Tohoku University Research Center for Policy Design Discussion Paper

TUPD-2025-005

Cross-border Partial Equity Ownership

Tomohiro Ara
Faculty of Economics and Business Administration, Fukushima University
Graduate School of Economics and Management, Tohoku University

Arghya Ghosh
School of Economics, UNSW Business School

Hodaka Morita
Faculty of Economics, Hitotsubashi University

Hiroshi Mukunoki
Faculty of Economics, Gakushuin University

March 2025

TUPD Discussion Papers can be downloaded from:

https://www2.econ.tohoku.ac.jp/~PDesign/dp.html

Cross-border Partial Equity Ownership*

Tomohiro Ara† Arghya Ghosh‡

Hodaka Morita§ Hiroshi Mukunoki¶

March 13, 2025

Abstract

Firms often form a cross-border alliance by partially owning the equity. When and why do firms have cross-border partial equity ownership (PEO)? Under which conditions should a government give approval for firms to form such PEO? To address the questions, this paper develops an international oligopoly model where one foreign firm forms cross-border PEO with one home firm. PEO helps firms adjust production by avoiding trade costs but decreases market competition inducing a rival firm to take aggressive actions. We find that when cost differences between cross-border alliance firms are moderate, they choose PEO in order to shift the output between them most effectively while alleviating a rival firm's aggressive actions. However, a government should ban this PEO from the viewpoint of welfare, since the negative effect of weakened competition dominates the positive effect of output shifting: only when cost differences are large, should a government approve cross-border PEO.

Keywords: Partial equity ownership, cross-border alliance, trade costs, oligopoly
JEL Classification Numbers: F12, F13

*This study is conducted as a part of the Project "Economic Policy Issues in the Global Economy" undertaken at the Research Institute of Economy, Trade and Industry (RIETI). We would like to thank participants of the RIETI DP seminar for their helpful comments.

†Faculty of Economics and Business Administration, Fukushima University, Fukushima 960-1296, Japan. *Email address*: tomohiro.ara@gmail.com

‡UNSW Business School, Sydney, NSW, 2052, Australia. *Email address*: a.ghosh@unsw.edu.au

§Faculty of Economics, Hitotsubashi University, Kunitachi, Tokyo 186-8601, Japan. *Email address*: hodaka.morita@gmail.com

¶Faculty of Economics, Gakushuin University, Mejiro 1-5-1, Toshima, Tokyo 171-8588, Japan. *Email address*: hiroshi.mukunoki@gakushuin.ac.jp

1 Introduction

An alliance among firms has become increasingly important in recent years. One form of such firm alliance is merger and acquisition (M&A), by which different firms merge into a single firm. Another form is partial equity ownership (PEO), by which a firm acquires a part of an alliance firm's equities but does not directly involve in the partner's managerial decisions. While both forms of firm alliance is observed in the real world, PEO is often more common than M&A. For example, the Japan Fair Trade Commission received 242 notifications of alliances in 2020. Among them, there were only 15 notifications of pure mergers, and 201 notifications were categorized into partial equity acquisitions. This indicates that PEO is one of the main competitive strategies for firms, which could be more important than M&A in practice.

Governments need to be careful about whether or not to give approval of PEO to firms. On the one hand, PEO can increase an alliance partner's profitability by inducing knowledge transfer (Ghosh and Morita, 2017). Moreover, PEO can increase cost-reducing R&D investments by inducing knowledge spillovers across firms (López and Vives, 2019). This kind of knowledge transfer and spillovers decrease firms' production costs and improves welfare. On the other hand, PEO can also increase the market power of alliance firms by weakening market competition, which may worsen welfare. These contrasting effects highlight the importance of PEO in competition policies. However, competition authorities usually overlook PEO between domestic firms because minor shareholdings do not come with control over other firms.[1]

In contrast, governments frequently intervene in cross-border PEO between domestic and foreign firms. This can be understood by noting that governments impose restrictions on foreign equity. Table 1 illustrates the Association of Southeast Asian Nations (ASEAN) foreign direct investment (FDI) regulatory restrictions. The table shows the following facts for a developing country (Brunei) and a developed country (Singapore). First, governments do not always ban foreign equity or permit full foreign equity, but set partial restrictions in many industries. Second, foreign equity is permitted up to 50% among them, which may reflect governments' preferences that foreign firms should not directly affect domestic firms' managerial decisions. Taken together, restrictions on cross-border PEO is one of the main competition policies for governments as well.

Table 1: ASEAN FDI Regulatory Restrictions, as of 2017

Industry	Brunei	Singapore
Distribution	No foreign equity allowed	None
Mining	No foreign equity allowed	None
Hotels & Restaurants	No foreign equity allowed	Foreign equity<50%
Fishery	Foreign equity<50%	None
Construction	Foreign equity<50%	None
Transport	Foreign equity<50%	Foreign equity<50%
Media	Foreign equity<50%	Foreign equity<50%
Financial Services	Foreign equity<50%	Foreign equity<50%
Agriculture	50%<Foreign equity<100%	None
Forestry	50%<Foreign equity<100%	None
Business Services	50%<Foreign equity<100%	Foreign equity<50%

Source: OECD (2024)

[1]Other than Ghosh and Morita (2017) and López and Vives (2019) described above, Reynolds and Snapp (1986) suggest that PEO could lead to less output and higher prices even if the ownership share is small. Jovanovic and Wey (2014) also show that the anti-competitive effect of partial ownership facilitates the competition authority's approval of subsequent takeovers.

A large number of papers have investigated the effect of mergers and cartels as a source of market power in the literature of industrial organization and international trade. However, there has been little research on PEO, despite its increasing importance in firms' competitive strategies and governments' competition policies. Furthermore, most existing models treat the equity level as exogenously given and do not explicitly explore the mechanism by which the equity level is chosen. This limitation is particularly serious in the trade literature, where multinationals enter the foreign markets either by opening a new plant (greenfield FDI) or by acquiring an existing plant (cross-border M&A). In turn, governments have trade policies on multinationals either by banning FDI or by allowing for FDI. These binary choices suggest that little is known about how globalization endogenously influences the equity ownership, which could be potentially partial, for firms and governments. This paper aims to fill this gap in the trade literature by adopting key insights into optimal equity ownership recently derived in the industrial organization literature.

When and why do firms have cross-border PEO? Under which conditions should a government give approval for firms to form such PEO? To address the questions, this paper develops an international oligopoly model in which one foreign firm forms PEO with one domestic firm to enter the domestic market. Our study describes the mechanism though which the equity level is endogenously determined by both cross-border alliance firms and a government. It also helps understand the interaction between trade liberalization and equity ownership. Our model builds on the seminal work of Ghosh and Morita (2017), who consider the role of PEO in facilitating knowledge transfer. They find that PEO increases an alliance partner's profits by inducing knowledge transfer, but it also reduces PEO firms' profits by inducing other firms' aggressive actions. This trade-off endogenously determines the equity level that maximizes alliance firms' profits. When the equity level is chosen in this way, PEO can benefit consumers and improve welfare. Although the analysis in Ghosh and Morita (2017) yields important policy implications for restrictions on PEO, they only consider a closed-economy model and hence cannot address the effect of globalization on equity ownership and its welfare consequence.

In contrast, we consider an open-economy model and examine the effect of trade costs on equity ownership that cross-border alliance firms endogenously choose. In our model, a foreign firm chooses to export or build a horizontal relationship with a domestic firm, whereby PEO enables a foreign firm to avoid trade costs and overcome its cost disadvantage. Thus PEO plays a role similar to that of "horizontal" FDI, that is, investments in production facilities abroad to serve consumers in a host country. Specifically, PEO induces a foreign firm to reduce its output because it cares about how its supply affects its partner's profits. In response, a partner firm increases its output. In this way, PEO shifts the output from a foreign firm to a domestic firm, improving production efficiency of cross-border alliance firms and raises their profits. Since the aggregate output of PEO firms decrease, however, it weakens market competition and another domestic firm takes aggressive actions, which decreases their profits. In equilibrium, this trade-off pins down the equity level that maximizes profits. A government also chooses the equity level that maximizes welfare by taking into account the trade-off between decreased competition and increased profitability. Thus our model is able to explain why the equity levels are different between firms and a government, and how globalization affects them.

The main findings are summarized as follows. When PEO is formed between domestic and foreign firms, market competition is weakened relative to the case without PEO, decreasing the aggregate quantity while increasing their joint profits. In addition, PEO shifts the quantity from the foreign firm to the domestic firm, while decreasing their total quantity. This output shifting between PEO firms leads to "partial tariff-jumping": namely, the foreign firm makes PEO to avoid trade costs by decreasing its quantity and, at the same time, acquiring part of the profits increases of the domestic firm.[2] However, as PEO decreases the total quantity of

[2]A seminal paper analyzing tariff-jumping FDI in an oligopoly model is Motta (1992), who shows whether tariff-jumping FDI is welfare-improving for host countries depends on the existence and the entries of domestic firms.

cross-border alliance firm and weakens market competition, another domestic firm takes aggressive actions and increases its quantity, decreasing PEO firms' joint profits. PEO firms choose the equity ownership level such that it strikes a balance between opposing effects in setting the equity level. If the foreign firm freely exports and all firms are identical, the foreign firm has no incentive to avoid trade costs while inducing another firm to take aggressive actions by PEO. Since the joint profits always decline, they set the equity level to zero and do not have PEO. This is similar to the "merger paradox," a well-known result in the literature of mergers.[3] If the foreign firm is inefficient by trade costs, however, the joint profits rise due to the cost-saving motive. Thus, they choose a strictly positive equity level and have PEO.

Though PEO benefits all firms, it hurts consumers by weakening competition and increasing market price. When evaluating welfare at the optimal equity level chosen by PEO firms, we find that the negative effect on consumers dominates the positive effect on firms and thus PEO worsens welfare. The finding does not always mean that a government should completely ban PEO. Assuming that a government chooses the equity level to maximize welfare, we find that this optimal level depends on cost differences between PEO firms. When cost differences are small, the cost-saving effect of PEO is limited, in which case the government should ban PEO. When cost differences are large, however, the positive effect can outweigh the negative effect, in which case the welfare-maximizing equity level is strictly positive. Hence our analysis yields policy implications such that allowing for PEO can be desirable even from the viewpoint of welfare despite its negative effect on consumers.

Related Literature

This study contributes to the literature on horizontal FDI. In particular, we focus on cross-border acquisitions of firms as a mode of entry into a host country's market. Some studies have considered cross-border M&A as a multinational's entry mode. For instance, Nocke and Yeaple (2007, 2008) extend the Helpman et al. (2004) model to include both greenfield FDI and cross-border M&A. They explore the factors that determine firms' choice between these two forms of FDI. By dividing firms' management resources into mobile and immobile capabilities, they find that multinationals tend to choose cross-border M&A when it is difficult to acquire the immobile capabilities (such as brand power) of local agents.[4] However, their models do not consider PEO as an entry mode. Additionally, M&A does not generate any competition effect because their studies employ a monopolistic competition model with a CES utility function. This study is distinct in that it considers PEO whose level is endogenously determined, and its effect on strategic interactions between firms.

Many studies have examined cross-border alliance in international oligopoly models but most of them have not focused on PEO. Neary (2007) shows that trade liberalization endogenously shapes comparative advantage through cross-border merger; however, merger is the only option to form alliance. Qiu (2010) studies strategic alliance as well as merger, where strategic alliance enables firms to reduce their distribution costs by sharing distribution networks. PEO in our study differs from Qiu (2010)'s strategic alliance, however, since PEO not only reduces distribution costs but also affects market competition in a host country. Beladi et al. (2009) and Ishikawa et al. (2009, 2011) investigate the role of joint ventures (JVs) in a host country's market. While PEO is similar to JVs, these studies take the equity share of JV firms as exogenously given. Finally, our study is also related to Ghosh and Mukunoki (2024) who explore partial cross-ownership (PCO) in international trade. Their model is richer in that they consider both cross-border and domestic PCOs, and examine the effect of trade liberalization on merger control policy. Like others, however, the equity level of PCO firms is exogenous in their study. To the best of our knowledge, this study is the first to uncover the mechanism through which the equity level is endogenously chosen by firms and governments in an international context.

[3] See Farrell and Shapiro (1990) and Salant et al. (1983).

[4] See Raff et al. (2012) for empirical evidence using the data on Japanese multinationals.

The rest of the paper is organized as follows. Section 2 describes a model setup by introducing cross-border PEO in a standard oligopoly framework. Section 3 explores the equilibrium analysis when firms make such PEO and characterizes the profit-maximizing equity level for firms. Section 4 conducts the welfare analysis and characterizes the welfare-maximizing equity level for a government in order to derive policy implications of cross-border PEO. Section 5 concludes.

2 Model

This section sets up an international oligopoly model in the presence of PEO between two cross-border alliance firms and explains the structure of equity ownership and the timing of moves in our model.

2.1 Setup

Consider a world consisting of two countries, Home and Foreign, with three firms selling a homogeneous good: two firms (firms 1 and 2) in Home and one firm (firm F) in Foreign, each producing the outputs, q_1, q_2, q_F, respectively. Consumers reside only in Home and thus a homogenous good is sold only in the Home market.[5] To obtain closed-form solutions of the equilibrium variables, we focus mainly on linear demand, $P = a - Q$, where $Q = q_1 + q_2 + q_F$ is the aggregate output in the Home market.

On the production side, firm $i(= 1, 2)$ incurs constant marginal costs c_i whereas firm F incurs constant marginal costs $c_F = c+t$, where t denote per-unit trade costs to ship q_F to the Home market. In the following, we first interpret t as transport costs that use real resources. It is possible to consider t as import tariffs, but the analysis becomes more complex because tariff revenues must be taken into account in welfare calculations. We will explain what happens to our main results if t are treated as import tariffs later (see Section 4.3).

2.2 Profits of PEO Firms

Firm F has the option of having cross-border PEO with either firm 1 or firm 2 to avoid transport costs t. To introduce this arrangement in our oligopoly setting, we assume that each firm owns a fraction of its partner's profits if the negotiation is successful between the firms involved in PEO. Specifically, we assume that firm F owns a fraction $\theta \in [0, 1]$ of firm i's equity, whereas firm i owns a fraction $\gamma \in [0, 1]$ of firm F's profits. Hence, if firms i and F agree to form PEO, they arrange their profits as follows:

$$\begin{aligned} \pi_i &= (1-\theta)(a - Q - c_i)q_i + \gamma(a - Q - c_F)q_F + r, \\ \pi_F &= (1-\gamma)(a - Q - c_F)q_F + \theta(a - Q - c_i)q_i - r, \end{aligned} \tag{1}$$

where r is the transfer from firm F to firm i, which can be positive or negative. (1) shows that if $\theta = \gamma = 0$, firms i and F are independent, where firm F serves the Home market by paying transport costs. In contrast, if $\theta >, \gamma > 0$, these firms share the equity with each other, where firm F can serve the Home market through a fraction θ of firm i's equity, avoiding transport costs. In that sense, PEO plays a role similar to "horizontal" FDI in our model. While most existing studies treat the equity levels as exogenously given, the present study aims to uncover the mechanism through which these levels are endogenously determined by firms.

We assume that there are exogenous threshold levels of equity, $\bar{\theta}, \bar{\gamma}$ that underpin firm i's and F's decisions. As for firm F, if $\theta \in [0, \bar{\theta}]$, firm F cannot have the majority of firm i's equity. Thus, firm i and firm F choose q_1

[5] Even if consumers reside in both countries, our results hold by assuming that two countries are symmetric and a homogeneous good is traded in segmented markets.

and q_F (whereas firm j chooses q_j) simultaneously and non-cooperatively to maximize the profits. In contrast, if $\theta \in (\bar{\theta}, 1]$, firm F has the majority of firm i's equity. This allows firm F to choose q_i and q_F (while firm j chooses q_j) simultaneously and non-cooperatively to maximize the profits. For simplicity, the threshold equity levels are given by $\bar{\theta} = \bar{\gamma} = 1/2$.[6] The main analysis focuses on the range of equity ownership, $\theta, \gamma \in [0, 1/2]$, relegating another range, $\theta, \gamma \in (1/2, 1]$, to Appendix A.4.

2.3 Timing of Moves

The model involves three stages of decisions. In Stage 1, firm F proposes PEO to either firm 1 or firm 2 first. In Stage 2, PEO firms negotiate both equity levels (θ, γ) and transfer (r). If firm F proposes to firm i first and they agree on these terms, they arrange the profits as in (1), while leaving firm j independent. Subsequently, three firms engage in Cournot competition, taking these equity levels and transfer as given, and they obtain the respective profits. In this case, the game ends in Stage 2. However, if the negotiation fails to agree, firm F proposes to firm j and negotiates the PEO terms in Stage 3. If PEO firms agree, they arrange their profits similarly to (1), while if they fail to agree, all firms are independent. In either case, the three firms engage in Cournot competition and they obtain the respective profits. Figure 1 shows the game tree and the associated profits of this game, where the profits in each stage are derived below. In summary, the timing of moves in our setting is as follows.

Stage 1 (PEO proposal) Firm F offers to either firm 1 first or firm 2 first.

Stage 2 (First negotiation) When firm F offers to firm i first, they negotiate the terms of PEO. If agreed, firms i and F arrange their profits as in (1) and compete for quantity. If not agreed, then firm F offers to firm $j(\neq i)$.

Stage 3 (Second negotiation) Firm j and firm F negotiate the terms of PEO. If agreed, firms j and F arrange their profits similarly to (1) and compete for quantity. If not agreed, all firms are independent and compete for quantity.

We mainly assume that firm i and firm F engage in generalized Nash bargaining in both Stages 2 and 3, where the equity levels $\theta, \gamma (\in [0, 1/2])$ and transfer $r (> 0)$ are determined efficiently in negotiation. However, this assumption is not crucial and our key results hold in a different bargaining framework.[7] In either case, the terms of PEO determined in bargaining are common knowledge for all firms and a government.

3 Equilibrium Analysis

This section analyzes the PEO equilibrium to show the mechanism through which equity ownership levels are endogenously determined. Since the non-PEO equilibrium and the associated profits $\pi_1^o, \pi_2^o, \pi_F^o$ in Figure 1 are not new to the literature, this section mainly considers the PEO equilibrium. In Section 3.1, we start by describing the PEO structure, that is, which firm owns which firm's equity, in order to maximize the profits. Noticing that $\pi_1^o, \pi_2^o, \pi_F^o$ serve as outside options in Stage-3 negotiation when firm i and firm F fail to agree, we consider the PEO equilibrium in Stage 3 and calculate the associated profits $\pi_1^*, \pi_2^*, \pi_F^*$ and $\pi_1^{**}, \pi_2^{**}, \pi_F^{**}$,

[6] In reality, the threshold levels might be also endogenously chosen by firms. We are agnostic on the precise determination of these bounds and treat $\bar{\theta}, \bar{\gamma}$ as exogenous (though θ, γ as endogenous) throughout the analysis.

[7] For example, we can assume that firm F first makes a take-it-or-leave-it offer to firm i in Stage 2. If the negotiation fails to agree, firm F and firm j then engage in generalized Nash bargaining in Stage 3. The equilibrium outcomes do not qualitatively change even in this bargaining setting.

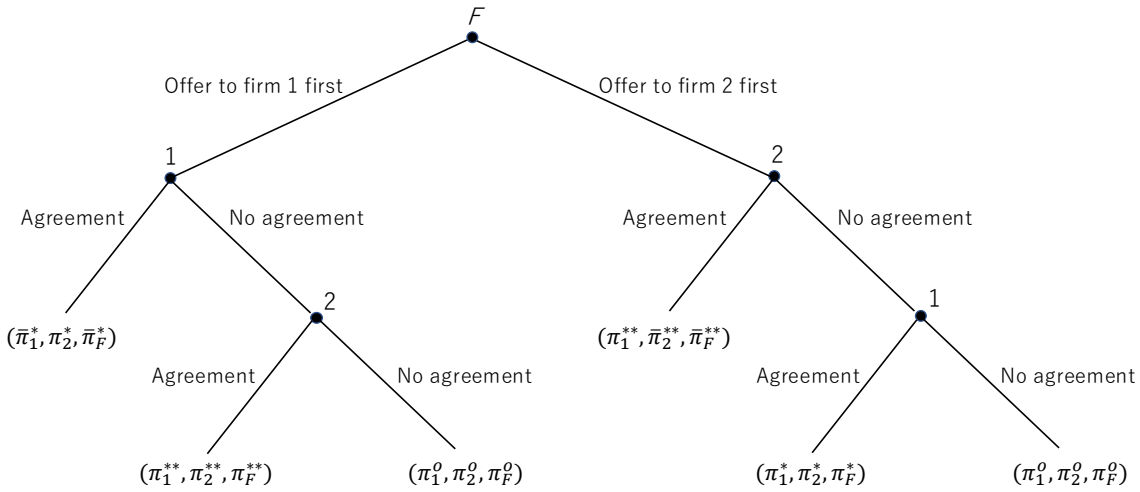

Figure 1: Timing of moves

by explaining the process through which firms endogenously choose the equity ownership level in Section 3.2. Finally, calculating the associated profits in Stage-2 negotiation, $\bar{\pi}_1^*, \bar{\pi}_F^*$ and $\bar{\pi}_2^{**}, \bar{\pi}_F^{**}$, we investigate firm F's proposal to either firm 1 or firm 2 first in Stage 1 in Section 3.3. The equilibrium concept we use in this study is the subgame perfect Nash equilibrium in pure strategies.

3.1 PEO Structure

Consider the Stage-3 subgame of PEO between firm i and firm F. As explained in Section 2.2, if firms involved in PEO have only a fraction θ, γ ($\leq 1/2$) of a partner's equity, it is not sufficient to affect management directly. Consequently, firms i and F choose q_i and q_F, respectively, to maximize the profits given by (1). By contrast, firm j is independent and chooses q_j to maximize its profits, $\pi_j = (a - Q - c_j)q_j$. In Cournot competition with PEO, each firm chooses its output taking as given not only rivals' outputs but also the equity ownership levels θ, γ. Thus, the outputs can be written as a function of these equity ownership levels.[8] Solving the standard profit-maximizing problem, we obtain the following equilibrium outputs:

$$\begin{aligned}
q_i(\theta,\gamma) &= \frac{(1-\gamma)[(1-\theta-\gamma)(a+c_j) - 3(1-\theta)c_i + (1-\theta+2\gamma)c_F]}{(4-\theta-\gamma)(1-\theta-\gamma)}, \\
q_j(\theta,\gamma) &= \frac{a + (1-\theta)c_i - (3-\theta-\gamma)c_j + (1-\gamma)c_F}{4-\theta-\gamma}, \\
q_F(\theta,\gamma) &= \frac{(1-\theta)[(1-\theta-\gamma)(a+c_j) + (1+2\theta-\gamma)c_i - 3(1-\gamma)c_F]}{(4-\theta-\gamma)(1-\theta-\gamma)}, \\
Q(\theta,\gamma) &= \frac{(3-\theta-\gamma)a - (1-\theta)c_i - c_j - (1-\gamma)c_F}{4-\theta-\gamma},
\end{aligned} \tag{2}$$

where $1-\theta-\gamma \geq 0$ under our restrictions, $\theta \leq 1/2$, $\gamma \leq 1/2$. Without PEO between firms i and F ($\theta = \gamma = 0$), the equilibrium outputs in (2) collapse to those in non-PEO equilibrium. However, with PEO ($\theta \geq 0, \gamma \geq 0$), these outputs are affected by equity ownership. Throughout the analysis, we restrict attention to the case in which each firm produces a positive amount of output in the PEO equilibrium, which arises for a sufficiently large market size measured by demand intercept a.

[8] Since the transfer r is also treated as constant in Cournot competition, the equilibrium outputs do not include it.

Next, we consider the Stage-2 subgame where firms i and F determine equity ownership levels using the equilibrium outputs. Assuming that they choose equity ownership levels so as to maximize their joint profits, we show that the cost difference between them plays a critical role in shaping the PEO structure. Note first that firm j's output rises in the PEO equilibrium compared with that in the non-PEO equilibrium, that is, $q_j(\theta, \gamma) \geq q_j(0, 0)$. This occurs because PEO reduces an acquiring firm's incentive to compete in the market, thereby weakening market competition and inducing firm j to take aggressive actions. For the same reason, the aggregate output $Q(\theta, \gamma)$ falls and the market price $P(\theta, \gamma) = a - Q(\theta, \gamma)$ rises.

On the contrary, firms i and F can shift their output by exploiting a cross-border alliance. This output shifting improves profitability, outweighing firm j's aggressive actions (otherwise, they would not form PEO). To increase joint profits, they choose equity ownership levels to shift the output most effectively, which in turn depends on the cost difference between firm i and firm F. If there is no cost difference ($c_i = c_F$), they are indifferent about how the outputs are produced between them, and thus any equity ownership level would not affect the output shifting at all. However, if firms i and F choose $\theta > 0,\ \gamma > 0$, firm j takes more aggressive actions that reduce their profits. Since only the negative effect remains in that case, firms i and F have no incentive to set a positive value of equity ownership levels and hence prefer to be independent ($\theta = \gamma = 0$). By contrast, if there is a cost difference between firm i and firm F ($c_i \neq c_F$), they find it profitable to shift the output from an inefficient firm to an efficient firm and choose equity ownership levels to induce such shifting most effectively. If firms i and F choose $\theta > 0,\ \gamma > 0$, they *bilaterally* own a fraction of the partner's equity for each other, which undermines the output shifting from one firm to another. To maximize the effect of output shifting between firms i and F while minimizing the aggressive actions of firm j, it is better to arrange equity ownership levels in such a way that only one firm *unilaterally* owns a fraction of another firm's equity. Lemma 1 summarizes the PEO structure that must vary with the cost difference between firm i and firm F (see Appendix A.1 for formal proof).

Lemma 1: *The PEO structure depends on the cost difference between firm i and firm F. More specifically,*

(i) If there is no cost difference ($c_i = c_F$), neither firm i nor firm F has another firm's equity ($\theta = \gamma = 0$).

(ii) If firm i is more efficient than firm F ($c_i \leq c_F$), firm F unilaterally owns firm i's equity ($\theta > 0, \gamma = 0$).

(iii) If firm F is more efficient than firm 1 ($c_i > c_F$), firm i unilaterally owns firm F's equity ($\theta = 0, \gamma > 0$).

Lemma 1 suggests that whenever a cost difference exists, an *inefficient* firm owns an *efficient* firm's equity in order to shift the output most effectively. To understand the intuition, consider the process through which θ rises gradually and exogenously from the equilibrium with $\theta = \gamma = 0$. This increase has different effects on the three firms. Firm F, which unilaterally acquires firm i's equity share, cares about how its supply affects firm i's profits. Through this channel, firm F has an incentive to decrease its output in the PEO equilibrium, $q_F(\theta, 0) \leq q_F(0, 0)$. In response, firm j takes more aggressive actions in the PEO equilibrium through PEO-driven decreased competition, $q_j(\theta, 0) \geq q_j(0, 0)$. For firm i, when $\theta > 0, \gamma = 0$, (1) shows that firm i's profits are given by $\pi_i = (1 - \theta)(a - Q - c_i)q_i + r$. Since PEO does not affect firm i's first-order condition of the profit-maximization problem with respect to q_i, firm i acts like an outsider in the market and also takes more aggressive actions in the PEO equilibrium, $q_i(\theta, 0) \geq q_i(0, 0)$. Hence, an increase in θ induces PEO firms to shift the output from firm F to firm i, which is profitable only if the cost difference is $c_i \leq c_F$. This shows why PEO firms choose $\theta > 0, \gamma = 0$ to shift their output effectively, as in Lemma 1(ii). In contrast, the intuition of Lemma 1(iii) follows from noticing that an increase in γ (from $\theta = \gamma = 0$) works in the opposite direction for output shifting, which is profitable only if $c_i > c_F$.

In what follows, the marginal costs are assumed to satisfy the following ordering in an initial equilibrium:

$$c_1 \leq c_2 \leq c_F = c + t. \tag{3}$$

Thus firm F is less efficient than firm 1 and firm 2 because firm F needs to pay t to serve the Home market. While the empirical literature reports that foreign exporters are typically more efficient than domestic firms (e.g., Bernard et al., 2007), we assume the ordering of (3) by setting c to be lower than c_i and keeping t high in an initial equilibrium, or by broadly interpreting t as distribution costs which foreign exporters cannot be easily overcome due to a lack of distribution channels. In the latter case, PEO reduces such costs by sharing distribution networks between cross-border alliance firms in the host country's market, which is again similar to the role played by horizontal FDI.

Noting that the ordering of (3) corresponds to Lemma 1(*ii*), we proceed under the equity ownership levels with $\theta > 0, \gamma = 0$. However, we have not addressed the optimal equity level of θ that maximizes the profits, which would differ when firm F has PEO with firm 1 or firm 2. Different formations of PEO, in turn, would generate different effects on equilibrium variables, including welfare. Further, when trade liberalization lowers firm F's marginal costs by reducing t, the PEO structure could change from $\theta > 0, \gamma = 0$ to $\theta = 0, \gamma > 0$ in view of Lemma 1(*iii*). Thus, on top of conventional pro-competitive effects, trade liberalization may generate additional effects on welfare by changing the equity levels, which have not been examined in previous studies where the equity levels are exogenously given. These are the key questions that we attempt to answer in the following analyses.

3.2 PEO Equilibrium

This section investigates the PEO equilibrium in which firm F makes PEO with firm i. The main objective of this section is to characterize the optimal equity level between cross-border alliance firms and to examine its consequences for PEO firms' negotiation in Figure 1.

PEO with Firms F and 1

Consider the Stage-3 subgame of PEO with firms F and 1 (PEO with firms F and 2 will be explored shortly). Substituting $\theta > 0, \gamma = 0$ into (2) and suppressing the second argument, the equilibrium outputs are given by

$$\begin{aligned} q_1^*(\theta) &= \frac{a - 3c_1 + c_2 + c_F}{4 - \theta}, \\ q_2^*(\theta) &= \frac{a + (1-\theta)c_1 - (3-\theta)c_2 + c_F}{4 - \theta}, \\ q_F^*(\theta) &= \frac{(1-\theta)(a + c_2) + (1 + 2\theta)c_1 - 3c_F}{4 - \theta}, \\ Q^*(\theta) &= \frac{(3-\theta)a - (1-\theta)c_1 - c_2 - c_F}{4 - \theta}, \end{aligned} \tag{4}$$

where a single asterisk is attached to variables in PEO equilibrium with firm 1. Using the equilibrium outputs in (4) for (1), the equilibrium profits can also be written as a function of θ:

$$\begin{aligned} \pi_1^*(\theta) &= (1-\theta)(q_1^*(\theta))^2 + r, \\ \pi_2^*(\theta) &= (q_2^*(\theta))^2, \\ \pi_F^*(\theta) &= [q_F^*(\theta) + \theta q_1^*(\theta)]q_F^*(\theta) + \theta(q_1^*(\theta))^2 - r. \end{aligned} \tag{5}$$

(4) shows that $q_1^*(\theta), q_2^*(\theta)$ are increasing in θ, while $q_F^*(\theta), Q^*(\theta)$ are decreasing in θ. The comparative statics results help understand the logic of Lemma 1 as to why firm F unilaterally owns a fraction of firm 1's equity. When $c_1 \leq c_F$, the equilibrium outputs satisfy $q_1^*(\theta) \geq q_F^*(\theta)$, reflecting that firm F pays large transport costs. However, if PEO is formed between these firms, it helps avoid this disadvantage by shifting the output from (inefficient) firm F to (efficient) firm 1 and improve profitability. Noting that PEO is a kind of horizontal FDI, the motive of cross-border PEO resembles that of "tariff-jumping" FDI, through we do not consider tariffs.

Optimal Equity

Next we characterize the optimal equity level from the profit-maximization viewpoint of firm 1 and firm F. However, we must first address whether firms 1 and F have incentive to form PEO. For this purpose, consider whether there exist a set of θ, r that simultaneously satisfy the participant conditions, $\pi_1^*(\theta) \geq \pi_1^o$, $\pi_F^*(\theta) \geq \pi_F^o$, where $\pi_1^*(\theta), \pi_F^*(\theta)$ are given in (5) which include r, while π_1^o, π_F^o are the profits in the non-PEO equilibrium. Hereafter, the superscript o is attached to variables in the non-PEO equilibrium.

As explained in Section 2.3, we assume efficient bargaining between PEO firms in deciding an equity level. This means that these firms are able to choose an equity level to maximize their joint profits in equilibrium. Let $\Pi^*(\theta) \equiv \pi_1^*(\theta) + \pi_F^*(\theta)$ denote the joint profits between firms 1 and F. Adding up the two inequalities, the participant conditions for PEO firms are rewritten as follows:

$$\Pi^*(\theta) \geq \pi_1^o + \pi_F^o. \tag{6}$$

Because the transfer r is cancelled out in $\Pi^*(\theta)$ from (5), we only need to see whether there exists θ such that the inequality in (6) holds in the equilibrium. The following observations stand out for (6). First, if $\theta = 0$, the joint profits are equal to the sum of the outside options, $\Pi^*(0) = \pi_1^o + \pi_F^o$. Second, if $\theta > 0$, then firm F shifts its output to firm 1 to avoid transport costs. Because this improves the production efficiency of PEO firms, the joint profits exceed the sum of the outside options $\Pi^*(\theta) > \pi_1^o + \pi_F^o$. These confirm that $\Pi^*(\theta)$ is greater than or equal to $\pi_1^o + \pi_F^o$. Since the inequality in (6) always holds, firms 1 and F have an incentive to form PEO in equilibrium. Finally, the extent to which the joint profits increase relative to the sum of outside options depends on the cost difference between firm 1 and firm F, because the difference is larger, the more significant the effect of output shifting on joint profits.

We now characterize the optimal equity level between firms 1 and F. To show that the characterization depends on cost differences, Figure 2 shows the $\Pi^*(\theta)$ curves along with the $\pi_1^o + \pi_F^o$ line, where the latter is independent of θ and thus is a flat line when the horizontal line is measured by θ. The $\Pi^*(\theta)$ curves coincide with the $\pi_1^o + \pi_F^o$ line at $\theta = 0$ and the $\Pi^*(\theta)$ curves are located above the $\pi_1^o + \pi_F^o$ line at $\theta > 0$. When the cost difference between firms 1 and F is small, the effect of output shifting on joint profits is limited. Thus, the increase in $\Pi^*(\theta)$ is small. In addition to that, PEO reduces market competition and induces firm 2 to take more aggressive actions. Reflecting these two opposing forces, $\Pi^*(\theta)$ is initially increasing and eventually decreasing in θ and becomes an inverted U-shaped curve. Hence, there is a unique interior solution of θ that maximizes $\Pi^*(\theta)$, which is denoted by θ^*. Furthermore, as the cost difference increases, PEO allows firms to avoid transportation costs more effectively. In Figure 2, a large cost difference shifts the $\Pi^*(\theta)$ curve rightward, and increases the optimal equity level θ^* accordingly.

When the cost difference between firms 1 and F is sufficiently large, the positive effect of output shifting can dominate the negative effect of more aggressive actions by firm 2. If this occurs in the PEO equilibrium, the $\Pi^*(\theta)$ curve is strictly increasing over the range of $\theta \in [0, 1/2]$. In Figure 2, this corresponds to the $\Pi^*(\theta)$ curve being strictly upward sloping for $\theta \in [0, 1/2]$, whereby the optimal equity level θ^* is greater than the

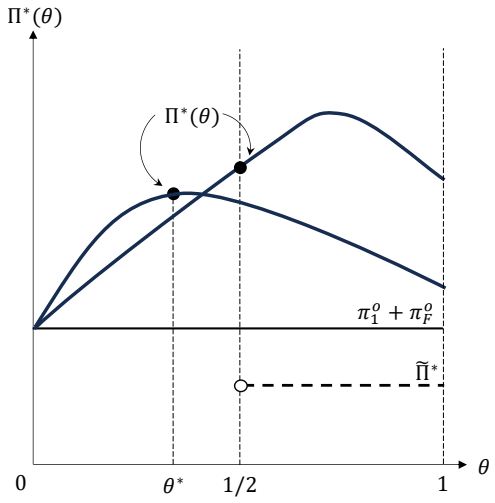

Figure 2: Profit-maximizing equity level

upper bound of 1/2. However, as long as we focus on the situation in which firms 1 and F choose the output separately, the equity level θ can take that bound at most, and the optimal equity level θ^* is a corner solution, i.e., $\theta^* = 1/2$.[9]

Proposition 1 provides more precise characterizations of the optimal equity level when firms 1 and F form PEO in the Stage-3 subgame equilibrium.

Proposition 1: *Let θ^* denote the profit-maximizing equity level that firm F owns firm 1' profits in Stage 3. For $c_F \in [c_1, \infty)$, an interior solution of the optimal equity level is given by*

$$\theta^* = \frac{8(c_F - c_1)}{a - c_1 + (c_2 - c_1) + 3(c_F - c_1)}, \tag{7}$$

where θ^* is strictly increasing in $c_F - c_1$.

Proposition 1 states that when c_F is strictly greater than c_1, the profit-maximizing equity level is strictly positive but strictly smaller than 1/2, and varies with exogenous parameters. In this sense, this indicates that firms 1 and F endogenously determine their equity ownership levels.

The intuition behind this proposition is as follows. When firm F owns a fraction θ of firm 1's equity, it helps circumvent firm F's cost disadvantage by shifting the output from inefficient firm F to efficient firm 1. This output shifting improves overall production efficiency of PEO firms, thereby increasing their joint profits. However, expecting that firm F decreases its output through PEO, which weakens market competition in the Home market, firm 2 takes more aggressive actions and increases its output. In contrast to the output shifting, this change in firm 2's actions decreases the joint profits of firms 1 and F. If $c_F = c_1$ so that there is no cost difference between these firms, the first channel is absent and hence PEO always decreases the joint profits similarly to the "merger paradox" (Farrell and Shapiro, 1990; Salant et al., 1983): the merger between firms 1 and F induces firm 2 to take more aggressive actions, decreasing the profitability of merged firms. Thus, the optimal equity level is zero ($\theta^* = 0$), without any cost difference.

[9]In equilibrium of $\theta \in (1/2, 1]$ where firm F chooses both q_1 and q_F, PEO firms have profits $\tilde{\Pi}^*$ in Figure 2 if they are merged; however, since $\tilde{\Pi}^*$ are smaller than $\Pi^*(\theta)$, an interior solution of θ^* continues to hold in such equilibrium (see Appendix A.4). The following analysis mainly considers $\theta \in [0, 1/2]$ by interpreting $\theta^* = 1/2$ as the maximum equity level.

In contrast, if firm F has a cost disadvantage over firm 1 ($c_F > c_1$), the first channel becomes operative. Since PEO increases the joint profits by shifting the output from firm F to firm 1, the output shifting motive of PEO firms leads the optimal equity level to be strictly positive ($\theta^* > 0$).[10] Further, firm F's marginal costs are higher, the more serious the cost disadvantage of firm F. As the output shift is more effective, these firms choose a higher equity level (θ^* is strictly increasing in $c_F - c_1$). Finally, if firm F's marginal costs are so high that the cost difference is large enough, the effect of output shifting is so large that the optimal equity level approaches the upper bound that firm F can take ($\theta^* = 1/2$).

Equilibrium Outcomes at Endogenous Equity Level

Now, we can evaluate the PEO equilibrium at the optimal equity level. Plugging (7) into (4) and suppressing the argument, the equilibrium outputs at $\theta = \theta^*$ are expressed as

$$q_1^* = \frac{a - 5c_1 + c_2 + 3c_F}{4}, \quad q_2^* = \frac{a - c_1 - 3c_2 + 3c_F}{4},$$
$$q_F^* = \frac{a + 7c_1 + c_2 - 9c_F}{4}, \quad Q^* = \frac{3a + c_1 - c_2 - 3c_F}{4}.$$

Relative to those in the non-PEO equilibrium, firm i's output rises and firm F's output falls ($q_i^* \geq q_i^o, q_F^* \leq q_F^o$). This decreases the aggregate output ($Q^* \leq Q^o$) because firm i considers firm j's aggressive actions in choosing its output q_i, and vice versa, whereby the increase in q_i^*, q_j^* is smaller than the decrease in q_F^*.

At the same time, however, PEO benefits all firms operating in the Home market. Relative to the profits in the non-PEO equilibrium, PEO firms can increase the joint profits by shifting the output from firm F to firm 1. Recalling that these firms engage in generalized Nash bargaining, the joint profits are distributed so that they obtain their outside options plus an additional surplus generated by PEO with bargaining power weights of β and $1-\beta$, respectively. The outside options are the profits in the non-PEO equilibrium π_1^o, π_F^o, while the surplus is defined as the difference in profits between the PEO and non-PEO equilibrium $\Delta^*(\theta) \equiv \Pi^*(\theta) - \pi_1^o - \pi_F^o$. Firm 2 can also increase the profits owing to the decreased competition by PEO. Using q_i^* and q_F^* above,

$$\begin{aligned}
\pi_1^* &= \underbrace{\left(\frac{a - 3c_1 + c_2 + c_F}{4}\right)^2}_{\pi_1^o} + \beta \underbrace{\frac{(c_F - c_1)^2}{2}}_{\Delta^*}, \\
\pi_2^* &= \left(\frac{a - c_1 - 3c_2 + 3c_F}{4}\right)^2, \\
\pi_F^* &= \underbrace{\left(\frac{a + c_1 + c_2 - 3c_F}{4}\right)^2}_{\pi_F^o} + (1 - \beta) \underbrace{\frac{(c_F - c_1)^2}{2}}_{\Delta^*}.
\end{aligned} \tag{8}$$

(8) indicates that firm 1's profits π_1^*, for example, consist of the outside options that it obtains when PEO is broken down π_1^o plus the surplus weighted by its bargaining power $\beta\Delta^*$. Because the relationship holds for firm F's profits π_F^*, both firms have sufficient incentive to form PEO in equilibrium. Notice that the surplus Δ^* is strictly increasing in cost difference $c_F - c_1$. This reflects Proposition 1, in that the output shifting from firm F to firm 1 is more effective, the larger is the cost difference between these firms. Finally, firm 2's profits π_2^* are also larger than those in the non-PEO equilibrium. This reflects the fact that firm 2 enjoys decreased competition by PEO as an outsider.

[10] This occurs even with a small cost difference $c_F - c_1 > 0$ in the baseline model with only one Foreign firm. If there are more than two Foreign firms, we can show that there exists a unique threshold, say $\tilde{c}$, such that $\theta^* > 0$ if and only if $c_F - c_1 > \tilde{c}$.

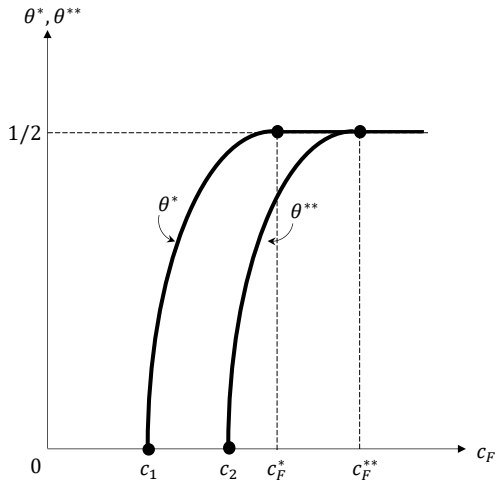

Figure 3: Comparison of profit-maximizing equity levels

PEO between Firms F and 2

What happens if firm F has PEO with (inefficient) firm 2? Since PEO between firms 2 and F has a limited effect of "output shifting," it increases the joint profits on a small scale. However, PEO has a limited effect of "anti-competition" in that case and thus induces firm 1's aggressive actions on a small scale, too. To examine this trade-off, we turn to characterizing the equilibrium of PEO with firm 2.

From Lemma 1, it directly follows that when firm 2 is more efficient than firm F (i.e., $c_2 \leq c_F$), firm F unilaterally owns a fraction θ of firm 2's equity. Since the PEO structure does not change (i.e., $\theta > 0, \gamma = 0$), the key difference mainly lies in the cost structure of PEO firms (i.e., $c_F - c_2 \leq c_F - c_1$). This suggests that the calculations should be similar to those in PEO with firms F and 1. Thus we only report the key outcomes in PEO with firms F and 2 below.

Let θ^{**} denote the profit-maximizing equity level that firm F owns firm 2' profits, where a double asterisk is attached to variables in PEO equilibrium with firm 2. An interior solution of the optimal equity level is

$$\theta^{**} = \frac{8(c_F - c_2)}{a - c_2 + (c_1 - c_2) + 3(c_F - c_2)},$$

where θ^{**} is strictly increasing in $c_F - c_2$. This shows that even when firm F makes PEO with firm 2, the optimal equity level is strictly positive but strictly smaller than 1/2. However, we find that for any c_F that satisfies (3), the optimal equity level is smaller when firm F has PEO with firm 2 than with firm 1 ($\theta^{**} \leq \theta^*$). A comparison of these optimal levels is shown in Figure 3. For example, if $c_F \in [c_F^*, c_F^{**}]$ where c_F^* and c_F^{**} are the thresholds at which $\theta^* = 1/2$ and $\theta^{**} = 1/2$, respectively, firm F owns the maximum equity of firm 1 ($\theta^* = 1/2$), whereas firm F owns a fraction of firm 2's equity ($\theta^{**} < 1/2$).

The intuition behind this difference is as follows. When firm F has a chance to make PEO with firm 2, the effect of "output shifting" is relatively limited, as noted above. Since PEO is able to shift the output from firm F to firm 2 less effectively, the increase in joint profits is smaller. Moreover, even though the extent to which PEO weakens market competition becomes milder, an increase in θ nevertheless induces (efficient) firm 1 to take more aggressive actions, which reduces the joint profit more significantly. Both of these considerations unambiguously discourage firms 2 and F from setting a higher equity level relative to the case in which firms 1 and F make PEO, that is, $\theta^{**} \leq \theta^*$.

Next, we evaluate the equilibrium variables at the optimal equity level $\theta = \theta^{**}$. Even when PEO occurs between firms 2 and F, they prefer PEO to merger and choose an interior solution of the optimal equity level. Using θ^{**}, we get the following results for the PEO equilibrium between firms F and 2. First, the "anti-competitive" effect of PEO on aggregate output and market price is smaller in this case, since the optimal equity level is lower. Second, the "output-shifting" effect of PEO on joint profits is also smaller because the cost difference is smaller. Assume that the joint profits are distributed by generalized Nash bargaining, as before, where firms 2 and F obtain the outside options plus the surplus due to PEO, $\Delta^{**}(\theta) \equiv \Pi^{**}(\theta) - \pi_2^o - \pi_F^o (> 0)$, with the same weights of bargaining power, β and $1-\beta$. Thus, firm 2's profits are $\pi_2^{**} = \pi_2^o + \beta\Delta^{**}(\theta)$. Evaluating this surplus at $\theta = \theta^{**}$, we can show that both firm 2 and firm F are better in the PEO equilibrium than in the non-PEO equilibrium ($\pi_2^{**} \geq \pi_2^o,\ \pi_F^{**} \geq \pi_F^o$). Thus, while PEO has a limited effect on the profits, these firms still have enough incentive to make PEO in equilibrium.

3.3 Negotiation

So far, we have shown that firm $i(= 1, 2)$ has incentive to make PEO with firm F in the Stage-3 subgame. Now, we turn to the Stage-2 subgame where firm i decides whether to agree with the first offer of firm F by taking account of the Stage-3 subgame equilibrium outcomes; and the Stage-1 subgame where firm F decides whether to propose to either firm 1 or firm 2 first.

As usual, we solve for the model's equilibrium backwards. For analytical simplicity, we assume that firm i has the same bargaining power, β, whereas firm F has the same bargaining power, $1-\beta$, in Stages 2 and 3.[11] Then, there is a unique equilibrium pass of negotiation, given in Lemma 2 (see Appendix A.2 for proof).

Lemma 2: *In the subgame-perfect Nash equilibrium, firm F first proposes PEO to firm 2 in Stage 1, but firm 2 does not agree on the negotiation in Stage 2, and firm 1 finally agrees on the negotiation in Stage 3, where each firm obtains the profits, π_1^*, π_2^*, π_F^*, respectively, given in (8).*

Figure 4 shows the equilibrium pass of negotiation when the market size is large. Consider firms' decisions in the left node of Figure 4. In Stage 3 where firm 2 decides whether to agree, firm 2 expects that PEO shifts the output and increases its profits relative to its outside options, $\pi_2^{**} \geq \pi_2^o$. Thus firm 2 agrees in Stage 3. Consider next Stage 2 where firm 1 decides whether to agree. Because the Stage-3 negotiation is successful, PEO firms' outside options rise to $\pi_1^{**} (\geq \pi_1^o)$ and $\pi_F^{**} (\geq \pi_F^o)$ in Stage 2. While they get the joint profits Π^* by setting $\theta = \theta^*$, the surplus by PEO reduces to $\tilde{\Delta}^* \equiv \Pi^* - \pi_1^{**} - \pi_F^{**} (\leq \Delta^*)$ due to increased outside options. Thus, firm 1's profits in Stage 2 are $\bar{\pi}_1^* = \pi_1^{**} + \beta\tilde{\Delta}^*$. To decide whether to agree in Stage 2, firm 1 compares the profits obtained as a partner of PEO, $\bar{\pi}_1^*$, and those as an outsider, π_1^{**}, which depends on market size. When the market size is large, the outside options in Stage 3 are large relative to the profits from making PEO in Stage 2 (as part of profits is distributed to firm F). Thus, firm 1 prefers to be an outsider and disagrees. The same negotiation occurs in Stages 2 and 3 of the right node. Finally, in Stage 1 where firm F compares π_F^* and π_F^{**}, it offers to firm 2 first because $\theta^* \geq \theta^{**}$ and thus $\pi_F^* \geq \pi_F^{**}$.[12]

Lemma 2 means that firm F always forms PEO with firm 1 in equilibrium. This result is not surprising because PEO with firm 1 is more cost-efficient and hence more profitable. Indeed, we can show that $\Pi^* \geq \Pi^{**}$ if and only if $c_1 \leq c_2$ and the joint profits are higher when firm F has PEO with firm 1 than with firm 2.

[11] This assumption is not crucial. Our results continue to hold even when firm F first makes a take-it-or-leave-it offer to firm i (which corresponds to full bargaining power of firm F) in Stage 2; if the negotiation fails to agree, firm F and firm j then engage in Nash bargaining (with bargaining power β and $1-\beta$, respectively) in Stage 3.

[12] As shown in Appendix A.2, the equilibrium pass of negotiation in Lemma 2 holds even when the market size is small.

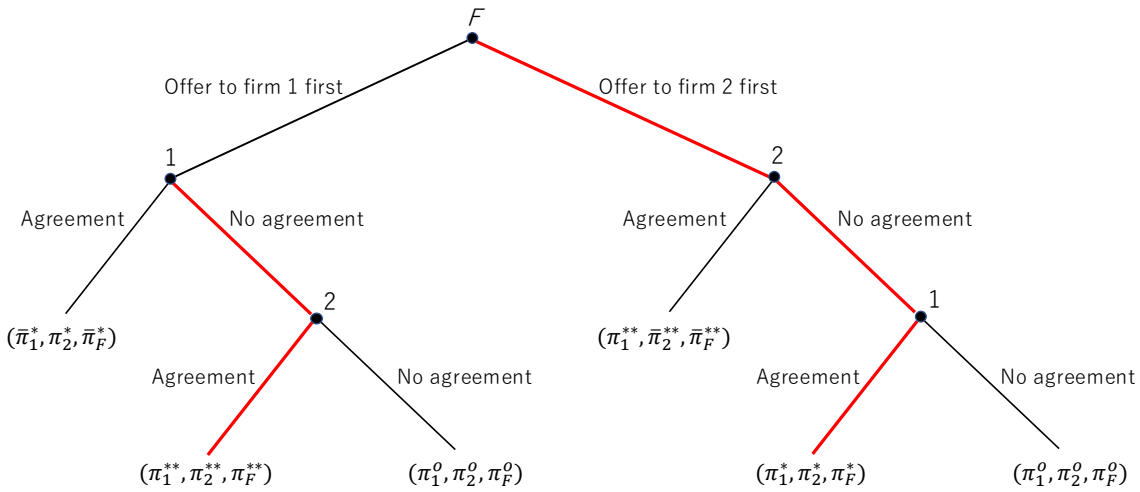

Figure 4: Equilibrium pass when market size is large

Despite that, firm F proposes to firm 2 first in order to eliminate firm 1's incentive to be an outsider of PEO, which improves firm F's bargaining position and contributes to increasing its profits in negotiation. This kind of strategic interactions among firms is at the heart of Lemma 2.

4 Welfare Analysis

Section 3 has shown the process through which cross-border alliance firms determine the equity level. While cross-border PEO helps firms avoid transport costs (similarly to tariff-jumping FDI), it weakens competition which hurts consumers (similarly to domestic merger). This section addresses whether or not the government should give approval for firms to form cross-border PEO by considering the effect on welfare.

4.1 Welfare without Government Intervention

Define first Home welfare, denoted by W below. In our model where the government does not impose tariffs, Home welfare consists of consumer surplus and Home firms' profits. Since the equilibrium outputs are written as a function of equity levels θ, γ (see (2)), the welfare components are also written as a function of θ, γ. Thus

$$W(\theta, \gamma) = CS(\theta, \gamma) + \pi_1(\theta, \gamma) + \pi_2(\theta, \gamma),$$

where $CS(\theta, \gamma) = [Q(\theta, \gamma)]^2/2$ under linear demand. Let W^o denote the Home welfare when PEO does not occur and all firms are independent. Then, $W^o = CS^o + \pi_1^o + \pi_2^o$ where CS^o and $\pi_1^o + \pi_2^o$ are consumer surplus and profits in the non-PEO equilibrium, respectively, which are obtained by setting $\theta = \gamma = 0$. Similarly, let W^* and W^{**} denote the Home welfare when firm F makes PEO with firm 1 and firm 2, respectively. Then, $W^* = CS^* + \pi_1^* + \pi_2^*$ and $W^{**} = CS^{**} + \pi_1^{**} + \pi_2^{**}$, which are evaluated at $\theta = \theta^*$ and $\theta = \theta^{**}$, respectively, while maintaining $\gamma = 0$. The exact expressions of Home welfare are shown in Appendix A.3.

Next, consider the effect of PEO on Home welfare by comparing W^o and W^*. On one hand, PEO decreases the aggregate quantity supplied to the Home market, which decreases consumer surplus. Thus, $CS^* \leq CS^o$. On the other hand, PEO allows for the output shifting from firm F to firm 1, which increases firm 1's profits; PEO also weakens competition in the Home market and increases firm 2's profits. Thus, $\pi_1^* \geq \pi_1^o$ and $\pi_2^* \geq \pi_2^o$.

Since Home consumers are worse off but Home firms are better off in the PEO equilibrium than those in the non-PEO equilibrium, the introduction of PEO can either improve or worsen the equilibrium Home welfare. This is a central trade-off of PEO for the Home welfare. The same trade-off applies to the PEO equilibrium between firms 2 and F to different degrees, due to different levels of optimal equity ownership.

Finally, compare the Home welfare across different regimes. Using the aggregate quantity and Home firms' profits in Section 3.2, we find that the Home welfare satisfies the following ranking in the presence of PEO, so long as least-efficient firm F produces in equilibrium (see Appendix A.3 for proof):

$$W^* \leq W^{**} \leq W^o.$$

The first inequality holds because PEO with firm 2 helps mitigate the negative impact of PEO on consumers as well as Home welfare through decreased competition. In contrast, the second inequality shows that despite the negative impact being softened, PEO with firm 2 harms the Home welfare relative to the non-PEO case. Thus, when the Home government needs to give approval for firm F to form PEO for some reason, it always prefers PEO with inefficient firm 2. As seen in Lemma 2, however, firm F always prefers to make PEO with efficient firm 1. Thus, when firms can freely choose the equity ownership level without facing any restrictions on foreign equity, they always lead to the worst outcome in terms of the Home welfare. The implications from our model are thus that the government has a strong incentive to intervene in forming cross-border PEO.

4.2 Welfare with Government Intervention

Having shown that PEO worsens the welfare, we next address whether the government should completely ban PEO in any case or allow firms to form cross-border PEO in some case, and if so, what kind of conditions is required to give approval of such PEO. To answer this question, we ask whether the government can choose the equity level to improve the welfare by allowing firm F to form PEO with firm 1.[13]

It is worth stressing that the negative impact of PEO on the Home welfare occurs in equilibrium at the profit-maximizing equity level, $\theta = \theta^*$. This result would not necessarily hold when the Home government can directly choose the equity level to maximize the Home welfare. To derive this welfare-maximizing equity level, let us consider a slightly different setting where the Home government chooses the equity level θ in Stage 1, taking as given the equilibrium outputs $q_1^*(\theta), q_2^*(\theta), q_F^*(\theta)$ in Stage 2. Then the consumer surplus is given by $CS^*(\theta) = [Q^*(\theta)]^2/2$, and the Home firms' profits are given by $\pi_1^*(\theta)$, $\pi_2^*(\theta)$, shown in (4) and (5), respectively. In this circumstance, θ is chosen to maximize the government's objective function, defined as follows:

$$\begin{aligned} G^*(\theta) &\equiv CS^*(\theta) + \pi_1^*(\theta) + \pi_2^*(\theta) \\ &= CS^*(\theta) + \pi_1^o + \beta\Delta^*(\theta) + \pi_2^*(\theta), \end{aligned}$$

where the second equality follows from recalling that firm 1's profits are distributed through Nash bargaining. Let θ_G^* denote the optimal equity level that the government chooses to maximize $G^*(\theta)$. If θ_G^* is always zero, the government should always completely ban PEO between firms 1 and F. If θ_G^* is strictly positive, however, the government should permit firms to form PEO between these firms, although the equity level chosen by the government would not be identical to that chosen by the firms.

Before characterizing this equity level, it is crucial to see whether the government indeed has incentive to permit PEO in equilibrium. For this purpose, we must check whether there exists an equity level θ such that

[13] We focus on this PEO formation here because firm F forms PEO with firm 1 without government intervention (Lemma 2). Our objective is to address whether the government has incentive to permit this PEO.

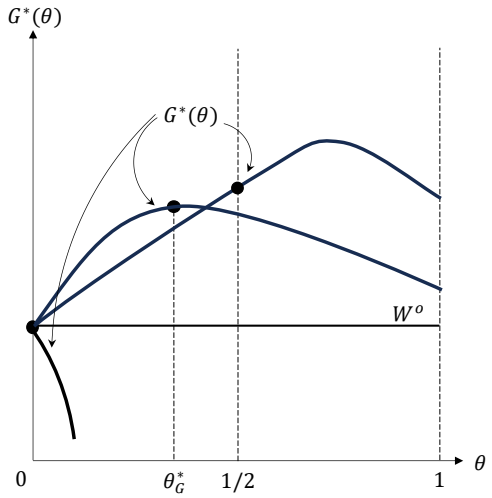

Figure 5: Welfare-maximizing equity levels

$G^*(\theta)$ is greater than W^o in the non-PEO equilibrium:

$$G^*(\theta) \geq W^o.^{14} \tag{9}$$

From the comparative statics results with respect to θ in Section 3, the following observations stand out. First, PEO generates the two opposing forces on welfare: the "output-shifting" effect that improves welfare and the "anti-competition" effect that worsens welfare. Second, the extent to which PEO affects welfare depends on the cost difference between firms 1 and F ($c_F - c_1$) through the above effects as well as the bargaining power of firm 1 (β) through the distribution of surplus in negotiation. To examine whether (9) holds in equilibrium, thus, we have to be careful about the values of both parameters.

Figure 5 illustrates the $G^*(\theta)$ curves for a large β case with different values of cost differences $c_F - c_1$ in the $(\theta, G^*(\theta))$ space. This figure is a counterpart to Figure 2, with the main difference being the Home welfare (instead of the joint profits) in the vertical axis. We have argued that PEO increases the joint profits due to the output shifting, but the extent to which the profits rise depends on the cost difference between firm 1 and firm F. In addition to this positive effect, the Home government also takes into account the negative effect of PEO on consumer surplus. Due to this additional consideration, the $G^*(\theta)$ curves are not necessarily located above the W^o line in Figure 5.

When the cost difference is small, PEO has a limited effect on profits, but nonetheless hurts consumers, and an increase in profits is dominated by a decrease in consumer surplus. Thus, for any equity level $\theta \in [0, 1/2]$, the Home welfare is always greater in the non-PEO equilibrium than in the PEO equilibrium ($W^o \geq G^*(\theta)$). In Figure 5, the outcome reflects that the $G^*(\theta)$ curve is always located below the W^o line for any $\theta \in [0, 1]$. As the inequality in (9) does not hold in this case, the government should completely ban cross-border PEO. Thus, $\theta_G^* = 0$. When the cost difference is large, the positive effect on profits dominates the negative effect on consumer surplus and $G^*(\theta)$ is located above the W^o line ($G^*(\theta) > W^o$). Although a small increase in θ is beneficial to welfare due to the output-shifting effect that raises the profits, a further increase in θ is harmful to welfare because of the anti-competition effect that reduces consumer surplus. Reflecting these two effects, the $G^*(\theta)$ curve becomes inverted U-shape which has a unique interior solution of θ that maximizes $G^*(\theta)$.

[14] Naturally, if $\theta = 0$, both consumer surplus and Home firms' profits reduce to those in the non-PEO equilibrium: $G^*(0) = W^o$.

Thus, $\theta_G^* > 0$. As the cost difference between firm 1 and firm F is larger, the output-shifting effect on welfare is more effective, shifting the $G^*(\theta)$ curve rightward. As a result of this, the interior solution of θ_G^* is strictly increasing in the cost difference $c_F - c_1$. Therefore, when the cost difference is sufficiently large, the optimal equity level takes a corner solution. Thus, $\theta_G^* = 1/2$.

This argument applies to the welfare-maximizing equity level with large β, but what happens for small β? To see the optimal equity level, consider a limiting case of $\beta = 0$ in which firm 1 has no bargaining power and all surplus is distributed to firm F in the Nash bargaining. Then, the government's objective function is $G^*(\theta) = CS^*(\theta) + \pi_1^o + \pi_2^*(\theta)$. In that case, however, firm 1 cannot enjoy the positive effect of output shifting, which reduces the government's incentive to give approval for firms to make PEO. In Figure 5, when $\beta = 0$, the $G^*(\theta)$ curve is always located below the W^o line, regardless of the cost difference $c_F - c_1 (\geq 0)$, and thus the welfare-maximizing equity level is always zero. This shows that there is a threshold of bargaining power β above which the positive effect is greater than the negative effect, whereby the optimal equity level is strictly positive.

Proposition 2 provides a more precise characterization of the optimal equity level for the welfare-maximizing government in this setting.

Proposition 2: *Let θ_G^* denote the welfare-maximizing equity level when firm F makes PEO with firm 1. For $c_F \in [c_1, \infty)$ and $\beta \in (3/4, 1]$, an interior solution of the optimal equity level is given by*

$$\theta_G^* = \frac{(3+8\beta)(c_F - c_1) - \{a - c_1 + (c_2 - c_1)\}}{3\beta(c_F - c_1) - (1-\beta)\{a - c_1 + (c_2 - c_1)\}}, \tag{10}$$

where θ_G^* is strictly increasing in $c_F - c_1$. If $\beta \in [0, 3/4]$, the optimal level is always zero.

Like the profit-maximizing equity level θ^* in (7), the welfare-maximizing equity level θ_G^* in (10) is strictly increasing in the cost difference $c_F - c_1$. Despite this similarity, there are several important differences between the two. First, while θ^* is always strictly positive, regardless of the cost difference, θ_G^* is not the case, and it is zero for a small cost difference. Setting $\theta_G^* = 0$, we get the threshold $c_F - c_1 = \{(a - c_1) + (c_2 - c_1)\}/(3 + 8\beta)$, above which $\theta_G^* > 0$. Second, while θ^* is independent of β, θ_G^* is dependent on β through the distribution of surplus, where $\theta_G^* > 0$ if and only if β is sufficiently large. This result can be confirmed by differentiating (10) with respect to $c_F - c_1$, which shows that the interior solution of θ_G^* is strictly increasing in $c_F - c_1$ if and only if $\beta > 3/4$. However, when $\beta \leq 3/4$, θ_G^* is decreasing in $c_F - c_1$ but this contradicts the fact that an increase in $c_F - c_1$ strengthens the government's incentive to allow for PEO. Thus, θ_G^* should not be decreasing in $c_F - c_1$, which explains why the interior solution does not arise unless β is sufficiently large.

From (10), we can obtain other comparative statics results. First, θ_G^* is strictly decreasing in a for $\beta > 3/4$, which implies that the government has less incentive to allow for PEO, the larger is the market size. Intuitively, as the market size is larger, the aggregate quantity rises relatively more than firms' quantity in equilibrium (see (4)), and consumer surplus rises relatively more than profits. As a result, when PEO is allowed for firms, a decrease in consumer surplus is greater than an increase in profits. Thus a larger market entails a more adverse welfare effect through consumer surplus, leading the government to set a more stringent equity level. Furthermore, θ_G^* is strictly decreasing in c_1, c_2 but it is strictly increasing in c_F for $\beta > 3/4$. For example, a decrease in c_2 intensifies the market competition and increases consumer surplus. Since the negative effect on welfare is softened, it provides more room to permit PEO. Though a decrease in c_F also intensifies the market competition, which positively affects welfare, it allows firms to exploit output shifting less effectively, which negatively affects welfare. In equilibrium, the latter dominates the former, leaving less room to permit PEO.

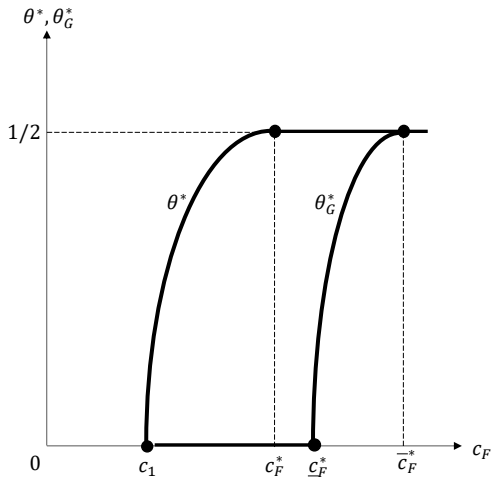

Figure 6: Comparison of profit- and welfare-maximizing equity levels

This implies that trade liberalization, which leads to pro-competitive effects by reducing $c_F = c + t$, may not always improve welfare in the presence of cross-border PEO for large trade costs.

Finally, we mention the welfare-maximizing equity level with (inefficient) firm 2, which is given by

$$\theta_G^{**} = \frac{(3+8\beta)(c_F - c_2) - \{(a - c_2 - (c_1 - c_2)\}}{3\beta(c_F - c_2) - (1-\beta)\{(a - c_2 - (c_1 - c_2)\}}.$$

A comparison with (10) shows that the optimal equity level for firm 2 is *larger* than that for firm 1 ($\theta_G^{**} \geq \theta_G^*$). This outcome occurs because PEO with firm 2 gives rises to a less severe decline in consumer surplus. Thus, for a given cost difference in (3), the government sets a less stringent equity level for firm 2 than for firm 1. This is in sharp contrast to the result for profit-maximizing equity levels, where the optimal equity level for firm 2 is *smaller* than that for firm 1 ($\theta^{**} \leq \theta^*$), as seen in Section 3.2.

4.3 Policy Implications

Now, we are ready to provide the policy implications from our analysis by comparing θ^* and θ_G^* given by Propositions 2 and 5. For this purpose, Figure 6 shows these optimal levels in the same space under which the bargaining power is large ($\beta > 3/4$) and the welfare-maximizing equity level is an interior solution ($\theta_G^* > 0$). From the figure, we can obtain the following observations.

First, the profit-maximizing equity level θ^* is located to the left of the welfare-maximizing equity level θ_G^*. Because $\theta_G^* \leq \theta^*$ for a given cost difference, this implies that the government does not permit the equity level required by PEO firms. Second, if the cost difference is small with $c_F \in (c_1, c_F^*)$, firms 1 and F make PEO by setting $\theta^* > 0$, while the government bans PEO by setting $\theta_G^* = 0$. This result is consistent with our analysis in Section 4.1, where the Home welfare in the PEO equilibrium is always smaller than that in the non-PEO equilibrium ($W^* \leq W^o$). Third, if the cost difference is large with $c_F \in (c_F^*, \infty)$, firm F owns the maximum level of firm 1's equity by setting $\theta^* = 1/2$, whereas the government permits PEO by setting $\theta_G^* < 1/2$, where firm F only owns a fraction of firm 1's equity. In Figure 6, θ_G^* is strictly positive but strictly smaller than $1/2$ between $c_F \in (\underline{c}_F^*, \bar{c}_F^*)$ where $\underline{c}_F^*$ and $\bar{c}_F^*$ are the thresholds at which $\theta_G^* = 0$ and $\theta_G^* = 1/2$, respectively. This is because the benefit of output shifting is larger than the cost of decreased competition in this range, whereby the government can improve the welfare by permitting PEO.

Proposition 3 provides a more precise comparison of the profit- and welfare-maximizing equity levels.

Proposition 3: *Regardless of bargaining power β, the profit-maximizing equity level θ^* is always greater than or equal to the welfare-maximizing equity level θ_G^* for $c_F \in [c_1, \infty)$.*

To draw the policy implications from this proposition, we consider an extended four-stage game as follows: In Stage 0, the Home government announces the maximum equity level that firms can take, say $\tilde{\theta} \in [0, 1/2]$; subsequently, three firms play the same game in Stages 1–3 in Figure 1. Also, assume that firms 1 and 2 have large bargaining power $\beta > 3/4$ so that Figure 6 applies to the equilibrium equity levels. Then, the optimal policy from the government perspective can be classified in terms of the cost difference between firm 1 and firm F: (i) when $c_F \leq \underline{c}_F^*$, the government should set $\tilde{\theta} = 0$ such that cross-border PEO is completely banned; and (ii) when $c_F > \underline{c}_F^*$, the government should set $\tilde{\theta} = \theta_G^*$ to permit cross-border PEO under the restriction that PEO firms choose $\theta < \tilde{\theta}$.

More broadly, our model offers the policy implications of cross-border alliance in globalization. It is often argued that foreign equity ownership, including cross-border PEO, could negatively affect welfare because of its anti-competitive force; thus, governments should restrict foreign equity ownership. Our model shows that this argument holds only when the cost difference is relatively small. When the difference is relatively large, in contrast, foreign equity ownership may increase welfare by helping firms shift the output across borders, avoiding transport costs and improving their overall production efficiency. Hence, if the government is able to choose an appropriate equity level in advance, foreign equity ownership could positively affect welfare despite the negative effect on consumer surplus associated with weakened competition.

It is also worth mentioning that a reduction in transport costs, which decreases the cost difference between cross-border alliance firms, leads to a decrease in the optimal equity levels, θ^* and θ_G^*, as shown in Figure 6. The intuition of this outcome is explained by observing that PEO is a kind of horizontal FDI, where one of the important differences between these two forms is whether a domestic firm is either partially or fully owned by a foreign firm. In existing trade models in the horizontal FDI literature, a foreign firm that enters a host country's market buys out the full ownership of a local firm ("brownfield FDI") or establishes a new plant ("greenfield FDI"). Though the trade-off between the two types of FDI has been extensively analyzed,[15] most of these studies treat the extent to which a foreign firm owns a domestic firm's equity as exogenously fixed. This study characterizes the optimal equity share of a foreign firm, which is endogenously determined by the cost difference that comes from transport costs, while keeping the tariff-jumping motive. Once the similarity is understood, it is not very surprising to see that θ^* and θ_G^* are decreasing in t. It is known that relative to export, FDI is profitable when firms incur large transport costs, and the same applies to cross-border PEO. As transport costs decline, the advantage of PEO diminishes and firms have a small incentive to form PEO. Further, the positive welfare effect of PEO is small and the government also has a small incentive to permit such PEO.

We conclude this section by briefly noting what happens if we treat t as import tariffs instead of transport costs. Clearly, this does not affect the profit-maximizing equity level θ^*, and hence Proposition 1 still holds. As for the welfare-maximizing equity level θ_G^* in Proposition 2, we need to consider tariff revenues as one of the welfare components. Thus, θ_G^* is chosen to maximize the following government's objective function:

$$G^*(\theta) = CS^*(\theta) + \pi_1^o + \beta\Delta^*(\theta) + \pi_2^*(\theta) + tq_F^*(\theta).$$

[15] See Nocke and Yeaple (2007, 2008) for the important contributions to the literature.

As seen above, trade liberalization decreases the optimal equity level for the government. Additionally, when t are treated as tariffs, trade liberalization lowers tariff revenues, which decreases the welfare-maximizing equity level further. Since $c_F = c + t$, however, trade liberalization can reverse the marginal cost ranking in (3) from $c_1 \leq c_F$ to $c_F < c_1$, changing the PEO structure from $\theta > 0, \gamma = 0$ to $\theta = 0, \gamma > 0$ in light of Lemma 1(*iii*). Thus PEO induces the output shifting from firm 1 to firm F, raising firm F's exports and thus tariff revenues, which in turn increases the welfare-maximizing equity level. Due to these additional channels associated with changes in tariff revenues and the PEO structure, the characterization of θ_G^* becomes more complex than that in Proposition 2. As a result, the relationship between θ^* and θ_G^* in Proposition 3 may change.

5 Conclusion

This study examines the optimal equity levels for cross-border alliance firms and a local government when a foreign firm enters a host country's market by partially owning a domestic firm's equity. While many studies have shown the increasing importance of firms' competitive strategy and governments' competition policy in the presence of such partial cross-border alliance, most existing models treat the equity level as exogenously given and do not explore the mechanism through which firms or governments endogenously choose the equity level. This study attempts to fill this important gap in the literature by explicitly analyzing this mechanism from the profit- and welfare-maximizing viewpoints. In particular, our model shows the channel through which cross-border alliance firms choose the equity level, as well as the possibility that allowing for PEO is beneficial even for the welfare-maximizing government, despite the negative effect on consumer surplus associated with weakened competition.

In our model, marginal costs of cross-border alliance firms play a key role in determining PEO structure. This implies that policymakers need to know these costs precisely so as to implement government restrictions on foreign equity ownership. In practice, however, it is often impossible to directly observe the marginal costs which makes it hard to apply our policy implications to the real world. One of the possible ways to deal with this problem is to look at the market share of a foreign firm that forms PEO with a domestic firm. Not only is the market share directly observable, but also the share is larger, the more efficient is the foreign firm. Thus, policymakers may refer to a foreign firm's market share in setting government restrictions on foreign equity, which would vary with industries and countries.

The current model is highly stylized, and the next step is to address the extent to which our results can be generalized. For example, we examined a simple oligopoly setup with two home firms and one foreign firm, but it is possible to employ a general number of these firms. In such an extension, though the main results do not change qualitatively, they do change quantitatively. For the profit-maximizing equity level (Proposition 1), we showed that even a small cost difference can induce firms to form PEO. However, if there is more than one foreign firm, market competition is tougher than in the baseline model; thus we need larger cost differences to make the profit-maximizing level positive. Similarly, for the welfare-maximizing equity level (Proposition 2), we showed that the bargaining power needs be large for the government to permit such PEO. However, if there is more than one foreign firm, intensified competition can alleviate the decreased consumer surplus by PEO, which lowers the threshold of bargaining power. These imply that the gap between optimal equity levels gets smaller and, in that sense, cross-border PEO is more likely to be desirable for both firms and a government in this extension. In addition to that, while the welfare-maximizing equity level is necessarily smaller than the profit-maximizing equity level in the baseline model, this relationship can be reversed in the extended model. We plan to present some of these generalizations in a future version of the paper.

A Appendix

A.1 Proof of Lemma 1

This appendix shows that when firm F proposes PEO to firm i, firm F always owns firm i's equity in PEO, but the opposite does not occur in equilibrium, as long as the marginal costs satisfy the ordering in (3). Our goal is to formally prove that (efficient) firm i has no incentive to own a fraction of (inefficient) firm F's equity whenever the equity ownership levels are restricted to $\theta, \gamma \in [0, 1/2]$. While we focus on the above case below, a similar proof applies to another case, in which the ordering of (3) is reversed by trade liberalization.

If PEO takes place between firm i and firm F, the profits of firms i, j, F are respectively given by

$$\begin{aligned}
\pi_i &= (1-\theta)(a-Q-c_i)q_i + \gamma(a-Q-c_F)q_F + r, \\
\pi_j &= (a-Q-c_j)q_j, \\
\pi_F &= (1-\gamma)(a-Q-c_F)q_F + \theta(a-Q-c_i)q_i - r.
\end{aligned}$$

Solving the standard profit-maximizing problem for a given θ and γ, we directly get the equilibrium outputs in (2). To ensure that the equilibrium outputs are positive, we assume that the market size measured by the demand intercept is large enough to satisfy

$$a > \underline{a} \equiv \frac{3(1-\gamma)c_F - (1-\gamma+2\theta)c_i - (1-\theta-\gamma)c_j}{1-\theta-\gamma}.$$

Note that PEO firms set the equity ownership levels to shift output effectively and maximize joint profits. Let $\Pi \equiv (a-Q-c_i)q_i + (a-Q-c_F)q_F$ denote the joint profits of firms i and firm F. Because the equilibrium outputs are written as a function of θ, γ, the equilibrium joint profits are also written as a function of θ, γ, denoted by $\Pi(\theta, \gamma)$. By substituting the equilibrium outputs, we obtain the joint profits. Clearly, PEO firms set θ, γ to maximize $\Pi(\theta, \gamma)$.

The equilibrium joint profits satisfy the following properties from the equilibrium outputs in (2) under the restriction of the market size $a > \underline{a}$. First, suppose $c_F = c_i$, such that the marginal costs of firm i and firm F are the same. Then, differentiating $\Pi(\theta, \gamma)$ and evaluating at $c_F = c_i$, we get

$$\left.\frac{\partial \Pi(\theta,\gamma)}{\partial \theta}\right|_{c_F=c_i} = \left.\frac{\partial \Pi(\theta,\gamma)}{\partial \gamma}\right|_{c_F=c_i} = -\frac{(\theta+\gamma)(a-2c_i+c_j)^2}{(4-\theta-\gamma)^3} < 0.$$

Since the introduction of PEO always decreases joint profits at $c_F = c_i$, neither firm has an incentive to make PEO; hence, we have $\theta = \gamma = 0$. Second, suppose $c_F \geq c_i$ as in (3). Then

$$\frac{\partial \Pi(\theta,\gamma)}{\partial \theta} - \frac{\partial \Pi(\theta,\gamma)}{\partial \gamma} = \frac{(c_F - c_i)\Gamma}{(1-\theta-\gamma)(4-\theta-\gamma)^2} > 0,$$

where

$$\begin{aligned}
\Gamma &\equiv 2(1-\theta-\gamma)(2-\theta-\gamma)(a+c_j) - (4+8\theta-3\theta^2-20\gamma+7\gamma^2+4\theta\gamma)c_F - (4+8\gamma-3\gamma^2-20\theta+7\theta^2+4\theta\gamma)c_i \\
&> 2(1-\theta-\gamma)(2-\theta-\gamma)(\underline{a}+c_j) - (4+8\theta-3\theta^2-20\gamma+7\gamma^2+4\theta\gamma)c_F - (4+8\gamma-3\gamma^2-20\theta+7\theta^2+4\theta\gamma)c_i \\
&= (c_F - c_i)(4-\theta-\gamma)(2+\gamma-3\theta) > 0.
\end{aligned}$$

Therefore, $\frac{\partial \Pi(\theta,\gamma)}{\partial \gamma} < 0$ when $\frac{\partial \Pi(\theta,\gamma)}{\partial \theta} = 0$, suggesting that $\gamma = 0$ when $0 \leq \theta \leq 1/2$.

A.2 Proof of Lemma 2

This appendix shows that, regardless of the market size, there exists a unique equilibrium pass in Lemma 2. As usual, we solve the model by backward induction. While the equilibrium profits in Stage-3 subgame when PEO occurs with firm 1 are (8), while those with firm 2 are given by

$$\begin{aligned}
\pi_1^{**} &= \left(\frac{a-3c_1-c_2+3c_F}{4}\right)^2, \\
\pi_2^{**} &= \underbrace{\left(\frac{a+c_1-3c_2+c_F}{4}\right)^2}_{\pi_2^o} + \beta \underbrace{\frac{(c_F-c_2)^2}{2}}_{\Delta^{**}}, \\
\pi_F^{**} &= \underbrace{\left(\frac{a+c_1+c_2-3c_F}{4}\right)^2}_{\pi_F^o} + (1-\beta) \underbrace{\frac{(c_F-c_2)^2}{2}}_{\Delta^{**}}.
\end{aligned} \tag{A.1}$$

Stage-2 Subgame

While firm i and firm F have bargaining power with β and $1-\beta$ across Stages 2 and 3, the profits obtained in the negotiation are different. In the Stage-2 negotiation between firms 1 and F, if they agree on the terms of PEO, they obtain the same joint profits Π^* as in Stage 3, because the cost and demand parameters are the same across different stages and they set the same equity level θ^* in Proposition 1. However, they obtain different outside options because, if the negotiation fails to agree, firms 2 and F always agree to form PEO in Stage 3, and they set the equity level θ^{**} in Section 3.2. Thus, the outside options that firm 1 and firm F obtain in Stage 2 are π_1^{**} and π_F^{**}, respectively, in (A.1), and the surplus generated by PEO in Stage 2 is given by $\tilde{\Delta}^* \equiv \Pi^* - \pi_1^{**} - \pi_2^{**}$. Since the surplus is distributed to firm 1 and firm F with the same weights of bargaining power β and $1-\beta$, firm 1 and firm F' profits in Stage 2 are respectively given by

$$\begin{aligned}
\bar{\pi}_1^* &= \pi_1^{**} + \beta(\Pi^* - \pi_1^{**} - \pi_F^{**}), \\
\bar{\pi}_F^* &= \pi_F^{**} + (1-\beta)(\Pi^* - \pi_1^{**} - \pi_F^{**}).
\end{aligned} \tag{A.2}$$

Similarly, when firm 2 and firm F negotiate the terms of PEO in Stage 2, if these firms agree, they set the equity level θ^{**} in Stage 2 and obtain the joint profits Π^{**}. However, if they fail to agree, firms 1 and F set the equity level θ^* in Stage 3. Consequently, firms 2 and F obtain the outside options π_2^* and π_F^* in (8), and the surplus is $\tilde{\Delta}^{**} \equiv \Pi^{**} - \pi_2^* - \pi_F^*$. Thus, firm 2 and firm F's profits in Stage 2 are respectively given by

$$\begin{aligned}
\bar{\pi}_2^{**} &= \pi_2^* + \beta(\Pi^{**} - \pi_2^* - \pi_F^*), \\
\bar{\pi}_F^{**} &= \pi_F^* + (1-\beta)(\Pi^{**} - \pi_2^* - \pi_F^*).
\end{aligned} \tag{A.3}$$

The equilibrium profits in Figure 1 are given by four sets of equations: (8) and (A.1) in the Stage-3 subgame and (A.2) and (A.3) in the Stage-2 subgame.

Stage-1 Subgame

Now, we turn to examining firm F's decision whether to propose to either firm 1 or firm 2 first in Stage 1. Firm F makes this decision by correctly expecting the profitability of PEO with firms 1 and 2 realized in Stages 2 and 3.

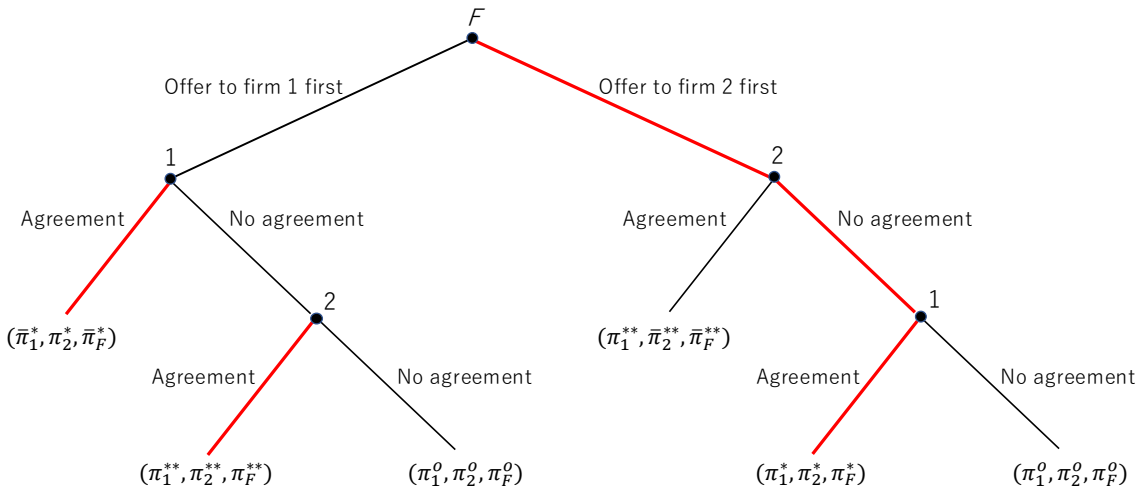

Figure 7: Equilibrium pass when market size is small

Consider first the Stage-3 subgame equilibrium. Firm 1 compares π_1^* and π_1^o to decide whether or not to agree on the terms of PEO. Due to the output shifting, we know that firm 1 has higher profits by forming PEO (i.e., $\pi_1^* \geq \pi_1^o$ in view of (8)); thus, firm 1 always agrees in Stage 3. Similarly, firm 2 compares π_2^{**} and π_2^o to decide whether or not to agree on the terms of PEO. Again, firm 2 has higher profits by forming PEO (i.e., $\pi_2^{**} \geq \pi_2^o$ in view of (A.1)); thus, firm 2 always agrees in Stage 3. Hence, both firm 1 and firm 2 agree with firm F in Stage 3.

Consider next the Stage-2 subgame equilibrium. Since firm 2 agrees to make PEO with firm F in Stage 3, firm 1 compares π_1^{**} and $\bar{\pi}_1^*$ to decide whether or not to agree in Stage 2, which are given in (A.1) and (A.2), respectively. However, the surplus generated by PEO in this stage, $\tilde{\Delta}^* = \Pi^* - \pi_1^{**} - \pi_F^{**}$, is not always positive because firms 1 and F can make PEO in Stage 3, which raises the outside options to $\pi_1^{**} + \pi_F^{**} (\geq \pi_1^o + \pi_F^o)$ in Stage 2. In fact, using (8) and (A.1) for (A.2) and rearranging, we find that $\tilde{\Delta}^* \leq 0$ if and only if

$$\beta \leq 1 + \frac{(a - c_1)(c_F - c_2) - 2(c_F - c_1)(c_2 - c_1)}{2(c_F - c_2)^2}.$$

Since β takes a value of one at most, the inequality is more likely, the larger the market size a. In this case, we have $\bar{\pi}_1^* \leq \pi_1^{**}$, whereby firm 1 disagrees to make PEO in Stage 2, as shown in Figure 4. Intuitively, when the market size is large, firm 1 obtains larger profits by being an outsider than by joining PEO. In contrast, when the market size is small, firm 1 obtains smaller profits as an outsider. Since it is better off by joining PEO, firm 1 agrees and the game ends in Stage 2, as in Figure 7.

Similarly, firm 2 compares $\bar{\pi}_2^{**}$ and π_2^* in Stage 2. In contrast to the PEO with firm 1, the surplus by PEO with firm 2 in this stage is always negative, $\tilde{\Delta}^{**} = \Pi^{**} - \pi_2^* - \pi_F^* \leq 0$ because an increase in joint profits at $\theta = \theta^{**}$ is smaller than an increase in joint profits at $\theta = \theta^*$ (see Section 3.2). Indeed, using (8) and (A.1) for (A.3) and rearranging, we find that $\tilde{\Delta}^{**} \leq 0$ if and only if

$$\beta \leq 1 + \frac{(a - c_2)(c_F - c_1) + 2(c_F - c_2)(c_2 - c_1)}{2(c_F - c_1)^2},$$

which holds under the ordering of marginal costs in (3). Since $\bar{\pi}_2^{**} \leq \pi_2^*$, firm 2 always disagrees in Stage 2, irrespective of market size.

Finally, consider the Stage-1 subgame equilibrium. When the market size is large (like Figure 4), firm F compares the profits between π_F^* and π_F^{**}. From (8) and (A.1), it follows that firm F obtains higher profits by forming PEO with firm 1 than with firm 2 ($\pi_F^* \geq \pi_F^{**}$) because PEO leads to output shifting more effectively. Thus, firm F proposes to firm 2 first. When the market size is small (like Figure 7), firm F compares the profits between $\bar{\pi}_F^*$ and π_F^*. From (8) and (A.2), it follows that $\pi_F^* \geq \bar{\pi}_F^*$.[16] Thus, firm F again proposes to firm 2 first. Hence, regardless of the market size, the game has a unique equilibrium pass such that firm F offers to firm 2 first, then firm 2 declines it, and firm 1 finally agrees to form PEO in Stage 3.

A.3 Equilibrium Welfare

This appendix derives the expressions of Home welfare in both the non-PEO and PEO equilibria. In particular, we show that the Home welfare is always smaller in the PEO equilibrium than in the non-PEO equilibrium, regardless of PEO formations.

Welfare in non-PEO Equilibrium

Consider first the Home welfare in the non-PEO equilibrium. In our model, the Home welfare is composed of consumer surplus (CS) and firms 1's and 2's profits, where $CS = Q^2/2$ with linear demand. If firms do not form cross-border PEO ($\theta = \gamma = 0$), aggregate output is $Q^o \equiv Q(0,0)$ in (2), while firm 1's and 2's profits are π_1^o and π_2^o in (8) and (A.1), respectively. Thus, the Home welfare in non-PEO equilibrium is given by

$$W^o = \underbrace{\frac{1}{2}\left(\frac{3a - c_1 - c_2 - c_F}{4}\right)^2}_{CS^o} + \underbrace{\left(\frac{a - 3c_1 + c_2 + c_F}{4}\right)^2}_{\pi_1^o} + \underbrace{\left(\frac{a + c_1 - 3c_2 + c_F}{4}\right)^2}_{\pi_2^o}. \tag{A.4}$$

Welfare in PEO Equilibrium

Consider next the Home welfare when firm F forms PEO with firm 1. From Lemma 1(ii), when the marginal costs satisfy $c_1 \leq c_F$, firm F unilaterally owns firm 1's equity, $\theta > 0, \gamma = 0$. Moreover, from Proposition 1, firms 1 and F choose the equity level θ^* in (7). Evaluating consumer surplus at $\theta = \theta^*$, the aggregate output is given by Q^*; thus the consumer surplus is $(Q^*)^2/2$. Regarding the Home firms' profits, in PEO with firm 1 and firm F, the equilibrium profits at $\theta = \theta^*$ are given by (8); thus, the Home firms' profits are $\pi_1^* + \pi_2^*$. Then the Home welfare in the PEO equilibrium at $\theta = \theta^*$ is given by

$$W^* = \underbrace{\frac{1}{2}\left(\frac{3a + c_1 - c_2 - 3c_F}{4}\right)^2}_{CS^*} + \underbrace{\left(\frac{a - 3c_1 + c_2 + c_F}{4}\right)^2 + \beta\frac{(c_F - c_1)^2}{2}}_{\pi_1^*} + \underbrace{\left(\frac{a - c_1 - 3c_2 + 3c_F}{4}\right)^2}_{\pi_2^*}. \tag{A.5}$$

Similarly, when firm F forms PEO with firm 2, the consumer surplus is $(Q^{**})^2/2$ and the Home firms' profits are π_1^{**}, π_2^{**} in (A.1). It then follows that the Home welfare in PEO equilibrium at $\theta = \theta^{**}$ is given by

$$W^{**} = \underbrace{\frac{1}{2}\left(\frac{3a - c_1 + c_2 - 3c_F}{4}\right)^2}_{CS^{**}} + \underbrace{\left(\frac{a + c_1 - 3c_2 + c_F}{4}\right)^2 + \beta\frac{(c_F - c_2)^2}{2}}_{\pi_1^{**}} + \underbrace{\left(\frac{a - 3c_1 - c_2 + 3c_F}{4}\right)^2}_{\pi_2^{**}}. \tag{A.6}$$

[16] We obtain this result by rewriting firm F's profits in (A.2) as $\bar{\pi}_F^* = \pi_F^o + (1-\beta)\Delta^{**} + (1-\beta)(\pi_1^o - \pi_1^{**} + \Delta^* - \Delta^{**} + \beta\Delta^{**})$ and comparing this with $\pi_F^* = \pi_F^o + (1-\beta)\Delta^*$ from (8).

Welfare Comparison

Given the equilibrium welfare derived above, we now compare the Home welfare across different regimes. From (A.4) and (A.5), PEO decreases aggregate output and increases market price, which decreases the consumer surplus. However, PEO leads to the output shifting from firm F to firm 1, which increases firm 1's profits, weakens market competition, and increases firm 2's profits as an outsider. Since the Home consumers are worse off, but two home firms are better off in PEO equilibrium relative to their counterparts in non-PEO equilibrium, the introduction of PEO can either improve or worsen the equilibrium welfare in (A.5) relative to that in (A.4). Subtracting (A.4) from (A.5),

$$W^* - W^o = -\frac{(c_F - c_1)[a + 5c_2 - 6c_F - 4\beta(c_F - c_1)]}{8}.$$

If the difference is positive (negative), PEO improves the (worsens) Home welfare relative to the non-PEO case. Note that the difference is zero in a special case where the cost difference between firms 1 and F disappears. Upon some rearrangement, it directly follows that $W^* \geq W^o$ if and only if $c_F \geq (a + 4\beta c_1 + 5c_2)/[2(3 + 2\beta)]$. Thus, the condition under which PEO improves the Home welfare requires a sufficiently large cost difference. This equilibrium outcome is intuitive: the larger is the cost difference, the greater is the output shifting and, thus, the larger the positive effect on welfare. Recall that for firm F to produce in equilibrium, we must assume that the market size is sufficiently large. From q_F^*, this condition is expressed in terms of firm F's marginal cost $c_F \leq (a + 7c_1 + c_2)/9$. Hence, we must check whether there is a parameterization of c_F that simultaneously satisfies two inequalities, $(a + \beta c_1 + 5c_2)/[2(3 + 2\beta)] \leq c_F \leq (a + 7c_1 + c_2)/9$, which does not exist for any $\beta \in [0, 1]$. Thus, if firm 1 and firm F choose the equity level to maximize their joint profits, the negative effect on consumer surplus dominates the positive effect on profits; hence, PEO worsens welfare ($W^* \leq W^o$). In other words, the Home government has an incentive to impose restrictions on cross-border PEO between firms 1 and F.

The effect of PEO on welfare, however, may change when firm F has PEO with (inefficient) firm 2, because the "anti-competitive" effect of PEO is softened relative to that in PEO with (efficient) firm 1. Furthermore, PEO still allows firms 2 and F to exploit "output shifting," although this is less effective. Thus, it is possible that the negative effect of PEO on welfare is dominated by the positive effect in equilibrium, in which case PEO is desirable even for the Home government. Subtracting (A.4) from (A.6),

$$W^{**} - W^o = -\frac{(c_F - c_2)[a + 5c_1 - 6c_F - 4\beta(c_F - c_2)]}{8}.$$

Similarly to the above case, we find that the difference is negative for any $\beta \in [0, 1]$ so long as firm F produces in the PEO equilibrium. Thus, the Home welfare is smaller in the PEO equilibrium between firms F and 2 than in the non-PEO equilibrium ($W^{**} \leq W^o$).

Finally, compare the Home welfare when firm F makes PEO with either firm 1 or firm 2. To explore this, subtracting (A.5) from (A.6), we immediately get

$$W^* - W^{**} = -\frac{(c_2 - c_1)(a + 4c_1 + 4c_2 - 9c_F)}{8}.$$

In this comparison, the parameter β is not entered because firms 1 and 2 have the same bargaining power. Under the parameter range that firm F produces in the PEO equilibrium, the difference is always negative and the Home welfare is smaller in the PEO equilibrium with firm 1 than with firm 2 ($W^* \leq W^{**}$). Taken together, we obtain the equilibrium welfare ranking in the main text.

A.4 Strategic Delegation

In the main text, we have characterized the optimal equity level within the range of $\theta \in [0, 1/2]$. This appendix shows the equilibrium outcomes if PEO firms are allowed to have $\theta \in (1/2, 1]$.

Recall that when the equity level is above $1/2$, firm F chooses both q_1 and q_F to maximize the joint profits. Because firm F is less efficient than firm 1, PEO firms may find it profitable to shut down firm F's production and concentrate only on firm 1's production. If only firm 1 produces in the range of $\theta \in (1/2, 1]$, the situation is a duopoly between firms 1 and 2 by merging firm F. Let $\tilde{q}_1^*$ and $\tilde{\Pi}^*$ denote the equilibrium output of firm 1 and the joint profits, respectively. Noting that the equilibrium outcomes become similar to merger outcomes,

$$\tilde{q}_1^* = \frac{a - 2c_1 + c_2}{3},$$
$$\tilde{\Pi}^* = \left(\frac{a - 2c_1 + c_2}{3}\right)^2.$$

Observe that the profits are independent of θ since firms F and 1 are merged into a single firm in this case. Reflecting this fact, $\tilde{\Pi}^*$ in the merger equilibrium is indicated by the broken line in Figure 2. From the figure, we have the following ordering in PEO firms' profits in the merger equilibrium:

$$\tilde{\Pi}^* \leq \pi_1^o + \pi_F^o \leq \Pi^*(\theta).$$

The first inequality shows that the joint profits are smaller than the sum of the outside options in the merger equilibrium. This outcome is similar to the "merger paradox." On the other hand, the second inequality shows that the joint profits are higher than the sum of outside options in PEO equilibrium even with $\theta \in (1/2, 1]$. This outcome reflects that PEO firms can shift the output, while restricting firm 2's aggressive actions by keeping firm F's production, which increases the joint profits as well. Thus, even if firm F owns the majority of firm 1's equity, it is better for firm F to allow firm 1 to produce freely while continuing to produce by itself. This result is similar to that of existing studies that explore interactions between strategic delegation and mergers (González-Maestre and López-Cuãt, 2001). Given that both firms 1 and F continue to produce even in equilibrium with $\theta \in (1/2, 1]$, the $\Pi^*(\theta)$ curve holds in all ranges of $\theta \in [0, 1]$ in Figure 2. Consequently, an interior solution of the optimal equity level θ^* applies to $\theta \in (1/2, 1]$ in the PEO equilibrium.

Next, consider the welfare-maximizing equity level when the Home government can choose $\theta \in (1/2, 1]$. The outcome of firms' strategic delegation is desirable for the government because all the three firms remain active after cross-border PEO, mitigating the negative effect of decreased competition on consumer surplus. Let $\tilde{W}^*$ denote the equilibrium welfare when firm F chooses to merge firm 1, which is given by

$$\tilde{W}^* = \frac{1}{2}\left(\frac{2a - c_1 - c_2}{3}\right)^2 + \beta\left(\frac{a - 2c_2 + c_2}{3}\right)^2 + \left(\frac{a + c_1 - 2c_2}{3}\right)^2.$$

Comparing this with (A.4), we find that the Home welfare is higher in the non-PEO equilibrium than in the merger equilibrium. From Figure 5, thus, we have the following ordering in Home welfare in this equilibrium:

$$\tilde{W}^* \leq W^o \leq W^*(\theta).$$

These inequalities show that, like firms 1 and F, the government prefers to choose an interior solution of the optimal equity level θ_G^* for all ranges of $\theta \in [0, 1]$. Hence, an interior solution of the optimal equity level θ_G^* also applies to $\theta \in (1/2, 1]$ in the PEO equilibrium.

References

Beladi H, Marjit S, Chakrabarti A. 2009. Tariff Jumping and Joint Ventures. *Southern Economic Journal* 75, 1256-1269.

Bernard AB, Jensen JB, Redding SJ, Schott SK. 2007. Firms in International Trade. *Journal of Economic Perspectives*, 21, 105-130.

Farrell J, Shapiro C. 1990. Horizontal Mergers: An Equilibrium Analysis. *American Economic Review* 80, 107-126.

Ghosh A, Morita H. 2017. Knowledge Transfer and Partial Equity Ownership. *RAND Journal of Economics* 48, 1044-1067.

Ghosh A, Mukunoki H. 2024. Partial Cross-ownership and Merger Control in International Trade. Mimeo.

González-Maestre M, López-Cuãt, J. 2001. Delegation and Mergers in Oligopoly. *International Journal of Industrial Organization*19,1263-1279.

Helpman E, Melitz MJ, Yeaple SR. 2004. Export versus FDI with Heterogeneous Firms. *American Economic Review* 94, 300-316.

Ishikawa J, Sugita Y, Zhao L. 2009. Corporate Control, Foreign Ownership Regulations and Technology Transfer. *Economic Record* 85, 197-209.

Ishikawa J, Sugita Y, Zhao L. 2011. Commercial Policy and Foreign Ownership. *Review of International Economics* 19, 300-312.

Jovanovic D, Wey C. 2014. Passive Partial Ownership, Sneaky Takeovers, and Merger Control. *Economic Letters* 125, 32-35.

Qiu LD. 2010. Cross-border Mergers and Strategic Alliances. *European Economic Review* 54, 818-831.

López AL, Vives X. 2019. Overlapping Ownership, R&D Spillovers, and Antitrust Policy. *Journal of Political Economy* 127, 2394-2437.

Motta M. 1992. Multinational Firms and the Tariff-jumping Argument: A Game Theoretic Analysis with Some Unconventional Conclusions. *European Economic Review* 36, 1557-1571.

Neary JP. 2007. Cross-border Mergers as Instruments of Comparative Advantage. *Review of Economic Studies* 74, 1229-1257.

Nocke V, Yeaple S. 2007. Cross-border Mergers and Acquisitions vs. Greenfield Foreign Direct Investment: The Role of Firm Heterogeneity. *Journal of International Economics* 72, 336-365.

Nocke V, Yeaple S. 2008. An Assignment Theory of Foreign Direct Investment. *Review of Economic Studies* 75, 529-557.

OECD. 2024. ASEAN FDI Regulatory Restrictions Database. `https://qdd.oecd.org/subject.aspx?Subject=ASEAN_INDEX`

Raff H, Ryan M, Stahler F. 2012. Firm Productivity and the Foreign-market Entry Decision. *Journal of Economics & Management Strategy* 21, 849-871.

Reynolds RJ, Snapp BR. 1986. The Competitive Effects of Partial Equity Interests and Joint Ventures. *International Journal of Industrial Organization* 4, 141-153.

Salant SW, Switzer S, Reynolds RJ. 1983. Losses from Horizontal Merger: the Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium. *Quarterly Journal of Economics* 98, 185-199.