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Excess Liquidity against Predation

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Abstract

To investigate why a firm may hold excess liquidity, we examine a duopoly competition in which a shallow-pocket entrant needs the financial support of an outside investor to pay for input costs and launch a business. We allow the investor to terminate the entry if they find the incumbent react too aggressively to the entry plan. However, such an exit option creates a threat of predation by a deep-pocket competitor. To avoid predation, the entrant must raise precautionary liquidity by taking out a loan both larger and further in advance than is actually needed. An entrant with little start-up capital will be less aggressive if the incumbent's capacity size is unverifiable, because the need to raise precautionary liquidity restricts the entrant's feasible capacity size.

Keywords: excess liquidity, predation, financial contract
JEL classification: G32; L12; D86

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1 Introduction

Firms tend to keep more liquidity than is actually needed, though such excess liquidity hoarding is costly in the presence of an interest rate spread between loans and deposits. Excess liquidity also worsens agency problems because it induces a soft budget constraint. Holmström and Tirole (1998) find there is a reason for firms to demand excess liquidity: to prepare for liquidity shocks in incomplete financial markets. On the other hand, empirical research by Hoberg et al. (2014) suggests that competitive pressure increases firms' cash holdings; Fresard (2010) empirically finds that larger cash holdings indeed help a firm gain larger market share. Motivated by these recent empirical findings, we theoretically uncover the mechanism behind the link between excess liquidity hoarding and product market competition.

We consider a duopoly competition between a *shallow-pocket entrant* who needs outside finance to launch the business and a *deep-pocket incumbent* who does not need it. We allow the outside investor to terminate the entrant's business if the incumbent reacts too aggressively. This financial asymmetry and the exit option result in threat of predation and a need for excess liquidity in our model; the logic is as follows. While the exit option should mitigate the risk of default, it also makes possibility for the incumbent to exclude the entrant by aggressive predation, which can be profitable only if the incumbent successfully monopolizes the market. To prevent predation, the investor needs to commit to financial support for the entrant even if it foresees operating losses due to predation. If we examine the cash flows of the entrant, this commitment should be monetized as *excess and up-front liquidity holdings* by means of precautionary cash holdings or credit lines: liquidity for cost payments should be secured earlier and should be more than actually needed. If predation is unverifiable, the entrant might falsify predation to reduce repayment to the investor, appealing to limited liability. When the entrant raises a sufficiently large mortgage, this reduces the entrant's incentive to falsely report predation by threat of liquidation. By doing so, the entrant demonstrates to the investor that it will not falsely report. If the entrant does not have enough assets for mortgage, the entrant cannot raise enough excess liquidity and must shrink the business.

Game theoretical analysis of predation dates back to Telser (1966). He argues, without developing a rigorous model, that a deep-pocket incumbent might prey on the shallow-pocket entrant through aggressive competition. However, he concludes that as long as the entrant's business is profitable in the absence of predation, a rational investor should agree to lend enough liquidity to the entrant so that the incumbent would be wholly discouraged from predation.[1]

Modern contract theory questions whether the entrant can finance such liquidity under financial imperfection. Bolton and Scharfstein (1990) (henceforth BS) is the seminal paper on the modern financial theory of predation in production market competition, followed by Poitevin (1989), Snyder (1996), Fernández-Ruiz (2004), Marquez (2010) and Khanna and Schroder (2010).[2]

[1]Precisely speaking, Telser (1966) predicts that a rational incumbent would try to buy out the entrant, instead of just giving up the monopoly profit. He also suggests that the incumbent should use the threat of predation to reduce the takeover bid of the entrant's company. However, since Telser's model takes the amount of precautionary liquidity ('reserve') as exogenously given, he concludes that an entrant should have plenty of liquidity to increase the takeover bid.

[2]Snyder (1996) introduces renegotiation, and Khanna and Schroder (2010) allow variable output/price levels

In the BS model, firms compete over two periods and the entrant must pay some fixed entry costs in the beginning of each period. They define predation as exclusion of the entrant from the second-period market. Due to financial imperfection, the entrant cannot finance the whole entry cost and needs some internal capital to continue doing business. The incumbent preys on the entrant in the first period, aiming to prevent him from earning enough money to meet his need for internal capital. In the BS model, the entrant demands only as much liquidity as needed for entry; thus the liquidity demand itself is exogenously determined regardless of competitive pressure. In contrast, it is endogenously determined in our model. Consequently, our results are more consistent with the empirical findings that connect product market competition and liquidity holding.

The rest of the introduction provides a more detailed summary of our model and a more extensive review of the related literature. Before this, we need to make a few remarks on terminology. To shorten their names, we call the shallow-pocket firm the "entrant" and the deep-pocket firm the "incumbent." In practice, an incumbent firm can easily obtain short-term trade credits or open accounts to buy factor inputs in order to delay payment until receiving the sales revenues. A new entrepreneur who has just started a business tends to have limited access to trade credits before establishing a history of credible transactions.[3] However, if we apply our model to competition between a financially weak small business owner who relies on their own cash to run the business and a large conglomerate that can subsidize a new enterprise with profits from its other businesses, the former would play the role of the "entrant" and the latter would play the role of the incumbent.[4]

Outline of the model. In this paper, we revisit Telser's argument and formalize the possibility that the incumbent preys on the entrant by diminishing future profitability of the entrant's business rather than by preventing him from raising enough capital to start or continue the business. Instead of having two periods like the BS model, we look into the details of timing of decisions and financing in one-shot duopoly competition.

The two firms choose strategies that affect profitability of each firm in the following product market competition. In short, we call such strategies "*capacity sizes*," referring to capacity-

in the BS model. Fernández-Ruiz (2004) is a version of adverse selection. Poitevin (1989) constructs a different model from these and investigates the entrant's choice between equity and debt financing in a one-shot game and also allows a variable output level. Since Poitevin (1989) is closest to our model, we provide a detailed comparison later in this section. Marquez (2010) also sheds light on the choice of financial methods, especially between bank loans and public debt financing, from the viewpoint of information and monitoring. See footnote 18.

[3]Trade credits such as "net 30" are indeed commonly used and work as a substitute of short-term bank loans especially for financially weak firms. But. if we regard foregone discounts in case of late payment as hidden interests of the credits, trade credits are much more expensive than bank loans; Danielson and Scott (2004) report that firms, including even small business owners, generally prefer bank loans. Young firms indeed tend to delay payments and thus miss discounts, according to Petersen and Rajan (1994). Besides, suppliers typically use credit scores such as Paydex to determine a customer's eligibility and limit of trade credits: see Kallberg and Udell (2003a) and Kallberg and Udell (2003b) about usage of Paydex in the U.S. A new entrepreneur has to start from small limit of credit until accumulating a good credit score; furthermore, the tendency of late payment makes it difficult: see Board of Governors of the Federal Reserve System (2012). Fisman and Love (2003) empirically find that trade credit does not help start up of new firms, though it helps growth of incumbent firms in countries with weak financial intermediaries.

[4]From empirical study on French business groups, Boutin et al. (2013) find that cash holding of an entrant's affiliated business group indeed encourages entry while that of the incumbent's group discourages its rival from entry.

constrained duopoly competition as we consider in Section 3. However, it can be any kind of precommitment activities, such as advertisement, R&D or cost-reducing investment. In our base model, a shallow-pocket firm chooses its capacity size first; then, after observing it, a deep-pocket firm chooses its own. Our propositions are robust to a change in timing structure from Stackelberg to Cournot where the entrant and incumbent would choose their capacity sizes simultaneously ; the Cournot version was studied in an older working paper (Zusai, 2012).

In the product market competition, the only deviation from a standard Stackelberg competition is to allow the shallow-pocket entrant to exit from the market before paying capacity costs, but after observing the deep-pocket incumbent's choice. The exit option allows the entrant to avoid incurring an operating loss if he finds that the incumbent is too aggressive.

But it generates threat of predation. By committing to excess capacity, the incumbent could lower profitability of the entrant's business and exclude him from the market. To prevent predation, the entrant needs to guarantee that the entrant will remain in the market even if the incumbent chooses predatory excess capacity. That is, the entrant should not use the exit option as long as doing so could increase the incumbent's profit.

We show that the incumbent's choice of excess capacity for the sake of monopolization is a strategic complement of the entrant's capacity, even if the incumbent's optimal capacity is a strategic substitute when the entrant remains in the market. The more aggressively the entrant is willing to compete in the market, the more the incumbent preys on the entrant. This is because the incumbent expects a lower profit in duopoly and the net benefit of predation becomes larger. Consequently, the entrant's demand for excess liquidity increases with his own capacity size.

The investor's commitment to letting the entrant remain in the market can be materialized as credit lines or precautionary liquidity to pay for capacity costs. The entrant needs to raise liquidity earlier than needed and keep it until the incumbent gives up predation. In addition, the liquidity has to cover capacity costs in case of predation. Therefore, the liquidity is *excess* liquidity hoarding in the senses of both timing and quantity.

So far we have not assumed any financial imperfection in the economy. Financial imperfection may stem from asymmetric information about the entrant's productivity, as in BS, or about the effort level, as in Fernández-Ruiz (2004). We present one new theoretical possibility that is more intrinsic to the threat of predation: even without uncertainty on the entrant's fundamentals (productivity) or the own choices (effort or the own capacity size), unverifiability of a competitor's action (the incumbent's predation) would prevent the entrant from raising enough *precautionary liquidity* at entry.

We find that this unverifiability indeed gives an advantage to the incumbent. The logic is as follows. We find that the optimal financial contract takes a form of a debt contract; so, we interpret the investment as a loan and the invetor as a lender. Given the limited liability constraint on repayment of the loan (promised monetary payment in the contract) and using predation as an excuse for operating losses, the entrant may be able to avoid the repayment of loans. Unverifiablity of predation is assumed in BS. In their model, the lender cannot distinguish operational loss due to predation from operating loss due to low productivity. We do not assume the uncertainty or unverifiability of productivity; once enough liquidity is raised, it is certain that there is no predation and the entrant earns enough profit to repay the loans. Instead, we assume that predation is unverifiable in court and the entrant actually suffers predatory loss.

We let the entrant voluntarily report the incumbent's capacity size. Thanks to common knowledge of the demand and cost structure, the lender can figure out the plausible range of capacity sizes under which the incumbent could still earn a greater profit by excluding the entrant from the market than in a duopoly equilibrium. If the reported capacity of the incumbent and the implied predatory loss are greater than plausible sizes, the lender can withdraw the long-term loan before production and force the entrant to exit from the market. Yet to prevent predation on the equilibrium path, the lender has to allow for continuation of the project as long as the reported capacity size of the entrant does not exceed the greatest plausible size even if it is not the equilibrium size. Otherwise, the incumbent would indeed execute plausible predation to get the lender to withdraw the loan and the entrant to exit.

Hence, the lender foresees the possibility of default due to a report of plausible predation. To avoid it, the lender would ask the entrant for collateral that could be used to punish the entrant for a (falsified) request for default on the loan. Thus, the entrant needs enough initial assets to raise the precautionary liquidity with a loan. This works as a borrowing constraint on the available amount of a loan at entry. Even if no interest is incurred on the long-term loan, a less capitalized entrant behaves less aggressively to reduce the needed precautionary liquidity within this borrowing constraint. The borrowing constraint is established as a necessary condition for *every* equilibrium in which predation is prevented. In any valid financial contract (including a debt contract), the entrant's total repayment should be kept constant, independent of the realized profit to prevent a falsified default. This is consistent with the empirical finding of Kjenstad and Su (2012) on the relation between competitive pressure and debt contracts.

More on related literature. Among the preceding literature on predation due to financial imperfection, Poitevin (1989) presents predatory excess supply and excess liquidity as a solution for an adverse selection problem. In his model, the entrant chooses either debt or equity to finance liquidity. Excess liquidity is raised by debt, which increases risk of bankruptcy and *stimulates* the incumbent's predation. This is what a high-productivity entrant wants. He raises the debt level so high that a low-productivity entrant cannot bear intensified predation; therefore, large debt is a signal of high productivity. A Poitevin's entrant first *wants to borrow* much despite risk as a signal of its productivity, while our entrant wishes to have enough liquidity to prevent predation *without relying on a loan*. Fresard (2010) empirically distinguishes the effects of cash holding on competition from the effects of debt holding, and confirms that the former has a significant impact on market shares distinct from the impact of the latter. So, it supports our model.

As there is no uncertainty in the cost and demand structure, our base model is a game of complete but imperfect information. In contrast, the preceding models of financial predation *à la* BS involve signaling about the entrant's hidden productivity or demand. Thus, they consider games of incomplete information. Some authors have constructed (nonfinancial) theories of predation under perfect and complete information: see Argenton (2010) and Fumagali and Motta (2013). Roth (1996) presents predation as a rationalizable strategy (in the sense of Bernheim and Pearce) in War of Attrition. Bevia et al. (2020) consider repeated Cournot competition with an ad-hoc constraint that requires a firm to achieve non-negative profit in *each* period. In the theoretical portion of their paper, Kjenstad and Su (2012) consider two-period Hotelling competition with the entrant having to pay some given amount of money to continue production. These models analyze how the threat of predation is realized in and/or affects repeated production market competition. Our model can be seen as another attempt to formalize a theory of predation in complete information games, paying more attention to financial decisions.

While our model can be readily extended to the case of incomplete information on the entrant's productivity, it is practically important that our results do *not need* such uncertainty or asymmetric information on the market fundamentals. Allegations of predatory pricing are often made from owners of small businesses.[5] Small businesses, such as local retailers, restaurants, and food manufacturers, do not involve large physical uncertainty or large investment.[6] We prove that the threat of predation distorts market outcomes even without physical uncertainty or fixed costs, as in such small businesses. These situations could not be captured by the incomplete information models *à la* BS.

Plan of the paper. The paper proceeds as follows. The next section presents the base model. After setting up the model, we see two benchmark cases in which there is no link between financial contract and the product market competition. In one case, the entrant can continue the business without relying on the investor; in the other case, the incumbent's capacity is determined exogenously. Then we see that, in the base model, the link creates threat of predation and demand for excess liquidity; further, we show that unverifiablity amplifies distortion in the product market. To see it more concretely, in Section 3, we consider the two-stage competition model with capacity constraints on production.

In Section 4, we discuss various extensions and modifications to the base model. For quantitative measure of excess liquidity, we materialize the entrant's need for outside finance by introducing the cash-in-advance constraint on the entrant's payment of capacity costs. Then, we briefly look at the working paper version of the model in which the two firms decide on capacity sizes simultaneously. Lastly, we relax the assumption that the entrant has full bargaining power. In Section 5 we discuss structural assumptions in the model and in the propositions. Section 6 concludes and technical proofs are given in the appendix.

[5] So are the most famous lawsuits of predatory pricing: Utah Pie Co. v. Continental Baking Co., 386 U.S. 685 (1967); William Inglis & Sons Baking Co. v. ITT Continental Baking Co., 688 F. 2d 1014 (9th Cir. 1981), *cert. denied*, 459 U.S. 825 (1982); A. A. Poultry Firms, Inc. v. Rose Acre Firms, Inc., 881 F. 2d 1396 (7th Cir. 1989), *cert. denied*, 494 U.S. 1019 (1990).

[6] The success of such local businesses depends mainly on how well the owner knows the local market and maintains his business, rather than on making costly and risky innovation. Taylor and Archer (1994) suggest ten principles and 273 *Kaizen* (improving) suggestions for a local retailer competing against giant supermarkets such as Walmart. The basic message there is to know the business environment, to keep good relationships with customers and to improve management on a daily basis. It is noteworthy that their banking strategies are to keep and share financial and business information with bankers and to help them monitor the business, as well as to arrange for credit lines before needing money but not to use up these lines.

2 The base model

We first describe a version of the model that formalizes the essential structure of the linkage between predation and excess liquidity holding. This version presents the logic of model in the clearest and most succinct way. For such expositional simplicity, first we assume that

- The entrant is the Stackelberg leader: In the production market competition, the entrant first chooses its capacity size. Observing this, the incumbent chooses its own.
- In a financial contract between the entrant and the investor, full bargaining power is given to the entrant: The entrant writes the financial contract such that it maximizes the entrant's share of the profit while guaranteeing non-negative profit for the investor.

These assumptions make the analysis more transparent. But the model is robust to changes in timing and bargaining power, as we confirm in the final section.

Product market competition

The production market consists of the two firms: the shallow-pocket entrant (firm E) and the deep-pocket incumbent (firm I). Each firm $i \in \{E, I\}$ chooses capacity size q^i from the feasible set $Q^i \subset \mathbb{R}_+ := [0, +\infty)$. Let $Q := Q^E \times Q^I$ and $\mathbf{q} := (q^E, q^I) \in Q$. In our base model, we let the entrant choose q^E first in period 0. Then the incumbent responds to the entry by choosing q^I in period 1. We allow the entrant to exit the market in period 2 after observing q^I and before incurring any cost.

Each active firm earns operating profit π^i in period 3 as determined by its chosen capacity. If the entrant stays in the market, each firm $i \in \{I, E\}$ earns the operating profit $\pi^i(q^i, q^j)$ from its own capacity q^i, given the opponent firm j's capacity q^j. If the entrant exits, only the incumbent earns the operating profit $\pi^I(q^I, 0)$.[7]

We assume that there is a unique maximizer $q^i_{\text{BR}}(q^j) \in Q^i$ of firm i's operating profit $\pi^i(q^i, q^j)$, given the opponent j's capacity size $q^j \in Q^j$. Let $\tilde{\pi}^E(q^E)$ be the entrant's profit following q^E and $q^I_{\text{BR}}(q^E)$:

$$q^i_{\text{BR}}(q^j) := \operatorname*{argmax}_{q^i \in Q^i} \pi^i(q^i, q^j), \quad \tilde{\pi}^E(q^E) := \pi^E(q^E, q^I_{\text{BR}}(q^E)).$$

We make four final assumptions: each firm's profit cannot increase if the rival chooses a greater capacity size; an inactive entrant's operating profit is zero regardless of the rivals' capacity size; q^I and q^E are strategic substitutes; and, the entrant's profit function is decreasing once its capacity exceeds the unique maximum size.

Assumption 1. i) For each $i \in \{E, I\}$, $j \neq i$ and each $q^j \in Q^j$, there exists a unique maximum point $q^i_{\text{BR}}(q^j) \in Q^i$ such that $\pi^i(q^i_{\text{BR}}(q^j), q^j) > \pi^i(q^i, q^j)$ for any $q^i \in Q^i \setminus \{q^i_{\text{BR}}(q^j)\}$.

[7] We prohibit the incumbent to change the capacity size after monopolizing the market. The incumbent can run the production facilities at a low operation rate after he succeeds at excluding the entrant from the market by setting a predatory large "capacity" q^I. Such a low operation rate with a large predatory capacity yields lower profit than at a high (efficient) rate with a smaller capacity. It just means in our model that q^I is not equal to $q^I_{\text{BR}}(0)$, the optimal monopoly capacity in the absence of the entrant.

ii) For each $i \in \{E, I\}$, $j \neq i$ and each $q^i \in Q^i$, we have $\pi^i(q^i, q_0^j) \geq \pi^i(q^i, q_1^j)$ if $q_0^j < q_1^j$ and $q^i \neq 0$.

iii) For each $i \in \{E, I\}$ and $j \neq i$, we have $0 \in Q^i$ and $\pi^i(0, q_j) = 0$ for any $q^i \in Q^i$.

iv) For each $i \in \{E, I\}$ and $j \neq i$, $q^i_{\mathrm{BR}}(q_j)$ cannot increase with q_j: i.e., $q^i_{\mathrm{BR}}(q_0^j) \geq q^i_{\mathrm{BR}}(q_1^j)$ if $q_0^j < q_1^j$.

v) For any $q^I \in Q^I$, we have $\pi^E(q_0^E, q^I) < \pi^E(q_1^E, q^I)$ if $q_0^E > q_1^E \geq q^E_{\mathrm{BR}}(q^I)$.

Financial contract

We consider a situation in which the shallow-pocket entrant needs the financial support of the outside investor to launch the business. One typical example is that the capacity cost needs to be paid before the entrant earns the sales revenue and thus the entrant needs to raise liquidity for this payment by a loan or a credit line. While we will consider this particular situation in Section 4, we abstract the cash flows regarding capacity costs in the base model; for now, we just assume that the continuation of the entrant's business needs agreement of the investor and the entrant raises precautionary liquidity B in period 1 from its own initial liquidity holding w_0 and the investor's investment (loan) $B - w_0$. If the business results in operating loss, the loss will be paid from B at the end of period 3. Then, the dividend (or the repayment of the loan) will be paid to the investor. For this, the entrant and the investor write the financial contract in period 0.

All the functions, the entrant's initial liquidity holding w_0, the overall precautionary liquidity holding B in period 1, and the entrant's capacity level q^E are assumed to be verifiable for the enforcement of the financial contract, while we assume that the incumbent's capacity level q^I is not. This makes the entrant's actual profit $\pi^E(q^E, q^I)$ also unverifiable. That is, after period 2, the entrant's actual liquidity holding $B + \pi^E(q^E, q^I)$ is not verifiable and thus the court cannot enforce repayment of the loan by the entrant. On the other hand, we assume that the entrant's exit from the market is verifiable. In this case, it is verifiable that the entrant has not spent any money and therefore still holds all his precautionary liquidity B. Thus, the court can enforce repayment up to B.

The financial contract is designed to get the entrant to report q^I voluntarily and truthfully, as well as to pay the investor. The contract determines the entrant's period-2 choice of whether to exit or stay based on the entrant's previous report of q^I. The entrant could write a contract that commits itself to stay regardless of q^I, which would nullify the threat of predation. However, we will see that the availability of such full commitment depends on the entrant's initial capital, not just on the profitability of the entrant's business.

To create an incentive for the entrant to voluntarily tell the truth, we assume that the entrant has a non-monetary asset V at the time of entry and mortgages it for the financial support of the investor. The mortgage may go into liquidation; the liquidation value $\underline{V}$ is assumed to be smaller than the private continuation value $\overline{V}$. We assume $\overline{V} > \underline{V} \geq 0$.[8]

[8]We assume that the liquidation value (the continuation value, resp.) is linear in the proportion of the asset that the lender (the entrant, resp.) takes over in period 4. But all of our propositions, especially the non-predation condition (NP$_*$), remain the same as long as its minimum is 0 and its maximum is $\underline{V}$ ($\overline{V}$, resp.)

Time line.

The events happen according to the following time line.

Period 0. The entrant and the investor write contract $\mathcal{C}$ (the list of terms in $\mathcal{C}$ will be summarized in Section 2.3). Then the investor chooses whether or not to accept it. We assume that contract $\mathcal{C}$ is made public and therefore is common knowledge for everyone in the economy, as well as being verifiable in court.

At the end of this period, the entrant chooses q^E from Q^E according to the contract. Besides, the investor may add precautionary liquidity and raise it to B, meaning that the entrant borrows $B - w_0$ from the investor.

Period 1. Observing $\mathcal{C}$ and q^E, the incumbent chooses q^I. Of course, this is not bound by the contract.

Period 2. The entrant announces a message at the beginning of period 2, after observing the incumbent's capacity size q^I in period 1. We presume that the set of available messages M consists of the set of the incumbent's capacity Q^I_S that will allow the entrant to stay and the set of messages M_0 that imply an intent to exit. Because of the revelation principle, this specification of the message space M does not limit the set of equilibrium outcomes.[9]

If any of the incumbent's capacity sizes in Q^I_S is announced, the entrant does not exercise its exit option and continues the business; the investor additionally lends liquidity to pay out the capacity costs, if needed. If any message in M_0 is announced, the exit option is exercised; the entrant quits the business before spending any of B.

Period 3. Every active firm makes money by utilizing its capacity; for example, they may produce and sell products. If staying in the market, the entrant earns sales revenue, pays the remaining capacity costs, and then repays the investor for the additional loan from period 2. At the end of this period, the entrant's liquidity holding is the operating profit $\pi^E(\mathbf{q})$ plus the precautionary liquidity B.

Period 4. Given message $m \in M$, the entrant pays $D(m)$ to the investor. In addition, the investor liquidates the proportion $\beta(m) \in [0, 1]$ of the mortgaged asset and earns the liquidation value $\beta(m)\underline{V}$. The entrant retains the rest of the asset and gains the private continuation value $(1 - \beta(m))\overline{V}$. We assume that the asset is divisible. That is, the proportion β can take any value in $[0, 1]$, not only 0 or 1.

In contrast to the monetary payment D, we define the **total payment** δ as the monetary payment D plus the entrant's loss of private value due to the liquidation of its mortgaged asset: $\delta(m) := D(m) + \beta(m)\overline{V}$.

[9] Zusai (2022) investigates the game behind a general form of a financial contract with an exit option and unverfiability of the outsider's strategy. To pin down the posterior belief about the outsider's strategy and thus its correlation with the agent's message in non-equilibrium outcomes, the author considers sequential equilibria in the game after the contracting party agrees on the contract and proves the revelation principle.

2.1 Benchmark

As a benchmark, we consider two situations where the financial contract and product market competition are, in some sense, not linked.

Benchmark equilibrium capacity profile: No exit option

As a benchmark, we consider a deep-pocket entrant. That is, the entrant commits to remaining in the market. Our game reduces to the standard Stackelberg competition: the benchmark capacity profile $\mathbf{q}_\dagger \in Q$ is determined by

$$q^E_\dagger := \operatorname*{argmax}_{q^E \in Q^E} \tilde{\pi}^E(q^E), \quad q^I_\dagger := q^I_{\mathrm{BR}}(q^E_\dagger). \tag{1}$$

Without the exit option, there is no threat of predation. We assume the existence of a unique equilibrium in the benchmark. Furthermore, we exclude the trivial case where neither firm can earn positive profit.

Assumption 2. Equation (1) uniquely determines capacity profile $\mathbf{q}_\dagger$ in the benchmark equilibrium. Further, $\pi^i(\mathbf{q}_\dagger) > 0$ for each $i \in \{I, E\}$.

Note that Assumptions 1-iii) and 2 jointly imply $q^i_\dagger > 0$ for each $i \in \{I, E\}$.

Benchmark optimal financial contract: q^I as exogenous shock

As the second benchmark, consider a situation in which a possible fluctuation in the entrant's profit comes from an *exogenous* shock rather than the rival's strategic choice. To keep the notation comparable, we let q^I be a random variable that follows distribution $\mathbb{P}^I$. Exogenous shock q^I is independent of q^E.

As in the base model, the entrant can exit from the market after observing q^I and this exit policy can be included in the financial contract. But, as q^I is determined exogenously, the presence of the exit option does not affect realization of q^I. Thus, there is no threat of predation.

In Appendix A we formulate the financial contract in this second benchmark case so that the entrant truthfully announces q^I and the monetary repayment inferred from the announcement of q^I is limited to the entrant's liquidity holding, given that the announcement is true. Then, we find that the optimal financial contract $\mathcal{C}_\ddagger$ is as follows:

$$\begin{aligned}
&Q^I_S := \{q^I \in Q^I \mid \pi^E(q^E, q^I) \geq 0\}, && \\
&D(\tilde{q}^I) = D(m_0) = B - w_0 && \text{for any } \tilde{q}^I \in Q^I_S, \\
&\beta(\tilde{q}^I) = \beta(m_0) = 0 && \text{for any } \tilde{q}^I \in Q^I_S.
\end{aligned}$$

This suggests that the continuation of the entrant's business is efficiently determined and solely based on profitability: the entrant can continue the business as long as the operating profit is positive and can quit if it is negative. As the operating loss never realizes, the loan is not needed at all; it can be any amount, even zero.[10] If borrowed, it is wholly repaid for sure. The monetary repayment is fixed at the amount of the loan. No threat of liquidity is needed to guarantee the whole repayment of the loan.

2.2 Stackelberg competition under threat of predation

The range of plausible predatory capacity sizes

The exit option leads to threat of predation because the entrant's exit allows the incumbent to monopolize the market and increases the incumbent's profit discontinuously. The incumbent therefore has an incentive to choose a capacity size greater than in the duopoly equilibrium capacity size: it will discourage the entrant from remaining in the market. In order to characterize the optimal decision for the entrant to exercise the exit option, we should first identify the range of the incumbent's plausible capacity sizes against which the entrant *has to* commit to remaining in the market in order to prevent predation.

We expect the incumbent to take the best response $q^I_{\mathrm{BR}}(q^E)$ to the entrant's choice of q^E, provided that the entrant remains in the market. This yields the duopoly profit of $\pi^I(q^I_{\mathrm{BR}}(q^E), q^E)$. On the other hand, *if* the incumbent could exclude the entrant from the market by choosing capacity size q^I_P, then the incumbent would gain the monopolizing profit $\pi^I(q^I_P, 0)$. If the latter is larger than the former and the entrant indeed exits after q^I, the incumbent should choose q^I_P rather than $q^I_{\mathrm{BR}}(q^E)$ in order to prey on the entrant. So, we can identify the range of the incumbent's plausible predatory capacity sizes from such comparison:[11]

$$Q^I_P(q^E) := \{q^I \in Q^I \mid \pi^I(q^I, 0) \geq \pi^I(q^I_{\mathrm{BR}}(q^E), q^E)\},\ \bar{q}^I_P(q^E) := \sup Q^I_P(q^E). \tag{2}$$

Too aggressive predatory capacity over the threshold $\bar{q}^I_P$ is not profitable for the incumbent because it makes the incumbent's profit less than it would be without predation. We call the threshold capacity size $\bar{q}^I_P(q^E)$ **the maximal plausible predation** and the entrant's loss due to this maximal plausible predation $-\pi^E(q^E, \bar{q}^I_P(q^E))$ **the maximal plausible predatory loss** $\bar{L}_P(q^E)$:

$$\bar{L}_P(q^E) = -\pi^E(q^E, \bar{q}^I_P(q^E)) = \sup\{-\pi^E(q^E, q^I) | q^I \in Q^I_P(q^E)\}. \tag{3}$$

To prevent the predation, the entrant needs to commit to remaining in the market as long as the incumbent's capacity size falls into the plausible range $Q^I_P(q^E)$. As long as $\bar{L}_P(q^E) > 0$, it implies that commitment to a wider range of $Q^I_P(q^E)$ is needed under threat of predation than commitment in the benchmark financial contract $\mathcal{C}_\ddagger$.

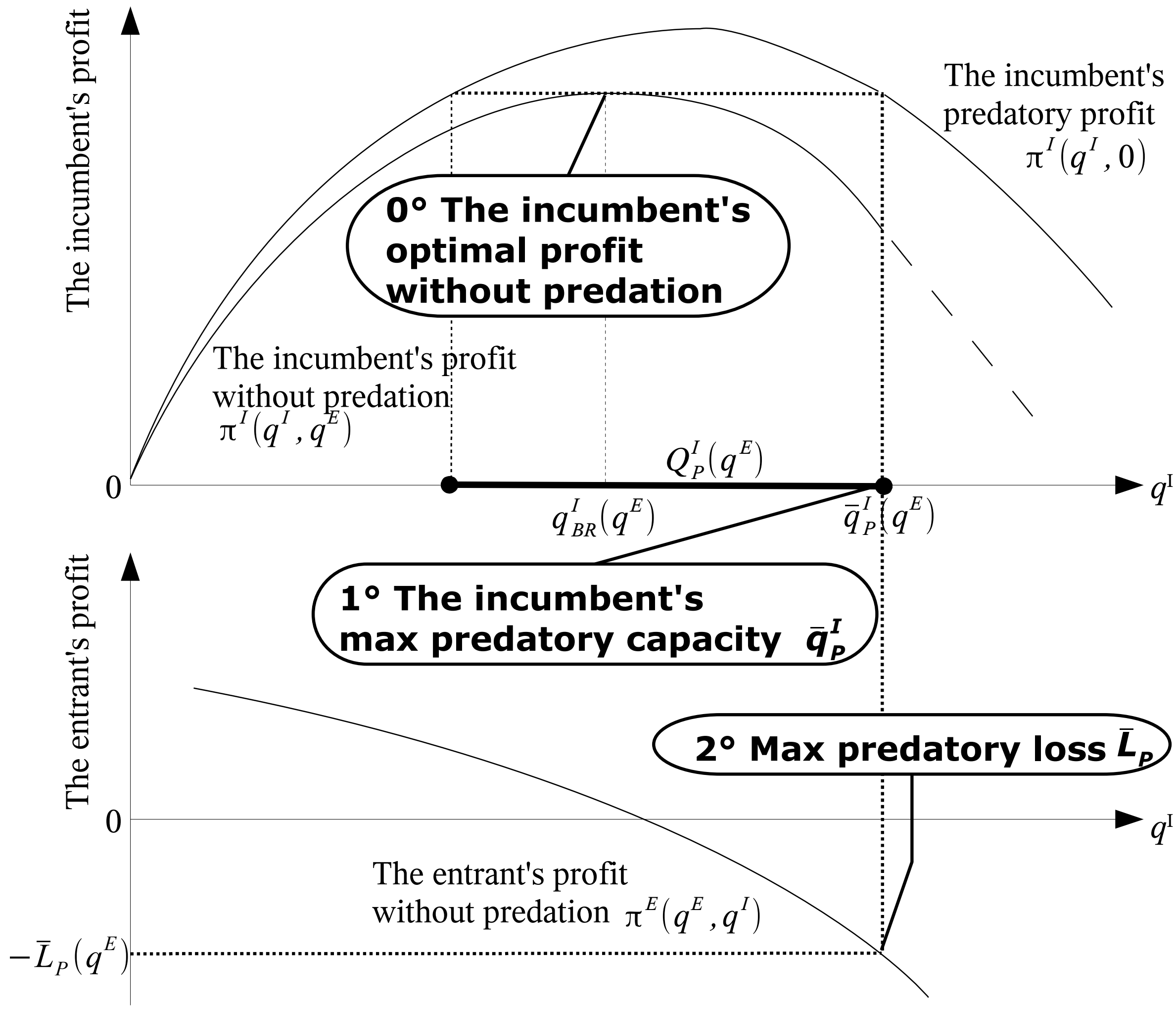
2.3 The optimal anti-predation financial contract

Components of the financial contract

The financial contract $\mathcal{C}$ consists of the following terms.

[10] Notice that, even *if* the liquidity holding must be maintained to be non-negative, the non-negativity constraint is satisfied with $B = 0$.

[11] Assumption 1-ii) guarantees that at least $q^I_{\mathrm{BR}}(q^E)$ belongs to $Q^I_P(q^E)$ and thus $Q^I_P(q^E)$ is not empty. Hence, we have $\bar{q}^I_P \geq q^I_{\mathrm{BR}}(q^E)$.



0° We want to see an equilibrium where the entrant prevents predation; the incumbent's equilibrium capacity maximizes the duopoly profit $\pi^E(q^E, q^I)$ given the entrant's q^E.

1° The incumbent benefits from predation if and only if the incumbent gets the entrant to exit the market by choosing a predatory capacity smaller than $\bar{q}_P^I(q^E)$.

2° $\bar{L}_P(q^E) := -\pi^E(q^E, \bar{q}_P^I(q^E))$ is thus the entrant's maximum plausible loss in the case of predation.

3° As long as the entrant can stay in the market, even suffering a loss of $\bar{L}_P(q^E)$, the incumbent does not prey on him.

Figure 1: The maximum plausible predatory loss $\bar{L}_P$.

- $B - w_0 \in \mathbb{R}$: the amount of the start-up investment (initial loan).
- $q^E_* \in Q^E$: the entrant's capacity.
- $M := Q^I_S \cup M_0$: the set of available messages.
  - ▷ $Q^I_S \subset Q^I$ is the set of announcements of the incumbent's capacity that allow the entrant to continue operating its business.
  - ▷ M_0 is the set of messages that terminate the entrant's business.
- $D : M \to \mathbb{R}$: the monetary payment policy.
- $\beta : M \to [0, 1]$: the liquidation policy.

The financial contract is designed to prevent predation and induce the entrant's truth-telling while guaranteeing non-negative net profits for both parties. Further, the limited liability condition restricts the feasible amount of the promised monetary payment. So the following four conditions constrain the terms of the financial contract under asymmetric information.

Anti-predation condition (AP$_*$): To prevent predation, the borrower is guaranteed to continue business under a plausible threat of predation: i.e., the borrower should be allowed to continue business even if the long-purse firm could make a positive extra profit by excluding the borrower from the market.

$$q^I_P \in Q^I_S \quad \text{for any } q^I_P \in Q^I_P(q^E_*), \text{ i.e., } Q^I_P(q^E_*) \subset Q^I_S. \tag{AP$_*$}$$

The incentive compatibility (truth telling) condition (IC$_*$): To let the entrant stay in the market, he could choose any message in Q^I_S, including the one that minimizes the total repayment. To guarantee truth telling, the *total* payment should not vary with message $\tilde{q}^I$ as long as $\tilde{q}^I \in Q^I_S$. To ensure incentive compatibility of the entrant's truthful reporting, the total payment must be constant, say $\underline{\delta}$, unless the intention of exit is not announced:

$$\delta(\tilde{q}^I) = \underline{\delta} \quad \text{for any } \tilde{q}^I \in Q^I_S. \tag{IC$^S_*$}$$

The entrant could choose to exit by sending message $m_0 \in M_0$. Doing so, the total payment changes from $\underline{\delta}$ to $\delta(m_0)$ while losing the operating profit. Incentive compatibility requires the entrant to voluntarily choose m_0 if $q^I \notin Q^I_S$ and to report the true q^I if $q^I \in Q^I_S$:

$$\pi^E(q^E_*, q^I) - \underline{\delta} \leq -\delta(m_0) \qquad \text{for any } q^I \notin Q^I_S, m_0 \in M_0 \tag{IC$^{S0}_*$}$$

$$\geq \qquad \text{for any } q^I \in Q^I_S, m_0 \in M_0 \tag{IC$^{0S}_*$}$$

Limited liability constraint (LL$_*$): We emphasize that $D(m)$ should be the actual effective amount, not just the face value, of the promised payment under any message m. When the entrant reports $m = \tilde{q}^I \in Q_S^I$ as the incumbent's capacity in period 2 and continues production, this report implies that the entrant's liquidity holding is $\pi^E(q^E, \tilde{q}^I) + B$ at the beginning of period 4. If the face value of $D(\tilde{q}^I)$ exceeds this amount, it cannot be fully repaid and the actual repayment is reduced to the amount of the entrant's liquidity holding.

As long as the borrower announces the message that allows him to stay in the market and this message is believed to be true, the promised monetary repayment cannot exceed the liquidity holding that the borrower should have at the time of repayment.

$$D(\tilde{q}^I) \leq \pi^E(q_*^E, \tilde{q}^I) + B \qquad \text{whenever } \tilde{q}^I \in Q_S^I. \tag{LL$_*^S$}$$

If the entrant exits the market by announcing $m_0 \in M_0$, the entrant's liquidity holding is B and it sets an upper bound on the repayment:

$$D(m_0) \leq B \qquad \text{whenever } m_0 \in M_0. \tag{LL$_*^0$}$$

Investor's participation condition (PC$_*^I$): The investor would not agree to the contract if he expects a loss. Then, to have the investor agree to the contrantct, the equilibrium dividend plus the liquidation value should cover the loan $B - w_0$. Given the equilibrium capacities $\mathbf{q}_*$, the condition reduces to

$$D(q_*^I) + \beta(q_*^I)\underline{V} \geq B - w_0. \tag{PC$_*^I$}$$

As we assume that the entrant has the full bargaining power, the optimal financial contract $\mathcal{C}^*$ is a solution of the constrained maximization of the entrant's profit subject to these four constraints:

$$\max_{\mathcal{C}} \tilde{\pi}^E(q_*^E) \text{ s.t. (AP}_*\text{), (IC}_*\text{), (LL}_*\text{), (PC}_*^I\text{)}. \tag{C$_*$}$$

The capacity profile $\mathbf{q}_*$ in non-predation equilibrium is determined as the entrant's optimal capacity q_*^E in this optimal anti-predation financial contract and the incumbent's best response to it, $q_*^I := q_{\text{BR}}^I(q_*^E)$.

Lower bound on collateral

The next theorem says that these four constraints jointly impose a non-trivial condition (NP$_*$) on the entrant's equilibrium capacity q_*^E. We look at the entrant's messaging strategies after observing the incumbent's equilibrium capacity q_*^I and a plausible predatory capacity $q_P^I \in Q_P^I(q_*^E)$. We call the condition (NP$_*$) **the non-predation condition**.

Theorem 1. *Consider an anti-predatory financial contract. Then, the entrant's equilibrium capacity size q_*^E must satisfy*

$$\overline{V} + w_0 \geq \bar{L}_P(q_*^E). \tag{NP$_*$}$$

Proof. Since $\overline{V} > \underline{V}$ and $\beta \geq 0$, the participation condition (PC$^I_*$) sets a lower bound on the total repayment after the equilibrium capacity size q^I_* is announced:

$$\delta(q^I_*) = D(q^I_*) + \beta(q^I_*)\overline{V} \geq B - w_0. \tag{4}$$

The limited liability condition (LL$_*$) sets an upper bound on the monetary dividend after a plausible predation q^I_P is announced:

$$D(q^I_P) \leq \pi^E(q^E_*, q^I_P) + B.$$

Since $\beta \in [0, 1]$ and $\overline{V} > 0$, this implies an upper bound on the total payment after q^I_P.

$$\delta(q^I_P) = D(q^I_P) + \beta(q^I_P)\overline{V} \leq \pi^E(q^E_*, q^I_P) + B + \overline{V}. \tag{5}$$

Finally, since $Q^I_P(q^E_*) \subset Q^I_S$, the incentive compatibility condition (IC$_*$) implies $\delta(q^I_P) = \underline{\delta} = \delta(q^I_*)$ for any $q^I_P \in Q^I_P(q^E_*)$. Combining the two bounds (4) and (5) with this, we obtain

$$\begin{aligned} \pi^E(q^E_*, q^I_P) + B + \overline{V} \geq \delta(q^I_P) = \delta(q^I_*) \geq B - w_0, \\ \therefore\ \overline{V} + w_0 \geq -\pi^E(q^E_*, q^I_P) \end{aligned} \tag{6}$$

for any $q^I_P \in Q^I_P(q^E_*)$. This implies

$$\overline{V} + w_0 \geq \sup\{-\pi^E(q^E_*, q^I_P) | q^I_P \in Q^I_P(q^E_*)\} = \bar{L}_P(q^E_*). \qquad \square$$

If (NP$_*$) is not satisfied, we have $\underline{\delta} < B - w_0$ as long as (LL$_*$) is imposed in the case of plausible predation; so the investor cannot expect to recoup the investment and thus refuse the investment.

(NP$_*$) is a necessary condition for a non-predatory equilibrium. With full bargaining power, the entrant anticipates the incumbent's non-predatory optimal capacity decision and designs the loan contract $\mathcal{C}$ so as to maximize his profit, subject to the non-predation condition (NP$_*$):

$$q^E_* = \underset{q^E \in Q^E}{\operatorname{argmax}} \left\{ \tilde{\pi}^E(q^E) \ \middle|\ \overline{V} + w_0 \geq \bar{L}_P(q^E) \right\}, \tag{7a}$$

$$q^I_* = q^I_{\mathrm{BR}}(q^E_*) = \underset{q^I \in Q^I}{\operatorname{argmax}}\, \pi^I(q^I, q^E_*). \tag{7b}$$

If the benchmark capacity $q^E_\dagger$ satisfies the non-predatory condition (NP$_*$), it is the optimum in (7a). Otherwise, the existence of the constrained optimum (7a) needs an additional assumption. For example, Q^E can be assumed to be a finite non-empty set. Or, if $Q^E = \mathbb{R}_+$, we may assume boundedness and continuity of $\bar{L}_P$ as well as continuity of $\tilde{\pi}^E$. These properties are guaranteed and the existence of the optimum are proven in Appendix B.1 for the continuous case with $Q^I = \mathbb{R}_+$, by assuming continuity of π^I and π^E, boundedness of the profitable capacity level for the incumbent in case of monopoly, and a technical topological property on the incumbent's monopoly profit function $\pi^I(\cdot, 0)$.

If the non-predatory condition (NP$*$) is binding at the optimum q^E_*, the proof of Theorem 1 suggests that both (4) and(5) are binding with $\underline{\delta} = B - w_0 = B + \overline{V} - \bar{L}^P(q^E_*)$. The former implies that (PC$^I_*$) binds with $\delta(q^I_*) = \underline{\delta} = B - w_0$. The latter implies that (LL$*$) binds at $\bar{q}^I_P(q^E_*)$ with $D(\bar{q}^I_P(q^E_*)) = B - \bar{L}^P(q^E_*)$ and $\beta(\bar{q}^I_P(q^E_*)) = 1$. One may notice that this does not determine the amount of the precautionary liquidity B. It is indeed observed from the limited liability constraint (LL$*$). *If* monetary repayment $D(\tilde{q}^I)$ needs to be non-negative, the liquidity holding at the end of period 3, $\pi^E(q^E_*, \tilde{q}^I) + B$, cannot be negative in order to pay it out.[12] To maintain it for all $\tilde{q}^I \in Q^I_P(q^E_*) \subset Q^I_S$, the precautionary liquidity B must satisfy

$$\inf_{\tilde{q}^I \in Q^I_P(q^E_*)} \pi^E(q^E_*, \tilde{q}^I) + B \geq 0, \quad \text{i.e., } B \geq \bar{L}_P(q^E_*).$$

Comparing with the benchmark financial contract (see footnote 10), this shows excess demand in precautionary liquidity. In Section 4, we look into the entrant's cash flows over periods to understand this requirement of the precautionary liquidity and its role.

Capacity profile $\mathbf{q}_*$ is implemented by a variation of a debt contract $\mathcal{C}_*$ as below.[13]

$$\begin{aligned}
&B - w_0 = \overline{V} \\
&Q^I_S = \{\tilde{q}^I \in Q^I \mid \pi^E(q^E_*, \tilde{q}^I) + B \geq 0\}, \qquad M = Q^I_S \cup \{m_0\}; \\
&D(m_0) = B, \qquad\qquad\qquad \beta(m_0) = 1; \\
&D(\tilde{q}^I) = \begin{cases} B - w_0 \\ \pi^E(q^E_*, \tilde{q}^I) + B \end{cases} \qquad \beta(\tilde{q}^I) = \begin{cases} 0 & \text{if } \pi^E(q^E_*, \tilde{q}^I) + B \geq B - w_0, \\ 1 - D(\tilde{q}^I)/\overline{V} & \text{if } \pi^E(q^E_*, \tilde{q}^I) + B \in [0, B - w_0). \end{cases}
\end{aligned}$$

Since $\tilde{\pi}^E(q^E_*)$ sets an upper bound on the entrant's attainable profit under an anti-predation financial contract, this contract $\mathcal{C}_*$ is indeed an optimal anti-predation financial contract, i.e., a solution of the optimization problem (C$*$), though it may not be the only one.

Under this contract $\mathcal{C}_*$, the entrant borrows as much as the *private* value $\overline{V}$ of the mortgaged start-up assets and repays the loan whenever he has enough liquidity to do so. If the entire loan is not repaid, the rest is covered by liquidation of the mortgaged assets. The total repayment δ is kept constant at $\overline{V}$ as long as the entrant continues business, regardless of the message. The entrant will continue the business even if it yields an operating loss, as long as the loss can be covered by precautionary liquidity B. To get the *entrant* choose the continuation of business against plausible predation, the entrant lets the initial lender take over all of his assets and liquidity when he quits.

Finally, our formulation of an anti-predatory financial contract allows the investor to fully commit to support the entrant's business, i.e., to set Q^I_S to the whole Q^I. But, then full com-

[12] Having said, we do not impose non-negativity constraint on $D(\cdot)$. So, theoretically, we allow negative monetary repayment $D < 0$; then, the entrant's liquidity holding can be negative at the end of period 3. But then we may wonder how the entrant can maintain negative balance of liquidity on its account; there should be someone who indeed lend money.

[13] As argued in footnote 9, the revelation principle that justifies our formulation of the financial contract establishes the connection between the optimal contract and a sequential equilibrium. In Appendix B.2, we verify the existence of sequential equilibrium with capacity profile $\mathbf{q}_*$ in the game after the debt contract $\mathcal{C}_*$ is accepted, assuming finiteness of Q.

mitment means that the entrant is guaranteed to continue the business under any predation—including the one that cannot be profitable for the incumbent even if the entrant is excluded. Thus, the investor must expect any predatory loss, i.e., $\pi^E(q^E_*, q^I)$ for any $q^I \in Q^I$, not only for $q^I \in Q^I_P(q^E_*)$. By the same token as in the proof of Proposition 1, we find that such a full commitment contract is feasible if and only if $\overline{V} + w_0$ covers $\sup\{\pi^E(q^E_*, q^I) \mid q^I \in Q^I\}$. Thus, we find that full commitment requires the entrant to have greater amount of initial capital than the contract just to block *plausible* predation.

2.4 Distortion in the product market

The non-predation condition generally restrains the entrant's capacity and makes it less aggressive. There might exist a trivial exception in which the maximal plausible predation does not respond to a change in the entrant's capacity. This occurs only because of the coarseness of the strategy space. To exclude this exception, we assume that the incumbent's feasible capacity set is "dense" enough.

Assumption 3. Compared with Q^E, the set Q^I is dense enough in the sense that, for any $q^E_0, q^E_1 \in Q^E$ such that $\pi^I(q^I_{\mathrm{BR}}(q^E_1), q^E_1) < \pi^I(q^I_{\mathrm{BR}}(q^E_0), q^E_0)$, there exists $q^I_2 \in Q^I$ such that

$$q^I_2 > \bar{q}^I_P(q^I_0) \qquad \text{and} \qquad \pi^I(q^I_{\mathrm{BR}}(q^E_1), q^E_1) < \pi^I(q^I_2, 0) < \pi^I(q^I_{\mathrm{BR}}(q^E_0), q^E_0).$$

Theorem 2. *Suppose that Assumptions 1, 2 and 3 hold. Then, the maximal predatory capacity $\bar{q}^I_P(q^E)$ increases with the entrant's capacity q^E. The maximal predatory loss $\bar{L}_P(q^E)$ also increases with q^E whenever q^E is greater than $q^E_\dagger$.*

Proposition 1. *If the entrant's start-up capital is so small that*

$$w_0 + \overline{V} < \bar{L}_P(q^E_\dagger),$$

the entrant cannot finance precautionary liquidity large enough to produce the benchmark capacity.

Suppose that the optimum q^E_* exists in the constrained maximization problem (7a)*. With Assumptions 1, 2, and 3, it follows that the entrant's equilibrium capacity size becomes smaller than the benchmark equilibrium capacity size $q^E_\dagger$. Consequently, the incumbent's capacity further expands.*

While the proof for a general case is given in Appendix C, here we see the essence of the logic by setting Q to $\mathbb{R}^2_+$ and assuming that π^E and π^I are strictly concave smooth functions as below so they satisfy Assumptions 1 and 3.[14]

> For each $i \in \{I, E\}$, profit function $\pi^i : \mathbb{R}^2_+ \to \mathbb{R}$ is C^2 and satisfies ${\pi^i}'_j < 0, {\pi^i}''_{ii} < 0, {\pi^i}''_{ij} < 0, {\pi^i}''_{jj} \leq 0, \pi^E(0, q^I) = 0$ and $\pi^I(0, q^E) = 0$ for each $i \in \{E, I\}$, $j \neq i$, and $\mathbf{q} \in \mathbb{R}^2_+$. (Here ${\pi^i}'_j := \frac{\partial \pi^i}{\partial q^j}, {\pi^i}''_{jk} := \frac{\partial^2 \pi^i}{\partial q^j \partial q^k}$.)

Since ${\pi^i}'_{ii} < 0$, we can uniquely determine the incumbent's optimal supply q^i to maximize $\pi^i(q^i, q^j)$ given q^j. First, the maximal plausible predation $\bar{q}^I_P(q^E)$ gets larger as the entrant

[14] To assure Assumption 2, we could further assume ${\pi^i}'_i(0, q^j) > 0$, for example. Then, the first order conditions for the benchmark equilibrium, ${\pi^I}'_I(\mathbf{q}_\dagger) = 0$ and $\tilde{\pi}^{E\prime}(q^E_\dagger) = {\pi^E}'_E(\mathbf{q}_\dagger) + {\pi^E}'_I(\mathbf{q}_\dagger) \cdot {q^I_{\mathrm{BR}}}'(q^E_\dagger) = 0$, can hold only in the interior of the domain, i.e., $\mathbb{R}^2_{++}$. But we do not need it in the rest of the argument.

chooses a larger q^E. An increase in q^E decreases the incumbent's optimal duopoly profit $\pi^I(q^I_{\mathrm{BR}}(q^E), q^E)$. This makes predation more attractive for the incumbent; thus a larger predatory capacity becomes profitable. That is, $\bar{q}^I_P(q^E)$ becomes larger. Analytically we obtain

$$\frac{d\bar{q}^I_P}{dq^E}(q^E) = \frac{{\pi^I}'_E(q^I_{\mathrm{BR}}(q^E), q^E)}{{\pi^I}'_I(\bar{q}^I_P(q^E), 0)} > 0$$

by differentiating $\pi^I(q^I_{\mathrm{BR}}(q^E), q^E) = \pi^I(\bar{q}^I_P(q^E), 0)$ with respect to q^E. (See (2).)

We can decompose into effects of a marginal increase in q^E on $\bar{L}_P$ as

$$\frac{d\bar{L}_P}{dq^E}(q^E) = \underbrace{-{\pi^E}'_E(\bar{q}^I_P(q^E), q^E)}_{\text{Direct effect}} \underbrace{-{\pi^E}'_I(\bar{q}^I_P(q^E), q^E) \times \frac{d\bar{q}^I_P}{dq^E}(q^E)}_{\text{Indirect effect}}.$$

The direct effect is the increase in $\bar{L}_P$ caused by the increase in the entrant's capacity, holding the incumbent's capacity fixed. The indirect effect is the increase caused by the change in the incumbent's maximal plausible predatory capacity $\bar{q}^I_P$.

The indirect effect is always positive because $\bar{q}^I_P(q^E)$ increases with q^E and ${\pi^E}'_I < 0$. The direct effect could be negative and thus the sign of the overall effects is ambiguous. However, the direct effect is positive whenever $q^E \geq q^E_\dagger$.[15] Therefore, the overall effect is positive. That is, if the entrant plans a greater capacity level, it needs to be prepared for a greater predatory loss.

Unless the entrant has enough start-up liquidity to protect himself from predation in the benchmark equilibrium, the non-predation condition (NP$_*$) effectively constrains the entrant's capacity at the optimum of (7a). The threat of predation indeed makes the entrant less aggressive. As a result, in a non-predation equilibrium, the entrant's capacity q^E is restricted by its start-up capital w_0 and the private value of his asset $\overline{V}$ through the non-predation condition.

3 Example: Two-stage product market competition

To see more concretely how the threat of predation affects the outcome of product market competition, we consider a two-stage competition with a capacity constraint on the actual quantity supplied, as in Dixit (1980) and Gabszewicz and Poddar (1997). In addition to the sequential decisions regarding capacity sizes in the base model, we now explicitly let each active firm decide on production levels in the second stage. With the assumption of a linear demand function, this second-stage competition identifies the gross revenue functions R^E and R^I numerically.

In the first stage, each firm $i \in \{E, I\}$ decides on *capacity size* q^i sequentially: the entrant

[15] The first order condition of maximization of $\tilde{\pi}^E$ should hold at the benchmark equilibrium: $0 = \tilde{\pi}^{E\prime}_E(q^E_\dagger) \equiv {\pi^E}'_E(\mathbf{q}_\dagger) + {\pi^E}'_I(\mathbf{q}_\dagger) \cdot {q^I_{\mathrm{BR}}}'(q^E_\dagger)$. Since ${\pi^E}'_I < 0$ and ${q^I_{\mathrm{BR}}}' < 0$ (by ${\pi^I}''_{IE} < 0$), this implies ${\pi^E}'_E(\mathbf{q}_\dagger) < 0$. On the other hand, since ${\pi^E}''_{EI} < 0$ and $\bar{q}^I_P(q^E_\dagger) > q^I_\dagger$ (by ${\pi^I}'_E < 0$ and ${\pi^I}''_{II} < 0$), the predatory capacity $\bar{q}^I_P(q^E_\dagger)$ decreases the entrant's marginal net profit ${\pi^E}'_E$ from that at the benchmark equilibrium, ${\pi^E}'_E(\mathbf{q}_\dagger)$. Hence ${\pi^E}'_E(\bar{q}^I_P(q^E_\dagger), q^E_\dagger) < {\pi^E}'_E(\mathbf{q}_\dagger) = 0$ and thus the direct effect is positive at $q^E = q^E_\dagger$. Further, consider the case of $q^E > q^E_\dagger$. Then, we have obtained $\bar{q}^I_P(q^E) > \bar{q}^I_P(q^E_\dagger)$. As ${\pi^E}'_{EE} < 0$ and ${\pi^E}'_{EI} < 0$, it implies that ${\pi^E}'_E(\bar{q}^I_P(q^E), q^E) < {\pi^E}'_E(\bar{q}^I_P(q^E_\dagger), q^E_\dagger) < 0$.

decides first and then, after observing q^E, the incumbent decides next. Here, we allow capacity size to be any positive real number, i.e., $Q^i = \mathbb{R}_+$. The capacity investment costs c^i per unit of capacity.

After observing the rival firm's capacity size, the two firms proceed to the second stage. Each firm i determines the actual *production level* $x^i \in \mathbb{R}_+$, which cannot be greater than the capacity size, i.e., $x^i \leq q^i$. In this paper, we assume that all the active firms decide on their production levels simultaneously. For simplicity, we assume that there is no marginal cost in addition to c^i.

Finally, the product price is determined by the total supply $X := x^I + x^E$ through the inverse demand function $P(X) = a - X$ with constant $a > 0$. Note that if there was no capacity constraint, cost for capacity investment, or threat of predation, then firm i's best response to anticipation of the opponent j's production x^j is $x^i_{\text{BR}}(x^j) := 0.5a - 0.5x^j$. The benchmark equilibrium production is then $x^E_0 = x^I_0 := a/3$.

We can identify the gross revenue $R^E(\mathbf{q})$ and $R^I(\mathbf{q})$ as the equilibrium gross profits in the second stage competition given capacity profile $\mathbf{q} \in Q$. According to Appendix D, firm i's gross revenue function $R^i : \mathbb{R}^2_+ \to \mathbb{R}$ is obtained as

$$R^i(q^i, q^j) = \begin{cases} a^2/9 & \text{if } q^i \geq x^i_0 \text{ and } q^j \geq x^j_0 \\ (a - q^i)q^i/2 & \text{if } q^i < x^i_0 \text{ and } q^j \geq x^j_{\text{BR}}(q^i) \\ (a - q^j)^2/4 & \text{if } q^j < x^j_0 \text{ and } q^i \geq x^i_{\text{BR}}(q^j) \\ (a - q^i - q^j)q^i & \text{otherwise.} \end{cases} \tag{8}$$

Then, i's net profit function $\pi^i : \mathbb{R}^2_+ \to \mathbb{R}$ is given by $\pi^i(\mathbf{q}) := R^i(\mathbf{q}) - c^i q^i$. We assume that a/c_E belongs to either of the two ranges,

$$a/c_E \leq 2 \qquad \text{or} \qquad a/c_E \geq 3(2 + \sqrt{2}), \tag{9}$$

each of which guarantees Assumption 1-v) for π^E. In Appendix D, we verify it from (9) and confirms that the rest of Assumption 1 hold for both π^E and π^I without further additional assumptions. Since R^i is a continuous function and thus so is π^i, this choice of the domain $Q = \mathbb{R}^2_+$ guarantees Assumption 3.

As in the benchmark production market competition, consider the case where the entrant cannot exit from the market after deciding on q^E. (But it is allowed to choose $x^E = 0$.) Then, the game reduces to the standard Stackelberg duopoly competition: the entrant's benchmark capacity sizes are thus $q^E_\dagger = a + c^I - 2c^E$ and $q^I_\dagger = a + c^E - 2c^I$. Assumption 2 is satisfied as long as both of them are positive, i.e., $a > \max\{2c^E - c^I, 2c^I - c^E\}$.

If the entrant can exit from the market after observing $\mathbf{q}$ but before paying $c^E q^E$, it makes a room for the incumbent to monopolize the market by excess capacity. Given q^E, the entrant's maximal predatory loss $\bar{L}_P(q^E)$ is the loss that occurs when the incumbent's capacity is set to the maximal predatory size $\bar{q}^I_P(q^E)$. As $\bar{q}^I_P(q^E) \geq x^I_{\text{BR}}(0) > x^I_{\text{BR}}(q^E)$, it falls to the second case in (8) if $q^E < x^E_0$. Then, the maximal predatory loss is $\bar{L}_P(q^E) = c^E q^E - (a - q^E)q^E/2$. This is positive if and only if $q^E > a - 2c^E$.

In particular, since $c^I > 0$ implies $q_\dagger^E > a - 2c^E$, the maximum predatory loss is positive at the benchmark equilibrium capacity profile $\mathbf{q}_\dagger$ whenever $q_\dagger^E > x_0^E$, or equivalently $a > 1.5(2c^E - c^I)$. In this case, Proposition 1 implies that if the entrant's start-up capital is insufficient to cover the maximum predatory loss $\bar{L}_P(q_\dagger^E) > 0$, the entrant's capacity shrinks while the incumbent's capacity expands. In particular, if $c_E < c_I$, it is more socially efficient to let the entrant produce more than the incumbent. Hence, if the entrant has more efficient production technology than the incumbent, then the threat of predation indeed yields a socially inefficient outcome in the product market—not only in the total amount of production but also in its allocation over producers.

4 Extensions and variations

4.1 Quantification of excess liquidity

In the base model, we left unspecified the reason why the entrant needs the investor's financial support. This leaves the question of how the amount of the initial investment $B - w_0$ is determined and why it is ever needed. To give a concrete idea without adding extra components to the model, we take a closer look at the entrant's cash flows. Production capacity is expanded by constructing new production facilities. The factor inputs necessary to expand capacity must be obtained at the time the facilities are constructed. We make explicit that the entrant must pay the factor input costs when the inputs are received. We include those payments into the time line of events as follows:

Period 2 If the entrant decides to launch the business, it must pay the capacity cost $C^E(\mathbf{q})$.

Period 3 The entrant receives the operating revenue $R^E(\mathbf{q})$ if it remains in the market.

This time line does not exclude the possibility that the entrant can postpone the payments or borrow a short-term loan until earning the sales revenue. We will consider this in Case 2 below.

The investor will keep the entrant's business by making up the difference between the capacity costs and the available short-term finance as long as the announced capacity size of the incumbent falls in Q_S^I.[16] The commitment to continuation of the entrant's business is implemented by committing to a credit line and/or lending a loan at the time of entry. Precautionary liquidity B should include the credit line and the loan, in addition to the entrant's own initial liquidity holding w_0.

Case 1: No additional loan. First, let us consider the case in which an additional loan from a new lender (or trade credit) is not allowed. Liquidity holding B thus must cover the entire capacity cost. For this, the investor finances a loan or grants the entrant a credit line. In this case, Q_S^I is determined from B as the range of the incumbent's capacity level for which the

[16]Of course, a truthful report of q^I is also needed to assess how large of an additional loan is required to continue business. Alternatively, we can say that the entrant first reports the amount of the additional loan he demands, from which q^I can be inferred. The revelation principle allows us to reduce the analysis of outcomes under such 'indirect' reporting of unverifiable information to outcomes under direct messaging (a direct mechanism).

entrant can pay the whole capacity cost by B, provided that the entrant's announcement tells the incumbent's true capacity size:

$$Q^I_S = \{\tilde{q}^I \in Q^I \mid C^E(q^E_*, \tilde{q}^I) \leq B\}.$$

The anti-predation condition (AP$_*$) requires precautionary liquidity B to satisfy

$$C^E(q^E_*, \tilde{q}^I) \leq B \qquad \text{for all } \tilde{q}^I \in Q^I_P(q^E_*).$$

Assume that C^E increases with q^I. Then, this condition reduces to

$$B \geq C^E(q^E_*, \bar{q}^I_P(q^E_*)).$$

Note that, if there were no threat of predation and thus the anti-predation condition was not imposed, it would be enough to raise B as much as needed to pay $C^E(\mathbf{q}_*)$.

As long as q^E_* satisfies the non-predation condition $\overline{V} + w_0 \geq \bar{L}_P(q^E_*)$, it is implementable with a debt contract such as

$$\begin{aligned}
&B - w_0 = \overline{V} + R^E(q^E_*, \bar{q}^I_P(q^E_*)) \\
&Q^I_S = \{\tilde{q}^I \in Q^I \mid C^E(q^E_*, \tilde{q}^I) \geq B\}, \qquad M = Q^I_S \cup \{m_0\}; \\
&D(m_0) = B, \qquad\qquad \beta(m_0) = 1; \\
&D(\tilde{q}^I) = \begin{cases} B - w_0 \\ \pi^E(q^E_*, \tilde{q}^I) + B \end{cases} \qquad \beta(\tilde{q}^I) = \begin{cases} 0 & \text{if } \pi^E(q^E_*, \tilde{q}^I) + B \geq B - w_0, \\ 1 - D(\tilde{q}^I)/\overline{V} & \text{if } \pi^E(q^E_*, \tilde{q}^I) + B \in [0, B - w_0). \end{cases}
\end{aligned}$$

The non-predation condition guarantees $C^E(q^E_*, \bar{q}^I_P(q^E_*)) \leq B$ and thus $Q^I_P(q^E_*) \subset Q^I_S$.

Case 2. Additional loan is allowed. Case 1 may be too restrictive for an entrepreneur; it is plausible that the entrepreneur finds a new lender for an additional loan if the investor will not provide a credit line or an additional loan. Here we consider the case where the entrepreneur can obtain an additional loan in period 3. Although it might seem possible that the entrepreneur would now have no need for the initial loan, this is not the case. Because of the threat of predation, the entrepreneur can only obtain a sufficiently large additional loan if he has already secured enough precautionary liquidity.

We have seen that the entrepreneur's assets upon entry V must be leveraged in the financial contract with the initial investor in order to eliminate the possibility of a false default due to asymmetric information. So, we suppose that, after the entrepreneur earns the sales revenue, it will repay the second lender before repaying the initial investor. The message to the initial investor is shared with the second lender. The mortgage on the collateral is kept by the initial investor.[17]

[17] As the initial investor takes the start-up asset as collateral, we assume that the additional lender has priority to be repaid from product sales. Although this financial structure is just an assumption,it captures situations in which the entrant puts up physical assets to start the business as collateral for an initial loan, and inventories and accounts receivable as collateral for an additional loan. Hart (1995, p.111) notes such a distinction between long-term and short-term loans, citing Dennis et al. (1988) and Dunkelberg and Scott (1985) as empirical evidence.

Fix $q^I \in Q^I_S$ such that the entrant will remain in the market. The entrant still needs to borrow the difference $C^E(q^E, \tilde{q}^I) - B$ between capacity costs and liquidity holding, while he can repay at most $R^E(q^E, \tilde{q}^I)$. The additional loan can be repaid fully if and only if

$$R^E(q^E, \tilde{q}^I) \geq C^E(q^E, \tilde{q}^I) - B, \qquad \text{i.e., } \pi^E(q^E, \tilde{q}^I) + B \geq 0$$

If this condition is met, the additional loan is available and the entrant can indeed launch the business. Thus, Q^I_S is determined from B as

$$Q^I_S = \{\tilde{q}^I \in Q^I \mid \pi^E(q^E_*, \tilde{q}^I) + B \geq 0\}.$$

As a benchmark, consider the case in which there is no threat of predation and both firms commit to the benchmark capacity sizes $\mathbf{q}_\dagger$. It is implementable as long as $B \geq -\pi^E(\mathbf{q}_\dagger)$. Under Assumption 2, the entrant should be able to earn a positive operating profit $\pi^E(\mathbf{q}_\dagger) > 0$. Therefore, the condition is satisfied even with $B = 0$; no precautionary liquidity is needed as long as there is an opportunity to take out a loan just when the entrant needs money to pay the capacity costs.

When there is threat of predation, the entrant needs to commit to maintaining the business even in cases that the incumbent reacts to the entry more aggressively with a larger capacity investment than $q^I_\dagger$. To keep the business when the incumbent chooses $\tilde{q}^I \in Q^I_S$, the above condition needs to hold for all $\tilde{q}^I \in Q^I_S$. To block all the plausible predatory capacity investments, precautionary liquidity B needs to meet

$$B \geq -\pi^E(q^E_*, \bar{q}^I_P(q^E_*)) = \bar{L}^P(q^E_*).$$

This is indeed implemented by equilibrium contract $\mathcal{C}_*$ in Theorem 1. In sum, even if the operation profit will be positive in the equilibrium outcome, the entrant needs to raise precautionary liquidity earlier and more than actually needed for payment of actual costs, as long as the maximal predatory loss is positive; we have seen that it is the case when the entrant has more efficient production technology than the incumbent in the two-stage competition model in the last section.

4.2 Exogneous costs to raise liquidity

The quantitative specification of excess liquidity allows us to embed our model into a financial macroeconomic model to see the implication of monetary policy on product market competition and entrepreneurship. Recall that, in models à la Bolton and Scharfstein (1990), the entrant needs only as much as to pay some fixed entry cost and thus the liquidity demand is determined exogenously, apart from the entrant's production.

In the base model, the investor's profit from the outside option (not investing in the entrant) is assumed to be zero. It does not significantly change the non-predatory condition even if we introduce a profitable outside option that yields risk-free interest rate r. However, now as the entrant needs to pay interest rB to the investor to meet its participation condition, the entrant raises precautionary liquidity only just as much as needed to meet the non-predation condition: i.e., now precautionary liquidity B is set just equal to $\bar{L}_P(q^E)$, if the uncommitted additional loan is allowed (Case 2 in Sec 4.1) and r is regarded as the spread between interest rates of short-term uncommitted loans and long-term committed loans. Thus, the entrant's optimal capacity level is determined by

$$\max_{q^E \in Q^E} \tilde{\pi}^E(q^E) - r\bar{L}_P(q^E) \qquad \text{s.t. } \bar{L}_P(q^E) \leq \bar{V} + w_0.$$

By solving this for given interest rate r, we identify the optimal capacity level of the entrant q^E_* and also the demand for precautionary liquidity $\bar{L}_P(q^E_*)$. By seeing it as a function of r, we obtain the liquidity demand function.

One possible interesting further extension in this direction is to combine this model with other types of liquidity demands for transactions and productions. The excess liquidity to block predation is kept only to show the entrant's financial healthiness and commitment to staying in the market. It does not contribute to production. When the economy's liquidity supply is limited, such demand for precautionary liquidity crowds out demand for the liquidity necessary for production and investment. Hence a policy that weakens threat of predation improves macroeconomic efficiency by releasing excess liquidity holdings. As argued by Holmström and Tirole (1998), the availability of credit lines might eliminate inefficiency in the monetary market.

When the entrant needs pracautionary liquidity to block predation, the relationship between the entrant's productivity and the entering decision may not be monotonic and may result in a negative correlation. While a higher productivity should yield a greater operating profit if predation is avoided, it comes with a greater capacity level of the entrant. This raises the incumbent's incentive of predation and thus the maximum plausible predatory loss becomes larger, as we saw in the example in Section 3. Hence, the entrant with a high productivity needs to raise more precautionary liquidity. When the market interest rate becomes higher, the production of such entrants are further more distorted due to the interest costs for excess liquidity. On the relationship between business cycle and firms' productivity, we could say that a slump with lower market interest rate may help highly productive firms by lowering the interest costs and suffering less from threat of predation. Lee and Mukoyama (2015) study the difference in the productivity distribution of entering firms between booms and slumps in U.S. business cycles. They found that the entering firms tend to have higher productivity in slumps than in booms; it is consistent with this prediction from our model. See the discussion on equity and debt financing in Section 5.

4.3 Simultaneous capacity choices

The model can be easily modified to capture a situation in which the incumbent makes its decision *at the same time* as the entrant makes its decision on q^E like a Cournot competition. Such a case is rigorously formulated and analyzed in an older working paper (Zusai, 2012). It only affects the quantitative identification of equilibrium capacity levels.

First, notice that even in this case, the incumbent bases its capacity choice on some guess of the entrant's q^E. Then, we reinterpret $Q^I_P(q^E)$ as the range of plausible predatory capacity sizes under the *guess* of q^E. We similarly reinterpret $\bar{L}_P(q^E)$. The derivation of the non-predatory condition (NP$_*$) remains the same, as long as the entrepreneur's equilibrium capacity choice q_*^E is properly modified to account for the change in timing. Under Cournot-like simultaneous capacity choices, equilibrium capacity profile $\mathbf{q}_C$ should be determined from

$$q_C^E = \operatorname*{argmax}_{q^E \in Q^E} \left\{ \pi^E(q^E, q_C^I) \mid \overline{V} + w_0 \geq \bar{L}_P(q^E) \right\} \tag{10}$$

$$q_C^I = \operatorname*{argmax}_{q^I \in Q^I} \pi^I(q^I, q_C^E). \tag{11}$$

The aforementioned working paper verifies that, with the proper reinterpretations of $Q_P^I(q^E)$ and $\bar{L}_P(q^E)$, every proposition in this current paper holds and the above system of equations indeed characterize the capacity profile in *any* sequential equilibrium of the game played by the entrepreneur, the investor, and the incumbent.

4.4 Bargaining power of the investor

For simplicity, the base model assigns all the bargaining power to the entrant. Now we allocate bargaining power to both sides of the contract, i.e., the entrant and the lender. We assume that, in the Nash bargaining problem, the entrant's bargaining power is $\gamma \in [0, 1]$ and the investor's is $1 - \gamma$. Then the optimal contract problem becomes

$$\begin{aligned}
\max_{q^E, D(\cdot), \beta(\cdot)} \quad & \{\overbrace{\tilde{\pi}^E(q^E) + B - D(q_{\mathrm{BR}}^I(q^E)) + (1 - \beta(q_{\mathrm{BR}}^I(q^E)))\overline{V} - w_0 - \overline{V}}^{\Pi_{NB}^E}\}^{\gamma} \\
& \times \{\underbrace{D(q_{\mathrm{BR}}^I(q^E)) + \beta(q_{\mathrm{BR}}^I(q^E))\underline{V} - (B - w_0)}_{\Pi_{NB}^L}\}^{1-\gamma} \\
\text{s.t.} \quad & \Pi_{NB}^E \geq 0, \qquad \qquad (\mathrm{PC}^E) \\
& \Pi_{NB}^L \geq 0, \qquad \qquad (\mathrm{PC}_L) \\
& (\mathrm{IC}_*), (\mathrm{LL}_*), \text{ and } (\mathrm{AP}_*).
\end{aligned}$$

In Appendix E, we verify that the benchmark capacity $q_\dagger^E$ is chosen and the benchmark equilibrium $\mathbf{q}_\dagger$ is implemented if and only if

$$(1 - \gamma)\tilde{\pi}^E(q_\dagger^E) + \bar{L}_P(q_\dagger^E) \leq \overline{V} + w_0. \tag{12}$$

The most crucial constraints are the limited liability constraints (LL$_*^S$) at $\tilde{q}^I = q_\dagger^I$ and at $\tilde{q}^I = \bar{q}_P^I(q_\dagger^E)$. Which of the limited liability constraints is binding depends on the size of collateral $\overline{V}$. Thus, which of the incumbent's strategies affects the entrant's choice of strategy also depends on the size of collateral $\overline{V}$. By setting the liquidation probability $\beta(\bar{q}_P^I)$ equal to 1 while keeping the net repayment constant for the incentive compatibility of truth telling, the limited liability on the monetary payment can be relaxed when the maximal predation $\bar{q}_P^I$ is reported, only as much as covered by the liquidation value of the collateral. If it is large enough to cover the predatory loss and raise the entrant's liquidity up to the equilibrium operating profit, the limited liability at $\bar{q}^I_P$ is not restrictive. Otherwise, the total net payment is restricted and it is bounded by the sum of the collateral's liquidation value and the entrant's liquidity holding after paying out the predatory loss. So, even if the investor exercises the full bargaining power to maximize its net profit in equilibrium, threat of predation affects the entrant's choice of strategy by limiting the feasible total repayment.

5 Discussion

In this section, we discuss structural assumptions underlying our model to clarify applicability of the theory presented here.

Commitment

The key aspect of our model's financial structure is the investor's *commitment* to keeping the entrant's business operating, though it can be conditioned on the incumbent's (truthfully reported) reaction to the entry. This point leads us to reconsider the meaning of the "entrant" in our model. The "entrant" cannot obtain the investor's full commitment, possibly because he is new to the industry and has yet to establish the creditworthiness and long-term relationship necessary to defer payment or get an unconditional advance draw. In some cases, our model is applicable by regarding an actual entrant as an "incumbent" in the model. For example, the actual entrant may be a large conglomerate that can subsidize the new business with profits from other enterprises or he may have unconditional support by the government or a large business group.

Unverifiability and monitoring

Let us consider the *unverifiability* of the incumbent's "capacity" q^I and the entrant's profit π^E. Note the distinction between unverifiability and unobservability. Even if q^I is unverifiable, the entrant may directly observe q^I or predict it with high accuracy by extensive marketing research. The entrant could even present marketing data about the rival's strategy and its impact on the entrant's own business to the lenders so as to convince them that the entry plan is profitable.

What we mean by unverifiability is that the outside investor cannot *legally* verify that such an observation or prediction coincides with the actual q^I (or π^E). Although this could be taken as an evidence in antitrust lawsuits, here we consider lawsuits to enforce the loan contract. To enforce repayment, the court needs to know whether the entrant actually has enough money to repay the loan. Furthermore, because the incumbent is a competitor against the entrant in the product market and a third party that cannot be bound by the entrant's loan contract, it is hard to expect that the incumbent would be willing to provide a verifiable evidence of the actual q^I to the entrant's lenders. According to our proposition, it would help the entrant finance its costs. So, the incumbent has an incentive to not release the information.

Financial contract $\mathcal{C}^*$ is designed to assure repayment without relying on the court for enforcement. Unverifiability prevents the court from enforcing full repayment of the loan.

Instead, the lender has to give the entrant an incentive to voluntarily repay the whole loan by using liquidation of collateral as a threat.

We do not insist that q^I or π^E is always unverifiable. Our propositions rather suggest that an entrant should make them verifiable for the sake of better financing. For example, in a Japanese "main bank system" (Hoshi et al., 1991), a borrower has its business activity monitored by "main banks" by using an account with the bank to execute all transactions and inviting a banker to be an accounting director.[18] This guarantees verifiability of the borrower's liquidity holding and enables the lender to enforce repayment of the whole loan.[19]

Equity versus debt financing

In our model, the "entrant" receives all remaining profits and assets after his loan repayments, while he devotes all of its start-up capital to the business. Therefore, the equity investors may be better regarded as parts of the "entrant," not as the outside investor in our model.

Lerner (1995) studies the disk drive industry from 1980 to 1988, seeing changes in equity financing as shocks to the entrant's financial strength. He tests whether price wars were triggered by entries of financially weak rivals.[20] Between 1980 and 1983, a venture company was able to easily raise start-up capital with equity finance. In this era of "capital market myopia," prices were wholly determined by the products' attributes, independent of the financial weakness of the entrants. In the period 1984–88, entrepreneurs suddenly faced difficulty in securing equity financing. Then, prices were significantly lower in the presence of financially weak rivals.

This empirical result is comparable with our propositions. In the early 1980s, "capital market myopia" enabled the entrants to raise enough start-up capital w_0. Thus, they satisfied the non-predation condition and could avoid predation. In the late 80s, the difficulty of securing equity financing forced the entrants to enter the industry with much less start-up capital. Hence, they could not obtain sufficient precautionary liquidity and, as a result, the incumbents were more aggressive.

6 Concluding remarks

We find that threat of predation creates the demand for excess precautionary liquidity that is not spent in equilibrium. This is consistent with empirical findings that generally report posi-

[18]Marquez (2010) argues that, provided that a bank can tell not only the realized profit but also fundamentals and potential profitability of the borrower's business, a bank loan can prevent predation more effectively than public debt financing.

[19]It would be easier for trade creditors to monitor the borrower's business and gather verifiable information as well as to enforce the repayment with threat of terminating supply (Petersen and Rajan, 1997). From Japanese database on small businesses, Tsuruta (2008) finds that trade credit lowers the interest rate of bank loans, possibly because trade creditors have good monitoring ability and weaken banks' informational advantage. But close monitoring is costly and thus may not be utilized for small and new customers who have yet made long-run relationship with suppliers. So, in a start-up stage of a small business where trade creditors rely on the credit score, trade credits may not significantly alleviate the informational problem.

[20]Lerner (1995) uses two criteria to identify a financially weak firm. First, the firm should specialize in disk drive manufacturing, which means the absence of internal financing from other business. Second, the firm's equity capital should be below the median of all samples. These are consistent with our definition of the "entrant with little start-up capital", as we discuss in this section.

tive relationship between competitive pressure and cash holding (Hoberg et al., 2014; Fresard, 2010). The need to raise precautionary liquidity by means of a long-term loan adds an extra marginal cost to the entrant's production, and thus makes the entrant less aggressive. Furthermore, we prove that if the incumbent's strategy (and thus the entrant's actual profit) is unverifiable, the entrant faces a restricted supply of excess liquidity and has to shrink its business further. However, close monitoring of the entrant's business by banks or trade creditors may alleviate this informational problem. Thus, empirical research must account for endogeneity between loan supply and liquidity demand under competitive pressure and pay attention to the entrant's relationship with banks and trade creditors.

Unlike the preceding models à la Bolton and Scharfstein (1990), the degree of predation is endogeneously determined as excess capacity in our model. All the key variables, such as the maximal predatory capacity, the maximal predatory loss, and the equilibrium capacity levels, are quantitatively identified from demand and cost structure of the production market, as visualized in Figure 1. This may be appealing to empirical researchers, as they do not have to find external conducts to damage rivals' business (e.g., excessive advertisement). Also, our model can be extended to a macroeconomic model to see how monetary policy affects entrepreneurship. Once we accept the idea of anti-predatory financial contract that the entrant has to raise precautionary liquidity to cover the maximal predatory loss, our quantitative analysis of product markets is in line with traditional price theory that focuses on the demand and cost structure. This would be more acceptable for practitioners in antitrust law and regulations, who may be still reluctant to apply game theoretic views on predation and adhere to traditional reasoning based on price theory such as Areeda-Turner rule.[21]

We have been looking at non-predatory equilibrium in which the entrant eventually enters the market even if it has small initial capital, possibly by taking a less aggressive strategy. One major criticism on preceding game theoretic analysis of predation is that predation is not frequently observed in reality (as far as critiques argue): see Elzinga and Mills (2001) for such criticism. Our focus on non-predatory equilibrium can defend against this criticism, though it is also possible to extend the model by adding a fixed cost to have the entrant give up the entry if it has to shrink capacity too much.[22] But our model points out that threat of predation itself generates distortion in the product market outcome even when the entrant actually enters the market. Our theory suggests the importance of preventive measures to reduce threat of predation (e.g. by easing equity finance for new enterprises).

Our model does not distinguish the forms of financing precautionary liquidity; it can be cash holding or credit line. But it may matter when we ask how to make the lender's commitment *observable and credible* to a rival firm. Cash or liquidity holding would be relatively easy

[21] Giocoli (2013) and Markovits (2016) document that the U.S. antitrust courts are still skeptical of predatory pricing claims. Elzinga and Mills (2001) pose negative responses to apply game theoretic views on predation and indeed strategic analysis to legal judgment.

[22] Even in our model, predation might happen after the entrant enters the market, if the entrant underestimates the maximum predation loss and thus did not raise enough liquidity or if the entrant just did not rationally predict the possibility that the incumbent may react to the entry by excessive predatory conduct. In the argument against the Areeda-Turner rule, Comanor and Frech (2015) raise a question on high order belief of rationality behind subgame perfection, citing experimental studies on Selten's chain-store paradox. Actually, when we justify our formulation of the anti-predation financial contract as the outcome of a sequentual equilibrium by revelation principle as in Zusai (2022), the entrant's choice is perturbed to pin down the belief on off-equilibrium path.

to observe. Basic balance sheet information may be disclosed formally and publicly in a large industry. It may be disclosed efficiently only to banks and suppliers, but such information can be shared to rival firms from mouth to mouth in a small local business community.

On the other hand, if the incumbent cannot know the entrant's cash holding, excess liquidity may not work effectively to deter predation. In such situations, the entrant may show off its financial strength in a costly way such as building large facilities or placing extensive advertisement. To shed light on the informational roles of cash holding in product market competition, it would be an interesting extension of our model to incorporate imperfect observation of the entrant's liquidity holding.

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A The benchmark optimal financial contract

The optimal contract is a solution of the following constrained maximization problem:

$$\max_{q^E,B,Q^I_S,D,\beta} \quad \Pi^E_\ddagger := (1-\mathbb{P}^I(Q^I_S))\{B+\overline{V}-\delta(m_0)\} + \int_{q^I\in Q^I_S}\{\pi^E(q^E,q^I)+B+\overline{V}-\delta(q^I)\}\mathbb{P}^I(dq^I)$$

$$\text{s.t.} \quad D(\tilde{q}^I)+\beta(\tilde{q}^I)\overline{V} =: \delta(\tilde{q}_0) = \underline{\delta} \quad \text{for any } \tilde{q}^I \in Q^I_S, \tag{IC$^S_\ddagger$}$$

$$\pi^E(q^E,q^I)-\underline{\delta} \le \delta(m_0) \quad \text{for any } \tilde{q}^I \in Q^I_S, \tag{IC$^{S0}_\ddagger$}$$

$$D(m_0)+\beta(m_0)\underline{V} =: \delta(m_0) \le \pi^E(q^E,q^I)-\underline{\delta} \quad \text{for any } \tilde{q}^I \notin Q^I_S, \tag{IC$^{0S}_\ddagger$}$$

$$D(\tilde{q}^I) \le \pi^E(q^E,\tilde{q}^I)+B \quad \text{for any } \tilde{q}^I \in Q^I_S, \tag{LL$^S_\ddagger$}$$

$$D(m_0) \le B, \tag{LL$^0_\ddagger$}$$

$$(1-\mathbb{P}^I(Q^I_S))\{D(m_0)+\beta(m_0)\underline{V}\} + \int_{q^I\in Q^I_S}\{D(\tilde{q}^I)+\beta(\tilde{q}^I)\underline{V}\}\mathbb{P}^I(dq^I) \ge B - w_0. \tag{PC$^L_\ddagger$}$$

The first three Incentive Compatibility constraints prevent the entrant from false announcement of q^I. The next two Limited Liability constraints restrict the monetary repayment to the liquidity holding inferred from the announcement. The last Participation Constraint guarantees a non-negative net surplus for the lender. We assign the full bargaining power to the entrant so the optimal contract maximizes the entrant's net profit while meeting all these constraints.

Now we solve this optimal financial contract problem. First, notice that, combined with (IC$^S_\ddagger$), (PC$^L_\ddagger$) is rewritten as

$$\begin{aligned}\mathbb{E}^I\delta &:= \mathbb{P}^I(Q^I_S)\underline{\delta} + (1-\mathbb{P}^I(Q^I_S))\underline{\delta}_0 \\ &\ge B - w_0 + \left[\int_{q^I\in Q^I_S}\beta(q^I)\mathbb{P}^I(dq^I) + (1-\mathbb{P}^I(Q^I_S))\beta(m_0)\right](\overline{V}-\underline{V}).\end{aligned} \tag{13}$$

The objective function is also rewritten as

$$\Pi^E_\ddagger = B + \overline{V} + \int_{q^I\in Q^I_S}\pi^E(q^E_\ddagger,q^I)\mathbb{P}^I(dq^I) - \mathbb{E}^I\delta.$$

We first find the "first-best" solution, ignoring the other constraints for a while; then, we verify that it indeed satisfies these constraints. To maximize $\Pi^E_\ddagger$ given Q^I_S, the expected total payment $\mathbb{E}^I\delta$ must be minimized while meeting (13). So the inequality constraint (13) must be binding at the first-best solution. Further, the RHS of (13) is minimized by having $\beta \equiv 0$, i.e.,

$$\beta(\tilde{q}^I) = 0 \text{ for all } \tilde{q}^I \in Q^I_S \qquad \text{and} \qquad \beta(m_0) = 0.$$

Thus, the expected total payment is $\mathbb{E}^I\delta = B - w_0$ at the first-best solution. The objective function now reduces to

$$\Pi^E_\ddagger = w_0 + \overline{V} + \int_{q^I\in Q^I_S}\pi^E(q^E_\ddagger,q^I)\mathbb{P}^I(dq^I).$$

This increases with the expected operating profit $\int_{q^I \in Q^I_S} \pi^E(q^E_\ddagger, q^I)\mathbb{P}^I(dq^I)$. With $q^E_\ddagger$ fixed, this is maximized by setting Q^I_S to

$$Q^I_S = \{q^I \in Q^I \mid \pi^E(q^E_\ddagger, q^I) \geq 0\}.$$

Finally, $q^E_\ddagger$ should be the solution of

$$\max_{q^E \in Q^E} \int_{q^I \in Q^I_S} \pi^E(q^E, q^I)\mathbb{P}^I(dq^I) = \int_{q^I \in Q^I} \max\{\pi^E(q^E, q^I), 0\}\mathbb{P}^I(dq^I).$$

The above characterized first-best solution indeed satisfies the remaining constraints with $D \equiv B - w_0$, i.e.,

$$D(\tilde{q}^I) = B - w_0 \text{ for all } \tilde{q}^I \in Q^I_S \qquad \text{and} \qquad D(m_0) = B - w_0.$$

It is immediate to see ($\text{LL}^0_\ddagger$) holds, i.e., $D(m_0) = B - w_0 \leq B$ by $w_0 \geq 0$. As $\tilde{q}^I \in Q^I_S$ implies $\pi^E(q^E_\ddagger, \tilde{q}^I) \geq 0$, we have $D(\tilde{q}^I) = B - w_0 \leq B + \pi^E(q^E_\ddagger, \tilde{q}^I)$; thus ($\text{LL}^S_\ddagger$) holds. Notice that, with $\beta \equiv 0$ and $D \equiv B - w_0$, we have $\delta(m_0) = B - w_0$ and ($\text{IC}^S_\ddagger$) is satisfied with $\underline{\delta} = B - w_0$. Then, ($\text{IC}^{0S}_\ddagger$) and ($\text{IC}^{S0}_\ddagger$) are satisfied with the above characterized Q^I_S. Therefore, the first-best solution is implementable with the benchmark financial contract. Note that any B satisfies all the constraints.

B Existence of equilibrium

B.1 Existence of the entrant's optimal capacity in continuous strategy space

We prove the existence of the entrant's optimal capacity for the case that the non-predation condition (NP_*) is effective, assuming continuity and a few additional properties. It is proven from basic topological theorems in Sydsæter et al. (2008), which we abbreviate by SH.

Assumption 4. Let $Q^E = Q^I = \mathbb{R}_+$ and π^I and π^E be continuous functions. Further, assume that (i) there exists $\bar{\bar{q}}^I$ such that $\pi^I(q^I, 0) < 0$ for any $q^I > \bar{\bar{q}}^I$ and (ii) for any $q^I_0 \in \int Q^I = (0, +\infty)$ there exists q^I_1 in its arbitrary small neighborhood such that $\pi^I(q^I_1, 0) < \pi^I(q^I_0, 0)$.

Theorem 3. *Suppose that Assumptions 1, 2 and 4 hold. Then, there exists an optimum in the constrained maximization problem* (7a) *even if the non-predation condition* (NP_*) *does not hold at* $q^E_\ddagger$.

Proof. First we confirm continuity of $\tilde{\pi}^E$. With parts iii) and iv) of Assumption 1, Assumption 4-i) implies that the domain of q^I in the maximization of $\pi^I(q^I, q^E)$ can be truncated to $[0, \bar{\bar{q}}^I]$. As it is compact and π^I is continuous, Assumption 1-i) implies that q^I_{BR} is a continuous function of q^E (SH, Theorem 13.4.1). With continuity of π^E, this implies continuity of $\tilde{\pi}^E$.

Now we prove continuity of $\bar{L}_P$. Continuity of π^I is succeeded to $\pi^I(q^I_{\text{BR}}(q^E), q^E)$ as a function of q^E. With continuity of $\pi^I(q^I, 0)$ in q^I, this implies that $Q^I_P(q^E) = \{q^I \in Q^I \mid \pi^I(q^I, 0) \geq \pi^I(q^I_{\text{BR}}(q^E), q^E)\}$ is a closed set. It is bounded by Assumption 4-i) and Assumption 1-i,iii), since $\pi^I(q^I_{\text{BR}}(q^E), q^E) > \pi^I(0, q^E) = 0 > \pi^I(q^I, 0)$ for any $q^I > \bar{\bar{q}}^I$ and thus $Q^I_P(q^E) \subset [0, \bar{\bar{q}}^I]$. Therefore, $Q^I_P : Q^E \rightrightarrows Q^I$ has a compact graph and thus it is an upper hemicontinuous correspondence.

Furthermore, for any $q^I_\infty \in Q^I_P(q^E)$, continuity of π^I and Assumption 4-ii) guarantee the existence of a sequence $\{q^I_t\}_{t\in\mathbb{N}}$ such that $q^I_t \to q^I_\infty$ as $t \to \infty$ and $\pi^I(q^I_t, 0) > \pi^I(q^I_{\mathrm{BR}}(q^E), q^E)$. That is, $Q^I_P(q^E)$ coincides with the closure of set $\{q^I \in Q^I \mid \pi^I(q^I, 0) > \pi^I(q^I_{\mathrm{BR}}(q^E), q^E)\}$. It implies that Q^I_P is a lower hemicontinuous correspondence (SH, p.506). Thus, it is a continuous correspondence.

Finally $Q^I_P(q^E)$ is nonempty for any q^E, since $\pi^I(q^I_{\mathrm{BR}}(0), 0) > \pi^I(q^I_{\mathrm{BR}}(q^E), 0) \geq \pi^I(q^I_{\mathrm{BR}}(q^E), q^E)$ by Assumption 1-i,ii) and thus $q^I_{\mathrm{BR}}(0) \in Q^I_P(q^E)$.

With continuity of $\pi^E(\cdot, q^I)$, these properties of Q^I_P guarantees well-definedness (the supremum being attained as the maximum) and continuity of $\bar{L}^P(q^E)$ by the maximum theorem (SH, Theorem 14.2.1)

Since the admissible set in maximization problem (7a) is the preimage of a closed set $(-\infty, \overline{V} + w_0]$ under continuous function $\bar{L}^P$, it is closed. Recall that we consider the case that the non-predatory condition (NP$_*$) does not hold at $q^E_\dagger$. Parts iii)–v) of Lemma 1 imply $\bar{L}^P(q^E) \geq \bar{L}^P(q^E_\dagger)$ for any $q^E \geq q^E_\dagger$. That is, the admissible set must be contained in the range $[0, q^E_\dagger]$, and thus it is bounded. As long as $\overline{V} + w_0 \geq 0$, $q^E = 0$ is admissible since $\bar{L}_P(0) = \sup\{-\pi^E(0, q^I) \mid q^I \in Q^I_P(0)\} = 0 \leq \bar{V} + w_0$ by Assumption 1-iii); thus, the admissible set is not empty.

With continuity of $\tilde{q}^E$, non-emptiness and compactness of the admissible set imply the existence of a maximum in (7a) by Weierstrass extreme value theorem. □

B.2 Existence of a sequential equilibrium in the finite case

If Q is a finite set, the existence of the entrant's optimal capacity in (7a) is immediately guaranteed. Further, as argued in footnote 13, here we verify the existence of a sequential equilibrium in the game after the optimal debt contract $\mathcal{C}_*$ is accepted. Actually, to apply the conventional definition of a sequential equilibrium in a finite game (e.g.), the strategy space Q should be finite. Extension of the concept to a continuous strategy space requires subtle treatment of topology embedded to the strategy space; see Myerson and Reny (2015)

Theorem 4. *Suppose that Assumptions 1 and 2 hold. Further, assume that Q^I and Q^E are finite subsets of $\mathbb{R}_+$. In this case, capacity profile $\mathbf{q}_*$, the solution of (7), is implemented as a sequential equilibrium in a game after contract $\mathcal{C}_*$ is accepted.*

Proof. Notice that $Q^I_P(q^E_*) \subset Q^I_S = \{q^I \in Q^I \mid \pi^E(q^E_*, \tilde{q}^I) + \overline{V} + w_0 \geq 0\}$, since q^E_* is the solution of (7a) and thus satisfies $-\pi^E(q^E_*, q^I) \leq \bar{L}_P(q^E_*) \leq \overline{V} + w_0$ as long as $q^I \in Q^I_P(q^E_*)$

Let the entrant's messaging strategy σ^M_* be

$$
\begin{aligned}
\sigma^M_*(q^I | q^I) &= 1 && \text{if } q^I \in Q^I_S, \\
\sigma^M_*(m_0 | q^I) &= 1 && \text{if } q^I \notin Q^I_S, \\
\sigma^M_*(\tilde{q}^I | q^I) &= 0 && \text{for any } q^I \in Q^I, \tilde{q}^I \in Q^I_S \setminus \{q^I\}.
\end{aligned}
$$

Here, for each $m \in M$ and $q^I \in Q^I$, $\sigma^M_*(m | q^I)$ is the probability that the entrant announces message m after observing q^I.

The (pure-strategy) capacity size profile $\mathbf{q}_*$ as in (7) and the messaging strategy σ^M_* constitute a sequential equilibrium with the following belief μ_* in the subgame after the entrant and the investor agree on contract $\mathcal{C}_*$: for each $q^I \in Q^I$, the belief μ_* is

$$\mu_*(q^I|\tilde{q}^I) = I(q^I, \tilde{q}^I) \quad \text{for each } \tilde{q}^I \in Q^I_S,$$

$$\mu_*(q^I|m_0) = (1 - I_S(q^I))/(\#Q^I - \#Q^I_S).$$

Here $I(q^I_1, q^I_2)$ is the indicator function for $q^I_1 = q^I_2$ and $I_S(q^I)$ is the one for $q^I \in Q^I_S$. Receiving message m, the investor believes that the incumbent has chosen q^I with probability $\mu_*(q^I|m)$.

Belief μ_* is consistent with a sequence of perturbed strategy profiles $\{\sigma_k\}_{k\in\mathbb{N}}$ such as

$$\begin{aligned}
\sigma^I_k(q^I) &:= \frac{1}{\sqrt{k}\#Q^I} + \left(1 - \frac{1}{\sqrt{k}}\right) I(q^I, q^I_*) && \text{for each } q^I \in Q^I;\\
\sigma^M_k(\tilde{q}^I|q^I) &:= \frac{1}{2k\#Q^I_S} + \left(1 - \frac{1}{k}\right) I(q^I, \tilde{q}^I) && \text{for each } q^I \in Q^I, \tilde{q}^I \in Q^I_S;\\
\sigma^M_k(m_0|q^I) &:= \frac{1}{2k} I_S(q^I) + \left(1 - \frac{1}{2k}\right)(1 - I_S(q^I)) && \text{for each } q^I \in Q^I.
\end{aligned}$$

Here, for each $q^I \in Q^I$, $\sigma^I_k(q^I)$ is the probability that the incumbent chooses q^I under the perturbed strategy profile σ_k.

Strategy profile σ_k induces Bayesian belief μ_k as follows. If $\tilde{q}^I \in Q^I_S \setminus \{q^I_*\}$, belief $\mu_k(\cdot|\tilde{q}^I)$ is given by

$$\begin{aligned}
&\mu_k(q^I|\tilde{q}^I)\\
&:= \frac{\sigma^I_k(q^I)\sigma^M_k(\tilde{q}^I|q^I)}{\sum_{\hat{q}^I \notin \{\tilde{q}^I, q^I_*\}} \sigma^I_k(\hat{q}^I)\sigma^M_k(\tilde{q}^I|\hat{q}^I) + \sigma^I_k(\tilde{q}^I)\sigma^M_k(\tilde{q}^I|\tilde{q}^I) + \sigma^I_k(q^I_*)\sigma^M_k(q^I_*|\tilde{q}^I)}\\
&= \frac{(\sqrt{k}\#Q^I)^{-1}(2k\#Q^I_S)^{-1}}{\frac{\#Q^I - 2}{\sqrt{k}\#Q^I \cdot 2k\#Q^I_S} + \frac{1}{\sqrt{k}\#Q^I}\left(\frac{1}{2k\#Q^I_S} + 1 - \frac{1}{k}\right) + \left(\frac{1}{\sqrt{k}\#Q^I} + 1 - \frac{1}{\sqrt{k}}\right)\frac{1}{2k\#Q^I_S}}\\
&= \left[\#Q^I - 2 + \left\{1 + 2(k-1)\#Q^I_S\right\} + \left\{1 + (\sqrt{k} - 1)\#Q^I\right\}\right]^{-1}\\
&= \left[\sqrt{k}\#Q^I + 2(k-1)\#Q^I_S\right]^{-1} \quad \text{for each } q^I \notin \{\tilde{q}^I, q^I_*\},\\
&\mu_k(\tilde{q}^I|\tilde{q}^I) = 1 - \sum_{q^I \neq \tilde{q}^I} \mu_k(q^I|\tilde{q}^I) = 1 - (\sqrt{k}\#Q^I - 1)\left[\sqrt{k}\#Q^I + 2(k-1)\#Q^I_S\right]^{-1},\\
&\mu_k(q^I_*|\tilde{q}^I) = \frac{\sigma^I_k(q^I_*)}{\sigma^I_k(q^I)}\mu_k(q^I|\tilde{q}^I) \qquad (\text{with any } q^I \notin \{\tilde{q}^I, q^I_*\}, \text{by } \sigma^M_k(\tilde{q}^I|q^I_*) = \sigma^M_k(\tilde{q}^I|q^I))\\
&= \left\{1 + (\sqrt{k} - 1)\#Q^I\right\}\left[\sqrt{k}\#Q^I + 2(k-1)\#Q^I_S\right]^{-1}.
\end{aligned}$$

$\mu_k(\cdot|q^I_*)$ is given by

$$\begin{aligned}
&\mu_k(q^I|q^I_*),\\
&:= \frac{\sigma^I_k(q^I)\sigma^M_k(\tilde{q}^I|q^I)}{\sum_{\hat{q}^I \neq q^I_*} \sigma^I_k(\hat{q}^I)\sigma^M_k(\tilde{q}^I|\hat{q}^I) + \sigma^I_k(\tilde{q}^I)\sigma^M_k(\tilde{q}^I|\tilde{q}^I)}\\
&= \frac{(\sqrt{k}\#Q^I)^{-1}(2k\#Q^I_S)^{-1}}{\frac{\#Q^I - 1}{\sqrt{k}\#Q^I \cdot 2k\#Q^I_S} + \left(\frac{1}{\sqrt{k}\#Q^I} + 1 - \frac{1}{\sqrt{k}}\right)\left(\frac{1}{2k\#Q^I_S} + 1 - \frac{1}{k}\right)}
\end{aligned}$$

$$
\begin{aligned}
&= \left[\#Q^I - 1 + \left\{1 + (\sqrt{k}-1)\#Q^I\right\}\left\{1 + 2(k-1)\#Q^I_S\right\}\right]^{-1} \\
&= \left[\sqrt{k}\#Q^I + 2(k-1)\#Q^I_S\left\{1 + (\sqrt{k}-1)\#Q^I\right\}\right]^{-1} \qquad \text{for each } q^I \neq q^I_*, \\
\mu_k(q^I_*|q^I_*) &= 1 - \sum_{q^I \neq q^I_*} \mu_k(q^I|q^I_*) \\
&= 1 - (\#Q^I - 1)\left[\sqrt{k}\#Q^I + 2(k-1)\#Q^I_S\left\{1 + (\sqrt{k}-1)\#Q^I\right\}\right]^{-1}.
\end{aligned}
$$

$\mu_k(\cdot|m_0)$ is given by

$$
\begin{aligned}
&\mu_k(q^I|m_0) \\
&= \sigma^I_k(q^I)\sigma^M_k(m_0|q^I) \\
&\times \left[\sigma^I_k(q^I_*)\sigma^M_k(m_0|q^I_*) + \sum_{\hat{q}^I \in Q^I_S \setminus \{q^I_*\}} \sigma^I_k(\hat{q}^I)\sigma^M_k(m_0|\hat{q}^I) + \sum_{\hat{q}^I \notin Q^I_S} \sigma^I_k(\hat{q}^I)\sigma^M_k(m_0|\hat{q}^I)\right]^{-1} \\
&= \frac{(\sqrt{k}\#Q^I)^{-1}\left(1 - \frac{1}{2k}\right)}{\left(\frac{1}{\sqrt{k}\#Q^I} + 1 - \frac{1}{\sqrt{k}}\right)\frac{1}{2k} + \frac{\#Q^I_S - 1}{\sqrt{k}\#Q^I \cdot 2k} + \frac{\#Q^I - \#Q^I_S}{\sqrt{k}\#Q^I}\left(1 - \frac{1}{2k}\right)} \\
&= \left[\frac{\left\{1 + (\sqrt{k}-1)\#Q^I\right\} + (\#Q^I_S - 1)}{2k-1} + \#Q^I - \#Q^I_S\right]^{-1} \\
&= \left[\frac{2k + \sqrt{k} - 2}{2k-1}\#Q^I - \frac{2k-2}{2k-1}\#Q^I_S\right]^{-1} \qquad \text{for each } q^I \notin Q^I_S, \\
&\mu_k(q^I|m_0) = \frac{\sigma^M_k(m_0|q^I)}{\sigma^M_k(m_0|\hat{q}^I)}\mu_k(\hat{q}^I|m_0) \qquad (\text{with any } \hat{q}^I \notin Q^I_S, \text{by } \sigma^I_k(q^I) = \sigma^I_k(\hat{q}^I)) \\
&= \frac{1/2k}{(1 - 1/2k)}\left[\frac{2k + \sqrt{k} - 2}{2k-1}\#Q^I - \frac{2k-2}{2k-1}\#Q^I_S\right]^{-1} \\
&= \left[(2k + \sqrt{k} - 2)\#Q^I - (2k-2)\#Q^I_S\right]^{-1} \qquad \text{for each } q^I \in Q^I_S \setminus \{q^I_*\}, \\
&\mu_k(q^I_*|m_0) = \frac{\sigma^I_k(q^I_*)}{\sigma^I_k(\hat{q}^I)}\mu_k(q^I|m_0) \qquad (\text{with any } \hat{q}^I \in Q^I_S \setminus \{q^I_*\}) \\
&= \frac{(\sqrt{k}\#Q^I)^{-1} + 1 - 2k^{-1}}{(\sqrt{k}\#Q^I)^{-1}}\left[(2k + \sqrt{k} - 2)\#Q^I - (2k-2)\#Q^I_S\right]^{-1} \\
&= \{1 + (\sqrt{k}-1)\#Q^I\}\left[(2k + \sqrt{k} - 2)\#Q^I - (2k-2)\#Q^I_S\right]^{-1}.
\end{aligned}
$$

Take the limits of these μ_k's as $k \to \infty$. For each $\tilde{q}^I \in Q^I_S \setminus \{q^I_*\}$, $\mu_k(\tilde{q}^I|\tilde{q}^I)$ converges to 1. $\mu_k(q^I_*|q^I_*)$ converges to 1. $\mu_k(q^I|m_0)$ converges to $(\#Q^I - \#Q^I_S)^{-1}$ for each $q^I \notin Q^I_S$. Therefore, $\mu_k \to \mu_*$.

Now we check sequential rationality. Any message $\tilde{q}^I \in Q^I_S$ implies $\mu_*(\tilde{q}^I|\tilde{q}^I) = 1$ and $\pi^E(q^E_*, \tilde{q}^I) + \overline{V} + w_0 \geq 0$. The total repayment followed by any such message $\tilde{q}^I \in Q^I_S$ is constant, i.e., $\delta(\tilde{q}^I) = \overline{V}$, which yields net profit $\pi^E(q^E_*, q^I) + B + \overline{V} - \delta(\tilde{q}^I) = \pi^E(q^E_*, q^I) + \overline{V} + w_0$ for the entrant at the end of period 4. In contrast, message m_0 yields the total repayment $\delta(m_0) = B + \overline{V}$ and thus the net profit $B + \overline{V} - \delta(m_0) = 0$. Truthful announcement $\sigma^M_*(q^I|q^I) =$

1 is the entrant's optimal messaging strategy, as long as $\pi^E(q^E_*, q^I) + \overline{V} + w_0 \geq 0$, i.e., $q^I \in Q^I_S$. Otherwise, the entrant chooses m_0 to exit.

In period 1, the incumbent could get the entrant to exit by setting q^I such that $\pi^E(q^E, q^I) + B < 0$, i.e., $q^I \notin Q^I_S$. Such $q^I \notin Q^I_S$ yields the predatory profit $\pi^I(q^I, 0)$, which is smaller than $\pi^I(\mathbf{q}_*)$ by $Q^I_P(q^E_*) \subset Q^I_S$. On the other hand, $q^I \in Q^I_S$ yields the duopoly profit $\pi^I(q^I, q^E_*)$, which is maximized at $q^I = q^I_*$. So q^I_* is the optimal output choice for the incumbent. $\square$

C Proof of Theorem 2 and Proposition 1

The theorems are obtained from the combination of the following claims.

Lemma 1. *Assume Assumptions 1, 2 and 3.*

i) $\pi^I(q^I_{BR}(q^E), q^E)$ *strictly decreases with* q^E.

ii) $\bar{q}^I_P(q^E)$ *strictly increases with* q^E *and* $\bar{q}^I_P(0) = q^I_{BR}(0)$.

iii) There is a unique capacity size $\underline{q}^E$ *such that*

$$q^E_{BR}(\bar{q}^I_P(q^E)) \begin{cases} > q^E & \text{if } q^E < \underline{q}^E, \\ \leq q^E & \text{if } q^E \geq \underline{q}^E. \end{cases}$$

Further, $\bar{L}_P(q^E)$ *strictly increases with* q^E *as long as* $q^E \geq \underline{q}^E$.

iv) Let $\mathbf{q}_C$ *be the (benchmark) Cournot equilibrium capacity profile in the sense that* $q^i_{BR}(q^j_C) = q^i_C$ *for each* $i \in \{E, I\}, j \neq i$. *Then,* $q^E_C > \underline{q}^E$.

v) In the benchmark Stackelberg equilibrium capacity profile $\mathbf{q}_\dagger$ *as defined in* (1), *we have* $q^E_\dagger > q^E_C$.

Proof of Lemma 1. Fix q^E_0 and q^E_1 such that $q^E_0 < q^E_1$ arbitrarily from Q^E.
Part i. Assumption 1-ii) implies

$$\pi^I(q^I_{\text{BR}}(q^E_1), q^E_1) \leq \pi^I(q^I_{\text{BR}}(q^E_1), q^E_0),$$

since $q^E_0 < q^E_1$. Further, Assumption 1-i) implies

$$\pi^I(q^I_{\text{BR}}(q^E_1), q^E_0) < \pi^I(q^I_{\text{BR}}(q^E_0), q^E_0).$$

Combining these, we obtain

$$q^E_0 < q^E_1 \qquad \Longrightarrow \qquad \pi^I(q^I_{\text{BR}}(q^E_1), q^E_1) < \pi^I(q^I_{\text{BR}}(q^E_0), q^E_0).$$

Part ii. Consider arbitrary $q^I \in Q^I_P(q^E_0)$; it means $\pi^I(q^I, 0) \geq \pi^I(q^I_{\text{BR}}(q^E_0), q^E_0)$. Then, by part i), we have $\pi^I(q^I, 0) > \pi^I(q^I_{\text{BR}}(q^E_0), q^E_0) > \pi^I(q^I_{\text{BR}}(q^E_1), q^E_1)$, which means $q^I \in Q^I_P(q^E_1)$. Thus,

$$q^E_0 < q^E_1 \qquad \Longrightarrow \qquad Q^I_P(q^E_0) \subset Q^I_P(q^E_1).$$

In particular, under Assumption 3, there exists $q_2^I \in Q^I$ such that $q_2^I > \bar{q}_P^I(q_0^E)$ and $\pi^I(q_{\rm BR}^I(q_1^E), q_1^E) < \pi^I(q_2^I, 0) < \pi^I(q_{\rm BR}^I(q_0^E), q_0^E)$. The latter inequality implies $q_2^I \notin Q_P^I(q_0^E)$; with the former, the definition of $\bar{q}_P^I(\cdot)$ implies $\bar{q}_P^I(q_1^E) \geq q_2^I > \bar{q}_P^I(q_0^E)$. When the entrant takes $q^E = 0$, Assumption 1-i) implies $Q_P^I(0) = \{q_{\rm BR}^I(0)\}$ and thus $\bar{q}_P^I(0) = q_{\rm BR}^I(0)$.

Part iii. Let the entrant take $\bar{q}^E := q_{\rm BR}^E(0)$. To prove $\bar{q}^E > 0$ by contradiction, now suppose $\bar{q}^E = 0$. Assumption 1-iii) implies $\pi^E(\bar{q}^E, 0) = 0$. Assumption 1-ii) implies, $\pi^E(\mathbf{q}_\dagger) < \pi^E(q_\dagger^E, 0) \leq \pi^E(q_{\rm BR}^E(0), 0) = \pi^E(\bar{q}^E, 0) = 0$; the latter weak inequality comes from the definition of $q_{\rm BR}^E$. But this contradict with Assumption 2. Hence we have $\bar{q}^E > 0$.

With $\bar{q}^E > 0$, part ii) of the current lemma implies $\bar{q}_P^I(\bar{q}^E) > \bar{q}_P^I(0) \geq 0$. By Assumption 1-iv), this further implies $\bar{q}^E = q_{\rm BR}^E(0) \geq q_{\rm BR}^E(\bar{q}_P^I(0)) > q_{\rm BR}^E(\bar{q}_P^I(\bar{q}^E))$.

As $q_{\rm BR}^E$ is a non-increasing function by Assumption 1-iv) and $\bar{q}_P^I$ is an increasing function by part ii), the composite $q_{\rm BR}^E(\bar{q}_P^I(\cdot))$ is a non-increasing function. Therefore, there is a unique $\underline{q}^E$ such that

$$q_{\rm BR}^E(\bar{q}_P^I(q^E)) \begin{cases} > q^E & \text{if } q^E < \underline{q}^E, \\ \leq q^E & \text{if } q^E \geq \underline{q}^E. \end{cases}$$

Now assume that $q_0^E \geq \underline{q}^E$. Then, we have $q_0^E \geq q_{\rm BR}^E(\bar{q}_P^I(q_0^E))$. Further, if $q_1^E > q_0^E$, then $q_1^E > q_0^E > q_{\rm BR}^E(\bar{q}_P^I(q_0^E))$ and thus

$$\pi^E(q_1^E, \bar{q}_P^I(q_0^E)) < \pi^E(q_0^E, \bar{q}_P^I(q_0^E)) = -\bar{L}_P(q_0^E)$$

by Assumption 1-v). Since $\bar{q}_P^I(q_0^E) < \bar{q}_P^I(q_1^E)$ by part ii), we have

$$-\bar{L}_P(q_1^E) = \pi^E(q_1^E, \bar{q}_P^I(q_1^E)) \leq \pi^E(q_1^E, \bar{q}_P^I(q_0^E))$$

by Assumption 1-ii). Combining these two equations, we obtain

$$q_1^E > q_0^E > \underline{q}^E \qquad \Longrightarrow \qquad \bar{L}_P(q_1^E) > \bar{L}_P(q_0^E).$$

Part iv. $\mathbf{q}_C$ satisfies $q_{\rm BR}^I(q_C^E) = q_C^I$. With part ii) of the current lemma, Assumption 1-iv) for $i = I$ implies $q_C^I = q_{\rm BR}^I(q_C^E) \leq q_{\rm BR}^I(0) = \bar{q}_P^I(0)$ and thus $q_C^I \leq \bar{q}_P^I(0) \leq \bar{q}_P^I(q_C^I)$ by part ii). This further implies $q_{\rm BR}^E(q_C^I) \geq q_{\rm BR}^E(\bar{q}_P^I(q_C^I))$ by Assumption 1-iv) for $i = E$. Recalling that $\mathbf{q}_C$ is a Cournot equilibrium in the above sense, we obtain $q_C^E = q_{\rm BR}^E(q_C^I) \geq q_{\rm BR}^E(\bar{q}_P^I(q_C^E))$. According to part iii), it must be the case $q_C^E \geq \underline{q}^E$.

Part v. We prove this part by contradiction; hypothetically assume that $q_C^E \geq q_\dagger^E$. From the definition of $q_{\rm BR}^E$, it immediately follows that

$$\pi^E(q_C^E, q_C^I) = \pi^E(q_{\rm BR}^E(q_C^I), q_C^I) \geq \pi^E(q_\dagger^E, q_C^I).$$

With the hypothesis $q_C^E \geq q_\dagger^E$, Assumption 1-iv) implies

$$q_C^I = q_{\rm BR}^I(q_C^E) \leq q_{\rm BR}^I(q_\dagger^E) = q_\dagger^I,$$

and further, by Assumption 1-ii),

$$\pi^E(q^E_\dagger, q^I_C) \geq \pi^E(q^E_\dagger, q^I_\dagger).$$

So we have $\pi^E(q^E_C, q^I_C) = \pi^E(q^E_C, q^I_{\text{BR}}(q^E_C)) \geq \pi^E(q^E_\dagger, q^I_\dagger) = \pi^E(q^E_\dagger, q^I_{\text{BR}}(q^E_\dagger))$. But, as $q^E_\dagger$ is supposed to be the unique maximum of $\pi^E(q^E, q^I_{\text{BR}}(q^E))$ by Assumption 2, this is a contradiction. Therefore, it must be the case $q^E_C < q^E_\dagger$. □

Proof of Theorem 2. Combining iv) and v) of the lemma, we can confirm $q^E_\dagger > \underline{q}^E$. Applying iii), we find that $\bar{L}_P(q^E)$ is increasing function of q^E in the range $q^E \geq q^E_\dagger$. □

Proof of Proposition 1. Recall that the equilibrium is characterized by (7); q^E_* is obtained as a solution of the constraint maximization problem (7a). If there were no constraint, $q^E_\dagger$ is the first best for the entrant, i.e., the global unconstrained maximizer of $\tilde{\pi}^E$. But, any $q^E \geq q^E_\dagger$ is not feasible in equilibrium under non-predation condition (NP$*$), since the assumption of Proposition 1 and Theorem 2 imply $w_0 + \overline{V} < \bar{L}_P(q^E_\dagger) \leq \bar{L}_P(q^E)$ for any $q^E \geq q^E_\dagger$. Hence, q^E_* in non-predation equilibrium must be smaller than $q^E_\dagger$ in order to meet the non-predation condition (NP$*$). □

D Two-stage product market competition

First, derive the reduced revenue functions R^E, R^I as in (8). There are four possible cases of $\mathbf{q}$ compared to $x^I_{\text{BR}}, x^E_{\text{BR}}$, as visualized in Figure 2.

Case 1: $q^E \geq x^E_0 = a/3$ and $q^I \geq x^I_0 = a/3$

Case 2: $q^E < x^E_0$ and $q^I \geq x^I_{\text{BR}}(q^E) = (a - q^E)/2$.

Case 3: $q^I < x^I_0$ and $q^E \geq x^E_{\text{BR}}(q^I) = (a - q^I)/2$.

Case 4: $q^i < x^i_0$ for some i and $q^j < x^j_{\text{BR}}(q^i)$ for the other $j \neq i$.

As in Figure 3, production levels (x^E, x^I) in the stage-2 equilibrium can be found for each case as follows.

Case 1: $x^E = x^E_0, x^I = x^I_0$.

Case 2: $x^E = q^E, x^I = x^I_{\text{BR}}(q^E)$.

Case 3: $x^I = q^I, x^E = x^E_{\text{BR}}(q^I)$.

Case 4: $x^E = q^E, x^I = q^I$.

Given q^I, the entrant's gross revenue function $R^E : \mathbb{R}^2_+ \to \mathbb{R}$ is identified as follows. See Figure 4. Replacing q^I and q^E, we obtain the formula for R^I.

- Suppose that $q^I > a/2 = x^I_{\text{BR}}(0)$.

$$R^E(q^E, q^I)$$

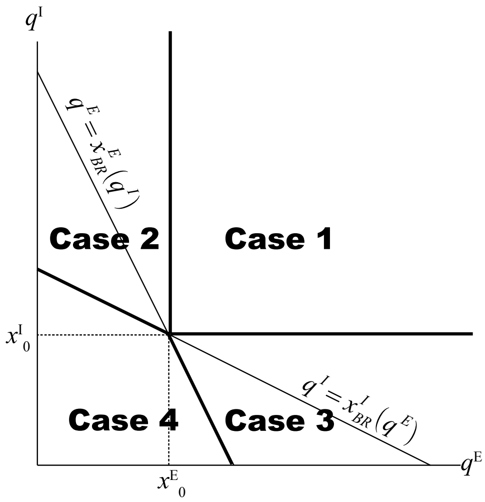


Figure 2: Cases

$$= \begin{cases} (a-q^E)q^E/2 & \text{if } q^E \le x^E_0, \quad \text{i.e., } q^E \in [0, a/3] \quad \text{(Case 2);} \\ a^2/9 & \text{if } q^I > x^E_0, \quad \text{i.e., } q^E > a/3 \quad \text{(Case 1).} \end{cases}$$

- Suppose that $q^I \in (a/3, a/2] = (x^I_0, x^I_{\text{BR}}(0)]$.

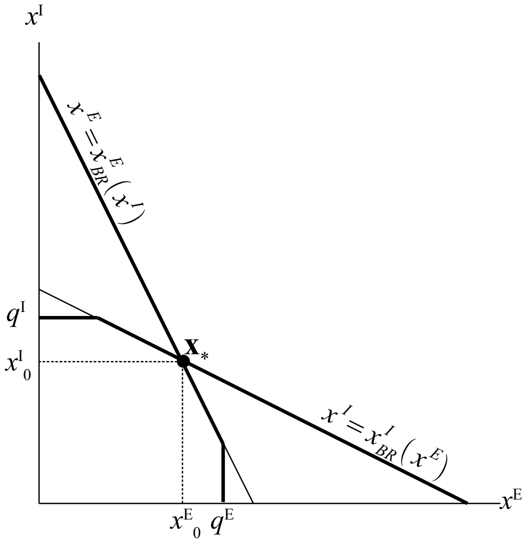
$$R^E(q^E, q^I)$$
$$= \begin{cases} (a-q^E-q^I)q^E & \text{if } q^I < x^I_{\text{BR}}(q^E), & \text{i.e., } q^E < a-2q^I & \text{(Case 4);} \\ (a-q^E)q^E/2 & \text{if } q^I \ge x^I_{\text{BR}}(q^E) \text{ and } q^E \le x^E_0, & \text{i.e., } q^E \in [a-2q^I, a/3] & \text{(Case 2);} \\ a^2/9 & \text{if } q^I > x^E_0, & \text{i.e., } q^E > a/3 & \text{(Case 1).} \end{cases}$$

- Suppose that $q^I \le a/3 = x^I_0$.

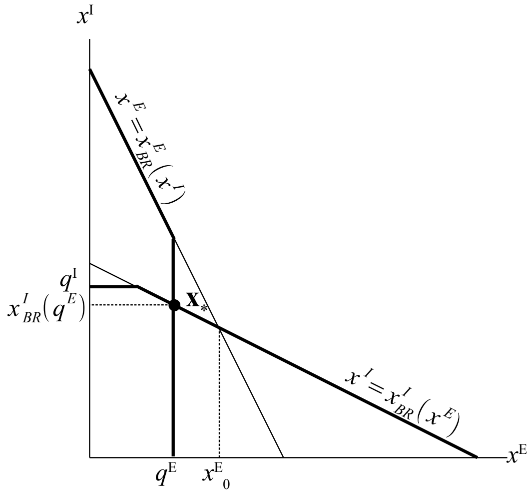
$$R^E(q^E, q^I) = \begin{cases} (a-q^E-q^I)q^E & \text{if } q^E \le x^E_{\text{BR}}(q^I) = (a-q^I)/2 \quad \text{(Case 4);} \\ (a-q^I)^2/4 & \text{if } q^E > x^E_{\text{BR}}(q^I) = (a-q^I)/2 \quad \text{(Case 3).} \end{cases}$$

Note that function R^E is continuous jointly in (q^E, q^I) in the whole domain $\mathbb{R}^2_+$ over these cases. Net profit function $\pi^i : \mathbb{R}^2_+ \to \mathbb{R}$ is given by $\pi^i(\mathbf{q}) = R^i(\mathbf{q}) - c^i q^i$. It is easy to confirm from the functional form and the graph that π^E satisfies all the parts of Assumption 1, except part v) for the second case $q^I \in (a/3, a/2]$ due to the subcase of $q^E > a/3$; so does π^I. The assumption (9) on a/c^E is made for π^E in the second case to meet Assumption 1-v). We check it below for each range in the assumption, focusing on π^E in the above second case $q^I \in (a/2, a/3]$. Note that, in this case, the left derivative of $R^E(\cdot, q^I)$ at $q^E = a - 2q^I$ is $3q^I - a$, while the right derivative at the point is $2q^I - a/2$. See Figure 5.

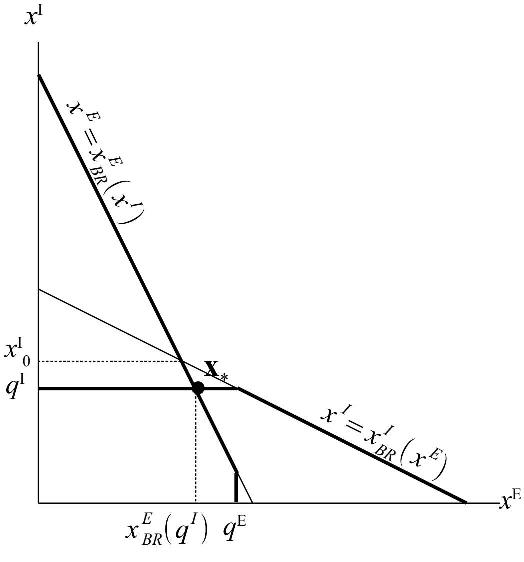
- Assume $a/c^E \le 2$. In this case, the optimal capacity level for E is found at $q^E = (a - q^I - c^E)/2$, which is smaller than $a - 2q^I$ as $q^I \le a/2$ and $c^E > a/2$, as long as $a > q^I + c^E$; otherwise, the optimum is $q^E = 0$. Since R^E in the range of $q^E \in [0.]a - 2q^I]$ is strictly concave in q^E, profit function $\tilde{\pi}^E(q^E, q^I) := R^E(q^E, q^I) - c^E q^E$ is a strictly concave function



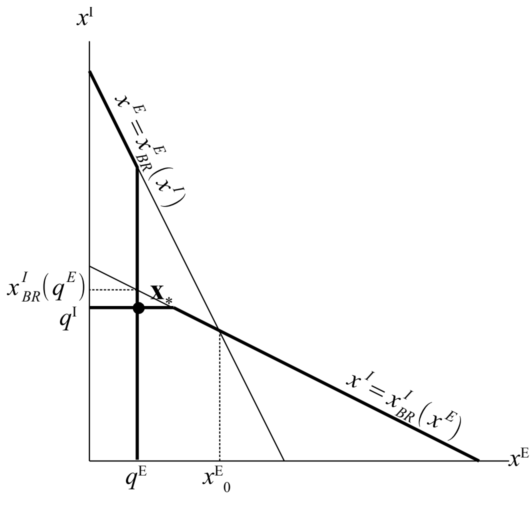
(a) Case 1.



(b) Case 2.

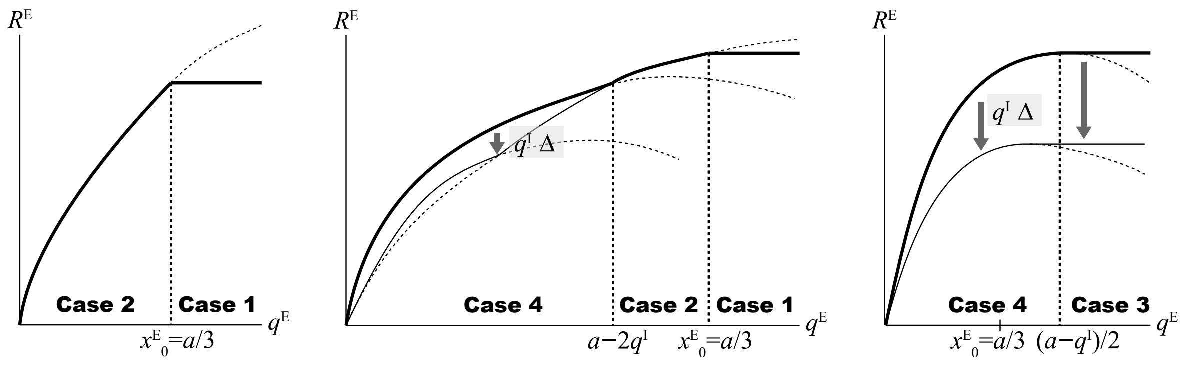


(c) Case 3.



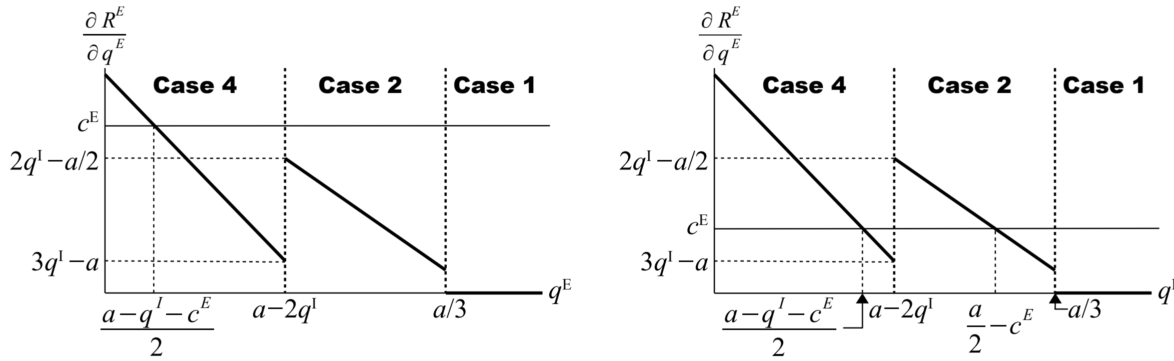
(d) Case 4, with $q^E < x_0^E$.

Figure 3: Stage-2 equilibrium in each case. Case 4 with $q^I < x_0^I$ can be drawn by replacing I and E in Figure (d). The bold lines illustrate the best response production level of each firm under capacity constraint in stage 2, while the thin lines are the graphs of best response without the capacity constraint.



(a) $q^I > a/2$. (b) $q^I \in (a/3, a/2]$. (c) $q^I \leq a/3$.

Figure 4: The gross revenue function R^E, illustrated by the bold curve. The thin curve represents the change in R^E when q^I becomes marginally greater. If $q^I > a/2$, marginal change in q^I does not change R^E.



(a) $a/c^E \leq 2$. (b) $a/c^E > 3(2+\sqrt{2})$.

Figure 5: The marginal revenue function $\partial R^E / \partial q^E$, illustrated by the bold curves. Here we assume $q^I \in [a/2, a/3]$.

of q^E with the peak at this optimum in this range of q^E. The assumption $a/c^E \leq 2$ guarantees that the right derivative of R^E at the threshold $2q^I - a/2$ is not greater than c^E, since the derivative cannot be greater than $a/2$ as $q^I \leq a/2$. As R^E is again strictly concave in q^E in the range of $q^E > a - 2q^I$, this tells that $\tilde{\pi}^E(q^E, q^I)$ decreases with q^E in this range. Thus, Assumption 1-v) is satisfied.

- Assume $a/c^E \geq 3(2+\sqrt{2})$. First, there are two points where the first order condition $R^{E\prime}_E = c^E$ holds, each of which lies in each range: $q^E = (a - q^I - c^E)/2 \in [0, a - 2q^I)$ (by $q^I \leq a/2$ and $c^E \leq a(2-\sqrt{2})/6 < a/2$) and $q^E = a/2 - c^E \in (a - 2q^I. + \infty)$ (by $q^I > a/3$ and $c^E \leq a(2-\sqrt{2})/6 < a/6$). The assumption $a/c^E \geq 3(2+\sqrt{2})$ guarantees that the latter point is the global optimum, since this assumption implies $\tilde{\pi}^E(a/2 - c^E, q^I) = (a/2 - c^E)^2/2 \geq \tilde{\pi}^E((a - q^I - c^E)/2, q^I) = (a - q^I - c^E)^2/4$. As R^E is strictly concave in q^E in the range of $q^E > a - 2q^I$, profit function $\tilde{\pi}^E$ is a strictly concave function of q^E in this range, with the peak at this global optimum. Thus, Assumption 1-v) is satisfied.

E Nash bargaining

Here we solve the Nash bargaining problem in Section 4.4 and verify that (12) is the condition to implement the benchmark equilibrium $\mathbf{q}_\dagger$.

Since $D(q^I_{\mathrm{BR}}(q^E)) + \beta(q^I_{\mathrm{BR}}(q^E))\overline{V} = \underline{\delta}$ by (IC$_*$) at $\tilde{q}^I = q^I_{\mathrm{BR}}(q^E) \in Q^I_P(q^E)$, the objective function reduces to

$$(\Pi^E_{NB})^\gamma(\Pi^L_{NB})^{1-\gamma} = \{\tilde{\pi}^E(q^E) - \underline{\delta} + B - w_0\}^\gamma\{\underline{\delta} - \beta(q^I_{\mathrm{BR}}(q^E))(\overline{V} - \underline{V}) - B + w_0\}^{1-\gamma}.$$

As $\overline{V} - \underline{V} > 0$, this decreases with $\beta(q^I_{\mathrm{BR}}(q^E))$. Thus, $\beta(q^I_{\mathrm{BR}}(q^E)) = 0$ at the optimum. Now the objective function further reduces to

$$(\Pi^E_{NB})^\gamma(\Pi^L_{NB})^{1-\gamma} = \{\tilde{\pi}^E(q^E) - \underline{\delta} + B - w_0\}^\gamma\{\underline{\delta} - B - w_0\}^{1-\gamma}. \tag{14}$$

With the above expression of $\underline{\delta}$, we obtain $D(q^I_{\mathrm{BR}}(q^E)) = \underline{\delta}$. (PC$_L$) reduces to

$$\underline{\delta} \geq B - w_0. \tag{15}$$

Evaluating (LL$_*$) at $\tilde{q}^I = q^I_{\mathrm{BR}}(q^E)$, we have

$$\underline{\delta} - B \leq \tilde{\pi}^E(q^E). \tag{16}$$

Evaluating (LL$_*$) at $\tilde{q}^I = \bar{q}^I_P(q^E)$, we have $\underline{\delta} - B \leq \beta(\bar{q}^I_P(q^E))\overline{V} - \bar{L}_P(q^E)$. With $\beta(\bar{q}^I_P(q^E)) = 1$, this implies

$$\underline{\delta} - B \leq \overline{V} - \bar{L}_P(q^E) \tag{17}$$

If there were no constraint, the maximum of this objective function would be attained by choosing the benchmark $\mathbf{q}^E_\dagger$ to maximize $\tilde{\pi}^E(q^E)$ and setting the value of $\underline{\delta} - B$ to meet the first order condition for maximization of the function in (14):

$$\frac{\tilde{\pi}^E(q^E_\dagger) - \underline{\delta} + B - w_0}{\underline{\delta} - B - w_0} = \frac{\gamma}{1-\gamma}, \quad \text{i.e., } \underline{\delta} - B = (1-\gamma)\tilde{\pi}^E(q^E_\dagger) - w_0.$$

As $\tilde{\pi}^E(q^E_\dagger) > 0$ by Assumption 2 and $\gamma \in [0,1]$, this satisfies (15). (16) also holds since $w_0 \geq 0$. Now, with this value of $\underline{\delta} - B$, (17) reduces to condition (12). So, it is the necessary condition to implement the benchmark $q^E_\dagger$ as the constrained optimum of the Nash bargaining problem.

Further, we can see that, once condition (12) holds, all the original constraints are satisfied by setting $B - w_0 = \overline{V} - (1-\gamma)\tilde{\pi}^E(q^E_\dagger)$, $M_0 = \{m_0\}$, $Q^I_S = \{\tilde{q}^I \in Q^I \mid \pi^E(q^E_\dagger, \tilde{q}^I) + B \geq 0\}$ and

$$D(m_0) = B, \qquad \beta(m_0) = 1,$$

$$D(\tilde{q}^I) = \begin{cases} \overline{V}, \\ \pi^E(q^E_\dagger, \tilde{q}^I) + B, \end{cases} \qquad \beta(\tilde{q}^I) = \begin{cases} 0 & \text{if } \pi^E(q^E_\dagger, \tilde{q}^I) + B \geq \overline{V}, \\ 1 - D(\tilde{q}^I)/\overline{V} & \text{otherwise} \end{cases}$$

for each $\tilde{q}^I \in Q^I_S$. Condition (12) guarantees (AP$_*$), (IC$^{S0}_*$) and $\beta(\bar{q}^I_P(q^E_\dagger)) \leq 1$.