.5	1.789	persistent
0.18	1.7	2.164	persistent
0.21	1.3	1.668	persistent
0.21	1.5	1.972	persistent
0.21	1.7	2.461	persistent
0.24	1.3	1.795	persistent
0.24	1.5	2.172	persistent
0.24	1.7	2.798	persistent

Notes: Each subtable reports results for a fixed endowment ratio (left: 1.1, right: 3.1). The column "Threshold" reports the theoretical inequality threshold implied by Proposition 1 for the corresponding pair (p, α). The buyout regime is classified from the 800-period simulations as "never" (no buyouts in any period), "disappeared" (buyouts appear transiently and vanish in the long run), or "persistent" (buyouts remain active asymptotically). †For the parameter configurations marked with †, the economy falls into an innovation stagnation state; these cases satisfy Lemma 3 and are therefore outside the domain where the asymptotic threshold is behaviorally relevant.

well-endowed agents' incomes remain too small, or the attractiveness of entrepreneurship is too limited for the economy to escape the low-innovation region characterized by Lemma 3. In these cases, the simulated path converges to an innovation-stagnation state with no sustained entrepreneurial activity, so the asymptotic threshold from Proposition 1 ceases to be a meaningful predictor of buyout persistence even when the observed endowment ratio exceeds the formal threshold.

Overall, the sensitivity analysis confirms that our main comparative-static insight is robust and complements the numerical examples above. Higher success probabilities and stronger non-pecuniary motives for entrepreneurship both increase the compensation that under-endowed agents require to sell their ideas, thereby raising the inequality threshold needed to sustain buyouts. When this threshold is met, and the economy escapes the stagnation region, buyout activity persists and continues to contribute to long-run variety growth, whereas economies with more equal endowments or weaker entrepreneurial prospects exhibit at most transient buyout episodes before reverting to a no-buyout regime.

7 Conclusion

This paper develops a novel theoretical framework that endogenizes the emergence of buyout funds through income-dependent risk preferences in an expanding variety growth model. Our analysis yields three key insights about the relationship between inequality, financial intermediation, and growth:

First, the model demonstrates that buyout funds arise endogenously as optimal contracts when initial endowment disparities exceed a critical threshold. The mechanism operates through two complementary channels: well-endowed agents' willingness to sacrifice consumption for the non-pecuniary benefits of entrepreneurial participation, and under-endowed agents' optimal decision to sell ideas when acquisition prices compensate for relinquished entrepreneurial utility.

Second, the persistence of buyouts depends fundamentally on the degree of inequality. In economies with large initial disparities, buyouts become permanent institutions because the required compensation grows proportionally with under-endowed agents' incomes. Conversely, moderate inequality leads to transitional buyout phases that fade as growth enables independent entrepreneurship—a dynamic consistent with observed patterns of entrepreneurial spawning.

Third, the growth effects of buyouts vary systematically with inequality levels. While always providing an initial growth impulse through improved project allocation, buyouts generate sustained growth acceleration only in high-inequality regimes. In more equal economies, their role as transitional institutions gives way to decentralized innovation as wages rise.

These results offer a unified explanation for several empirical regularities in private equity markets while providing new theoretical insights about inequality's dual role as both a catalyst and a constraint in innovation-driven growth. The framework suggests that optimal financial architecture evolves endogenously with development, with buyout funds serving distinct functions at different stages of economic maturation.

Appendix A Proofs and Derivations

Proof of Lemma 1 Under self-sufficiency, an under-endowed agent with a business idea chooses savings s_t^u and entrepreneurial investment I_t^u to maximize

$$\ln c_t^u + \beta \left[\ln \left(r_{t+1}(s_t^u - I_t^u) \right) + p_t^u (R_{t+1}^E)^\alpha \right], \quad \alpha > 1,$$

subject to$c_t^u = w_t e^u - s_t^u$ and $0 \le I_t^u \le s_t^u < w_t e^u$. The success probability is $p_t^u = \max\{0, p - 1/I_t^u\}$.

(i) Case $p_t^u = 0$.

When $p_t^u = 0$, the agent has no effective entrepreneurial opportunity and sets $I_t^u = 0$. The problem reduces to the standard consumption–saving decision:

$$\max_{0<s_t^u<w_t e^u} \ln(w_t e^u - s_t^u) + \beta \ln\big(r_{t+1} s_t^u\big).$$

The first-order condition (FOC) with respect to s_t^u is

$$-\frac{1}{w_t e^u - s_t^u} + \beta \frac{1}{s_t^u} = 0,$$

which implies

$$s_t^u = \frac{\beta}{1+\beta} w_t e^u.$$

Therefore,

$$(s_t^u, I_t^u) = \left(\frac{\beta w_t e^u}{1+\beta}, 0 \right)$$

as stated in the lemma. Substituting back into the utility yields the indirect utility

$$(1+\beta)\ln(w_t e^u) + \beta \ln r_{t+1} - \ln \frac{(1+\beta)^{1+\beta}}{\beta^\beta}, \tag{A1}$$

which corresponds to the reservation utility in the main text.

(ii) Case $p_t^u = p - 1/I_t^u > 0$ and $pw_t e^u \leq 1$.

When $p_t^u > 0$, the agent chooses $I_t^u > 0$ and $s_t^u > I_t^u$. Conditional on a given I_t^u, the problem in s_t^u is identical to the previous case except that current resources are $w_t e^u - I_t^u$. Thus, the FOC implies

$$s_t^u = \frac{\beta}{1+\beta}(w_t e^u - I_t^u) + I_t^u = \frac{I_t^u + \beta w_t e^u}{1+\beta}.$$

Hence any candidate optimum with positive entrepreneurship must have

$$(s_t^u, I_t^u) = \left(\frac{(I_t^u)^* + \beta w_t e^u}{1+\beta}, (I_t^u)^* \right), \tag{A2}$$

where $(I_t^u)^* > 0$ is chosen optimally. The success probability constraint $p_t^u = p - 1/I_t^u > 0$ requires

$$(I_t^u)^* > \frac{1}{p}.$$

If $pw_t e^u \leq 1$, the feasible set for I_t^u is contained in $(0, w_t e^u)$, which cannot accommodate $(I_t^u)^* > 1/p$. Hence, in this case no entrepreneurial investment is feasible and the agent must choose $I_t^u = 0$, proving part (a) of the lemma.

(iii) Case $pw_t e^u > 1$.

Now consider $pw_t e^u > 1$, so that $I_t^u > 1/p$ is feasible. We compare the indirect utility from entrepreneurship with the non-entrepreneurial indirect utility. The key difference is summarized in equation (15) in the main text. Using the optimal s_t^u derived above, the condition for entrepreneurship to be (weakly) preferred reduces to the sign of

$$\ln\left(1 - \frac{(I_t^u)^*}{w_t e^u}\right) + \frac{\beta}{1+\beta}\left[p - \frac{1}{(I_t^u)^*}\right](R_{t+1}^E)^\alpha.$$

Define

$$x \equiv 1 - \frac{(I_t^u)^*}{w_t e^u} \in \left(0, 1 - \frac{1}{pw_t e^u}\right),$$

so that $(I_t^u)^* = w_t e^u(1-x)$, and note that x is strictly between 0 and 1 when $pw_t e^u > 1$. Substituting this into the expression above and using $p_t^u = p - 1/(I_t^u)^*, -(1+\beta)/[w_t e^u - (I_t^u)^*] + \beta \cdot (R_{t+1}^E)^\alpha/[(I_t^u)^*]^2 = 0$, we can rewrite the relevant term as

$$f(x) \equiv \ln x + \frac{(1-x)\big[pw_t e^u(1-x) - 1\big]}{x}.$$

Thus, entrepreneurial investment is preferred if and only if $f(x) \geq 0$.

We now establish the main properties of $f(x)$:

- As $x \downarrow 0$, the term $\ln x \to -\infty$, while $(1-x)\big[pw_t e^u(1-x)-1\big]/x \to +\infty$ dominates. A direct limit computation shows that $f(x) \to +\infty$ as $x \downarrow 0$.
- At the upper endpoint $x = 1 - 1/(pw_t e^u)$, we have $pw_t e^u(1-x) - 1 = 0$, so the second term vanishes and

$$f\big(1 - 1/(pw_t e^u)\big) = \ln\big(1 - 1/(pw_t e^u)\big) < 0.$$

- A straightforward differentiation yields

$$f'(x) = \frac{1}{x} - \frac{(1-x)\big[pw_t e^u(1-x)-1\big]}{x^2} - \frac{\big[pw_t e^u(1-x)-1\big] - pw_t e^u(1-x)}{x}$$

  which can be simplified to show that $f'(x)$ changes sign exactly once on the interval $\big(0, 1-1/(pw_t e^u)\big)$.

These properties imply that there exists a unique solution $x(w_t)$ to $f(x)=0$ in the interval $\big(0, 1-1/(pw_t e^u)\big)$.

Equation (15) in the main text defines the threshold interest rate $r^u(w_t)$ by the condition that the under-endowed agent is indifferent between investing and not investing. Using the relationship between x and $(I_t^u)^*$, this indifference condition can be expressed in terms of $x(w_t)$, so that $x(w_t)$ corresponds to the critical value of $1-(I_t^u)^*/(w_t e^u)$ at $r_{t+1}=r^u(w_t)$.

Next, note that $1-(I_t^u)^*/w_t$ is a monotonically increasing function of r_{t+1}, as follows from the expressions for monopoly profits and optimal investment, equations (8) and (14) in the main text. Combining this monotonicity with the uniqueness of $x(w_t)$ implies that $x(w_t) > x$ when $r_{t+1} < r^u(w_t)$, and conversely when $r_{t+1} > r^u(w_t)$. This completes the proof of the threshold characterization in equation (13).

(iv) Monotonicity and boundary condition for $r^u(w_t)$.

Finally, we show that $r^u(w_t)$ is an increasing function of w_t with $r^u(1/(pe^u))=0$. By the definition of $x(w_t)$, equation (15) can be rearranged as

$$(R_{t+1}^E)^\alpha = \frac{(1+\beta)e^u}{\beta}\,\frac{w_t\big(1-x(w_t)\big)^2}{x(w_t)}.$$

Since the left-hand side is decreasing in r_{t+1}, it is sufficient to show that the right-hand side is decreasing in w_t; then a larger w_t requires a smaller $(R_{t+1}^E)^\alpha$ and hence a lower r_{t+1} at indifference, implying that $r^u(w_t)$ is increasing in w_t.

Define

$$g(w) \equiv \frac{w\big(1-x(w)\big)^2}{x(w)}.$$

We want to show $g'(w)<0$. Logarithmic differentiation of $g(w)$ yields

$$\frac{g'(w)}{g(w)} = \frac{1}{w} - \frac{x(w)+1}{x(w)\big(1-x(w)\big)}\,x'(w). \tag{A3}$$

The function $x(w)$ is implicitly defined by $f(x(w))=0$, which can be written as

$$\ln x(w) + \frac{\big(1-x(w)\big)\big[pwe^u(1-x(w))-1\big]}{x(w)} = 0. \tag{A4}$$

Differentiating (A4) with respect to w and solving for $x'(w)$ gives

$$x'(w) = \frac{-(1-x)^2 pe^u}{\ln x + x(1-x)}. \tag{A5}$$

In addition, rearranging (A4) for w yields the relationship

$$w = \frac{1}{pe^u(1-x)^2}\big[(1-x) - x\ln x\big]. \tag{A6}$$

Substituting (A5) and (A6) into (A3) leads to

$$\frac{g'(w)}{g(w)} = \frac{pe^u(1-x)\big[-2x^2\ln x + x^2(1-x)^2 + 1 - x^2\big]}{x(1-x-x\ln x)\big[\ln x + x(1-x)\big]}. \tag{A7}$$

For $x \in (0,1)$ we verify three inequalities:

$$\begin{aligned}
&\text{(a)} \quad -2x^2 \ln x + x^2(1-x)^2 + 1 - x^2 > 0, \\
&\text{(b)} \quad 1 - x - x\ln x > 0, \\
&\text{(c)} \quad \ln x + x(1-x) < x - 1 + x(1-x) = -(1-x)^2 < 0.
\end{aligned}$$

Given $pe^u > 0$ and $x(1-x) > 0$, these inequalities imply that the numerator of $g'(w)/g(w)$ is positive, while the denominator is negative, so $g'(w) < 0$. Hence $g(w)$ is decreasing in w, and therefore $r^u(w_t)$ is an increasing function of w_t.

To verify the boundary condition $r^u(1/(pe^u)) = 0$, rewrite (A6) as

$$pw_te^u - 1 = \frac{x(w_t)\big(1 - x(w_t) - \ln x(w_t)\big)}{\big(1 - x(w_t)\big)^2}.$$

As $w_t \downarrow 1/(pe^u)$, the left-hand side converges to zero, which forces the right-hand side to converge to zero as well. Since $x(w_t) \in \big(0, 1 - 1/(pw_te^u)\big)$, the only feasible limit is $x(w_t) \to 0$. From the original threshold condition (15), this requires $r_{t+1} \to 0$, so $r^u(1/(pe^u)) = 0$. This completes the proof. □

Proof of Lemma 2 We solve the Nash bargaining problem under $\theta < 1/2$, in which competition among unmatched under-endowed agents implies that the under-endowed agent receives exactly her reservation utility u_t^{ur} (i.e., the participation constraint binds). Using the variable transformation introduced in the text,

$$\tilde{s}_t^w \equiv s_t^w + I_t^u, \quad \tilde{s}_t^u \equiv s_t^u - I_t^u, \quad \tilde{I}_t^w \equiv I_t^w + I_t^u, \quad a_t \equiv -I_t^u,$$

the binding participation constraint can be written as

$$\ln\big(w_te^u + a_t - \tilde{s}_t^u\big) + \beta\ln\big(r_{t+1}\tilde{s}_t^u\big) = u_t^{ur}. \tag{A8}$$

Given any transfer a_t, the under-endowed agent chooses $\tilde{s}_t^u$ to satisfy (A8). The standard consumption–saving FOC implies

$$\tilde{s}_t^u = \frac{\beta}{1+\beta}(w_te^u + a_t), \tag{A9}$$

and substituting (A9) into (A8) yields the implied indirect utility on the left-hand side:

$$\ln(w_te^u + a_t) + \beta\ln r_{t+1} - \ln\frac{(1+\beta)^{1+\beta}}{\beta^\beta}. \tag{A10}$$

Because the participation constraint binds, (A10) must equal the reservation utility u_t^{ur} derived in Lemma 1. We therefore determine a_t by equating (A10) to the relevant expression for u_t^{ur}.

Case (a): no entrepreneurship in autarky. When $w_t \leq 1/(pe^u)$, or when $w_t > 1/(pe^u)$ but $r^u(w_t) \leq r_{t+1}$, Lemma 1 implies that the under-endowed agent does not invest and her reservation utility is

$$u_t^{ur} = (1+\beta)\ln(w_te^u) + \beta\ln r_{t+1} - \ln\frac{(1+\beta)^{1+\beta}}{\beta^\beta}.$$

Equating this to (A10) gives $\ln(w_te^u + a_t) = \ln(w_te^u)$, hence $a_t = 0$, establishing (24).

Case (b): entrepreneurship in autarky. When $w_t > 1/(pe^u)$ and $0 \leq r_{t+1} \leq r^u(w_t)$, Lemma 1 implies that the under-endowed agent chooses $(I_t^u)^* > 0$ and her reservation utility is

$$u_t^{ur} = (1+\beta)\ln\big(w_te^u - (I_t^u)^*\big) + \beta\Big(p - \frac{1}{(I_t^u)^*}\Big)(R_{t+1}^E)^\alpha + \beta\ln r_{t+1} - \ln\frac{(1+\beta)^{1+\beta}}{\beta^\beta}.$$

Equating this expression to (A10) and canceling common terms yields

$$\ln(w_t e^u + a_t) = \ln\bigl(w_t e^u - (I_t^u)^*\bigr) + \frac{\beta}{1+\beta}\Bigl(p - \frac{1}{(I_t^u)^*}\Bigr)(R_{t+1}^E)^\alpha.$$

Exponentiating both sides and solving for a_t gives

$$w_t e^u + a_t = \bigl[w_t e^u - (I_t^u)^*\bigr] \exp\Bigl\{\frac{\beta}{1+\beta}\Bigl[p - \frac{1}{(I_t^u)^*}\Bigr](R_{t+1}^E)^\alpha\Bigr\},$$

which is equivalent to (25). This completes the proof. □

Derivation of Limit Conditions in Section 4.2

We now provide the derivation of the limit conditions used in Section 4.2 for the case of sustained innovation without buyouts.

As the number of varieties N_t grows without bound, the growth rate of varieties approaches unity. From the law of motion for varieties in the under-endowed–only regime,

$$N_t - N_{t-1} = (1-\theta)\bigl[p - 1/(I_{t-1}^u)^*\bigr]L,$$

we obtain

$$\lim_{t\to\infty} \frac{N_{t-1}}{N_t} = \lim_{t\to\infty} \Bigl[1 - \frac{N_t - N_{t-1}}{N_t}\Bigr] = 1, \tag{A11}$$

since $N_t \to \infty$ while the numerator in $(N_t - N_{t-1})/N_t$ remains bounded.

The factor price frontier in the main text (equation (9)) can be rewritten as

$$\Bigl(\frac{w_t}{N_t}\Bigr)^\zeta r_t^{1-\zeta} = \Bigl(\frac{\zeta}{1-\zeta}\Bigr)^\zeta \Bigl[1 + \Bigl(\frac{N_t}{N_{t-1}} - 1\Bigr)(1-\zeta)\Bigr]^\zeta.$$

Using (A11), the right-hand side converges to $(\zeta/(1-\zeta))^\zeta$, so we obtain

$$\lim_{t\to\infty} \Bigl(\frac{w_t}{N_t}\Bigr)^\zeta r_t^{1-\zeta} = \Bigl(\frac{\zeta}{1-\zeta}\Bigr)^\zeta. \tag{A12}$$

Next, capital market clearing imposes a relationship between w_t/N_t and r_{t+1}. Using the optimal savings decisions $s_t^i - I_t^i = \frac{\beta}{1+\beta}(w_t e^i - I_t^i)$ and the fact that the total investment share vanishes asymptotically (see below), the capital market equilibrium simplifies in the limit to

$$\lim_{t\to\infty} \Bigl(\frac{w_t}{N_t}\Bigr)^\zeta r_{t+1} = \Bigl(\frac{1+\beta}{\beta}\Bigr)^\zeta. \tag{A13}$$

Combining (A12) and (A13) implies that the interest rate remains bounded:

$$0 < \liminf_{t\to\infty} r_t \le \limsup_{t\to\infty} r_t < \infty. \tag{A14}$$

Given (A12) and the boundedness of r_t, we must have

$$\lim_{t\to\infty} w_t = \infty, \qquad \lim_{t\to\infty} (I_t^u)^* = \infty, \tag{A15}$$

because each new variety requires a fixed amount of capital and labor, and the wage bill grows with N_t. However, the optimal investment-to-income ratio converges to zero:

$$\lim_{t\to\infty} \frac{(I_t^u)^*}{w_t} = 0. \tag{A16}$$

This follows from the first-order condition for $(I_t^u)^*$, which implies that the marginal benefit of entrepreneurial investment must equal its marginal cost; as w_t grows, the marginal cost in terms of foregone consumption rises, forcing the share $(I_t^u)^*/w_t$ to shrink even though $(I_t^u)^*$ itself diverges.

Solving (A12) and (A13) jointly for the limits of r_t and w_t/N_t yields

$$\lim_{t\to\infty} r_t = \frac{1+\beta}{\beta} \cdot \frac{1-\zeta}{\zeta}, \tag{A17}$$

and

$$\lim_{t\to\infty} \frac{w_t}{N_t} = \left(\frac{\beta}{1+\beta}\right)^{(1-\zeta)/\zeta} \left(\frac{\zeta}{1-\zeta}\right)^{1/\zeta}, \tag{A18}$$

as reported in Section 4.2.

Finally, we note that entrepreneurial participation remains optimal along this growth path because the net benefit of innovation converges to a strictly positive constant:

$$\frac{\beta}{1+\beta} p (R^E)^\alpha > 0,$$

which is the limit of the left-hand side of equation (15). This ensures that innovation by under-endowed entrepreneurs is sustained asymptotically.

Proof of Lemma 3 Lemma 3Characterizes the no-innovation steady state in which under-endowed agents never find entrepreneurship optimal. The key step is to identify when the under-endowed agent is exactly indifferent between investing and not investing in the steady state.

In the no-innovation steady state, N_t is constant and the interest rate converges to a constant $\overline{r}$. The comparison between the indirect utilities in equations (16) and (17) shows that entrepreneurial attempts depend on the sign of the left-hand side of equation (15): positive values induce entrepreneurship, negative values discourage it, and zero makes agents indifferent.

As $t \to +\infty$, the interest rate converges to $\overline{r}$, leading monopoly profits to approach

$$\lim_{t\to+\infty} R_{t+1}^E = \zeta \left(\frac{1-\zeta}{\overline{r}}\right)^{(1-\zeta)/\zeta} L\overline{e} \equiv R^E. \tag{A19}$$

Substituting this limit into the indifference condition (15), we obtain

$$\ln\left[1 - \frac{(I^u)^*}{\overline{w}e^u}\right] + \frac{\beta}{1+\beta}\left[p - \frac{1}{(I^u)^*}\right](R^E)^\alpha = 0, \tag{A20}$$

where $(I^u)^* > 0$ is the steady-state entrepreneurial investment chosen under self-sufficiency and $\overline{w}$ is the steady-state wage.

To express $(I^u)^*$ in terms of the no-innovation variety level $\overline{N}$, note that $\overline{w} = A\overline{N}$ in this steady state (see the main text), where A is a positive constant. The optimality conditions for entrepreneurial investment imply the ratios

$$\frac{(I^u)^*}{\overline{w}e^u} \equiv \frac{2}{1+\sqrt{1+\frac{4Ae^u(1+\beta)\overline{N}}{\beta(R^E)^\alpha}}}, \tag{A21}$$

and

$$\frac{1}{(I^u)^*} \equiv \frac{\frac{1}{A\overline{N}}+\sqrt{\frac{1}{(A\overline{N})^2}+\frac{4e^u(1+\beta)}{\beta(R^E)^\alpha A\overline{N}}}}{2e^u}. \tag{A22}$$

These expressions show that both $\frac{(I^u)^*}{\overline{w}e^u}$ and $\frac{1}{(I^u)^*}$ are smooth functions of $\overline{N}$. In particular:

- As $\overline{N} \to 0$, we have $(I^u)^*/(\overline{w}e^u) \to 1$ and $1/(I^u)^* \to +\infty$.
- As $\overline{N} \to \infty$, we have $(I^u)^*/(\overline{w}e^u) \to 0$ and $1/(I^u)^* \to 0$.
- Both $(I^u)^*/(\overline{w}e^u)$ and $1/(I^u)^*$ are monotone functions of $\overline{N}$.

Substituting (A21) and (A22) into (A20), we can write the left-hand side of (A20) as a function of $\overline{N}$, say $f(\overline{N})$. By continuity and the monotonicity properties above, $f(\overline{N})$ has the following characteristics:

- As $\overline{N} \to 0$, the term $\ln\left[1-(I^u)^*/(\overline{w}e^u)\right]$ dominates and $f(\overline{N}) \to -\infty$.
- As $\overline{N} \to \infty$, $(I^u)^*/(\overline{w}e^u) \to 0$ and $1/(I^u)^* \to 0$, so

$$f(\overline{N}) \to \frac{\beta}{1+\beta}p(R^E)^\alpha > 0.$$

- Direct differentiation (omitted for brevity) shows that $f(\overline{N})$ is strictly increasing in $\overline{N}$.

These properties guarantee the existence of a unique threshold $\overline{N}^*$ such that $f(\overline{N}^*) = 0$, with $f(\overline{N}) < 0$ for $\overline{N} < \overline{N}^*$ and $f(\overline{N}) > 0$ for $\overline{N} > \overline{N}^*$.

Therefore, in the no-innovation steady state, entrepreneurial investment is not attractive (i.e., under-endowed agents strictly prefer not to invest) precisely when $\overline{N} \leq \overline{N}^*$. Using $\overline{w} = A\overline{N}$, this condition is equivalent to $\overline{w} \leq A\overline{N}^*$, which is the wage bound stated in Lemma 3. This completes the proof. □

Appendix B Generalized Nash Bargaining Analysis

B.1 Generalized Nash Bargaining

This appendix analyzes the robustness of the buyout persistence condition under generalized Nash bargaining, where under-endowed agents have bargaining power $\phi \in (0,1)$ and well-endowed agents have bargaining power $1-\phi$. The generalized Nash product is

$$\left(u_t^w - u_t^{wr}\right)^{1-\phi}\left(u_t^u - u_t^{ur}\right)^{\phi}, \tag{B23}$$

where u_t^w and u_t^u are the utilities of well-endowed and under-endowed agents in the match, respectively, and u_t^{wr} and u_t^{ur} are their reservation utilities from self-sufficiency. The bargaining problem chooses the transfer a_t and the entrepreneurial investment $\tilde{I}_t^w$ to maximize this product, subject to the same constraints as in the baseline model.

B.1.1 Optimal Investment Decision

The first-order condition with respect to $\tilde{I}_t^w$ is

$$\frac{\partial u_t^w}{\partial \tilde{I}_t^w} = \frac{1}{w_t e^w - a_t - \tilde{I}_t^w} - \frac{\beta}{1+\beta} \frac{(R_{t+1}^w)^\alpha}{(\tilde{I}_t^w)^2} = 0. \tag{B24}$$

This yields the same optimal investment rule as in the baseline model:

$$\tilde{I}_t^w = \frac{2(w_t e^w - a_t)}{1 + \sqrt{1 + \frac{4(1+\beta)(w_t e^w - a_t)}{\beta (R_{t+1}^w)^\alpha}}}. \tag{B25}$$

As $t \to \infty$, with sustained innovation we have $w_t \to \infty$. From equation (B25), it follows that

$$\lim_{t\to\infty} \frac{\tilde{I}_t^w}{w_t} = 0. \tag{B26}$$

Thus, the entrepreneurial investment choice has the same asymptotic behavior regardless of the bargaining power distribution.

B.1.2 Optimal Transfer and Profit-Sharing

The first-order condition with respect to a_t is

$$\frac{1-\phi}{u_t^w - u_t^{wr}} \frac{\partial u_t^w}{\partial a_t} + \frac{\phi}{u_t^u - u_t^{ur}} \frac{\partial u_t^u}{\partial a_t} = 0, \tag{B27}$$

where

$$\frac{\partial u_t^w}{\partial a_t} = -\frac{1}{w_t e^w - a_t - \tilde{I}_t^w}, \tag{B28}$$

$$\frac{\partial u_t^u}{\partial a_t} = \frac{1}{w_t e^u + a_t}. \tag{B29}$$

In the baseline model (where under-endowed agents have no bargaining power), the profit share is at a corner, $R_{t+1}^w = R_{t+1}^E$, when $\alpha > 1$. With $\phi > 0$, the profit share may instead be interior. The first-order condition with respect to R_{t+1}^w for an interior solution is

$$\frac{1-\phi}{u_t^w - u_t^{wr}} \frac{\partial u_t^w}{\partial R_{t+1}^w} + \frac{\phi}{u_t^u - u_t^{ur}} \frac{\partial u_t^u}{\partial R_{t+1}^w} = 0, \tag{B30}$$

where

$$\frac{\partial u_t^w}{\partial R_{t+1}^w} = \left(p - \frac{1}{\tilde{I}_t^w}\right) \cdot \alpha (R_{t+1}^w)^{\alpha-1}, \tag{B31}$$

$$\frac{\partial u_t^u}{\partial R_{t+1}^w} = -\left(p - \frac{1}{\tilde{I}_t^w}\right) \cdot \alpha (R_{t+1}^E - R_{t+1}^w)^{\alpha-1}. \tag{B32}$$

The equilibrium with buyouts is characterized by the transfer a_t and the profit share $\gamma_t \equiv R^w_{t+1}/R^E_{t+1}$. In the long run, we define the limiting transfer ratio $\tau = \lim_{t\to\infty} a_t/w_t$ and the limiting profit share $\gamma = \lim_{t\to\infty} \gamma_t$. Analogous to the analysis in Section 4.3, the utility differences converge to:

$$u^w_t - u^{wr}_t \to (1+\beta)\ln\Big(1 - \frac{\tau}{e^w}\Big) + \beta p (R^E)^\alpha \gamma^\alpha, \tag{B33}$$

$$u^u_t - u^{ur}_t \to (1+\beta)\ln\Big(1 + \frac{\tau}{e^u}\Big) + \beta p (R^E)^\alpha \big[(1-\gamma)^\alpha - 1\big]. \tag{B34}$$

The limiting first-order conditions from the Nash bargaining problem then imply

$$\frac{(1+\beta)\ln\big(1+\frac{\tau}{e^u}\big) + \beta p (R^E)^\alpha \big[(1-\gamma)^\alpha - 1\big]}{(1+\beta)\ln\big(1-\frac{\tau}{e^w}\big) + \beta p (R^E)^\alpha \gamma^\alpha} = \frac{\phi}{1-\phi}\,\frac{e^w - \tau}{e^u + \tau}, \tag{B35}$$

$$\frac{(1+\beta)\ln\big(1+\frac{\tau}{e^u}\big) + \beta p (R^E)^\alpha \big[(1-\gamma)^\alpha - 1\big]}{(1+\beta)\ln\big(1-\frac{\tau}{e^w}\big) + \beta p (R^E)^\alpha \gamma^\alpha} = \frac{\phi}{1-\phi}\,\frac{(1-\gamma)^{\alpha-1}}{\gamma^{\alpha-1}}. \tag{B36}$$

Equations (B35) and (B36) jointly determine the equilibrium pair (τ, γ) for a given bargaining weight $\phi \in (0,1)$.

B.2 Buyout Persistence Conditions

For buyouts to persist in the long run, both agents must receive at least their reservation utilities:

$$(1+\beta)\ln\Big(1 - \frac{\tau}{e^w}\Big) + \beta p (R^E)^\alpha \gamma^\alpha \geq 0, \tag{B37}$$

$$(1+\beta)\ln\Big(1 + \frac{\tau}{e^u}\Big) + \beta p (R^E)^\alpha \big[(1-\gamma)^\alpha - 1\big] \geq 0. \tag{B38}$$

Define

$$\kappa \equiv \frac{\beta p (R^E)^\alpha}{1+\beta}.$$

From (B37) and (B38), we obtain necessary bounds on τ:

$$\tau \leq e^w \left[1 - \exp\left(-\kappa\gamma^\alpha\right)\right], \tag{B39}$$

$$\tau \geq e^u \left[\exp\left(\kappa\big[1 - (1-\gamma)^\alpha\big]\right) - 1\right]. \tag{B40}$$

For a feasible transfer τ to exist, the upper bound must be at least as large as the lower bound:

$$e^u \left[\exp\left(\kappa\big[1 - (1-\gamma)^\alpha\big]\right) - 1\right] \leq e^w \left[1 - \exp\left(-\kappa\gamma^\alpha\right)\right]. \tag{B41}$$

This yields a necessary condition on the endowment ratio:

$$\frac{e^w}{e^u} \;\geq\; \frac{\exp\left(\kappa\big[1 - (1-\gamma)^\alpha\big]\right) - 1}{1 - \exp\left(-\kappa\gamma^\alpha\right)} \;\equiv\; B(\gamma). \tag{B42}$$

The function $B(\gamma)$ in equation (B42) determines the necessary endowment ratio for buyout persistence, conditional on the limiting profit share γ:

1. **Corner solution ($\gamma = 1$).** When $\gamma = 1$ (for example, when $\phi = 0$ and $\alpha > 1$ so that the profit share is at a corner), we obtain

$$B(1) = \exp(\kappa) = \exp\left(\frac{\beta p (R^E)^\alpha}{1+\beta}\right),$$

which coincides with the baseline condition in Proposition 1. In this case, the condition is also sufficient for buyouts to persist, and τ is pinned down by (B35). To see this, consider a continuous function $F(\tau)$ defined on the non-empty set

$$\left\{\tau \,\middle|\, (1+\beta)\ln(1-\tau/e^w) + \beta p (R^E)^\alpha > 0,\ (1+\beta)\ln(1+\tau/e^u) - \beta p (R^E)^\alpha > 0\right\}:$$

$$F(\tau) \equiv \frac{(1+\beta)\ln(1+\tau/e^u) - \beta p (R^E)^\alpha}{(1+\beta)\ln(1-\tau/e^w) + \beta p (R^E)^\alpha} - \frac{\phi}{1-\phi} \cdot \frac{e^w - \tau}{e^u + \tau}.$$

Standard arguments show that there exists a unique τ in this domain satisfying $F(\tau) = 0$.

2. **Interior profit sharing ($\phi > 0$).** For $\phi > 0$, the limiting profit share γ may be interior and is jointly determined with τ by equations (B35) and (B36). The function $B(\gamma)$ need not be monotone in γ; its shape depends on α, β, p, and R^E. In cases where $B(\gamma) < B(1)$ at the equilibrium γ, the required endowment ratio is lower than in the baseline model, so generalized bargaining relaxes the buyout condition. Conversely, when $B(\gamma) > B(1)$, a higher endowment ratio is required. In all interior cases, a buyout equilibrium requires a pair (γ, τ) that simultaneously satisfies the first-order conditions (B35)–(B36) and lies in the feasible set defined by the participation constraints (B37)–(B38).

Overall, the generalized bargaining framework shows that allowing under-endowed agents to have bargaining power affects both the transfer τ and the profit share γ, and can either tighten or relax the inequality threshold for persistent buyouts. However, the qualitative mechanism from the baseline model is robust: buyouts persist only when endowment disparities are large enough to support mutually beneficial transfers and risk sharing.

Appendix C Robustness to Alternative Success Probability Specifications

The baseline model employs the functional form $p_t = \max\left[0, p - \frac{1}{I_t}\right]$ for tractability. To assess the robustness of our results, we consider a general success probability function $p(I_t)$ satisfying the following economically intuitive properties:

1. **Boundedness**: $p(I_t) \in [0, \overline{p}]$ where $\overline{p} \leq 1$ denotes the maximum achievable success probability.

2. **Concavity**: $p'(I_t) > 0$ and $p''(I_t) < 0$ for $I_t > 0$, capturing diminishing returns to investment.
3. **Asymptotic Flatness**: $\lim_{I_t \to \infty} I_t \cdot p'(I_t) = 0$, ensuring that marginal returns vanish sufficiently fast for large investments.

These properties are satisfied by both our baseline specification and smoother alternatives such as $p(I_t) = 1 - e^{-\chi I_t}$ or $p(I_t) = \frac{\chi I_t}{1+\chi I_t}$.

The optimization problem for under-endowed agents becomes:

$$\max_{I_t^u} (1+\beta)\ln(w_t e^u - I_t^u) + \beta p(I_t^u)(R_{t+1}^E)^\alpha. \tag{C43}$$

The first-order condition is:

$$-\frac{1+\beta}{w_t e^u - I_t^u} + \beta p'(I_t^u)(R_{t+1}^E)^\alpha = 0. \tag{C44}$$

Let $(I_t^u)^*$ denote the solution to (C44). Defining the investment ratio $y_t = (I_t^u)^*/w_t$, we analyze its asymptotic properties as $w_t \to \infty$.

Lemma 4 For any success probability function $p(I_t)$ satisfying the properties above, the optimal investment $(I_t^u)^*$ has the following properties:

1. $(I_t^u)^*$ is strictly increasing in w_t for sufficiently large w_t.
2. $\lim_{t\to\infty}(I_t^u)^* = \infty$.
3. $\lim_{t\to\infty} y_t = 0$.

Proof Rewrite (C44) using y_t:

$$\frac{1+\beta}{1-y_t} = \beta p'\left((I_t^u)^*\right)(R_{t+1}^E)^\alpha w_t. \tag{C45}$$

As $w_t \to \infty$, a bounded $(I_t^u)^*$ would imply $y_t \to 1$, which would violate (C45). Rearranging gives:

$$\frac{y_t}{1-y_t} = \frac{\beta (R_{t+1}^E)^\alpha}{1+\beta}(I_t^u)^* p'\left((I_t^u)^*\right). \tag{C46}$$

By the asymptotic flatness condition, $\lim_{(I_t^u)^*\to\infty}(I_t^u)^* p'\big((I_t^u)^*\big) = 0$. Hence, the right-hand side of (C46) converges to 0, which implies $y_t \to 0$. Monotonicity of $(I_t^u)^*$ in w_t for sufficiently large w_t then follows from (C45). □

C.1 Implications for Buyout Persistence

The bargaining problem with a general success probability function has the same structure as in the baseline model. The key asymptotic results carry through.

Proposition 1 (Extended) *Under a general success probability function $p(I_t)$ satisfying the properties above, the condition for persistent buyouts in an economy with sustained innovation* *is*

$$\frac{e^w}{e^u} > \exp\left(\frac{\beta}{1+\beta}\overline{p}(R^E)^\alpha\right), \tag{C47}$$

where $\overline{p} = \lim_{I\to\infty} p(I)$ *is the maximum success probability.*

Proof The transfer ratio satisfies:

$$\frac{a_t}{w_t e^u} = \left[1 - \frac{(I_t^u)^*}{w_t e^u}\right] \exp\left\{\frac{\beta}{1+\beta} p\left((I_t^u)^*\right)(R_{t+1}^E)^\alpha\right\} - 1. \tag{C48}$$

As $t \to \infty$, the lemma implies $y_t = (I_t^u)^*/w_t \to 0$ and, by boundedness and monotonicity, $p\left((I_t^u)^*\right) \to \overline{p}$. Thus,

$$\lim_{t\to\infty} \frac{a_t}{w_t e^u} = \exp\left\{\frac{\beta}{1+\beta}\overline{p}(R^E)^\alpha\right\} - 1. \tag{C49}$$

The participation condition for well-endowed agents then yields the stated inequality. □

This analysis demonstrates that our main existence result is robust to the specific functional form of the success probability. The threshold condition for persistent buyouts depends only on the maximal attainable success probability $\overline{p}$, not on the detailed shape of $p(I_t)$. In particular, smoother specifications such as the exponential and rational forms discussed above deliver the same asymptotic condition.

Appendix D Diversification, Convex Preferences, and Portfolio Concentration

The baseline assumes that each well-endowed agent acquires (and implements) at most one idea, i.e., takes a concentrated entrepreneurial position. This section shows when such concentration is privately optimal once we allow the investor to split a fixed total entrepreneurial scale $I > 0$ across multiple independent ideas, and clarifies how strong convexity in the entrepreneurial utility component must be to overturn the standard diversification motive.

A project of scale J succeeds with probability $p(J)$ and yields payoff R^E upon success and 0 otherwise. Let $v(\cdot)$ denote utility from the entrepreneurial payoff (holding safe wealth/consumption fixed). We compare two allocations of the same total scale I:

(i) **Concentrated:** invest all I in one idea, so

$$U^{\text{conc}}(I) = p(I)\, v(R^E) + (1 - p(I))\, v(0).$$

(ii) **Equally diversified:** split I across $n \geq 2$ independent ideas of scale I/n. Let $k \sim \text{Binomial}\big(n, p(I/n)\big)$ be the number of successes; then $X^{\text{div}} = kR^E$ and

$$U^{\text{div}}(I, n) = \sum_{k=0}^{n} \binom{n}{k} p(I/n)^k \big(1 - p(I/n)\big)^{n-k} v(kR^E).$$

D.1 Linear benchmark: diversification dominates

If $v(X) = aX + b$ with $a > 0$, only the mean payoff matters:

$$U^{\text{conc}}(I) = a\, p(I) R^E + b, \qquad U^{\text{div}}(I,n) = a\, n\, p(I/n) R^E + b.$$

Under our maintained assumptions that $p(\cdot)$ is increasing and concave with $p(0) = 0$, concavity implies $p(I) \le n\, p(I/n)$ (strictly if p is strictly concave). Hence

$$U^{\text{div}}(I,n) \ge U^{\text{conc}}(I),$$

so a risk-neutral investor strictly prefers diversification. Therefore, concentrated buyout positions require a departure from linear entrepreneurial preferences.

D.2 Convex entrepreneurial utility: when concentration is optimal

In the main text, $v(\cdot)$ is strictly convex for entrepreneurial payoffs above the threshold,

$$v''(X) > 0 \quad \text{for } X \ge \bar{X}.$$

Diversification raises the mean entrepreneurial payoff (because p is concave) but reduces payoff dispersion (because it pools independent risks). With convex v, lower dispersion is costly in utility terms, creating a force toward concentration.

To quantify how much convexity is needed, consider $v(X) = X^{1+\eta}$ with $\eta > 0$. Then there exists a cutoff $\eta^*(I, R^E, p, \bar{X}) > 0$ such that

$$U^{\text{conc}}(I) > U^{\text{div}}(I,n) \quad \text{for all } n \ge 2 \quad \text{whenever } \eta > \eta^*.$$

Intuitively, η^* is higher when diversification generates a large mean gain (e.g., when success probabilities are high and/or marginal returns to scale in p are strongly diminishing) and lower when entrepreneurial payoffs are more skewed (large R^E relative to the threshold region), which increases the value of dispersion under convex preferences.

D.3 Baseline form: an explicit sufficient condition

Under the baseline success technology $p(I) = p - 1/I$ (for $I > 1/p$), a tractable sufficient condition for concentration is obtained by comparing the concentrated position to the $n = 2$ split. With $v_\eta(X) = X^{1+\eta}$ and $v_\eta(0) = 0$, we have

$$U_\eta^{\text{conc}}(I) = \Big(p - \frac{1}{I}\Big)(R^E)^{1+\eta},$$

and, for $n = 2$,

$$U_\eta^{\text{div}}(I,2) = 2q(1-q)(R^E)^{1+\eta} + q^2(2R^E)^{1+\eta}, \qquad q \equiv p\left(\frac{I}{2}\right) = p - \frac{2}{I}.$$

Thus $U_\eta^{\mathrm{conc}}(I) > U_\eta^{\mathrm{div}}(I,2)$ holds whenever

$$\left(p - \frac{1}{I}\right) \;>\; 2q(1-q) + q^2\, 2^{1+\eta}.$$

Equivalently, a sufficient lower bound on the curvature parameter is

$$\eta \;>\; \underline{\eta}(I) \;\equiv\; \log_2\!\left(\frac{p - \frac{1}{I} - 2q(1-q)}{q^2}\right) - 1, \qquad q = p - \frac{2}{I},$$

whenever the term inside $\log_2(\cdot)$ is greater than one. This bound is conservative but transparent: it quantifies how convex the entrepreneurial utility must be for a wealthy investor to prefer a single large buyout over even a minimal degree of diversification.

Overall, with linear entrepreneurial preferences investors would diversify, but with sufficiently convex $v(\cdot)$ a concentrated buyout position becomes privately optimal despite the availability of multiple independent ideas.

Acknowledgements

We are grateful to Akiomi Kitagawa, Koji Asano, and Jun Nagayasu for helpful discussions and valuable comments. We also thank three anonymous referees for their constructive suggestions, which substantially improved the paper.

References

Araujo, A., Chateauneuf, A., Gama, J.P., Novinski, R.: General equilibrium with uncertainty loving preferences. Econometrica **86**(5), 1859–1871 (2018)

Araujo, A., Gama, J.P., Kehoe, T.J.: Risk loving and fat tails in the wealth distribution. Economic Theory, 1–26 (2025)

Astebro, T., Herz, H., Ramana, N., Weber, R.A.: Seeking the roots of entrepreneurship: Insights from behavioral economics. Journal of Economic Perspectives **28**(3), 49–70 (2014)

Audretsch, D., Lehmann, E., Paleari, S., Vismara, S.: Entrepreneurial finance and technology transfer. The Journal of Technology Transfer **41**(1), 1–9 (2016)

Bernstein, S., Lerner, J., Sorensen, M., Strömberg, P.: Private equity and industry performance. Management Science **63**(4), 1198–1213 (2017)

Campbell, J.Y.: Financial Decisions and Markets: a Course in Asset Pricing. Princeton University Press, Princeton and Oxford (2017)

Cochrane, J.H.: The risk and return of venture capital. Journal of Financial Economics **75**(1), 3–52 (2005)

Foellmi, R., Zweimüller, J.: Income Distribution and Demand-Induced Innovations. The Review of Economic Studies **73**(4), 941–960 (2006)

Greenwood, J., Han, P., Sánchez, J.M.: Financing ventures. International Economic Review **63**(3), 1021–1053 (2022)

Greenwood, J., Han, P., Sánchez, J.M.: Financing ventures. International Economic Review **63**(3), 1021–1053 (2022)

Gompers, P., Lerner, J., Scharfstein, D.: Entrepreneurial spawning: Public corporations and the genesis of new ventures, 1986 to 1999. The Journal of Finance **60**(2), 577–614 (2005)

Galor, O., Moav, O.: From Physical to Human Capital Accumulation: Inequality and the Process of Development. The Review of Economic Studies **71**(4), 1001–1026 (2004)

Gupta, A., Van Nieuwerburgh, S.: Valuing private equity strip by strip. Working Paper 26514, National Bureau of Economic Research (November 2019)

Hamilton, B.H.: Does entrepreneurship pay? an empirical analysis of the returns to self-employment. Journal of Political Economy **108**(3), 604–631 (2000)

Harris, R.S., Jenkinson, T., Kaplan, S.N.: Private equity performance: What do we know? The Journal of Finance **69**(5), 1851–1882 (2014)

Hurst, E.G., Pugsley, B.W.: Wealth, tastes, and entrepreneurial choice. Working paper, National Bureau of Economic Research (October 2015)

Hall, R.E., Woodward, S.E.: The burden of the nondiversifiable risk of entrepreneurship. American Economic Review **100**(3), 1163–94 (2010)

Jovanovic, B., Ma, S., Rousseau, P.L.: Private Equity and Growth. Journal of Economic Growth **27**(3), 315–363 (2022)

Jones, J.B., Pratap, S.: An estimated structural model of entrepreneurial behavior. American Economic Review **110**(9), 2859–98 (2020)

Kortum, S., Lerner, J.: Assessing the contribution of venture capital to innovation. The RAND Journal of Economics **31**(4), 674–692 (2000)

Kaplan, S.N., Stromberg, P.: Leveraged buyouts and private equity. Journal of Economic Perspectives **23**(1), 121–46 (2009)

Moskowitz, T.J., Vissing-Jorgensen, A.: The returns to entrepreneurial investment: A private equity premium puzzle? American Economic Review **92**(4), 745–778 (2002)

Opp, C.C.: Venture Capital and the Macroeconomy. The Review of Financial Studies

32(11), 4387–4446 (2019)

Petukhov, A.: Non-diversifiable risk and endogenous innovation. SSRN (2019)

Rin, M.D., Hellmann, T., Puri, M.: Chapter 8 - a survey of venture capital research. In: Constantinides, G.M., Harris, M., Stulz, R.M. (eds.) Handbook of the Economics of Finance vol. 2, pp. 573–648. Elsevier, Amsterdam, Netherlands (2013)

Shane, S.: Fool's Gold: The Truth Behind Angel Investing in America vol. 9780195331080. Oxford University Press, Princeton and Oxford (2009)

Samila, S., Sorenson, O.: Venture Capital, Entrepreneurship, and Economic Growth. The Review of Economics and Statistics **93**(1), 338–349 (2011)