Tohoku University Policy Design Lab Discussion Paper

TUPD-2022-005

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March 2022

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Non-point source pollution control policy for stochastic but constant environmental damage

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Abstract The interest of this paper is in the area of non-point source pollution, especially that produces a certain large environmental damage in a stochastic manner. Most previous studies on non-point source pollution control policies have implicitly assumed that the abatement efforts of economic agents as a whole can be estimated by assuming environmental damages whose magnitude depends on the abatement efforts of economic agents. When assuming environmental damage that occurs at a certain magnitude in a stochastic manner, the policies proposed by previous studies may achieve efficiency but do not prevent collusion. Therefore, this paper designs new efficiency and collusion-proof policies that work even when not only individual abatement efforts cannot be observed, but furthermore, when the total abatement effort cannot be estimated. Our policy only requires an honest reportig as a whole. By remaining room for adjustment in the reported amount of individual economic agents, our policy is also shown to achieve an equal burden among eocnomic agents.

Keywords: Non-point source pollution control policy; stochastic constant environmental damage; self-reporting; relative evaluation; collusion-proof; equal burden.

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1 Introduction

In environmental economic research, non-point source pollution is characterized as pollution in which the behavior of individual economic agents is unobservable and is highly affected by the uncertainty of natural factors. Among them, it is particularly difficult to apply traditional environmental economic policies to non-point source pollution due to the moral hazard problem caused by the unobservable behavior of individual economic agents (Holmstöm,1982; Segerson; 1988). Policies to control non-point source pollution require a fundamental rethinking of the design of environmental policies, and various approaches have proposed policies that can achieve efficiency without requiring information on the behavior of individual economic agents (Xepapadeas, 1992; Hansen, 1998; Horan, Shortle and Abler, 1998; Shortle, Horan and Abler, 1998; Hansen and Romstad, 2007).

The interest of this paper is in the area of non-point source pollution, especially that produces a certain large environmental damage in a stochastic manner. Prior studies have defined the magnitude of environmental damage as dependent on the behavior of economic agents. This is a very natural definition, and similar definitions are widely used in environmental economic research, not only in the area of non-point source pollution research. However, when we look at actual environmental problems, there are many environmental problems, especially those cause extensive damage, that once they occur, the magnitude of the damage cannot be controlled. In recent years, such environmental problems have been on the increase due to the effects of climate change. The only thing we can do for such environmental problems is to make efforts to prevent the occurrence of environmental damage.

Can we apply the non-point source control policies, which have been developed by previous studies, to environmental damage of a certain magnitude? If not, how can the policies proposed by previous studies be modified? This is the research question of this paper.

The structure of this paper is as follows. First, in Section 2, as a reference to clarify the problem of this paper, we review the non-point source pollution control policies that assume environmental damage dependent on abatement efforts. In Section 3, we confirm that the policies proposed by previous studies can achieve efficiency but cannot prevent collusion when assuming environmental damage that occurs at a certain magnitude in a stochastic manner. Section 4 and Section 5 propose a new non-point source pollution control policy that improves on the policies proposed in the previous studies: the efficiency policy in Section 4 and the collusion-proof policy in Section 5, respectively. As an efficiency policy, this paper proposes a policy that introduces self-reporting. Within Sections 4 and 5, it is shown that the proposed policies work effectively without requiring the honest declaration of individual economic agents for both the efficiency policy and the collusion-busting policy. Finally, in Section 6, we summarize the paper and discuss the limitations and remaining issues of our analysis.

2 Constant environmental damage whose probability of occurence depends on abatement efforts

Consider polluting economic activities conducted by N risk-neutral economic agents $i = 1 \dots N$. Let x_i be the abatement effort of economic agent i and $X = \sum_{j=1}^{N} x_j$ be the total abatement effort of economic agents. The pollution assumed in this paper is non-point source pollution. Therefore, throughout this paper, policy makers cannot observe abatement efforts of individual economic agents $\{x_j\}_{j=1}^{N}$.

In this section, we consider again the same problem as in the previous section, assuming constant environmental damage whose probability of occurence depends on abatement efforts.

Let $D > 0$ be the environmental damage of constant magnitude and $P(X) \in [0, 1]$ be the probability of occurence of environmental damage, and assume $P'(\cdot) < 0$. The only difference from the previous section is that the expected environmental damage is specified and expressed as $P(X)D$.

Let $C_i(x_i)$ be the abatement cost function of economic agent i and assume $C'(x_i) > 0$ and $C''(x_i) > 0$.

From the above, we can formulate the expected social cost minimization problem as following:

$$\min_{\{x_j\}_{j=1}^{N}} \left(\sum_{j=1}^{N} C_j(x_j) + P(X)D \right). \tag{1}$$

The first-order conditions for this problem are

$$C_i' + P'(X)D = 0 \text{ for } i = 1 \dots N. \tag{2}$$

The solution to this problem is denoted by superscript $*$. In this case, the expected value of the total burden on economic agents is

$$\sum_{j=1}^{N} C_j(x_j^*) + P(X^*)D. \tag{3}$$

In designing environmental policies in the subsequent paper, (6) is the criterion for evaluating the total burden on economic agents. That is, if the total burden on economic agents exceeds (6) when an environmental policy is introduced, such a policy is undesirable because it would be an excessive burden on economic agents.

Without any environmental policy in place, private economic agents will not willingly pay the abatement costs and will choose zero as their abatement effort. The simplest non-point pollution control policy that can provide appropriate pollution abatement incentives to economic agents is to make each economic agent bear the full environmental damage. That is, each economic agent is required to pay the realized value of $D(X, \varepsilon)$ when environmental damage occurs.

The problem faced by economic agent i after such policy is introduced is

$$\min_{x_i} \ (\ C_i(x_i) + P(X)D\ )\,. \tag{4}$$

The first-order condition for this problem is

$$C_i' + P'(X)D = 0. \tag{5}$$

From (2) and (5), we see that the first-order conditions are equal. While this approach can achieve efficiency through a very simple policy, it is known to have two major problems. The first problem is the excessive burden of economic agents. The expected value of the total burden on economic agents is expressed as

$$\sum_{j=1}^{N} C_j(x_j^*) + NP(X^*)D. \tag{6}$$

Since everyone bears the full environmental damage, the expected value of the tax should be equal to the actual environmental damages times the number of economic agents. Needless to say, (6) exceeds (3). As the number of economic agents increases, the deviation from (3) increases, and the policy become less realistic.

The second problem, often pointed out, is that economic agents have an incenteive to collude when introducing the full burden of environmental damage. When the magnitude of environmental damage depends on the abatement efforts of economic agents, their abatement efforts reduce not only their own burden but also that of all other economic agents. By colluding to minimize the total cost of several economic agents, the economic agents can then reduce the overall cost of the economic agents participating in the collution compared to before the collusion.

In this paper, to understand the workings of the collusion-proof policy, we consider only the collusion of N eocnomic agents as a collusion. The problem of collusion when the full burden of environmental damage is introduced is

$$\min_{\{x_j\}_{j=1}^{N}} \sum_{j=1}^{N} (\ C_j(x_j) + P(X)D\ ). \tag{7}$$

The first-order conditions for this problem is

$$C_i' + P'(X)D + (N-1)P'(X)D = 0 \text{ for } i = 1 \ldots N. \tag{8}$$

The difference between (2) and (8) is the presence of the third term on the left-hand side of (8). The third term on the left-hand side of (8) represents the magnitude of the reduction in the burden on $N-1$ economic agents other than eocnomic agents i due to the additional abatement efforts of eocnomic agent i. Because the marginal benefit of the abatement effort of economic agents i increase by this term, eocnomic agent i choose an excess abatement effort over x_i^*. The collusion of economic agents will result in a loss of the efficiency of the policy.

To prevent collusion, the policy should be modified so that the burden does not decrease even if excessive abatement efforts are chosen. For example, the following procedure can be used to decouple excessive abatement efforts from reduced burdens.

At first glance, it appers that collusion prevention is possible in a simple way, but this approach assumes that the total abatement efforts of economic agents can be estimated. And this is only possible when the magnitude of environmental damage depends on the abatement efforts of economic agents. Only when the environmental damage and the abatement efforts of economic agents is one to one relationship can X be estimated backward from the observed ε and $D(X, \varepsilon)$ realizations. By assuming a abatement effort-depndent environmental damage function, it can be said that the previous studies assumed that policy makers can estimate the total abatement efforts of economic agents.

The first-order conditions for expected social cost minimization problem are then

$$C_i' + P'(X)D = 0 \text{ for } i = 1 \ldots N. \tag{9}$$

The expected value of the total burden on economic agents is

$$\sum_{j=1}^{N} C_j(x_j^*) + P(X^*)D. \tag{10}$$

Since the environmental damage is only specified, the results shown in the previous section regarding (i) that the full burden of environmental damage can achieve efficiency, (ii) that the full burden of environmental damage is an excessive burden on economic agents, and (iii) that efficiency is undermined by the collusion of economic agents are similarly established. The qeustion is whether collusion can be prevented in the same way as in the previous section. If the abatement efforts of economic agents can be estimated, the same approach as in the previous section can be applied, but if they cannot be estimated, another approach must be required.

Here, the realized value of environmental damage that the policy maker can observe is always D. Therefore, even if the environmental damage can be observed, the total abatement efforts of economic agents cannot be estimated from it. Since the probability of occurrence of environmental damage depended on the abatement efforts of economic agents, if the true probability of occurrence could be known, the total abatement efforts of economic agents could be estimated from it. However, once an occurrence is observed, it is not possible to know to what extent the actual probability of occurrence has descreased due to abatement efforts. If the frequency of occurrence is very high, it may be possible to obtain an approximation of the true probability of occurrence within a short period of time, but at least as far as the environmental issues on which this paper focuses, a long period of time is required.

3 Non-point source pollution control policy based on self-reported abatement efforts

In this section, we propose a new non-point source pollution control policy based on self-reported abatement efforts. The policy maker requires economic agents to self-report their abatement efforts. Let r_i be the reported amount of economic agents i and $R = \sum_{j=1}^{N} r_j$ be the total reported amount of economic agents. This reported amount does not necessarily coincide with the true abatement efforts of economic agents. The policy maker can design a policy using the newly available reported amount of eonomic agents $\{r_j\}_{j=1}^{N}$ and the relative reported amount $\{\frac{r_j}{R}\}_{j=1}^{N}$, as follows.

Efficiency policy (P1):

1. Pay an incentive of t_i to all eoconomic agent $i = 1, \ldots, N$ for every one unit of reported amount.
2. In case of environmental damage, collect a lump-sum tax $\{\bar{T}_j\}_{j=1}^{N}$ from only one economic agent.
3. The economic agent that bears the lump-sum tax is detemined stochastically according to the relative reported amount $\{\frac{r_j}{R}\}_{j=1}^{N}$.

The structure of the efficiency policy $(P1)$, in which only one economic agent bears a large burden, may be similar to the all-or-nothing policy proposed by Xepapadeas (1992). In the policy of Xepapadeas (1992), only one eocnomic agents determied stochastically bore the full burden of environmental damage in order to achieve both efficiency and an appropriate burden on economic agents at the same time. However, the policy proposed by Xepapadeas (1992) differed in that the economic agents bearing the burden were chosed randomly, whereas in our policy, the economic agents with large relative reported amount have a higher probability of being chosen. Moreover, as will be shown in the proposition that followm, our policy can adjust not only the total burden but also the individual burdens.

The expected value of the burden on economic agent i due to introducing the efficiency policy

$(P1)$ is

$$E[T_i] = t_i r_i + P(X)\frac{r_i}{R}\bar{T}_i. \tag{11}$$

The problem faced by the economic agent i is

$$\min_{x_i, r_i} \ (\ C_i(x_i) + E[T_i] \ ) \, . \tag{12}$$

First let us see that the efficiency policy $(P1)$ can achieve efficiency for the case when economic agents do not consider collusion. In the proposition, it is shown that the achievement of efficiency does not require an honest report. Consequently, a new policy parameter will be added: what level should be targeted as the amount of self-report. Here, denoting $\{g_j\}_{j=1}^N$ be the targets of reported amount from economic agents, $G = \sum_{j=1}^N g_j$ be the sum of the individual target and $G_{-i} = \sum_{j \neq i}^N g_j$ be the sum of the individual target other than economic agent i.

Proposition 1. *The efficiency policy $(P1)$ choosing the policy parameters as*

$$\begin{aligned} t_i\left(\{g_j\}_{j=1}^N\right) &= -P(X^*)\frac{G_{-i}}{Gg_i}D \textit{ for } i = 1, \ldots, N, \\ \bar{T}_i\left(\{g_j\}_{j=1}^N\right) &= \frac{G}{g_i}D \textit{ for } i = 1, \ldots, N \end{aligned}$$

can achieve the efficiency under the given targets of reported amount $\{g_j\}_{j=1}^N$.

Proof. The first-order conditions for the problem (12) are

$$C_i' + \frac{\partial E[T_i]}{\partial x_i} = 0, \tag{13}$$

$$\frac{\partial E[T_i]}{\partial r_i} = 0, \tag{14}$$

where

$$\frac{\partial E[T_i]}{\partial x_i} = P'(X)\frac{r_i}{R}\bar{T}_i = 0, \tag{15}$$

$$\frac{\partial E[T_i]}{\partial r_i} = t_i + P(X)\frac{R_{-i}}{R^2}\bar{T}_i, \tag{16}$$

and $R_{-i} = \sum_{j \neq i} r_j$ is the sum of the reported amount other than economic agent i. For (9) and (13) to coincide, we must have

$$\bar{T}_i = \frac{R}{r_i}D. \tag{17}$$

Substituting (17) into (14) and rearranging for t_i, we obtain

$$t_i = -P(X)\frac{R_{-i}}{Rr_i}D. \tag{18}$$

When choosing $\{\bar{T}_i\}_{i=1}^N$ according to (17), efficiency can be achieved independent of the reported amount. Therefore, the policy parameters of the efficiency policy $(P1)$ to achieve the efficiency can be expressed as

$$t_i(\{g_j\}_{j=1}^N) = -P(X^*)\frac{G_{-i}}{Gg_i}D, \tag{19}$$

$$\bar{T}_i(\{g_j\}_{j=1}^N) = \frac{G}{g_i}D. \tag{20}$$

□

Next, let us see that the efficiency policy $(P1)$ does not excessively burden economic agents. Let $E[T_i(\{g_j\}_{j=1}^N)]$ be the expected value of the burden on economic agent i when the efficiency policy $(P1)$ is introduced and the policy parameters are chosen as (19) and (20).

Proposition 2. *The efficient policy* $(P1)$ *choosing policy parameters as* (19) *and* (20) *can achieve appropriate burden of economic agents. In other words, it satisfies that*

$$\sum_{j=1}^N E[T_j(\{g_j\}_{j=1}^N)] = P(X^*)D,$$

under the given targets of reported amount.

Proof. Substituting $x_i = x_i^*$, $r_i = g_i$, (19) and (20) into $E[T_i(\{g_j\}_{j=1}^N)]$, we obtain

$$E[T_i(\{g_j\}_{j=1}^N)] = -P(X^*)\frac{G_{-i}}{Gg_i}Dg_i + P(X^*)\frac{g_i}{G}\frac{G}{g_i}D = P(X^*)D\frac{g_i}{G}. \tag{21}$$

Summing this over the entire eocnomic agents, we obtain

$$\sum_{j=1}^N E[T_j(\{g_j\}_{j=1}^N)] = P(X^*)D\frac{\sum_{j=1}^N g_j}{G} = P(X^*)D\frac{G}{G} = P(X^*)D. \tag{22}$$

□

By proposition 1 and proposition 2, both efficiency and an appropriate burden on economic agents can be achieved under any given reported amount. It is not the case, then, that the targets of reported amount $\{g_j\}_{j=1}^N$ has no effect anywhere. From (21), we know that the targets of reported amount $\{g_j\}_{j=1}^N$ changes the expected value of the burden on individual economic agents. Therefore, the target of reported amount $\{g_j\}_{j=1}^N$ allows the policy maker to adjust the expected value of the burden on individual economic agents while keeping the expected value of the burden on economic agents as a whole at an appropriate level.

One special allocation is the equal burden among economic agents achieved by determing $\{g_j\}_{j=1}^N$ to satisfy

$$E[T_i(\{g_j\}_{j=1}^N)] = \frac{\sum_{j=1}^N E[T_j]}{N} \text{ for } i = 1, \ldots, N. \tag{23}$$

Proposition 3. *The efficient policy* $(P1)$ *choosing policy parameters as* (19) *and* (20)*, and choosing the targets of reported amount to satisfy* (23) *can simultaneously achieve efficiecny and equal burden.*

4 Policy amendments to prevent collusion

The propositions about the properties of the efficiency policy $(P1)$, derived in the previous section, did not consider collusion-proofness. In fact, if economic agents collude, all the propositions of the previous section would not hold. Therefore, in this section, we attempt to modify the policy to prevent collusion. Specifically, we add the collusion-proof policy $(P2)$ to the efficiency policy $(P1)$, proceeding as follows. The information available to the policymaker for designing the policy is the individual abatement efforts and their sum, X_i^* and X^*; the individual reported amount and their sum, r_i and R; and the observations about the occurrence of environmental damages. Policy makers can know whether environmental damage has occurred or not. The occurrence or non-occurrence is denoted by a random variable n that takes on a value of either 0 or 1. Using all of this available information, we can design a collusion-proof policy as follows.

Collusion-proof policy (P2):

1. The policy maker continues to observe whether environmental damage occurs for a certain period of tim (e.g., one year).
2. Calculate the square of the diviation between the efficient probability of occurrence for society as a whole, and the realized value of n.
3. Calculate the square of the deviation between the self-report based probability of occurrence and the realized value of n.
4. Collect he difference between 3. and 4. multiplied by the policy parameter α_i from economic agent i.
5. Furthermore, collect the difference between the total efficient abatement effort and the total reported amount multiplied by another policy parameter β_i from economic agent i.

From the above procedure, the additional burden V_i of economic agent i due to the collusion-proof policy ($P2$) can be expressed as

$$V_i = \alpha_i \left\{ (P(X^*) - n)^2 - (P(R) - n)^2 \right\} + \beta_i (X^* - R)^2 . \tag{24}$$

Since, n is a random variable, V_i is also a random variable. The expected value of V_i can be calculated as follows

$$E[V_i] = \alpha_i \left\{ (P(X^*) - P(X))^2 - (P(R) - P(X))^2 \right\} + \beta_i (X^* - R)^2 . \tag{25}$$

When the collusion-proof policy ($P2$) is added, the problem faced by economic agent i is

$$\min_{x_i, r_i} \left(C_i(x_i) + E[T_i] + E[V_i] \right) . \tag{26}$$

The problem of the collusion of all economic agents is

$$\min_{\{x_j\}_{j=1}^N, \{r_j\}_{j=1}^N} \sum_{j=1}^{N} \left(C_j(x_j) + E[T_j] + E[V_j] \right) . \tag{27}$$

The addition of the collusion-proof policy ($P2$) would be useless if it undermines the properties of the efficiency policy ($P1$) shown in the previous section when economic agents do not collude. Therefore, let us first focus on the problem (26) and see whether the propositions in the previous section can be replicated when the efficiency policy $P(1)$ and the collusion-proof policy ($P2$) are introduced simultaneously.

Proposition 4. *The efficient policy* ($P1$) *choosing the policy parameters as*

$$\begin{aligned} t_i(\{g_j\}_{j=1}^N, \alpha_i, \beta_i) =& - P(X)\frac{G_{-i}}{Gg_i}\left\{D + 2\alpha_i\left(P(G) - P(X^*)\right)\right\} \\ & - 2\alpha_i P'(G)\left(P(G) - P(X^*)\right) + 2\beta_i\left(X^* - G\right) \text{ for } i = 1, \ldots, N, \\ \bar{T}_i(\{g_j\}_{j=1}^N, \alpha_i) =& \frac{G}{g_i}\left\{D + 2\alpha_i\left(P(G) - P(X^*)\right)\right\} \text{ for } i = 1, \ldots, N \end{aligned}$$

and the collusion policy ($P2$) *can achieve efficiency under the given targets of reported amount* $\{g_j\}_{j=1}^N$ *and the given policy parameters of collusion-proof policy* ($P2$) $\{\alpha_j\}_{j=1}^N$ *and* $\{\beta_j\}_{j=1}^N$.

Proof. The first-order conditions for the problem (26) are

$$C_i' + \frac{\partial E[T_i]}{\partial x_i} + \frac{\partial E[V_i]}{\partial x_i} = 0, \tag{28}$$

$$\frac{\partial E[T_i]}{\partial r_i} + \frac{\partial E[V_i]}{\partial r_i} = 0, \tag{29}$$

where

$$\frac{\partial E[V_i]}{\partial x_i} = -2\alpha_i P'(X)\left(P(R) - P(X^*)\right), \tag{30}$$

$$\frac{\partial E[V_i]}{\partial r_i} = 2\alpha_i P'(R)\left(P(R) - P(X)\right) - 2\beta_i\left(X^* - R\right). \tag{31}$$

For (9) and (28) to coincide, we must have

$$\bar{T}_i = \frac{R}{r_i}\left\{D + 2P'(X)\alpha_i\left(P(R) - P(X^*)\right)\right\}. \tag{32}$$

Substituting (32) into (29) and rearranging for t_i, we obtain

$$t_i = -P(X)\frac{R_{-i}}{Rr_i}\left\{D + 2\alpha_i\left(P(R) - P(X^*)\right)\right\} - 2\alpha_i P'(R)\left(P(R) - P(X)\right) + 2\beta_i\left(X^* - R\right). \tag{33}$$

Again, the achievement of efficiency does not depend on the reported amount. In addition, it does not depend on the policy paramters of the collusion-proof policy ($P2$). Therefore, the policy parameters of the efficiency policy $(P1)$ to achieve the efficiency can be expressed as

$$t_i(\{g_j\}_{j=1}^N, \alpha_i, \beta_i) = - P(X)\frac{G_{-i}}{Gg_i}\{D + 2\alpha_i (P(G) - P(X^*))\} \\ - 2\alpha_i P'(G)(P(G) - P(X^*)) + 2\beta_i (X^* - G) \tag{34}$$

$$\bar{T}_i(\{g_j\}_{j=1}^N, \alpha_i) = \frac{G}{g_i}\{D + 2\alpha_i (P(G) - P(X^*))\}. \tag{35}$$

□

Proposition 5. *When sum of the targets of the reported amount is set to be equal to the sum of the efficient abatement efforts, the policy parameters of the efficiency policy* $(P1)$ *that achieves efficiency when only the efficiecny policy* $(P1)$ *is introduced coincide with those when efficient policy* $(P1)$ *and the collusion-proof policy* $(P2)$ *are introduced under the given policy parameters of the collusion-proof policy* $(P2)$*.*

In other words, if $G = X^*$*, then it satisfies*

$$t_i(\{g_j\}_{j=1}^N)|_{G=X^*} = t_i(\{g_j\}_{j=1}^N, \alpha_i, \beta_i)|_{G=X^*},$$
$$\bar{T}_i(\{g_j\}_{j=1}^N)|_{G=X^*} = \bar{T}_i(\{g_j\}_{j=1}^N, \alpha_i)|_{G=X^*}$$

under given $\{\alpha_j\}_{j=1}^N$ *and* $\{\beta_j\}_{j=1}^N$*.*

Proof. When $G = X^*$, (34) and (35) are

$$t_i(\{g_j\}_{j=1}^N, \alpha_i, \beta_i)|_{G=X^*} = -P(X)\frac{G_{-i}}{Gg_i}D, \tag{36}$$

$$\bar{T}_i(\{g_j\}_{j=1}^N, \alpha_i)|_{G=X^*} = \frac{G}{g_i}D. \tag{37}$$

(36) and (37) are equal to (19) and (20) respectively. □

Proposition 6. *Set the sum of the targets of reported amount to the sum of their total efficient abatement efforts. Then, the efficient policy* $(P1)$ *choosing policy parameters as* (36) *and* (37)*, and the pollution-proof policy* $(P2)$ *can achieve appropriate burden of economic agents under given policy parameters of collusion-proof policy* $(P2)$*.*

In other words, if $G = X^*$*, then it satisfies that*

$$\sum_{j=1}^{N} E[T_j(\{g_j\}_{j=1}^N, \alpha_i, \beta_i)|_{G=X^*}] = P(X^*)D$$

under given $\{\alpha_j\}_{j=1}^N$ *and* $\{\beta_j\}_{j=1}^N$*.*

Proof. From Proposition 4, $X = X^*$, because efficiency could be achieved by adding the collusion-proof policy ($P2$) to the efficiency policy ($P1$). Now, setting $G = X^*$ for the sum of the targets of reported amount, and then we have $X = R = X^*$. Substituting $X = R = X^*$ into (25), the expected value of the additional burden due the collusion-proof policy ($P2$) is

$$E[V_i]|_{X=R=X^*} = 0 \text{ for } i = 1, \ldots, N. \tag{38}$$

What remains is the expected value of the burden of the efficiency policy ($P1$), $E[T_i]$, but when $G = X^*$ from proposition (5), the policy parameters of the efficiency policy ($P1$) that achieves efficiency with and without the collusion-proof policy ($P2$) did not change. Thus, the magnitude of $E[T_i]$ does not change with the addition of the collusion-proof policy ($P2$). Naturally, the size of the sum, $\sum_{j=1}^{N} E[T_j]$, also remains unchanged. From Proposition 2, since the expected value of the total burden of economic agents when only the efficiency policy ($P1$) is introduced coincides with the expected value of environmental damage, the expected value of the total burden of economic agents after adding the collusion-proof policy ($P2$) also coincides with the expected value of environmental damage. □

Comparing propositions 2 and 6 in the previous section, we find that the economic agents were able to achieve an appropriate burden at any target of reported amount in proposition 2, whereas in proposition 6, which considers collusion, the appropriate burden on economic agents could not be achieved unless the sum of targets was the sum of efficient abatement efforts. However, there is still room to adjust the targets of reported amount $\{g_j\}_{j=1}^{N}$ to the extent that the condition $G = X^*$ is satisfied. This room for adjustment is sufficient to achieve equal burden among economic agents.

Proposition 7. *Set the sum of the targets of reported amount to the sum of their total efficient abatement efforts. Then, the efficient policy* ($P1$) *choosing policy parameters as* (19) *and* (20), *and the pollution-proof policy* ($P2$) *can simultaneously achieve efficiecny and equal burden under the given policy parameters of collusion-proof policy* ($P2$).

Proof. When the efficiency policy (P1) and the collusion-proof policy ($P2$) are introduced and (34) and (35) are chosen as policy parameters of the efficiency policy (P1) to achieve efficiency, the expected value of the burden on economic agent i is

$$E[T_i] = P(X^*)D\frac{g_i}{X^*}. \tag{39}$$

The only difference in the magnitude of the burden among economic agents is g_i. Therefore, equal burden can be achieved by setting the total target of reported amount to X^* and dividing it equally among N eocnomic agents to determine $g_i = \frac{X^*}{N}$. □

Finally, let us see that the collusion-proof policy $(P2)$ can infact prevent collusion. The first-order conditions of the problem (27) are

$$C_i' + \frac{\partial E[T_i]}{\partial x_i} + \frac{\partial E[V_i]}{\partial x_i} + \sum_{j \neq i}\left(\frac{\partial E[T_j]}{\partial x_i} + \frac{\partial E[V_j]}{\partial x_i}\right) = 0, \tag{40}$$

$$\frac{\partial E[T_i]}{\partial r_i} + \frac{\partial E[V_i]}{\partial r_i} + \sum_{j \neq i}\left(\frac{\partial E[T_j]}{\partial r_i} + \frac{\partial E[V_j]}{\partial r_i}\right) = 0, \tag{41}$$

where

$$\frac{\partial E[T_j]}{\partial x_i} = P'(X)\frac{r_j}{R}\bar{T}_j, \tag{42}$$

$$\frac{\partial E[T_j]}{\partial r_i} = -P(X)\frac{r_j}{R^2}\bar{T}_j, \tag{43}$$

$$\frac{\partial E[V_j]}{\partial x_i} = -2\alpha_j P'(X)\left(P(X^*) - P(R)\right), \tag{44}$$

$$\frac{\partial E[V_j]}{\partial r_i} = -2\alpha_j P'(R)\left(P(R) - P(X)\right) - 2\beta_j\left(X^* - R\right). \tag{45}$$

Denote x_i and r_i as $\{x_i^{\#}\}_{i=1}^N$ and $\{r_i^{\#}\}_{i=1}^N$ that satisfy (40) and (41). The fourth term in the (40) left-hand side and the third term in the (41) left-hand side are the new terms added by collusion, respectively. (42) represents the effect of the additional abatement effort on reducing the probability of environmental damage, thereby reducing the expected value of the burden on economic agents other than economic agent i. (43) represents the effect of additional self-reporting reducing the expected value of the burden on all other economic agents by reducing the probability of economic agents other than economic agent i paying lump-sum taxes. The addition of these terms distorts the incentives for both abatement efforts and self-reporting.

On the other hand, (44) and (45) represent the effect of additional abatement efforts and self-reporting by economic agent i on the additional burden of the collusion-proof policy $(P2)$ of economic agents other than economic agent i. (44) and (45) contain the policy parameters α_i

and β_i of the collusion-proof policy $(P2)$. The following proposition shows that by choosing α_i and β_i appropriately, the fourth term in (40) left-hand side and the third term in (41) left-hand side, which represent the distortion of incentives due to collusion, can be cancelled out.

Proposition 8. *The efficient policy* $(P1)$ *and the collusion-proof policy* $(P2)$ *can achieve efficiency with or without collusion of economic agents under the given targets of reported amount* $\{g_j\}_{j=1}^N$ *by choosing policy parameters as*

$$\begin{aligned} t_i(\{g_j\}_{j=1}^N, \alpha_i^*, \beta_i^*) =& - P(X)\frac{G_{-i}}{Gg_i}\{D + 2\alpha_i^*(P(G) - P(X^*))\} \\ &- 2\alpha_i^* P'(G)(P(G) - P(X^*)) + 2\beta_i^*(X^* - G) \text{ for } i = 1, \ldots, N, \\ \bar{T}_i(\{g_j\}_{j=1}^N, \alpha_i^*) =& \frac{G}{g_i}\{D + 2\alpha_i^*(P(G) - P(X^*))\} \text{ for } i = 1, \ldots, N, \end{aligned}$$

and $\{\alpha_j^*, \beta_j^*\}_{j=1}^N$ *satisfying* (46) *and* (47).

Proof. By Proposition 4, the efficient policy (P1) can achieve efficiency when not condiering collusion under the given policy parameters of the collusion-proof policy (P2), $\{\alpha_j, \beta_j\}_{j=1}^N$. The change in the first-order conditions of the economic agents problem due to collusion is the addition of the last term in (40) and (41). If these additional terms can be eliminated by properly choosing $\{\alpha_j, \beta_j\}_{j=1}^N$, then efficiency can be achieved even if economic agents collude. We show that one can actually choose such $\{\alpha_j, \beta_j\}_{j=1}^N$.

The last term in (40) and (41) include $\{\bar{T}_j, \alpha_j, \beta_j\}_{j=1}^N$ as policy parameters. Substituting the efficient policy parmeter $\{\bar{T}_j\}_{j=1}^N$ when not considering collusion, $\{\bar{T}_j(\{g_j\}_{j=1}^N, \alpha_j)\}_{j=1}^N$ into $\{\bar{T}_j\}_{j=1}^N$ in the last terms in (40) and (41). Then, there we can find $\{\alpha_j, \beta_j\}_{j=1}^N$ satisfying

$$\sum_{j \neq i} \left(\left. \frac{\partial E[T_j]}{\partial x_i} \right|_{\bar{T}_j = \bar{T}_j(\{g_j\}_{j=1}^N, \alpha_j)} + \frac{\partial E[V_j]}{\partial x_i} \right) = 0 \text{ for } i = 1, \ldots, N, \tag{46}$$

$$\sum_{j \neq i} \left(\left. \frac{\partial E[T_j]}{\partial r_i} \right|_{\bar{T}_j = \bar{T}_j(\{g_j\}_{j=1}^N, \alpha_j)} + \frac{\partial E[V_j]}{\partial r_i} \right) = 0 \text{ for } i = 1, \ldots, N, \tag{47}$$

because there are 2N parameters for 2N eauations. Let $\{\alpha_i^*, \beta_j^*\}_{j=1}^N$ be a paremeter set $\{\alpha_i, \beta_j\}_{i=1}^N$ satisfying (46) and (47). Corresponding polciy parmeters of the efficiency policy (P1) are $\{t_j(\{g_j\}_{j=1}^N, \alpha_j^*, \beta_j^*), \bar{T}_j(\{g_j\}_{j=1}^N, \alpha_j^*)\}_{j=1}^N$.

Proposition 9. *Introduce the efficient policy (P1) and the collusion-proof policy (P2) as Proposition 8 and set the sum of the targets of reported amount to the sum of their total efficient abatement efforts ($G = X^*$). Then, economic agents either choose not to collude or all collude.*

Proof. Assume that the efficient policy (P1) and the collusion-proof policy (P2) are both introduced as Proposition 8. First, let us calculate the total burden on economic agents when they all collude. By Proposition 8, $\{x_j\}_{j=1}^N = \{x_j^*\}_{j=1}^N$ under the policy parameters (34), (35) and $\{\alpha_j^*, \beta_j^*\}_{j=1}^N$ for any $\{g_j\}_{j=1}^N$. By Proposition 5 and Proposition 6, if $G = X^*$, then the total burden on ecnomic agents equals to the optimal expected social cost:

$$\sum_{j=1}^{N} \left(C_j(x_j^*) + E[T_j(\{g_j\}_{j=1}^N, \alpha_i, \beta_i)|_{G=X^*}] + E[V_j]|_{X=R=X^*}\right) = \sum_{j=1}^{N} C_j(x_j^*) + P(X^*)D. \tag{48}$$

Thus, the total burden is equal when economic agents do not collude and when they all collude under the policy parameters (34), (35), $\{\alpha_j^*, \beta_j^*\}_{j=1}^N$ and $G = X^*$.

Next, let us consider the total burden on economic agents when one collusion except for collusion among all is chosen. We denote the solution to the problem and the corresponding burden by superscript #. Then, it must be

$$\sum_{j=1}^{N} \left(C_j(x_j^{\#}) + E[T_j^{\#}] + E[V_j^{\#}]\right) \geq \sum_{j=1}^{N} C_j(x_j^*) + P(X^*)D, \tag{49}$$

because the collusion among all minimizes the total burden on economic agents. Economic agents can improve Nash by either choose not to collude or by all collude. This inequality holds for any collusion except for collusion among all.

□

5 Concluding remarks

This paper proposes efficiency and collusion-proof policies that can work even if the total abatement efforts cannot be estimated as well as individual abatement efforts, assuming stochastic and constant environmental damage caused by non-point source pollution. The policy proposed in this paper can achieve the three goals of efficiency, appropriate burden on economic agents, and collusion-proof. These properties could be shown if economic agents as a whole made honest reporting and did not require honest reporting by individual economic agents. By remaining room for adjustment in the reported amount of individual economic agents, our proposed policy is also shown to achieve an equal burden.

Finally, we discuss the limitations of our analysis in this paper. The most significant limitation of the analysis in this paper is that only the collusion of all economic agents is considered as collusion. In general, heterogeneous N economic agents can be divided into various collusion groups. Therefore, collusion-proof policy is required to eliminate incentives for any collusion groups. Among the policies proposed in this paper, further elaborate policy design remains an issue, especially for the collusion-proof policy.

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