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Two-sided Heterogeneity: New Implications for Input Trade

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Abstract

How different are the impacts of trade cost on trade flows between intermediate goods and final goods? How large are the welfare gains from trade for intermediate goods relative to final goods? To address these questions, we develop a heterogeneous firm model in which selection into exporting and importing play a key role in industry productivity of vertically-related sectors. We show that the impact of trade cost on trade flows is is greater for intermediate goods than for final goods, due to an extra adjustment through the extensive margin. We also find that the impact of trade cost on welfare is greater for intermediate goods than for final goods if and only if the domestic input share is smaller than the domestic output share.

Keywords: Two-sided heterogeneity, input trade, selection

JEL Classification Numbers: F12, F14

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1 Introduction

How different are the impacts of trade cost on trade flows between intermediate goods and final goods? How large are the welfare gains from trade for intermediate goods relative to final goods? Though the recent trade literature has devoted enormous effort to developing new trade models that match with empirical evidence, few theoretical work has explored a distinctive feature of intermediate good trade that is absent in final good trade when the two types of goods are costly traded subject to selection. The present paper tries to fill this important gap in the literature by deriving a gravity equation of intermediate goods and by relating the trade elasticity obtained from that gravity equation with the welfare gains from trade.

There is mounting evidence suggesting that intermediate good trade has been growing faster and its share in world trade is larger than final good trade, due to “outsourcing” or “offshoring” that fragments production processes across the globe (Hummels et al., 2001; Hanson et al., 2005; Johnson and Noguera, 2012). Some recent empirical work has revealed that when firms import intermediate goods from foreign countries, firm importing displays a number of the same performance differences as firm exporting. A series of work by Bernard et al. (2007, 2012, 2018a) unveil empirical regularity on firm importing that, just as in exporters, importers are larger and more productive than non-importers within the same industries, and only a small fraction of firms import. Another line of empirical work has also documented that the impact of trade liberalization is quite different between intermediate goods and final goods. For example, Amiti and Konings (2007) find that input tariff reductions increase industry productivity twice greater than output tariff reductions in Indonesia; Topalova and Khandelwal (2011) similarly find that firms’ gains from input tariff reductions can be ten times greater than those from output tariff reductions in India. Further, input tariff reductions give rise to different productivity gains from output tariff reductions by expanding technological possibilities of firms, as shown by Goldberg et al. (2010).

This paper develops a heterogeneous firm model in which selection into exporting and importing play a key role in industry productivity. In our model, an industry is composed of two production sectors (i.e., upstream and downstream sectors) in which the former sector produces and exports intermediate goods by using labor, and the latter sector imports intermediate goods to produce and export final goods by using intermediates and labor. While intermediate good suppliers in the upstream sector are modeled in a similar way to Melitz (2003), one of main departures from Melitz (2003) is that final good firms in the downstream sector incur additional fixed cost when using intermediate goods imported from the foreign market. As a result, selection occurs not only in the upstream sector but also in the downstream sector, which allows us to capture the empirical pattern that only more productive firms access imported intermediate goods. This framework also allows us to show different impacts of tariff reductions on resource reallocations in vertically-related sectors, which could help us explain why input tariff reductions can increase industry productivity more than output tariff reductions. For the sake of parsimony, we focus on two symmetric countries; nonetheless, our model identifies a new channel through which selection in the downstream and upstream sectors jointly affects the equilibrium outcomes in a consistent manner with empirical evidence.

Our first contribution is in showing that the impact of trade cost on trade flows is greater for intermediate goods than for final goods. The result is most clearly seen in a special case of a Pareto distribution with free entry in the vertically-related sectors. Let T_j denote the value of trade between country 1 and country 2 where the subscript j is either final goods ($j = X$) or intermediate goods ($j = M$). Then T_j is given by

$$T_j = \psi_j \times \frac{Y_1^a \times Y_2^b}{\tau_j^{\varepsilon_j}},$$

where ψ_j is a constant term, Y_1, Y_2 are each country's GDP and τ_j is trade barriers between the two countries. As with a usual gravity equation, the value of trade is positively affected by sizes of exporting and importing countries but is negatively affected by trade barriers for both types of goods. In that sense, the gravity equation applies to intermediate goods let alone final goods. We find, however, that the elasticity of the value of trade with respect to trade barriers is *endogenously* greater for intermediate goods than for final goods, i.e., $\varepsilon_M > \varepsilon_X$. The difference in the trade elasticity arises through which trade barriers have a different impact on selection into exporting and importing in the vertically-related sectors.

In the case of intermediate good trade, reductions in trade cost allow not only intermediate good suppliers to export intermediate goods more easily, but also final good firms to import these goods that are used for their production more easily. As new and less productive suppliers (firms) start exporting (importing) in the upstream (downstream) sector, the effect on trade flows is magnified in the current setting relative to a single production sector setting. In the case of final good trade, in contrast, such reductions induce only new final good firms to start exporting since intermediate good suppliers do not import final goods for their production. This suggests that there exists an extra adjustment in the set of importers (extensive margin) in intermediate good trade that is absent in final good trade, which elevates the trade elasticity of intermediate goods relative to that final goods.[1] The finding could help us better understand why intermediate good trade has been growing faster than final good trade in globalization (Hummels et al., 2001; Hanson et al., 2005; Johnson and Noguera, 2012). To demonstrate our contribution most sharply, we focus on bilateral trade flows between symmetric countries; however, our result would hold for those flows between asymmetric countries by introducing an outside good that equalize wages.

Our second contribution is in showing that the impact of trade cost on welfare is greater for intermediate goods than for final goods if and only if the domestic input share is smaller than the domestic output share. In our vertical production model where both final goods and intermediate goods are costly traded subject to selection, trade liberalization can magnify the standard selection effect relative to a single production model: resource reallocations occur in the two production sectors, forcing the least productive firms (suppliers) to exit the downstream (upstream) sector. As in Melitz (2003), the welfare gains from trade liberalization are directly related to selection that occurs in the vertically-related sectors. However, trade liberalization in final goods and that in intermediate goods trigger such two-sided reallocations on a different scale, which makes it ambiguous to compare the absolute magnitude of welfare changes. Given complex interactions between the two production sectors, it is generally difficult to figure out economic factors that generate different welfare changes associated with trade liberalization in final goods and intermediate goods separately. We find however that welfare changes are again clearly seen in a special case of a Pareto distribution with free entry in the two production sectors. Let λ_j denote the domestic share where j is either final goods ($j = X$) or intermediate goods ($j = M$). Then welfare changes associated with trade cost τ_j are

$$\widehat{W} = \widehat{\lambda}_j^{-\frac{1}{\varepsilon_j}},$$

where a "hat" is proportional changes in a variable. Thus, welfare changes associated with trade liberalization in each type of goods can be captured by only the the domestic share and the trade elasticity estimated from the gravity equation above. In other words, the welfare formula by Arkolakis et al. (2012) applies even in the

[1] In the model with roundabout production, it is known that the presence of importing leads to an expansion of both exporters and importers (Gopinath and Neiman, 2014; Blaum, 2019). While this channel works in our model with vertical production in the sense that reductions in output trade cost indirectly induce entry of final good exporters and raise the extensive margin elasticity (relative to that in a single production sector model), the direct effect is strong enough that $\varepsilon_M > \varepsilon_X$.

presence of two-sided heterogeneity of trade.[2] Though the result may not be very surprising by itself, note that not only is the trade elasticity ε_j but also the domestic share λ_j is far away from equality between final goods and intermediate goods: the share of intermediate goods is greater than that of final goods in the real world. This means that the welfare evaluation holding the two domestic shares equal might lead to wrong understanding of globalization where fragmentation of production processes plays a prominent role in improving welfare in each country. The finding that the welfare gains from trade are greater for intermediate goods under rapidly rising input trade is also consistent with empirical evidence that input tariff reductions increase industry productivity more than output tariff reductions (Amiti and Konings, 2007; Topalova and Khandelwal, 2011), because such productivity improvement is typically associated with the higher welfare gains from input trade liberalization than from output trade liberalization.

This paper contributes to the literature that examines the impact of imported input on industry productivity, trade flows and welfare gains in a setup with input-output linkages. Antràs et al. (2017) develop a multi-country sourcing model in which more productive final good firms import intermediate goods from a larger number of countries. Under the condition that final goods are non-tradable, they find, like ours, that the aggregate trade elasticity with respect to variable trade cost tend to be higher than the firm-level trade elasticity in a gravity equation of intermediate good trade. However, they employ the Eaton-Kortum (2002) framework for sourcing intermediate goods, which implies that the upstream sector is characterized by perfect competition and selection into the export market is not operative for intermediate good suppliers. Further, they do not address differences in the welfare gains from trade between intermediate goods and final goods, which is one of the main focuses in our paper.

Bernard et al. (2018b) develop a model of two-sided heterogeneity in terms of productivity in intermediate good suppliers and final good firms. Under the condition that final goods are non-tradable, they find the negative degree assortivity among final good firms and intermediate good suppliers, i.e., more productive intermediate good suppliers tend to match with less productive final good firms. While intermediate good suppliers self-select into exporting by incurring fixed trade cost, selection of final good firms is made by matching without paying fixed trade cost. In practice, however, such fixed cost is empirically relevant for explaining why a small fraction of firms import (Kasahara and Lapham 2013; Halpern et al. 2015). Abstracting from matching between final good firms and intermediate good suppliers, we show that trade liberalization gives rise to two-sided reallocations as a result of self-selection into exporting and importing, which turns out to be crucial to derive the difference in the gravity structure of intermediate good trade and final good trade.

Our result of the welfare gains from intermediate good trade is related to that in Melitz and Redding (2014). They show that when non-traded final goods are produced from traded intermediate goods, the welfare gains from trade liberalization are magnified by raising domestic productivity. They consider, however, only perfectly competitive markets in every production sector in which all the firm-level variables do not play a key role in resource reallocations by trade liberalization. This paper, while allowing final goods to be produced from traded intermediate goods, focuses on two-sided heterogeneity and identifies a potential channel through which trade liberalization in intermediate goods can have a different impact from that in final goods due to selections in the two production sectors.

[2] There are two classes of models with input-output linkages of production. First is the model with "roundabout" production in which output is sold to consumers as final goods and also to firms as intermediate goods. This modeling approach is often employed in the recent literature, including Arkolakis et al. (2012, section IV). Second is the model with "vertical" production in which final (intermediate) goods are produced in the downstream (upstream) sector. This modeling approach used in this paper is found in the IO literature and is applied to the trade literature; see for example Ishikawa and Spencer (1999), Ghosh and Morita (2007), and Ara and Ghosh (2016). The difference in the modeling approach has a non-trivial consequence for trade flows and welfare gains through selection in the vertically-related sectors.

Table 1 – Gravity in China's imports

	Overall			Final			Intermediate		
	Total	Extensive	Intensive	Total	Extensive	Intensive	Total	Extensive	Intensive
Distance	-0.758	-0.549	-0.208	-0.689	-0.508	-0.181	-0.797	-0.574	-0.223
	(0.015)	(0.007)	(0.010)	(0.023)	(0.010)	(0.016)	(0.020)	(0.009)	(0.013)
Tariff	-0.148	-0.085	-0.064	-0.106	-0.072	-0.034	-0.184	-0.100	-0.084
	(0.018)	(0.007)	(0.014)	(0.022)	(0.008)	(0.016)	(0.029)	(0.011)	(0.023)
No. of obs	576,509	576,509	576,509	220,693	220,693	220,693	354,976	354,976	354,976
Adj. R^2	0.403	0.498	0.389	0.443	0.478	0.446	0.372	0.510	0.343

Source: Ara and Zhang (2020, Tables 3 and 4)

Notes: Standard errors clustered at product-level are in brackets. Product and year fixed effects are included. All results are statistically significant at the 1% level.

The current paper is most closely related to Ara and Zhang (2020). In that paper, we extend our setup to a multiple-industry, asymmetric-country framework in order to empirically investigate its theoretical prediction. While the two papers have a similar flavor, the scope of the papers is different. Ara and Zhang (2020) focus on country asymmetry to allow for potential differences in trade cost across countries, but complexity of the model leads them to study a special case where either type of goods is only tradable. In contrast, we develop a more general model in which both final goods and intermediate goods are costly traded and address the role played by the interaction between these two types of goods in the welfare gains from trade. Nevertheless, one of the main findings is the same: the trade elasticity is greater for intermediate goods than for final goods. Below we provide some evidence supporting this theoretical prediction.

To assess empirically this pattern, we first divide China's imports in terms of U.S. dollar into final goods and intermediate goods by applying the UN Broad Economic Categories classification to the China Customs database (at the 6-digit HS product level) in 2000-2007. For each product imported from each trading partner in each year, the value of total imports of two types of goods is further decomposed into the number of importing firms with positive trade flows (extensive margin) and the average import value conditional on positive trade flows (intensive margin). We then estimate a gravity equation of our model derived under the condition that only either type of goods are tradable in order to see how the trade elasticity with respect to variable trade cost (distance and tariff) is different across the two types of goods. We also estimate the gravity of overall goods without distinguishing among final goods and intermediate goods.

Table 1 reports the estimates for the trade elasticities that are decomposed into the extensive and intensive margins for each type of goods where all variables are measured in logs. In the estimates of overall goods, the negative relationship between variable trade cost and the total import value is largely accounted for by the extensive margin, which accords well with the previous findings (e.g., Bernard et al., 2007). However, comparing the estimates between the two types of goods, the coefficient of variable trade cost on the total import value is greater for intermediate goods than for final goods, while retaining the major role of the extensive margin in the gravity for both types of goods. To check whether there is a statistically significant difference in the trade elasticity between the two types of goods, we regress the gravity equation with an input dummy, and find that the coefficient difference is significant only for the extensive margin (see Table 5 in Ara and Zhang (2020)). These pieces of evidence highlight a new prediction of our model: the trade elasticity is significantly greater for intermediate goods than for final goods due mainly to the extensive margin.

2 Model

Consider a model in which two symmetric countries costly trade both input and output. There is one industry composed of upstream and downstream sectors in which suppliers produce differentiated input and firms produce differentiated output (using input) in monopolistically competitive markets. Labor is only a factor of production and each country is endowed with L units of labor which is chosen as a numeraire of the model.

2.1 Consumers

Consumers' preference is represented by a CES utility function with elasticity $\sigma > 1$:

$$U = \left(\int_i q_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}},$$

where q_i is output quantity produced by firm i. Utility maximization yields consumers' output demand:

$$q_i = p_i^{-\sigma} P^{\sigma-1} R,$$

where p_i is an output price, R is consumers' aggregate output expenditure and

$$P = \left(\int_i p_i^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}$$

is the price index associated with the output bundle. Defining an aggregate output $Q \equiv U$, we have $PQ = R$.

2.2 Firms

Firms' technology is linear combination of firms' productivity φ_i and input bundle x_i:

$$q_i = \varphi_i x_i.$$

The productivity level φ_i is randomly drawn from a fixed distribution $G(\varphi_i)$ with unbounded upper support, while the input bundle x_i is produced by a CES production function with elasticity σ:

$$x_i = \left(\int_v x_{Div}^{\frac{\sigma-1}{\sigma}} dv + \mathbb{1}_{Mi} \int_v x_{Miv}^{\frac{\sigma-1}{\sigma}} dv \right)^{\frac{\sigma}{\sigma-1}},$$

where x_{Div} and x_{Miv} are domestic and imported input quantity used by firm i and provided by supplier v,[3] and $\mathbb{1}_{Mi}$ is an indicator function which takes the value of one if firm i uses imported input and zero otherwise. To facilitate the analysis below, we follow Bernard et al. (2018b) in assuming that only input is used for output production and the elasticity of substitution between input in firms' technology is identical with the elasticity of substitution between output in consumers' preference, but these simplifications would not affect the qualitative results of the paper. Cost minimization yields firm i's input demand for variety v:

$$x_{Div} = p_{Dv}^{-\sigma} c_i^{\sigma-1} e_i,$$
$$x_{Miv} = p_{Mv}^{-\sigma} c_i^{\sigma-1} e_i,$$

[3]The subscripts i and v are attached to relevant variables to firms and suppliers respectively.

where p_{Dv} and p_{Mv} are domestic and imported input prices set by supplier v (common to all firms i) and

$$c_i = \left(\int_v p_{Dv}^{1-\sigma} dv + \mathbb{1}_{Mi} \int_v p_{Mv}^{1-\sigma} dv \right)^{\frac{1}{1-\sigma}},$$
$$e_i = \int_v e_{Div} dv + \mathbb{1}_{Mi} \int_v e_{Miv} dv,$$

where $e_{Div} = p_{Dv} x_{Div}$ and $e_{Miv} = p_{Mv} x_{Miv}$ are domestic and imported input expenditure incurred by firm i and provided by supplier v. A few points are in order for this specification. First, substituting x_{Div} and x_{Miv} into the CES production function and rearranging, firm i's total input expenditure e_i is expressed as

$$e_i = \frac{c_i}{\varphi_i} q_i. \tag{1}$$

Thus, firm i's input expenditure increases with its output quantity q_i but decreases with its productivity level φ_i. Second, from the input pricing rules set by suppliers and selection into exporting among suppliers, it follows that the price index associated with the input bundle (referred to as firm i's unit cost hereafter) is expressed as

$$c_i^{1-\sigma} = c_D^{1-\sigma}(1 + \mathbb{1}_{Mi} \tau_M^{1-\sigma} \Delta) \tag{2}$$

where $c_D^{1-\sigma} = \int_v p_{Dv}^{1-\sigma} dv$, τ_M is variable trade cost of input and Δ is the market share of exporting suppliers defined later. To understand this, suppose that τ_M is sufficiently high that no supplier profitably exports. Then $\Delta = 0$ and the unit cost is the same across all firms. Evidence suggests however that firms using both domestic and imported input have a cost advantage over firms using only domestic input (Halpern et al., 2015). Further, even if τ_M is not prohibitively high, there is selection into exporting in the upstream sector and not all suppliers export. Then $\Delta < 1$ and the unit cost of importing firms is lower than that of non-importing firms. Intuitively, firms using both domestic and imported input can exploit a "love-of-variety" effect for output production and raise their production efficiency.

Given the sourcing strategy and associated unit cost, firm i's domestic profit is given by

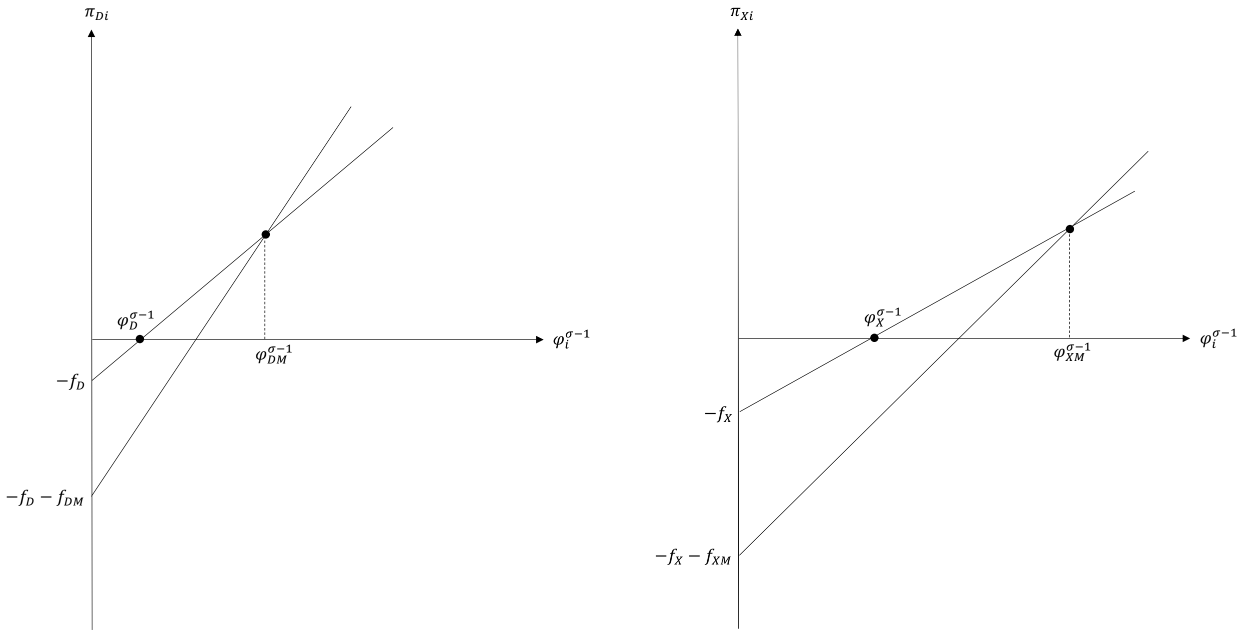
$$\pi_{Di} = \left(p_{Di} - \frac{c_i}{\varphi_i} \right) p_{Di}^{-\sigma} P^{\sigma-1} R - f_{Di},$$

where f_{Di} is firm i's fixed production cost that satisfies $f_{Di} = f_D + \mathbb{1}_{Mi} f_{DM}$. If firm i sources input only from the domestic market, it incurs fixed cost of domestic sourcing: $f_{Di} = f_D$. In contrast, if firm i also sources input from abroad, it incurs *additional* fixed cost of foreign sourcing: $f_{Di} = f_D + f_{DM}$. This assumption follows from the firm importing literature in which importing firms incur higher fixed sourcing cost than non-importing firms to serve the domestic market (e.g., Antràs and Helpman, 2004). Firm i chooses its domestic output price p_{Di} to maximize domestic profit. Observing that firm i takes the term $P^{\sigma-1}R$ as given, profit maximization yields the following pricing rule for domestic output:

$$p_{Di} = \frac{\sigma}{\sigma - 1} \frac{c_i}{\varphi_i}.$$

This in turn gives us the following expression of domestic revenue:

$$r_{Di} = p_{Di} q_{Di} = \sigma B c_i^{1-\sigma} \varphi_i^{\sigma-1}$$



(a) Domestic profit

(b) Export profit

Figure 1 – Domestic and export profit in downstream sector

where

$$B = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} P^{\sigma-1} R$$

is the index of output market demand. Domestic profit is

$$\pi_{Di} = \frac{r_{Di}}{\sigma} - f_{Di} = Bc_i^{1-\sigma}\varphi_i^{\sigma-1} - f_{Di}.$$

Comparing the unit cost c_i and fixed cost f_{Di}, firm i chooses its domestic sourcing strategy so that

$$\pi_{Di} = \max\left\{0,\ Bc_D^{1-\sigma}\varphi_i^{\sigma-1} - f_D,\ B(1+\tau_M^{1-\sigma}\Delta)c_D^{1-\sigma}\varphi_i^{\sigma-1} - f_D - f_{DM}\right\}.$$

Figure 1(a) draws domestic profit. In $(\varphi_i^{\sigma-1}, \pi_{Di})$ space, the slope of π_{Di} is $Bc_D^{1-\sigma}$ for non-importing firms while $Bc_D^{1-\sigma}(1+\tau_M^{1-\sigma}\Delta)$ for importing firms, reflecting that variable profit is greater for importing firms due to lower unit cost. However, the intercept of π_{Di} is $-f_D$ for non-importing firms while $-f_D - f_{DM}$ for importing firms, reflecting that fixed cost is greater for importing firms due to additional sourcing cost. From this tradeoff, there are productivity cutoffs at which domestic profit of importing firms exceed that of non-importing firms, namely $\varphi_{DM}^{\sigma-1} > \varphi_D^{\sigma-1}$. This ensures that a fraction of firms above $\varphi_{DM}^{\sigma-1}$ use both domestic and imported input while others between $\varphi_D^{\sigma-1}$ and $\varphi_{DM}^{\sigma-1}$ use only domestic input to serve the domestic market.

Among operating firms in the domestic market, more efficient firms above some cutoff can enter the export market. *Additional* profit from exporting is given by

$$\pi_{Xi} = \left(p_{Xi} - \frac{\tau_X c_i}{\varphi_i}\right) p_{Xi}^{-\sigma} P^{\sigma-1} R - f_{Xi},$$

where f_{Xi} is firm i's fixed trade cost that satisfies $f_{Xi} = f_X + \mathbb{1}_{Mi} f_{XM}$. The fixed cost structure is similar to that in the domestic market in that importing firms incur higher fixed sourcing cost than non-importing firms to serve the export market. On top of fixed trade cost, exporting firms also incur variable trade cost of output τ_X and hence unit cost is higher for exporting firms than for domestic firms.[4] Profit maximization yields the following pricing rule for exported output:

$$p_{Xi} = \frac{\sigma}{\sigma - 1} \frac{\tau_X c_i}{\varphi_i}.$$

This in turn gives us the following expression of export revenue:

$$r_{Xi} = p_{Xi} q_{Xi} = \sigma B (\tau_X c_i)^{1-\sigma} \varphi_i^{\sigma-1}.$$

Export profit is

$$\pi_{Xi} = \frac{r_{Xi}}{\sigma} - f_{Xi} = B(\tau_X c_i)^{1-\sigma} \varphi_i^{\sigma-1} - f_{Xi}.$$

Comparing the unit cost $\tau_X c_i$ and fixed cost f_{Xi}, firm i chooses its export sourcing strategy so that

$$\pi_{Xi} = \max\left\{0,\ B(\tau_X c_D)^{1-\sigma}\varphi_i^{\sigma-1} - f_X,\ B(1+\tau_M^{1-\sigma}\Delta)(\tau_X c_D)^{1-\sigma}\varphi_i^{\sigma-1} - f_X - f_{XM}\right\}.$$

Figure 1(b) draws export profit. Comparison to Figure 1(a) reveals a similar pattern between the domestic and export markets: there are productivity cutoffs at which export profit of importing firms exceed that of non-importing firms, namely $\varphi_{XM}^{\sigma-1} > \varphi_X^{\sigma-1}$. This ensures that a fraction of exporting firms above $\varphi_{XM}^{\sigma-1}$ use both domestic and imported input, which emerges only if they incur higher fixed cost to serve the export market.

To characterize the equilibrium of the downstream sector, we need to identify the productivity cutoffs. From Figure 1 and the domestic and export profit above, these cutoffs are given by

$$\begin{aligned} B c_D^{1-\sigma} \varphi_D^{\sigma-1} &= f_D, \\ B(\tau_X c_D)^{1-\sigma} \varphi_X^{\sigma-1} &= f_X, \\ B(\tau_M c_D)^{1-\sigma} \Delta \varphi_{DM}^{\sigma-1} &= f_{DM}, \\ B(\tau_X \tau_M c_D)^{1-\sigma} \Delta \varphi_{XM}^{\sigma-1} &= f_{XM}. \end{aligned} \tag{3}$$

From the productivity cutoff φ_c for $c \in \{D, X, M, XM\}$, we have the following selection patterns that arise in the downstream sector. First, it follows from (3) that

$$\left(\frac{\varphi_X}{\varphi_D}\right)^{\sigma-1} = \frac{\tau_X^{\sigma-1} f_X}{f_D}, \qquad \left(\frac{\varphi_{XM}}{\varphi_{DM}}\right)^{\sigma-1} = \frac{\tau_X^{\sigma-1} f_{XM}}{f_{DM}}. \tag{4}$$

Assuming variable trade cost τ_X and fixed trade cost f_X, f_{XM} are large so that $\varphi_X > \varphi_D$ and $\varphi_{XM} > \varphi_{DM}$, (4) means firm selection into exporting, which holds not only for firms using domestic input but also for firms using imported input; thus not all importing firms export. Moreover,

$$\left(\frac{\varphi_{DM}}{\varphi_D}\right)^{\sigma-1} = \frac{1}{\Delta} \frac{\tau_M^{\sigma-1} f_{DM}}{f_D}, \qquad \left(\frac{\varphi_{XM}}{\varphi_X}\right)^{\sigma-1} = \frac{1}{\Delta} \frac{\tau_M^{\sigma-1} f_{XM}}{f_X}. \tag{5}$$

[4] The subscripts X and M are attached to relevant variables to output trade and input trade respectively.

Assuming variable trade cost τ_M and fixed trade cost f_{DM}, f_{XM} are large so that $\varphi_{DM} > \varphi_D$ and $\varphi_{XM} > \varphi_X$, (5) means firm selection into importing, which holds not only for the domestic market in Figure 1(a) but also for the export market in Figure 1(b); thus not all exporting firms import. These selection patterns accord well with empirical evidence that firm selection is ubiquitous for both exporting and importing (Bernard et al., 2007, 2012, 2018a). As a result, the productivity cutoffs in the downstream sector satisfy

$$\varphi_D < \min\{\varphi_{DM}, \varphi_X\} < \varphi_{XM}.$$

The ranking of the productivity cutoffs means that, among operating firms, those with the lowest productivity between φ_D and $\min\{\varphi_{DM}, \varphi_X\}$ use only domestic input and sell their output in only the domestic market, whereas those with the highest productivity above φ_{XM} use both domestic and imported input and sell their output in both the domestic and export markets.

In addition to the zero profit cutoff condition, we impose a free entry condition. Upon incurring fixed entry cost f_E, a mass of entrants M_E observe their productivity level φ_i drawn from a fixed distribution $G(\varphi_i)$. Then firm i decides whether to enter the downstream sector by choosing markets from which to source input as well as to which to provide output, or to exit without producing. Obviously the former outcome occurs whenever φ_i is greater than the domestic productivity cutoff φ_D in (3). Hence the free entry condition is defined as

$$\int_{\varphi_D}^{\infty} \pi_i dG(\varphi_i) = f_E,$$

where the left-hand side is expected profit among operating firms which is equivalent to $\frac{1}{M_E}\int_i \pi_i di$. Using the productivity cutoffs in (3), this condition can be expressed as

$$f_D J(\varphi_D) + f_X J(\varphi_X) + f_{DM} J(\varphi_{DM}) + f_{XM} J(\varphi_{XM}) = f_E, \tag{6}$$

where $J(\varphi_c) = \int_{\varphi_c}^{\infty} \left[\left(\frac{\varphi_i}{\varphi_c}\right)^{\sigma-1} - 1\right] dG(\varphi_i)$ is a strictly decreasing function of φ_c. It is worth emphasizing that the zero profit cutoff condition (3) and the free entry condition (6) cannot characterize the downstream sector. As in (3), the import productivity cutoffs φ_{DM} and φ_{XM} are affected by the market share of input exporters Δ that is endogenously determined in the upstream sector. This implies that any trade shocks that induce changes in Δ in the upstream sector has an impact on firm selection in the downstream sector through the availability of input used to output production by firms.

We conclude this section by deriving the domestic output share as well as the domestic input share. First, from the fact that the aggregate revenue of firms equals the aggregate expenditure of consumers, the domestic output share (from a viewpoint of consumers) is given by

$$\lambda_X = \frac{\int_i r_{Di} di}{\int_i r_i di} = \frac{1}{1 + \tau_X^{1-\sigma} \Lambda_X}, \tag{7}$$

where

$$\Lambda_X = \frac{V(\varphi_X) + \tau_M^{1-\sigma} \Delta V(\varphi_{XM})}{V(\varphi_D) + \tau_M^{1-\sigma} \Delta V(\varphi_{DM})}$$

and $V(\varphi_c) = \int_{\varphi_c}^{\infty} \varphi_i^{\sigma-1} dG(\varphi_i)$ is a strictly decreasing function of φ_c. It can be shown that the numerator and denominator of Λ_X are proportional to $\int_i r_{Xi} di$ and $\int_i r_{Di} di$ respectively. Following Melitz and Redding (2015), Λ_X is referred to as the market share of exporting firms, though we recognize that there are lots of definitions of the market share in the literature. From this, if variable trade cost of input τ_M is sufficiently high so that no supplier profitably exports ($\Delta = 0$), the domestic output share (7) collapses to that in the plain Melitz model.[5] Second, from the fact that aggregate revenue of suppliers equals aggregate expenditure of firms, the domestic input share (from a viewpoint of firms) is given by

$$\lambda_M = \frac{\int_i e_{Di} di}{\int_i e_i di} = \frac{1}{1 + \tau_M^{1-\sigma} \Delta \Lambda_M}, \tag{8}$$

where

$$\Lambda_M = \frac{V(\varphi_{DM}) + \tau_X^{1-\sigma} V(\varphi_{XM})}{V(\varphi_D) + \tau_X^{1-\sigma} V(\varphi_X)}.$$

It can be shown that the numerator and denominator of Λ_M are proportional to $\int_i e_{Mi} di$ and $\int_i e_{Di} di$ respectively, and Λ_M is referred to as the market share of importing firms. Not surprisingly, if variable trade cost of input τ_M is sufficiently high so that no supplier profitably exports ($\Delta = 0$) and no firm profitably imports ($\Lambda_M = 0$), the domestic input share (8) collapses to unity. To make the analysis more interesting, we restrict the range of τ_M under which $\Delta < 1$ and $\Lambda_M < 1$.

2.3 Suppliers

Suppliers' technology is represented by a linear cost function of labor that involves fixed cost and marginal cost where the latter cost is inversely related to productivity:

$$l_v^p = k + \frac{x_v}{\phi_v}.$$ [6]

All suppliers incur the same fixed cost k but it varies with the markets to which suppliers provide their input. The productivity level ϕ_v is randomly drawn from a fixed distribution $G(\phi_v)$ with unbounded upper support, while $x_v = \int_i x_{iv} di$ is total input quantity provided by supplier v where x_{iv} is firm i's input demand for variety v as in the last section. These features of input production implies that the upstream sector is also characterized by a monopolistically competitive market and self-selection in exporting among suppliers as in the downstream sector.

Given the production technology and firms' input demand, supplier v's domestic profit is given by

$$\pi_{Dv} = \left(p_{Dv} - \frac{1}{\phi_v} \right) p_{Dv}^{-\sigma} \int_i c_i^{\sigma-1} e_i di - k_D,$$

where k_D is suppliers' fixed production cost which is common for all operating suppliers in the domestic market. Supplier v chooses its domestic price p_{Dv} to maximize domestic profit. Observing that supplier v takes the term $\int_i c_i^{\sigma-1} e_i di$ as given, profit maximization yields the pricing rule for domestic input:

$$p_{Dv} = \frac{\sigma}{\sigma - 1} \frac{1}{\phi_v}$$

and the following expression of domestic revenue:

$$r_{Dv} = p_{Dv} x_{Dv} = \sigma A \phi_v^{\sigma-1}$$

[5] See, for example, eq (18) in Melitz and Redding (2015) who study the special case without importing.

[6] The superscript p is attached to stress production worker. Labor is also used for entry which is denoted by l_v^e as shown later.

where

$$A = \frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}} c_D^{\sigma-1} \int_i e_{Di} di$$

is the index of input market demand. Domestic profit is

$$\pi_{Dv} = \frac{r_{Dv}}{\sigma} - k_D = A\phi_v^{\sigma-1} - k_D.$$

Supplier v chooses its domestic production strategy so that

$$\pi_{Dv} = \max\left\{0,\ A\phi_v^{\sigma-1} - k_D\right\}.$$

Like Figure 1(a), domestic profit can be drawn in $(\phi_v^{\sigma-1}, \pi_{Dv})$ space where the slope of π_{Dv} is A and the intercept is $-k_D$. In contrast to firms who decide not only to serve the domestic market but also to import input from abroad in their domestic strategy, suppliers decide only to serve the domestic market in their domestic strategy as they do not import firms' output to produce their input in the presence of vertical linkages between upstream and downstream sectors. As a result, there is a unique productivity cutoff at which domestic profit of operating suppliers is zero, namely $\phi_D^{\sigma-1}$. This ensures that a fraction of suppliers above $\phi_D^{\sigma-1}$ provide domestic input to firms while others below $\phi_D^{\sigma-1}$ immediately exit.

Among operating suppliers in the domestic market, more efficient suppliers above some cutoff can enter the export market. *Additional* profit from exporting is given by

$$\pi_{Mv} = \left(p_{Mv} - \frac{\tau_M}{\phi_v}\right) p_{Mv}^{-\sigma} \int_i c_i^{\sigma-1} e_i di - k_M,$$

where k_M is suppliers' fixed export cost which is common for all exporting suppliers. On top of fixed trade cost, exporting suppliers also incur variable trade cost of input τ_M.[7] Clearly trade cost structure is similar to firms in the downstream sector. Profit maximization yields the pricing rule for exported input:

$$p_{Mv} = \frac{\sigma}{\sigma-1}\frac{\tau_M}{\phi_v}.$$

Note that the input prices satisfy $p_{Mv} = \tau_M p_{Dv}$ just like the output prices satisfy $p_{Xi} = \tau_X p_{Di}$. As seen in (2), this gives rise to firms' unit cost in such a way that a markup on differentiated input (set by suppliers) is fully passed through to the output price bundled from differentiated input (set by firms), i.e., double marginalization takes place between firms and suppliers. Combining the input pricing rule with the imported input demand, we get the following expression of export revenue:

$$r_{Mv} = p_{Mv} x_{Mv} = \sigma A \tau_M^{1-\sigma} \Lambda_M \phi_v^{\sigma-1}.$$

It is important to see that the market share of importing firms Λ_M enters the revenue expression of exporting suppliers. This is because, so long as τ_M is not prohibitively high, trade cost limits the set of importing firms to which exporting suppliers sell their input. As a result of this additional channel operating through selection in importing among firms, variable trade cost of input τ_M affects supplier's export revenue r_{Mv} not only through supplies' shipment (i.e., through $\tau_M^{1-\sigma}$) directly, but also through firms' market share who profitably make use

[7] Recall that the subscript M is attached to relevant variables to input trade in this paper.

of imported input (i.e., through Λ_M) indirectly. Export profit is

$$\pi_{Mv} = \frac{r_{Mv}}{\sigma} - k_M = A\tau_M^{1-\sigma}\Lambda_M\phi_v^{\sigma-1} - k_M.$$

Supplier v chooses its export strategy so that

$$\pi_{Mv} = \max\left\{0,\ A\tau_M^{1-\sigma}\Lambda_M\phi_v^{\sigma-1} - k_M\right\}.$$

Like Figure 1(b), export profit can be drawn in $(\phi_v^{\sigma-1}, \pi_{Dv})$ space and there is a unique productivity cutoffs, namely $\phi_M^{\sigma-1}$, above which suppliers profitably export input to the foreign market.

To characterize the equilibrium of the upstream sector, we identify the productivity cutoffs of suppliers. From the domestic and export profit above, these cutoffs are given by

$$\begin{aligned} A\phi_D^{\sigma-1} &= k_D, \\ A\tau_M^{1-\sigma}\Lambda_M\phi_M^{\sigma-1} &= k_M. \end{aligned} \tag{9}$$

From the productivity cutoff ϕ_c for $c \in \{D, M\}$, we have the following selection pattern in the upstream sector:

$$\left(\frac{\phi_M}{\phi_D}\right)^{\sigma-1} = \frac{1}{\Lambda_M}\frac{\tau_M^{\sigma-1}k_M}{k_D}. \tag{10}$$

Assuming variable trade cost τ_M and fixed trade cost k_M are large (while keeping $\Lambda_M < 1$) so that $\phi_M > \phi_D$, (10) means supplier selection into exporting. Note the similarity between (5) and (10) in the selection patterns. Whereas (5) imposes firm selection into importing in the downstream sector, (10) imposes supplier selection into exporting in the upstream sector.

In addition to the zero profit cutoff condition, we impose a free entry condition. Upon incurring fixed entry cost k_E, a mass of entrants N_E observe their productivity level ϕ_v drawn from a fixed distribution $G(\phi_v)$. Then supplier v decides whether to enter the upstream sector by choosing markets to which to provide input or to exit without producing, and the former outcome occurs whenever ϕ_v is greater than the domestic productivity cutoff ϕ_D in (9). Hence the free entry condition is defined as

$$\int_{\phi_D}^{\infty} \pi_v dG(\phi_v) = k_E,$$

where the left-hand side is equivalent to $\frac{1}{N_E}\int_v \pi_v dv$. Using the productivity cutoffs in (9), we get

$$k_D J(\phi_D) + k_M J(\phi_M) = k_E, \tag{11}$$

where the functional form of $J(\phi_c)$ is the same as that of $J(\varphi_c)$. Note that (9) and (11) cannot characterize the upstream sector. The export productivity cutoff ϕ_M is affected by the market share of output importers Λ_M which means that the range of input exporters depends on the range of output importers. As a result of this, the impact of trade cannot be examined without taking into account the interaction between the upstream and downstream sectors.

We conclude this section by deriving the domestic input share. Since suppliers' input revenue equals firms' input expenditure, the domestic input share (8) is alternatively defined as the ratio of the aggregate revenue of domestic input to the aggregate revenue of total input earned by suppliers. Using the domestic and export revenue above, the domestic input share (8) is given by

$$\lambda_M = \frac{\int_v r_{Dv} dv}{\int_v r_v dv} = \frac{1}{1 + \tau_M^{1-\sigma} \Delta \Lambda_M},$$

where

$$\Delta = \frac{V(\phi_M)}{V(\phi_D)}.$$

We can show that the numerator and denominator of Δ are proportional to $\int_v r_{Mv} dv$ and $\int_v r_{Dv} dv$ respectively, and Δ is referred to as the market share of exporting suppliers. Notice that $\Delta < 1$ so long as supplier selection into exporting (10) holds. The expression of unit cost (2) follows from this market share Δ and the pricing rules set by suppliers p_{Dv}, p_{Mv}.

2.4 Economy

To close the model, we impose the labor market clearing condition. Noting that labor is used in both downstream and upstream sectors, the condition is expressed as

$$\int_i l_i di + \int_v l_v dv = L,$$

where $l_i = l_i^e + l_i^p$ and $l_v = l_v^e + l_v^p$ denote labor used for entry and production by firm i and supplier v respectively. Substituting labor used by firms and suppliers, the labor market clearing condition is simply expressed as

$$R = L.$$

Moreover, aggregate labor used in each production sector is given by

$$\int_i l_i di = \frac{L}{\sigma}, \quad \int_v l_v dv = \left(\frac{\sigma - 1}{\sigma} \right) L,$$

and hence the labor allocation between the two production sectors is exogenously fixed. Using the labor market clearing condition as well as the zero profit cutoff and free entry conditions, the mass of entrants M_E, N_E can be written as a function of labor endowment L and the productivity cutoffs φ_c, ϕ_c.

Welfare per worker (equivalent to real wage) is expressed as

$$W = \sigma^{\frac{2}{1-\sigma}} \left(\frac{\sigma - 1}{\sigma} \right)^{\frac{2\sigma-1}{\sigma-1}} L^{\frac{2}{\sigma-1}} (f_D k_D)^{-\frac{1}{\sigma-1}} \varphi_D \phi_D \lambda_M^{\frac{1}{\sigma-1}}. \tag{12}$$

As is standard in the literature, welfare rises with country size L and falls with fixed production cost f_D, k_D. More important however are the sufficient statistics for welfare in the presence of vertical linkages: when both input and output are costly traded subject to selection, welfare is endogenously determined not only by the domestic productivity cutoff of firms φ_D in the downstream sector but also by the domestic productivity cutoff of suppliers ϕ_D and the domestic input share λ_M in the upstream sector.

This completes the characterization of the model. The next section solves for the equilibrium to address the impact of trade on resource reallocations, trade flows and welfare gains.

3 Equilibrium

Since there are the eight equilibrium conditions in the model ((3), (6), (9), (11)), these eight conditions jointly provide implicit solutions for the following eight unknowns:

$$\varphi_D,\ \varphi_X,\ \varphi_{DM},\ \varphi_{XM},\ B,\ \phi_D,\ \phi_M,\ A,$$

where the labor market clearing condition is omitted by choosing labor as a numeraire of the model. Once these unknowns are determined, the other endogenous variables can be written as a function of them.

3.1 Resource Reallocations

We start with examining the impact of trade liberalization on the productivity cutoffs. Recall that the market share of exporting suppliers Δ and that of importing firms Λ_M enter the zero profit cutoff conditions in (3) and (9) respectively. As a result, changes in the equilibrium variables by trade liberalization depend critically on how these market shares are affected by such liberalization. For example, totally differentiating $\Delta = V(\phi_M)/V(\phi_D)$, changes in the market share of exporting suppliers in the upstream sector are given by

$$d\ln\Delta = -\theta_M d\ln\phi_M + \theta_D d\ln\phi_D,$$

where $\theta_c \equiv -d\ln V(\phi_c)/d\ln\phi_c$ can be thought of as the extensive margin elasticity in the upstream sector.[8] By definition, the extensive margin elasticity θ_c is a function of the productivity cutoff ϕ_c, and changes in Δ come not only from changes in ϕ_c directly but also from changes in θ_c indirectly. To make the following analysis as simple as possible, we hereafter restrict our attention to a subset of the general productivity functions where the extensive margin elasticity satisfies $\theta_c = \theta$ for any c. This means that the extensive margin elasticity is the same across all productivity cutoffs taking a constant value regardless of suppliers' global status. In a similar vein, let $\vartheta_c \equiv -d\ln V(\varphi_c)/d\ln\varphi_c$ denote the extensive margin elasticity in the downstream sector and we assume that $\vartheta_c = \theta$ for any c. Admittedly, this is a restrictive assumption but it holds under one of the most commonly-used distributions: Pareto. Specifically, if φ_i and ϕ_v are distributed Pareto with a common shape parameter γ, the extensive margin elasticities in the two production sectors are given by

$$\theta_c = \vartheta_c = \gamma - (\sigma - 1) \equiv \theta. \tag{13}$$

Although recent empirical work reports that the extensive margin elasticities are less likely to be constant (e.g., Bas et al., 2017), the Pareto distribution is nonetheless a good approximation of observed micro-level data and we follow the standard practice in the literature.

With this restriction, we first consider the impact of input trade liberalization on the productivity cutoffs. Differentiating and solving the equilibrium conditions simultaneously yields the following expressions of changes in the domestic productivity cutoffs φ_D, ϕ_D (see Appendix A.1 for proof):

$$\begin{aligned} d\ln\varphi_D &= -\left(\frac{(\sigma-1)(1-\lambda_M)}{\sigma-1-\theta}\right) d\ln\tau_M, \\ d\ln\phi_D &= -\left(\frac{(\sigma-1)(1-\lambda_M)}{\sigma-1-\theta}\right) d\ln\tau_M. \end{aligned} \tag{14}$$

[8] See Arkolakis et al. (2012, p.110) for the single production sector model.

(14) shows that reductions in τ_M increase φ_D in the downstream sector as well as ϕ_D in the upstream sector, forcing the least productive firms and suppliers to exit the respective production sector if and only if

$$\sigma - 1 > \theta. \tag{15}$$

In the inequality, $\sigma - 1$ is the (common) intensive margin elasticities under the CES utility/production functions, whereas θ is the (common) extensive margin elasticities in the two production sectors under the distributional assumption. Thus, (15) requires that the extensive margin elasticities are not too large relative to the intensive margin elasticities. We impose the inequality in the analysis below, since (14) implies that less productive firms and suppliers otherwise enter the respective production sector by input trade liberalization, which would be less likely in reality.

It is possible to see the shift in the other productivity cutoffs. In the upstream sector, from the free entry condition (11), it follows that the export productivity cutoff ϕ_M shifts in the opposite directions to ϕ_D, and hence input trade liberalization generates the Melitz-type resource reallocations:

$$\frac{d\ln\phi_D}{d\ln\tau_M} < 0 < \frac{d\ln\phi_M}{d\ln\tau_M}.$$

In the downstream sector, from selection into exporting (4), the export productivity cutoff φ_X (φ_{XM}) change proportionately to φ_D (φ_{DM}). Moreover, from the free entry condition (6), the import productivity cutoff φ_{DM} (φ_{XM}) shifts in the opposite direction to φ_D (φ_X). Together with (14), we have

$$\frac{d\ln\varphi_D}{d\ln\tau_M} = \frac{d\ln\varphi_X}{d\ln\tau_M} < 0 < \frac{d\ln\varphi_{DM}}{d\ln\tau_M} = \frac{d\ln\varphi_{XM}}{d\ln\tau_M}.$$

This means that resource reallocations arise even within exporting firms: more productive firms sourcing input from multiple markets expand by input trade liberalization, whereas less productive firms sourcing input from only a single market shrink by such liberalization. As a result, firms that simultaneously export and import are more likely to benefit from input trade liberalization, magnifying the effect of initial productivity differences and leading to sales concentration toward these most globalized firms (Bernard et al., 2018a).

Lemma 1: *If both input and output are costly traded subject to selection under (15),*

(i) Input trade liberalization gives rise to resource reallocations not only in the upstream sector but also in the downstream sector.

(ii) Input trade liberalization simultaneously induces such reallocations even within exporting firms: more (less) productive exporting firms sourcing input from multiple markets (a single market) expand (shrink).

Let us next investigate the impact of output trade liberalization. As shown in Appendix A.2, reductions in τ_X have the following impact on the domestic productivity cutoffs φ_D, ϕ_D:

$$\begin{aligned} d\ln\varphi_D &= -\left(1 - \lambda_X + \frac{\theta(\mu_D - \mu_M)(1-\lambda_M)}{\sigma - 1 - \theta}\right) d\ln\tau_X, \\ d\ln\phi_D &= -\left(\frac{(\sigma-1)(\mu_D - \mu_M)(1-\lambda_M)}{\sigma - 1 - \theta}\right) d\ln\tau_X, \end{aligned} \tag{16}$$

where μ_D (μ_M) is the domestic output share of firms using only domestic input (both domestic and imported input).[9] (16) shows that reductions in τ_X increase ϕ_D in the upstream sector as well as φ_D in the downstream sector, forcing the least productive suppliers and firms to exit in each production sector if and only if (15) and $\mu_D - \mu_M > 0$ where the latter condition is equivalent to

$$\frac{V(\varphi_{XM})}{V(\varphi_{DM})} > \frac{V(\varphi_X)}{V(\varphi_D)}. \tag{17}$$

Noticing that $V(\varphi_c)$ is proportional to the aggregate output produced by firms above the productivity cutoff φ_c, $V(\varphi_{XM})/V(\varphi_{DM})$ is the output share of exporting firms conditional on also importing, whereas $V(\varphi_X)/V(\varphi_D)$ is the output share of exporting firms conditional on producing. The inequality would be likely to hold in the circumstance where exporting and importing exhibit some complementarity: firms engaging in both exporting and importing increment their output relatively more than firms engaging in only one of these global activities. Based on the recent empirical finding that intense importers tend to be also intense exporters (Blaum, 2019), we will assume that not only is (15) but also (17) is satisfied in the following analysis.

We can see the shift in the other productivity cutoffs. On the one hand, reductions in τ_X increase ϕ_D but decrease ϕ_M, giving rise to resource reallocations from less productive suppliers to more productive suppliers in the upstream sector, just as in reductions in τ_M. On the other hand, from selection into importing (5) and the free entry condition (6), reductions in τ_X increase φ_D and φ_{DM} but decrease φ_X and φ_{DM} in such a way that

$$\frac{d\ln\varphi_D}{d\ln\tau_X} = \frac{d\ln\varphi_{DM}}{d\ln\tau_X} < 0 < \frac{d\ln\varphi_X}{d\ln\tau_X} = \frac{d\ln\varphi_{XM}}{d\ln\tau_X}.$$

This means that, in contrast to input trade liberalization, resource reallocations arises within importing firms: more productive firms providing output to multiple markets expand by output trade liberalization, while less productive firms providing output in only a single market shrink by such liberalization. Despite this difference, however, firms that simultaneously export and import are more likely to benefit from output trade liberalization (Bernard et al., 2018a). While the finding looks similar to the existing result in the literature exploring firms' export and import decisions, it operates through different channels. For example, Bernard et al. (2018a) focus on strategic market power across a small number of global firms where input is produced under perfect competition. In contrast, we focus on endogenous selection across measure-zero producers in the vertically-related sectors where input is produced under imperfect competition.

Lemma 2: *If both output and input are costly traded subject to selection under (15) and (17),*

(i) *Output trade liberalization gives rise to resource reallocations not only in the downstream sector but also in the upstream sector.*

(ii) *Output trade liberalization simultaneously induces such reallocations even within importing firms: more (less) productive importing firms providing output to multiple markets (a single market) expand (shrink).*

The difference in the impacts of trade shows that input trade liberalization may require less strict conditions than output trade liberalization to trigger the resource reallocations in the vertically-related sectors: input trade

[9] $\mu_D = \frac{1}{1+\tau_X^{1-\sigma}\frac{V(\varphi_X)}{V(\varphi_D)}}$ and $\mu_M = \frac{1}{1+\tau_X^{1-\sigma}\frac{V(\varphi_{XM})}{V(\varphi_{DM})}}$. Note that these always satisfy $1 > \mu_D - \mu_M$.

liberalization requires only (15) while output trade liberalization requires both (15) and (17). More importantly, the difference in (14) and (16) illustrates potential channels through which input trade liberalization has more significant impacts on the trade-induced reallocations than output trade liberalization found by empirical work (e.g., Amiti and Konings, 2007; Topalova and Khandelwal, 2011). In fact, the comparison of the impact on the domestic productivity cutoff in the *upstream* sector reveals that

$$\left|\frac{d\ln\phi_D}{d\ln\tau_M}\right| > \left|\frac{d\ln\phi_D}{d\ln\tau_X}\right|,$$

and input trade liberalization always gives rise to greater resource reallocations among suppliers. As for changes in the domestic productivity cutoff in the *downstream* sector,

$$\left|\frac{d\ln\varphi_D}{d\ln\tau_M}\right| > \left|\frac{d\ln\varphi_D}{d\ln\tau_X}\right| \quad \Longleftrightarrow \quad (\sigma-1-\theta)(\lambda_X-\lambda_M)+\theta(1-\mu_D+\mu_M)(1-\lambda_M)>0.$$

Hence, the sufficient condition for this inequality is $\lambda_X \geq \lambda_M$, i.e., the domestic output share is greater than or equal to the domestic input share. This would be satisfied in current globalization where the input trade share is larger than the output trade share in the world trade volumes.

3.2 Trade Flows

Having shown the impact of trade on resource reallocations in the vertically-related sectors, let us turn to the impact on trade flows. We will continue to examine the impact of input trade liberalization and output trade liberalization separately, but note that trade liberalization in either type of goods affects trade flows of both types of goods. For example, input trade liberalization affects not only input trade flows directly but also output trade flows indirectly. Thus we first analyze the direct effect of trade liberalization on each type of goods, and then analyze the indirect effect of such liberalization.

Consider first the impact of input trade liberalization. To see the sensitivity of input trade flows to changes in variable trade cost of input, we derive the (full) trade elasticity with respect to variable trade cost below. Following Melitz and Redding (2015) and using the domestic input share (8), this trade elasticity is given by

$$\begin{aligned}\varepsilon_M &= -\frac{d\ln\left(\frac{1-\lambda_M}{\lambda_M}\right)}{d\ln\tau_M}\\ &= \underbrace{(\sigma-1)}_{\text{Intensive margin elasticity}} + \underbrace{\left(-\frac{d\ln\Delta}{d\ln\tau_M}\right)}_{\substack{\text{Exporter extensive margin elasticity}\\ \text{in upstream sector}}} + \underbrace{\left(-\frac{d\ln\Lambda_M}{d\ln\tau_M}\right)}_{\substack{\text{Importer extensive margin elasticity}\\ \text{in downstream sector}}}.\end{aligned}$$

The fact that the extensive margin elasticity stems from the upstream and downstream sectors indicates that reductions in variable trade cost of input allow not only suppliers to export input more easily, but also firms to import input that are used for their production more easily. In other words, the impact of trade on input trade flows can be magnified in the presence of vertical linkages, due to additional entry that takes place in the respective production sector. Moreover, with a constant extensive margin elasticity $\theta_c = \vartheta_c = \theta$, the exporter and importer extensive margin elasticities are the same for each other taking a constant value at

$$-\frac{d\ln\Delta}{d\ln\tau_M} = -\frac{d\ln\Lambda_M}{d\ln\tau_M} = \frac{\theta(\sigma-1)}{\sigma-1-\theta}.$$

Summing up the three terms, the input trade elasticity is expressed as

$$\varepsilon_M = \frac{(\sigma - 1)(\sigma - 1 + \theta)}{\sigma - 1 - \theta}. \tag{18}$$

As for output trade, using the domestic output share (7), the trade elasticity with respect to variable trade cost of output is given by

$$\begin{aligned}\varepsilon_X &= -\frac{d \ln \left(\frac{1-\lambda_X}{\lambda_X}\right)}{d \ln \tau_X} \\ &= \underbrace{(\sigma - 1)}_{\text{Intensive margin elasticity}} + \underbrace{\left(-\frac{d \ln \Lambda_X}{d \ln \tau_X}\right)}_{\substack{\text{Exporter extensive margin elasticity} \\ \text{in downstream sector}}}.\end{aligned}$$

Since suppliers do not import output produced by firms in input production, output trade liberalization does not induce suppliers to enter and there is no importer extensive margin elasticity in the upstream sector. However, this does not mean that output trade liberalization has no impact on suppliers in the upstream sector at all. An expansion of exporting firms (by output trade liberalization) triggers an expansion of importing firms, which in turn leads to an expansion of exporting suppliers through changes in the market demand A, B. In other words, the output trade elasticity can be also magnified relative to that in the single production sector setting due to the joint interaction between the vertically-related sectors. Although the result seems natural, (16) shows that output trade liberalization leads to resource reallocations in the two production sectors if and only if (17) holds, which is related to the extensive margin. In fact, the exporter extensive margin elasticity is

$$-\frac{d \ln \Lambda_X}{d \ln \tau_X} = \theta \left[1 + \left(\frac{\sigma - 1 + \theta}{\sigma - 1 - \theta}\right)(\eta_D - \eta_X)(\mu_D - \mu_M)\right],$$

where η_D (η_X) is the domestic (foreign) input share of firms using only domestic input, satisfying $\eta_D - \eta_X > 0$ under (17).[10] Hence the output trade elasticity is

$$\varepsilon_X = \sigma - 1 + \theta \left[1 + \left(\frac{\sigma - 1 + \theta}{\sigma - 1 - \theta}\right)(\eta_D - \eta_X)(\mu_D - \mu_M)\right]. \tag{19}$$

As shown by Chaney (2008), the output trade elasticity can be decomposed into the intensive margin elasticity $\sigma - 1$ and the extensive margin elasticity θ when firm heterogeneity is present. While the decomposition applies, the extensive margin elasticity can be larger in the multi-production sector model than in the single production sector model. Obviously, so long as (17) holds, the exporter extensive margin elasticity has an additional term which captures the feedback effect from an expansion of the upstream sector production to an expansion of the downstream sector production triggered by output trade liberalization. In this way, the impact of output trade liberalization on output trade flows can be magnified with vertical production, even though such liberalization does not directly induce entry of suppliers in the upstream sector.

Which trade elasticity is greater when both input and output are costly traded subject to selection? Simple comparison of the trade elasticities in (18) and (19) immediately reveals that

$$\varepsilon_M > \varepsilon_X.$$

[10] $\eta_D = \frac{1}{1+\tau_M^{1-\sigma}\Delta\frac{V(\varphi_{DM})}{V(\varphi_D)}}$ and $\eta_X = \frac{1}{1+\tau_M^{1-\sigma}\Delta\frac{V(\varphi_{XM})}{V(\varphi_X)}}$. Note that these always satisfy $1 > \eta_D - \eta_X$.

Thus, we can say that the trade elasticity is always greater for input than for output. This finding accords with the widely-known empirical fact that input trade has been growing faster than output trade in the real world (e.g., Hummels et al., 2001; Hanson et al., 2005; Johnson and Noguera, 2012).

The finding suggests that the gravity structure is drastically different between input trade and output trade. Suppose that φ_i and ϕ_v are distributed Pareto with a scale parameter $\varphi_{\min} = \phi_{\min} = 1$ and a shape parameter γ. Using (13), output trade flows $R_X = \int_i r_{Xi} di$ and input trade flows $E_M = \int_v r_{Mv} dv$ can be decomposed into

$$R_X = \underbrace{\frac{1}{\eta_X} \frac{\sigma(\sigma - 1 + \theta)}{\theta} f_X}_{\text{Average sales per firm}} \times \underbrace{\left(\frac{1}{\varphi_X}\right)^{\sigma - 1 + \theta} M_E}_{\text{Mass of firms}},$$

and

$$E_M = \underbrace{\frac{\sigma(\sigma - 1 + \theta)}{\theta} k_M}_{\text{Average sales per supplier}} \times \underbrace{\left(\frac{1}{\phi_M}\right)^{\sigma - 1 + \theta} N_E}_{\text{Mass of suppliers}}.$$

Moreover, it can be shown that the mass of entrants in each production sector is proportional to country size L but independent of the productivity cutoffs φ_c, ϕ_c under the distribution:

$$M_E = \frac{\sigma - 1}{\sigma(\sigma - 1 + \theta)} \frac{L}{f_E}, \quad N_E = \frac{(\sigma - 1)^2}{\sigma^2(\sigma - 1 + \theta)} \frac{L}{k_E}.$$

Substituting φ_X from (3) and ϕ_M from (9) as well as M_E, N_E derived above, we can express these trade flows as a gravity equation form:

$$\begin{aligned} R_X &= \frac{\psi_X}{\eta_X} L B^{\frac{\sigma-1+\theta}{\sigma-1}} (\tau_X c_D)^{-(\sigma-1+\theta)} f_X^{-\frac{\theta}{\sigma-1}}, \\ E_M &= \psi_M \Lambda_M^{\frac{\sigma-1+\theta}{\sigma-1}} L A^{\frac{\sigma-1+\theta}{\sigma-1}} \tau_M^{-(\sigma-1+\theta)} k_M^{-\frac{\theta}{\sigma-1}}, \end{aligned} \tag{20}$$

where ψ_X and ψ_M are some constant term.[11] As in a usual gravity equation, trade flows in either type of goods are a function of exporting country size L, importing country demand B, A, and bilateral trade barriers, both variable τ_X, τ_M and fixed f_X, k_M. The functional form is very similar to the gravity equation in Chaney (2008) in terms of the elasticity of trade flows with respect to trade barriers; however, output trade flows include the foreign input share of firms using only domestic input η_X while input trade flows include the market share of output importers Λ_M, which work to elevate the elasticity of trade flows relative to that in the single production sector model. For example, applying the Pareto distribution to the market share of output exporters, we get

$$\Lambda_M^{\frac{\sigma-1+\theta}{\sigma-1}} = \left[\tau_M^{-(\sigma-1+\theta)} \left(\frac{\mu_D}{\mu_M}\right)^{\frac{\sigma-1}{\theta}} \left(\frac{f_{DM}}{f_D}\right)^{-1} \left(\frac{k_M}{k_D}\right)^{-\frac{\theta}{\sigma-1}}\right]^{\frac{\theta}{\sigma-1-\theta}}.$$

which is of course negatively affected by both variable and fixed trade cost. Thus variable trade cost τ_M decrease input trade flows not only through supplies' shipment with elasticity $\sigma - 1 + \theta (= \gamma)$ directly as in Chaney (2008), but also through firms' market share who profitably use imported input with elasticity $\frac{\theta(\sigma-1+\theta)}{\sigma-1-\theta}$ indirectly by changes in Λ_M. The same claim applies to output trade flows in the sense that they are indirectly affected by changes in η_X.

[11] $\psi_X = \frac{\sigma-1}{\theta f_E}$ and $\psi_M = \frac{(\sigma-1)^2}{\sigma\theta k_E}$.

To obtain the trade elasticity from the gravity equation (20), it follows from the domestic shares λ_X, λ_M that the trade elasticities can be alternatively written as follows:[12]

$$\varepsilon_X = -\frac{d \ln(R_X/R_D)}{d \ln \tau_X}, \quad \varepsilon_M = -\frac{d \ln(E_M/E_D)}{d \ln \tau_M}.$$

Applying Pareto to $R_D = \int_i r_{Di} di$, $E_D = \int_v r_{Dv} dv$ and using Λ_M, we can show that the closed-form solutions of ε_X and ε_M are the same expressions in (19) and (18) respectively. Defining the trade elasticities with respect to fixed trade cost f_X, k_M similarly, these elasticities are also greater for input trade than for output trade. Hence the model predicts that, when estimating the elasticity of the value of trade with respect to trade barriers from the gravity equation, the trade elasticity is *endogenously* greater for input trade than for output trade.

The result on trade flows is our first main proposition of the paper.

Proposition 1: *If trade liberalization induces resource reallocations in the vertically-related sectors, the impact of trade barriers on trade flows is greater for input trade than for output trade.*

It is worth stressing that our result does *not* come from a CES production function where output is produced by a variety of input. As is evident from the above decompositions, the result comes from the difference in the impact of trade liberalization on the extensive margin: reductions in variable trade cost induce entry of new firms and suppliers into the respective production sector in a different way. From this reason, it is possible to empirically test our theoretical prediction taking account of the different impact on the extensive margin. In a companion paper (Ara and Zhang, 2020), we study this channel by estimating the gravity equation derived in a multiple-industry, asymmetric-country setting under the Pareto distribution. As overviewed in the Introduction, we find empirical support for the theoretical prediction in China's imports.

We conclude this section by briefly mentioning the effect of input (output) trade barriers on output (input) trade flows. The trade elasticities capturing this indirect effect are defined as

$$\tilde{\varepsilon}_X = -\frac{d \ln \left(\frac{1-\lambda_X}{\lambda_X} \right)}{d \ln \tau_M}, \quad \tilde{\varepsilon}_M = -\frac{d \ln \left(\frac{1-\lambda_M}{\lambda_M} \right)}{d \ln \tau_X}.$$

It is clear that both $\tilde{\varepsilon}_X$ and $\tilde{\varepsilon}_M$ have no intensive margin elasticity, implying that trade cost have the impact on each trade flows only through changes in the market share of firms and suppliers, i.e., the extensive margin elasticity. Using the impact of trade liberalization on the productivity cutoffs in Lemmas 1 and 2, we get

$$\begin{aligned} \tilde{\varepsilon}_X &= (\sigma - 1) \left(\frac{\sigma - 1 + \theta}{\sigma - 1 - \theta} \right) (\eta_D - \eta_X), \\ \tilde{\varepsilon}_M &= (\sigma - 1) \left(\frac{\sigma - 1 + \theta}{\sigma - 1 - \theta} \right) (\mu_D - \mu_M). \end{aligned} \tag{21}$$

Thus, so long as there is some complementarity between exporting and importing under (17), both $\tilde{\varepsilon}_X$ and $\tilde{\varepsilon}_M$ have the positive value. As a result, input trade liberalization not only increases input trade flows directly, but also increases output trade flows indirectly, and vice versa.

[12]This is similar to that in Arkolakis et al. (2012) who consider the "partial" trade elasticity focussing only on the direct effect of variable trade cost, while we consider the "full" trade elasticity taking into account all effects of that cost. Though these elasticities are generally different, they are the same when the extensive margin elasticity is constant. See Melitz and Redding (2015).

3.3 Welfare Gains

We finally examine welfare implications of trade liberalization in the presence of the vertically-related sectors. Totally differentiating (12), we know that changes in welfare per worker W can be captured by changes in the domestic productivity cutoffs φ_D, ϕ_D as well as the domestic input share λ_M in this model:

$$d \ln W = d \ln \varphi_D + d \ln \phi_D + \left(\frac{1}{\sigma - 1} \right) d \ln \lambda_M. \tag{22}$$

Consider first the impact of input trade liberalization on welfare. On the one hand, the impact of variable trade cost of input on the domestic productivity cutoffs are given by (14). Then (22) shows that reductions in this trade cost improve welfare by raising the domestic productivity cutoffs in the two production sectors. The welfare changes come from the Melitz-type resource reallocations among firms and suppliers seen in Lemma 1: input trade liberalization magnifies the standard selection effect by inducing the least productive firms as well as the least productive suppliers to exit the respective production sector so long as condition (15) is satisfied. This two-sided selection magnifies the welfare gains from trade relative to the single production sector setting. On the other hand, totally differentiating the domestic input share in (8) and noting the definition of ε_M in the last section, the impact of variable trade cost of input on the domestic input share is given by

$$d \ln \lambda_M = (1 - \lambda_M) \varepsilon_M d \ln \tau_M.$$

Then (22) shows that reductions in this trade cost *deteriorate* welfare by reducing the domestic input share. The reason for the welfare changes is explained as follows. Using φ_D in (3) and the output market demand B, welfare is defined as an inverse of the output price index:

$$\frac{1}{P} = \left(\frac{\sigma - 1}{\sigma} \right) \left(\frac{L}{\sigma f_D} \right)^{\frac{1}{\sigma - 1}} \frac{\varphi_D}{c_D}.$$

Thus, for given φ_D, welfare is negatively affected by the unit cost of firms using only domestic input c_D because the higher is this unit cost, the less efficient are these firms and the higher is the output price index. Note that input trade liberalization endogenously affects the unit cost by changing the range of input available to firms. Using ϕ_D in (9) and the input market demand A, the unit cost is expressed as

$$\frac{1}{c_D} = \left(\frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma}{\sigma - 1}} \left(\frac{L}{\sigma k_D} \right)^{\frac{1}{\sigma - 1}} \phi_D \lambda_M^{\frac{1}{\sigma - 1}}.$$

Thus, for given ϕ_D, the unit cost is negatively affected by the domestic input share λ_M because the higher is this share, the more available is input for firms using only domestic input. This means that when there is selection into importing among firms, input trade liberalization can have a negative impact on welfare by increasing the unit cost of firms who cannot access imported input. However, this negative effect is always dominated by the positive effect from selection. Substituting ε_M in (18) and summing up the three terms in (22), the changes in welfare with respect to input trade liberalization are simply given by

$$d \ln W = -(1 - \lambda_M) \ln \tau_M, \tag{23}$$

which indicates that input trade liberalization is always welfare-enhancing. More important is that the elasticity of welfare with respect to input trade barriers is equivalent to the foreign input share $1 - \lambda_M$.

Next we examine the impact of output trade liberalization on welfare. The impact of variable trade cost of output on the domestic productivity cutoffs are given by (16). Then, from Lemma 2, (22) shows that reductions in this trade cost have a similar impact as above in the sense that such reductions improve welfare by raising the domestic productivity cutoffs in the two production sectors, so long as conditions (15) and (17) are satisfied. On the other hand, the impact on the domestic input share operates through which output trade liberalization indirectly increases input trade flows, due to the complementarity between exporting and importing under (17). Observing that the corresponding trade elasticity is $\tilde{\varepsilon}_M$ in (21), the impact of variable trade cost of output on the domestic input share is given by

$$d\ln\lambda_M = (1-\lambda_M)\tilde{\varepsilon}_M d\ln\tau_X.$$

Then (22) shows that reductions in this trade cost deteriorate welfare by reducing the domestic input share, which is explained by noting the impact of τ_X on c_D. While each of the welfare changes is qualitatively similar between input trade and output trade, the magnitude of these changes is different between them because the domestic productivity cutoffs φ_D, ϕ_D rise on a different scale (see (14) and (16)), which in turn gives a different impact on the domestic input share λ_M. Substituting $\tilde{\varepsilon}_M$ in (21) and summing up the three terms in (22), the changes in welfare with respect to output trade liberalization are simply given by

$$d\ln W = -(1-\lambda_X)\ln\tau_X. \tag{24}$$

It is important to note again that the elasticity of welfare with respect to output trade barriers is equivalent to the foreign output share $1-\lambda_X$.

We are ready to compare the changes in welfare by input trade liberalization and output trade liberalization. It follows immediately from (23) and (24) that

$$\left|\frac{d\ln W}{d\ln\tau_M}\right| > \left|\frac{d\ln W}{d\ln\tau_X}\right| \quad \Longleftrightarrow \quad \lambda_X > \lambda_M$$

Thus, the welfare gains from input trade liberalization are greater than those from output trade liberalization if and only if the domestic output share is greater than the domestic input share. Recall that this is the sufficient condition under which input trade liberalization induces greater changes in the domestic productivity cutoff in the downstream sector than output trade liberalization. In this sense, this welfare result is consistent with the resource-reallocation result in Lemmas 1 and 2.

We conclude this section by relating our welfare result to the Arkolakis et al. (2012) welfare formula. Using the relationship between the changes in the domestic input share and the changes in variable trade cost, we can express the changes in welfare by input trade liberalization (23) as

$$d\ln W = -\frac{d\ln\lambda_M}{\varepsilon_M}.$$

Thus, the welfare changes by input trade liberalization can be captured by only their two sufficient statistics: the domestic input share λ_M and the input trade elasticity ε_M, even in the presence of the vertically-related sectors. Similarly, totally differentiating the domestic output share in (7) and noting the definition of ε_X, the impact of variable trade cost of output on the domestic output share is given by

$$d\ln\lambda_X = (1-\lambda_X)\varepsilon_X d\ln\tau_X.$$

Using this relationship, the changes in welfare by output trade liberalization (24) are expressed as

$$d \ln W = -\frac{d \ln \lambda_X}{\varepsilon_X}.$$

Defining proportional changes of a variable by a "hat" (e.g., $\widehat{x} = dx/x$), the changes in welfare associated with trade liberalization in each type of goods are given by

$$\widehat{W} = \widehat{\lambda}_M^{-\frac{1}{\varepsilon_M}} = \widehat{\lambda}_X^{-\frac{1}{\varepsilon_X}}. \tag{25}$$

Therefore, the welfare formula by Arkolakis et al. (2012) applies here: conditional on the two sufficient statistics, the welfare gains from trade are the same between input trade and output trade. More importantly, our welfare comparison displays the main emphasis of Arkolakis et al. (2012) in a very clear manner. Holding the domestic share equal between input and output ($\lambda_X = \lambda_M$), Lemmas 1 and 2 show that input trade liberalization always has a greater effect on the two-sided resource reallocations than output trade liberalization. Given this difference in the impact of trade, it is natural to imagine that the welfare gains from trade are greater for input trade than for output trade. The welfare expression (25) indicates that this is not the case. As stressed in the last section, the fact that the trade elasticities are different between the two types of trade reflects that there exists an extra adjustment in the extensive margin for input trade that is absent for output trade. However, conditional on the two sufficient statistics, this extra margin only affects the composition of the welfare gains from trade, not their total size.

Though useful, the equivalence in the welfare changes by trade liberalization in input and output holds under the condition that the domestic share is equal between input and output. There is however mounting evidence suggesting that input trade has been growing faster and its share in world trade is larger than output trade (Hummels et al., 2001; Hanson et al., 2005; Johnson and Noguera, 2012). This piece of evidence implies that the welfare evaluation holding the domestic share in input and output equal might lead to wrong understanding of globalization where fragmentation of production processes plays a prominent role in improving welfare in each country. The finding that the welfare changes are greater for input trade than output trade under rapidly rising input trade is also consistent with empirical evidence that input tariff reductions increase industry productivity more than output tariff reductions (Amiti and Konings, 2007; Topalova and Khandelwal, 2011), because such productivity improvement is typically associated with the higher welfare gains from input trade liberalization than from output trade liberalization.

Besides this caveat, (25) depends on a constant extensive margin elasticity which makes the mass of entrants in each production sector invariant to any trade shocks. If the extensive margin elasticity is variable, however, trade liberalization affects welfare from changes in the mass of entrants, yielding an additional welfare channel. Given recent empirical work reporting a non-constant extensive margin elasticity (Bas et al., 2017), we should be careful about interpreting our welfare result.[13]

The result on welfare gains is our second main proposition of the paper.

Proposition 2: *If trade liberalization induces resource reallocations in the vertically-related sectors, the impact of trade barriers on welfare gains is greater for input trade than for output trade if and only if the domestic output share is greater than the domestic input share.*

[13]See Head et al. (2014) and Melitz and Redding (2015) for welfare implications with a variable extensive margin elasticity.

4 Conclusion

This paper has presented a heterogeneous firm model in which selection into exporting and importing play a key role in industry productivity of vertically-related sectors. We show that reductions in trade cost increase trade flows for intermediate goods more than for final goods, due to an extra adjustment through the extensive margin. We also find that reductions in trade cost induce greater welfare changes for intermediate goods than for final goods if and only if the domestic output share is greater than the domestic input share. These findings could help us to obtain better understanding about rapidly rising growth of intermediate good trade and large productivity gains associated with trade liberalization in intermediate goods reported by empirical work. One of broader policy implications from this paper is that the difference in the trade elasticities between final goods and intermediate goods is crucial for understanding the mechanism that generates the difference in the welfare gains from trade.

To highlight an extra adjustment through the extensive margin in intermediate good trade, we have resorted to a two-symmetric-country setting. While our model can be extended to a many-symmetric-country setting, it is challenging to develop a many-asymmetric-country setting where reductions in trade cost potentially have an asymmetric impact on trading partners, but such an extension could provide an important channel through which to magnify a positive correlation between productivity levels of firms and the number of source countries. It is also interesting to explore the impact of country asymmetry on the trade pattern between final goods and intermediate goods, and general-equilibrium consequences of such specialization patterns for the welfare gains. Does a larger country host disproportionally more final good firms and become a net exporter of final goods in vertical linkages? Does unilateral trade liberalization in final goods induce agglomeration of final good firms in a liberalizing country and give rise to the greater welfare gains there relative to unilateral trade liberalization in intermediate goods? We leave these questions to feature work.

A Proofs

A.1 Proof of Lemma 1

To show the derivation of (14), we first derive changes by variable trade cost of input in the downstream sector. Taking the log and differentiating the zero profit cutoff conditions (4) and (5) with respect to τ_M,

$$\begin{aligned} d\ln\varphi_X - d\ln\varphi_D &= 0, \\ d\ln\varphi_{XM} - d\ln\varphi_{DM} &= 0, \\ d\ln\varphi_{DM} - d\ln\varphi_D &= -\frac{1}{\sigma-1}d\ln\Delta + d\ln\tau_M, \\ d\ln\varphi_{XM} - d\ln\varphi_X &= -\frac{1}{\sigma-1}d\ln\Delta + d\ln\tau_M. \end{aligned} \tag{A.1}$$

Using the definition of Δ and $\theta_c = \theta$,

$$d\ln\Delta = -\theta(d\ln\phi_M - d\ln\phi_D).$$

Moreover, differentiating the free entry condition (6) with respect to τ_M,

$$\sum_c f_c J'(\varphi_c)\varphi_c d\ln\varphi_c = 0.$$

Solving this for $d\ln\varphi_{DM}$ and $d\ln\varphi_{XM}$ by using $d\ln\varphi_X = d\ln\varphi_D$ and $d\ln\varphi_{XM} = d\ln\varphi_{DM}$ from (A.1),

$$d\ln\varphi_{DM} = -\alpha d\ln\varphi_D, \tag{A.2}$$

where

$$\alpha \equiv \frac{f_D J'(\varphi_D)\varphi_D + f_X J'(\varphi_X)\varphi_X}{f_{DM}J'(\varphi_{DM})\varphi_{DM} + f_{XM}J'(\varphi_{XM})\varphi_{XM}}.$$

Just like (3) and (6) cannot characterize the levels in the downstream sector, (A.1) and (A.2) cannot characterize the changes in the downstream sector through the changes in the market share $d\ln\Delta$.

Next we calculate changes by variable trade cost of input in the upstream sector. Taking the log and differentiating the zero profit cutoff condition (10) with respect to τ_M,

$$d\ln\phi_M - d\ln\phi_D = -\frac{1}{\sigma-1}d\ln\Lambda_M + d\ln\tau_M. \tag{A.3}$$

Using the definition of Λ_M and $\vartheta_c = \theta$ as well as $d\ln\varphi_X = d\ln\varphi_D$ and $d\ln\varphi_{XM} = d\ln\varphi_{DM}$ from (A.1),

$$d\ln\Lambda_M = -\theta(d\ln\varphi_{DM} - d\ln\varphi_D).$$

Moreover, differentiating the free entry condition (11) with respect to τ_M,

$$\sum_c k_c J'(\phi_c)\phi_c d\ln\phi_c = 0.$$

Solving this for $d\ln\phi_M$, we have

$$d\ln\phi_M = -\beta d\ln\phi_D, \tag{A.4}$$

where

$$\beta \equiv \frac{k_D J'(\phi_D)\phi_D}{k_M J'(\phi_M)\phi_M}.$$

Note the similarity between (A.1) and (A.3) as well as (A.2) and (A.4).

Finally, we solve for the changes in the economy by variable trade cost of input by taking account of the joint interaction between the two production sectors. Substituting (A.2) and (A.4) into the third equation of (A.1), and substituting (A.2) and (A.4) into (A.3) respectively,

$$\begin{aligned}
-(\alpha+1)d\ln\varphi_D &= -\frac{\theta}{\sigma-1}(\beta+1)d\ln\phi_D + d\ln\tau_M,\\
-(\beta+1)d\ln\phi_D &= -\frac{\theta}{\sigma-1}(\alpha+1)d\ln\varphi_D + (\sigma-1) + d\ln\tau_M.
\end{aligned}$$

Solving these for $d\ln\varphi_D$ and $d\ln\phi_D$ yields

$$\begin{aligned}
d\ln\varphi_D &= -\left(\frac{\sigma-1}{(\sigma-1-\theta)(\alpha+1)}\right)d\ln\tau_M,\\
d\ln\phi_D &= -\left(\frac{\sigma-1}{(\sigma-1-\theta)(\beta+1)}\right)d\ln\tau_M.
\end{aligned}$$

It remains to show $\frac{1}{\alpha+1} = \frac{1}{\beta+1} = 1-\lambda_M$. Differentiating $J(\varphi_c) = \int_{\varphi_c}^{\infty}\left[\left(\frac{\varphi_i}{\varphi_c}\right)^{\sigma-1} - 1\right]dG(\varphi_i)$ with respect to φ_c,

$$J'(\varphi_c) = -\left(\frac{\sigma-1}{\varphi_c}\right)[J(\varphi_c) + 1 - G(\varphi_c)].$$

From $V(\varphi_c) = \int_{\varphi_c}^{\infty}\varphi_i^{\sigma-1}dG(\varphi_i)$, it follows that $J(\varphi_c) + 1 - G(\varphi_c) = \varphi_c^{1-\sigma}V(\varphi_c)$ and hence

$$J'(\varphi_c) = -(\sigma-1)\varphi_c^{-\sigma}V(\varphi_c), \tag{A.5}$$

which also holds for suppliers' productivity cutoff ϕ_c. Substituting (A.5) into the definition of α and β above, and subsequently using the zero profit cutoff conditions (3) and (9),

$$\alpha = \beta = \frac{1}{\tau_M^{1-\sigma}\Delta\Lambda_M}. \tag{A.6}$$

The result follows immediately from using (A.6) in the definition of λ_M.

A.2 Proof of Lemma 2

To show the derivation of (16), we closely follow the steps in Appendix A.1. Taking the log and differentiating the zero profit cutoff condition (4) and (5) with respect to τ_X,

$$\begin{aligned}
d\ln\varphi_X - d\ln\varphi_D &= d\ln\tau_X,\\
d\ln\varphi_{XM} - d\ln\varphi_{DM} &= d\ln\tau_X,\\
d\ln\varphi_{DM} - d\ln\varphi_D &= -\frac{1}{\sigma-1}d\ln\Delta,\\
d\ln\varphi_{XM} - d\ln\varphi_X &= -\frac{1}{\sigma-1}d\ln\Delta,
\end{aligned} \tag{A.7}$$

where $d \ln \Delta$ is the same expression in Appendix A1. Moreover, differentiating the free entry condition (6) with respect to τ_X also yields the same expression in Appendix A.1. Solving this for $d \ln \varphi_{DM}$ and $d \ln \varphi_{XM}$ by noting $d \ln \varphi_X = d \ln \varphi_D + d \ln \tau_X$ and $d \ln \varphi_{XM} = d \ln \varphi_{DM} + d \ln \tau_X$ in (A.7),

$$d \ln \varphi_{DM} = -\alpha d \ln \varphi_D - \gamma d \ln \tau_X, \tag{A.8}$$

where

$$\gamma \equiv \frac{f_X J'(\varphi_X)\varphi_X + f_{XM} J'(\varphi_{XM})\varphi_{XM}}{f_{DM} J'(\varphi_{DM})\varphi_{DM} + f_{XM} J'(\varphi_{XM})\varphi_{XM}}.$$

As for the equilibrium in changes in the upstream sector, taking the log and differentiating the zero profit cutoff condition (10) with respect to τ_X,

$$d \ln \phi_M - d \ln \phi_D = -\frac{1}{\sigma - 1} d \ln \Lambda_M, \tag{A.9}$$

where

$$d \ln \Lambda_M = -\theta(d \ln \varphi_{DM} - d \ln \varphi_D) - (\sigma - 1 + \theta)(\mu_D - \mu_M) d \ln \tau_X.$$

Moreover, differentiating the free entry condition (11) with respect to τ_X yields

$$d \ln \phi_M = -\beta d \ln \phi_D. \tag{A.10}$$

Finally, substituting (A.8) and (A.10) into the third equation of (A.7), and substituting (A.8) and (A.10) into (A.9) respectively,

$$\begin{aligned} -(\alpha + 1) d \ln \varphi_D &= -\frac{\theta}{\sigma - 1}(\beta + 1) d \ln \phi_D + \gamma d \ln \tau_X, \\ -(\beta + 1) d \ln \phi_D &= -\frac{\theta}{\sigma - 1}(\alpha + 1) d \ln \varphi_D - \frac{\theta\gamma - (\sigma - 1 + \theta)(\mu_D - \mu_M)}{\sigma - 1} d \ln \tau_X. \end{aligned}$$

Solving for $d \ln \varphi_D$ and $d \ln \phi_D$ yields

$$\begin{aligned} d \ln \varphi_D &= -\left(\frac{\gamma(\sigma - 1 - \theta) + \theta(\mu_D - \mu_M)}{(\sigma - 1 - \theta)(\alpha + 1)}\right) d \ln \tau_X, \\ d \ln \phi_D &= -\left(\frac{(\sigma - 1)(\mu_D - \mu_M)}{(\sigma - 1 - \theta)(\beta + 1)}\right) d \ln \tau_X. \end{aligned}$$

Moreover, using (A.5) into the definition of γ above,

$$\gamma = (1 - \lambda_X)(\alpha + 1).$$

The result follows immediately from using this and (A.6) in the definition of λ_M.

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S Supplementary Note (Not for Publication)

This Note contains detailed derivations for the expressions omitted in the main text due to the space constraint. The same section names are attached to the following sections where the detailed derivations are required.

S.1 Firms

To derive firm i's input expenditure in (1), substituting x_{Div} and x_{Miv} into the CES production function,

$$x_i = \frac{e_i}{c_i}.$$

Rewriting this as $e_i = c_i x_i$ and using firm i's technology $q_i = \varphi_i x_i$ yields (1).

To derive firm i's unit cost in (2), note that $c_D^{1-\sigma} = \int_v p_{Dv}^{1-\sigma} dv$ can be expressed as

$$c_D^{1-\sigma} = N_E \int_{\phi_D}^{\infty} p_{Dv}^{1-\sigma} dG(\phi_v).$$

Using the input pricing rule p_{Dv} and the definition of $V(\phi_c)$, we get

$$c_D^{1-\sigma} = N_E \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} V(\phi_D).$$

Similarly, $c_M^{1-\sigma} = \int_v p_{Mv}^{1-\sigma} dv$ can be expressed as

$$c_M^{1-\sigma} = N_E \left(\frac{\sigma\tau_M}{\sigma-1}\right)^{1-\sigma} V(\phi_M).$$

Substituting these into $c_i^{1-\sigma} = c_D^{1-\sigma} + \mathbb{1}_{Mi} c_M^{1-\sigma}$ and using $\Delta = V(\phi_M)/V(\phi_D)$ yields (2).

The free entry condition can be expressed as

$$\int_{\varphi_D}^{\infty} \pi_{Di} dG(\varphi_i) + \int_{\varphi_X}^{\infty} \pi_{Xi} dG(\varphi_i) = f_E.$$

From domestic profit π_{Di}, the expected profit from the domestic market is given by

$$\int_{\varphi_D}^{\infty} \pi_{Di} dG(\varphi_i) = \int_{\varphi_D}^{\varphi_{DM}} \left(B c_D^{1-\sigma} \varphi_i^{\sigma-1} - f_D\right) dG(\varphi_i) + \int_{\varphi_{DM}}^{\infty} \left(B(1+\tau_M^{1-\sigma}\Delta) c_D^{1-\sigma} \varphi_i^{\sigma-1} - f_D - f_{DM}\right) dG(\varphi_i).$$

Rearranging and substituting φ_D and φ_{DM} in (3) into the above equality,

$$\int_{\varphi_D}^{\infty} \pi_{Di} dG(\varphi_i) = f_D \int_{\varphi_D}^{\infty} \left[\left(\frac{\varphi_i}{\varphi_D}\right)^{\sigma-1} - 1\right] dG(\varphi_i) + f_{DM} \int_{\varphi_{DM}}^{\infty} \left[\left(\frac{\varphi_i}{\varphi_{DM}}\right)^{\sigma-1} - 1\right] dG(\varphi_i).$$

Similarly, the expected profit from the export market is given by

$$\int_{\varphi_X}^{\infty} \pi_{Xi} dG(\varphi_i) = f_X \int_{\varphi_X}^{\infty} \left[\left(\frac{\varphi_i}{\varphi_X}\right)^{\sigma-1} - 1\right] dG(\varphi_i) + f_{XM} \int_{\varphi_{XM}}^{\infty} \left[\left(\frac{\varphi_i}{\varphi_{XM}}\right)^{\sigma-1} - 1\right] dG(\varphi_i).$$

Using the definition of $J(\varphi_c)$ yields (6).

To derive the domestic output share in (7), note that $R = \int_i r_i di$ is

$$R = \int_i r_{Di} di + \int_i r_{Xi} di.$$

This aggregate revenue can be expressed as

$$R = M_E \int_{\varphi_D}^{\infty} r_{Di} dG(\varphi_i) + M_E \int_{\varphi_X}^{\infty} r_{Xi} dG(\varphi_i).$$

From domestic revenue r_{Di}, the aggregate revenue from the domestic market $R_D = \int_i r_{Di} di$ is given by

$$R_D = M_E \int_{\varphi_D}^{\varphi_{DM}} \left(\sigma B c_D^{1-\sigma} \varphi_i^{\sigma-1}\right) dG(\varphi_i) + M_E \int_{\varphi_{DM}}^{\infty} \left(\sigma B (1 + \tau_M^{1-\sigma} \Delta) c_D^{1-\sigma} \varphi_i^{\sigma-1}\right) dG(\varphi_i).$$

Using the definition of $V(\varphi_c)$ and rearranging, this revenue is rewritten as

$$R_D = M_E \sigma B c_D^{1-\sigma} (V(\varphi_D) + \tau_M^{1-\sigma} \Delta V(\varphi_{DM})). \tag{S.1}$$

Similarly, the aggregate revenue from the export market $R_X = \int_i r_{Xi} di$ can be expressed as

$$R_X = M_E \sigma B (\tau_X c_D)^{1-\sigma} (V(\varphi_X) + \tau_M^{1-\sigma} \Delta V(\varphi_{XM})). \tag{S.2}$$

Substituting these in $\lambda_X = R_D/R$ yields (7).

To derive the domestic input share in (8), note that $E = \int_i e_i di$ is

$$E = \int_i e_{Di} di + \int_i e_{Mi} di,$$

where $e_{Di} = \int_v e_{Div} dv$ and $e_{Mi} = \int_v e_{Miv} dv$. This aggregate expenditure can be expressed as

$$E = M_E \int_{\varphi_D}^{\infty} e_{Di} dG(\varphi_i) + M_E \int_{\varphi_{DM}}^{\infty} e_{Mi} dG(\varphi_i).$$

Note that $e_{Div} = p_{Dv}^{1-\sigma} c_i^{\sigma-1} e_i$ and $e_{Miv} = p_{Mv}^{1-\sigma} c_i^{\sigma-1} e_i$ where e_i is written from (1) as

$$e_i = \frac{c_i}{\varphi_i} (q_{Di} + \mathbb{1}_{Xi} \tau_X q_{Xi}),$$

where $\mathbb{1}_{Xi}$ is an indicator function which takes the value of one if firm i exports their output and zero otherwise. Substituting consumers' output demand q_{Di}, q_{Xi} into e_i and subsequently using this in e_{Div}, e_{Miv},

$$\begin{aligned} e_{Div} &= (\sigma - 1) B p_{Dv}^{1-\sigma} \varphi_i^{\sigma-1} (1 + \mathbb{1}_{Xi} \tau_X^{1-\sigma}), \\ e_{Miv} &= (\sigma - 1) B p_{Mv}^{1-\sigma} \varphi_i^{\sigma-1} (1 + \mathbb{1}_{Xi} \tau_X^{1-\sigma}). \end{aligned} \tag{S.3}$$

Recalling that $\int_v p_{Dv}^{1-\sigma} dv = c_D^{1-\sigma}$ and $\int_v p_{Mv}^{1-\sigma} dv = c_M^{1-\sigma}$, aggregation of e_{Div}, e_{Miv} yileds

$$\begin{aligned} e_{Di} &= (\sigma - 1) B c_D^{1-\sigma} \varphi_i^{\sigma-1} (1 + \mathbb{1}_{Xi} \tau_X^{1-\sigma}), \\ e_{Mi} &= (\sigma - 1) B c_M^{1-\sigma} \varphi_i^{\sigma-1} (1 + \mathbb{1}_{Xi} \tau_X^{1-\sigma}). \end{aligned}$$

Then, the aggregate expenditure of domestic input $E_D = \int_i e_{Di} di$ is expressed as

$$E_D = M_E(\sigma - 1)Bc_D^{1-\sigma}(V(\varphi_D) + \tau_X^{1-\sigma}V(\varphi_X)). \tag{S.4}$$

Similarly, the aggregate expenditure of imported input $E_M = \int_i e_{Mi} di$ is expressed as

$$E_M = M_E(\sigma - 1)Bc_M^{1-\sigma}(V(\varphi_{DM}) + \tau_X^{1-\sigma}V(\varphi_{XM})). \tag{S.5}$$

Substituting these and (2) into $\lambda_M = E_D/E$ yields (8).

S.2 Suppliers

Using (S.3), (S.4) and (S.5), supplier v's revenue $r_{Dv} = \int_i e_{Div} di$ and $r_{Mv} = \int_i e_{Miv} di$ is given by

$$r_{Dv} = p_{Dv}^{1-\sigma} c_D^{\sigma-1} E_D,$$
$$r_{Mv} = p_{Mv}^{1-\sigma} c_M^{\sigma-1} E_M.$$

Moreover, it follows from (S.4) and (S.5) that

$$E_M = \left(\frac{c_M}{c_D}\right)^{1-\sigma} \Lambda_M E_D,$$

and hence

$$r_{Mv} = p_{Mv}^{1-\sigma} \Lambda_M c_D^{\sigma-1} E_D.$$

Using the input pricing rule p_{Dv}, p_{Mv} and the definition of A yields the expression of r_{Dv}, r_{Mv}. The zero profit cutoff condition (9) follows immediately from that expression.

The free entry condition can be expressed as

$$\int_{\phi_D}^{\infty} \pi_{Dv} dG(\phi_v) + \int_{\phi_M}^{\infty} \pi_{Mv} dG(\phi_v) = k_E,$$

where the first (second) term is expected profit from the domestic (export) market. From domestic profit π_{Dv}, the expected profit from the domestic market is given by

$$\int_{\phi_D}^{\infty} \pi_{Dv} dG(\phi_v) = \int_{\phi_D}^{\infty} \left(A\phi_v^{\sigma-1} - k_D\right) dG(\phi_v).$$

Substituting ϕ_D in (9) into the above equality,

$$\int_{\phi_D}^{\infty} \pi_{Dv} dG(\phi_v) = k_D \int_{\phi_D}^{\infty} \left[\left(\frac{\phi_v}{\phi_D}\right)^{\sigma-1} - 1\right] dG(\phi_v).$$

Similarly, the expected profit from the export market is given by

$$\int_{\phi_M}^{\infty} \pi_{Mv} dG(\phi_v) = k_M \int_{\phi_M}^{\infty} \left[\left(\frac{\phi_v}{\phi_M}\right)^{\sigma-1} - 1\right] dG(\phi_v).$$

Using the definition of $J(\phi_c)$ yields (11).

To derive the domestic input share in (8), note that $E = \int_v r_{Dv} dv + \int_v r_{Mv} dv$ can be expressed as

$$E = N_E \int_{\phi_D}^{\infty} r_{Dv} dG(\phi_v) + N_E \int_{\phi_M}^{\infty} r_{Mv} dG(\phi_v),$$

where the first (second) term is aggregate revenue from the domestic (export) market. From domestic revenue r_{Dv}, the aggregate revenue from the domestic market $E_D = \int_v r_{Dv} dv$ is given by

$$E_D = N_E \int_{\phi_D}^{\infty} \left(\sigma A \phi_v^{\sigma-1}\right) dG(\phi_v).$$

Using the definition of $V(\phi_c)$, this revenue is rewritten as

$$E_D = N_E \sigma A V(\phi_D). \tag{S.6}$$

Similarly, the aggregate revenue from the export market $E_M = \int_v r_{Mv} dv$ can be expressed as

$$E_M = N_E \sigma A \tau_M^{1-\sigma} \Lambda_M V(\phi_M). \tag{S.7}$$

Substituting these in $\lambda_M = E_D/E$ yields (8).

S.3 Economy

To define the labor market clearing condition of the economy, let us first consider the aggregate amount of labor used for entry and production in the downstream sector. Denoting this by $L_i = \int_i l_i^e di + l_i^p di$,

$$L_i = M_E f_E + M_E \int_{\varphi_D}^{\infty} f_{Di} dG(\varphi_i) + M_E \int_{\varphi_X}^{\infty} f_{Xi} dG(\varphi_i).$$

Every entrant incurs fixed entry cost f_E, and firm i above φ_D (φ_X) also incurs fixed production (export) cost f_{Di} (f_{Xi}) to serve the domestic (export) market. Note that production worker is used only for fixed cost in the downstream sector since firms purchase input from the market and hence labor is not used for variable cost (i.e., for transforming input into output). From domestic profit π_{Di}, export profit π_{Xi} and the free entry condition (6) in the downstream sector, L_i is expressed as

$$L_i = R - E. \tag{S.8}$$

We also consider the aggregate amount of labor in the upstream sector. Denoting this by $L_v = \int_v l_v^e dv + l_v^p dv$,

$$L_v = N_E k_E + N_E \int_{\phi_D}^{\infty} l_{Dv}^p G(\phi_v) + N_E \int_{\phi_M}^{\infty} l_{Mv}^p dG(\phi_v),$$

where $l_{Dv}^p = k_D + x_{Dv}/\phi_v$ and $l_{Mv}^p = k_M + \tau_M x_{Mv}/\phi_v$. Every entrant incurs fixed entry cost k_E, and supplier v above ϕ_D (ϕ_M) also employ labor used for production l_{Dv}^p (l_{Mv}^p) to serve the domestic (export) market. Thus, production worker is used for both variable cost and fixed cost in the upstream sector. From domestic profit π_{Dv}, export profit π_{Mv} and the free entry condition (11) in the upstream sector, L_v is expressed as

$$L_v = E. \tag{S.9}$$

Substituting (S.8) and (S.9) into $L_i + L_v = L$, we get the standard labor market condition $R = L$. Moreover, noting $R = R_D + R_X$ in (S.1) and (S.2) as well as $E = E_D + E_M$ in (S.4) and (S.5),

$$E = \left(\frac{\sigma - 1}{\sigma}\right) R. \tag{S.10}$$

Substituting this into (S.8) and (S.9) gives us aggregate labor allocated to each production sector.

To get the mass of entrants in each production sector, we first consider the mass of entrants in the downstream sector. Substituting φ_c in (3) into (S.1) and (S.2), $R = R_D + R_X$ can be expressed as

$$R = M_E \sigma \sum_c f_c \varphi_c^{1-\sigma} V(\varphi_c).$$

Moreover, using $R = L$ and rewriting this gives us the mass of entrants in the downstream sector:

$$M_E = \frac{L}{\sigma \sum_c f_c (\varphi_c)^{1-\sigma} V(\varphi_c)}. \tag{S.11}$$

Similarly, substituting ϕ_c in (9) into (S.6) and (S.7), $E = E_D + E_M$ can be expressed as

$$E = N_E \sum_c k_c \phi_c^{1-\sigma} V(\phi_c).$$

Moreover, using (S.10) and rewriting this gives us the mass of entrants in the upstream sector:

$$N_E = \left(\frac{\sigma - 1}{\sigma}\right) \frac{L}{\sigma \sum_c k_c (\phi_c)^{1-\sigma} V(\phi_c)}. \tag{S.12}$$

Regarding welfare per worker defined as $W \equiv U/L$, it follows from $Q \equiv U$ and $PQ = R = L$ that

$$W = \frac{Q}{L} = \frac{1}{P}.$$

Noting that $w = 1$, welfare per worker is equivalent to the real wage. To get the output price index, using φ_D in (3) and the definition of the output market demand B,

$$\frac{1}{P} = \left(\frac{\sigma - 1}{\sigma}\right) \left(\frac{L}{\sigma f_D}\right)^{\frac{1}{\sigma-1}} \frac{\varphi_D}{c_D},$$

which depends not only on firms' domestic productivity cutoff φ_D but also on firms' unit cost c_D, both are an endogenous variable of the model. Using ϕ_D in (9), (S.10) and the definition of the input market demand A,

$$\frac{1}{c_D} = \left(\frac{\sigma - 1}{\sigma}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{L}{\sigma k_D}\right)^{\frac{1}{\sigma-1}} \phi_D \lambda_M^{\frac{1}{\sigma-1}}.$$

Combining these two expressions,

$$W = \left(\frac{\sigma - 1}{\sigma}\right)^{\frac{2\sigma-1}{\sigma-1}} \left(\frac{L}{\sigma f_D}\right)^{\frac{1}{\sigma-1}} \left(\frac{L}{\sigma k_D}\right)^{\frac{1}{\sigma-1}} \varphi_D \phi_D \lambda_M^{\frac{1}{\sigma-1}}.$$

Rewriting this gives us the welfare expression in (12).

S.4 Trade Flows

Under the Pareto distribution, $J(\varphi_c) = \int_{\varphi_c}^{\infty} \left[\left(\frac{\varphi_i}{\varphi_c} \right)^{\sigma-1} - 1 \right] dG(\varphi_i)$ and $V(\varphi_c) = \int_{\varphi_c}^{\infty} \varphi_i^{\sigma-1} dG(\varphi_i)$ are simple power functions of the productivity cutoff φ_c:

$$\begin{aligned} J(\varphi_c) &= \frac{\sigma-1}{\theta} \frac{1}{(\varphi_c)^{\sigma-1+\theta}}, \\ V(\varphi_c) &= \frac{\sigma-1+\theta}{\theta} \frac{1}{(\varphi_c)^{\theta}}. \end{aligned} \tag{S.13}$$

To compute the mass of entrants M_E, N_E under the Pareto distribution, using (S.13) in the mass of entrants in the downstream sector (S.11),

$$M_E = \frac{L}{\frac{\sigma(\sigma-1+\theta)}{\theta} \left[\sum_c f_c \left(\varphi_c \right)^{-(\sigma-1+\theta)} \right]}.$$

Further, applying the Pareto distribution to the free entry condition (6),

$$\frac{\sigma-1}{\theta} \left[\sum_c f_c(\varphi_c)^{-(\sigma-1+\theta)} \right] = f_E.$$

Combining the two expressions, we have that the mass of entrants is proportional to labor endowment L:

$$M_E = \frac{\sigma-1}{\sigma(\sigma-1+\theta)} \frac{L}{f_E}. \tag{S.14}$$

Similarly, using (S.13) in the mass of entrants in the upstream sector (S.12) and applying the Pareto distribution to the free entry condition (11), we also have

$$N_E = \frac{(\sigma-1)^2}{\sigma^2(\sigma-1+\theta)} \frac{L}{k_E}. \tag{S.15}$$

To derive the gravity equation (20), let output trade flows $R_X = \int_i r_{Xi} di$ decompose into the average sales per firm and the mass of firms:

$$\begin{aligned} R_X &= \frac{1}{1-G(\varphi_X)} \int_{\varphi_X}^{\infty} r_{Xi} dG(\varphi_X) \;\; \times \;\; [1-G(\varphi_X)] M_E \\ &= \frac{1}{\eta_X} \frac{\sigma(\sigma-1+\theta)}{\theta} f_X \;\; \times \;\; \left(\frac{1}{\varphi_X} \right)^{\sigma-1+\theta} M_E, \end{aligned}$$

where the second equality follows from using (S.2), (S.13) and the definition of η_X. This decomposition in turn can be rearranged as

$$\begin{aligned} R_X &= \frac{1}{\eta_X} \frac{\sigma(\sigma-1+\theta)}{\theta} f_X \;\; \times \;\; (\varphi_X)^{-(\sigma-1+\theta)} \left(\frac{\sigma-1}{\sigma(\sigma-1+\theta)} \frac{L}{f_E} \right) && \text{(using (S.14))} \\ &= \frac{1}{\eta_X} \left(\frac{\sigma-1}{\theta f_E} \right) L f_X \left(\frac{f_X}{B(\tau_X c_D)^{1-\sigma}} \right)^{-\frac{\sigma-1+\theta}{\sigma-1}} && \text{(using (3))} \\ &= \frac{\psi_X}{\eta_X} L B^{\frac{\sigma-1+\theta}{\sigma-1}} (\tau_X c_D)^{-(\sigma-1+\theta)} f_X^{-\frac{\theta}{\sigma-1}}. \end{aligned}$$

Similarly, input trade flows are

$$\begin{aligned}
E_M &= \frac{\sigma(\sigma-1+\theta)}{\theta}k_M \;\times\; \phi_M^{\sigma-1+\theta}\left(\frac{(\sigma-1)^2}{\sigma^2(\sigma-1+\theta)}\frac{L}{k_E}\right) && \text{(using (S.15))}\\
&= \left(\frac{(\sigma-1)^2}{\sigma\theta k_E}\right)Lk_M\left(\frac{k_M}{A\tau_M^{1-\sigma}\Lambda_M}\right)^{-\frac{\sigma-1+\theta}{\sigma-1}} && \text{(using (9))}\\
&= \psi_M\Lambda_M^{\frac{\sigma-1+\theta}{\sigma-1}}LA^{\frac{\sigma-1+\theta}{\sigma-1}}\tau_M^{-(\sigma-1+\theta)}k_M^{-\frac{\theta}{\sigma-1}}.
\end{aligned}$$

This gives us the gravity equation expressions in (20).

To get the output trade elasticity from the gravity equation, applying (S.13) to R_D in (S.1) and subsequently dividing R_X derived above by R_D,

$$\frac{R_X}{R_D} = \tau_X^{-(\sigma-1+\theta)}\frac{\eta_D}{\eta_X}\left(\frac{f_X}{f_D}\right)^{-\frac{\theta}{\sigma-1}}.$$

Taking the log and differentiating R_X/R_D with respect to τ_X,

$$\varepsilon_X = \sigma-1+\theta-\left(\frac{d\ln(\eta_D/\eta_X)}{d\ln\tau_X}\right).$$

Moreover, from the definition of η_D,η_X, it follows that

$$-\left(\frac{d\ln(\eta_D/\eta_X)}{d\ln\tau_X}\right) = \theta\left(\frac{\sigma-1+\theta}{\sigma-1-\theta}\right)(\eta_D-\eta_X)(\mu_D-\mu_M).$$

This gives us the same expression of the output trade elasticity in (19).

To get the input trade elasticity from the gravity equation, applying (S.13) to E_D in (S.6) and subsequently dividing E_M derived above by E_D,

$$\frac{E_M}{E_D} = \tau_M^{-(\sigma-1+\theta)}\Lambda_M^{\frac{\sigma-1+\theta}{\sigma-1}}\left(\frac{k_M}{k_D}\right)^{-\frac{\theta}{\sigma-1}}.$$

Moreover, noting that $\Lambda_M = \frac{\mu_D V(\varphi_{DM})}{\mu_M V(\varphi_D)}$ and solving for Λ_M and Δ,

$$\begin{aligned}
\Lambda_M^{\frac{\sigma-1+\theta}{\sigma-1}} &= \left[\tau_M^{-(\sigma-1+\theta)}\left(\frac{\mu_D}{\mu_M}\right)^{\frac{\sigma-1}{\theta}}\left(\frac{f_{DM}}{f_D}\right)^{-1}\left(\frac{k_M}{k_D}\right)^{-\frac{\theta}{\sigma-1}}\right]^{\frac{\theta}{\sigma-1-\theta}},\\
\Delta^{\frac{\sigma-1+\theta}{\sigma-1}} &= \left[\tau_M^{-(\sigma-1+\theta)}\frac{\mu_D}{\mu_M}\left(\frac{f_{DM}}{f_D}\right)^{-\frac{\theta}{\sigma-1}}\left(\frac{k_M}{k_D}\right)^{-1}\right]^{\frac{\theta}{\sigma-1-\theta}}.
\end{aligned} \tag{S.16}$$

Taking the log and differentiating E_M/E_D with respect to τ_M,

$$\varepsilon_M = \sigma-1+\theta+\left(\frac{\sigma-1+\theta}{\sigma-1}\frac{d\ln\Lambda_M}{d\ln\tau_M}\right).$$

While Λ_M in (S.16) depends not only on τ_M directly but also on μ_D/μ_M indirectly, it follows from the definition of μ_D,μ_M that $\frac{d\ln(\mu_D/\mu_M)}{d\ln\tau_M}=0$ and only the direct changes of τ_M remains. This gives us the same expression of the input trade elasticity in (18).

To show the trade elasticities capturing the indirect effect, note that

$$\tilde{\varepsilon}_X = -\frac{d\ln(R_X/R_D)}{d\ln\tau_M} = -\frac{d\ln\Lambda_X}{d\ln\tau_M},$$
$$\tilde{\varepsilon}_M = -\frac{d\ln(E_M/E_D)}{d\ln\tau_X} = -\frac{d\ln\Delta}{d\ln\tau_X} - \frac{d\ln\Lambda_M}{d\ln\tau_X},$$

which shows that there is only the extensive margin elasticity. Applying (S.13) to $\Lambda_X = \frac{\eta_D V(\varphi_X)}{\eta_X V(\varphi_D)}$,

$$\Lambda_X = \frac{\eta_D}{\eta_X}\left(\frac{\tau_X^{\sigma-1} f_X}{f_D}\right)^{-\frac{\theta}{\sigma-1}},$$

while the closed-form solutions of Δ and Λ_M are given in (S.16). It can be easily shown that

$$-\frac{d\ln\Lambda_X}{d\ln\tau_M} = \left(\frac{(\sigma-1)(\sigma-1+\theta)}{\sigma-1-\theta}\right)(\eta_D - \eta_X),$$
$$-\frac{d\ln\Delta}{d\ln\tau_X} = \left(\frac{\theta(\sigma-1)}{\sigma-1-\theta}\right)(\mu_D - \mu_M),$$
$$-\frac{d\ln\Lambda_M}{d\ln\tau_X} = \left(\frac{(\sigma-1)^2}{\sigma-1-\theta}\right)(\mu_D - \mu_M),$$

This gives us the expressions of the trade elasticities in (21).