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Optimal Tariffs when the Trade Elasticity Varies

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Abstract

We show that the variable nature of the trade elasticity provides new policy implications of optimal tariffs. Conditional on the two sufficient statistics for welfare, the optimal level of import tariffs is the same across different trade models with a constant trade elasticity, but more generally it depends on the micro structure that makes the trade elasticity variable. Using analytical solutions of comparative statics with respect to trade liberalization and country size, we also quantitatively compare the difference in bilateral and unilateral impacts of these competitive pressures on optimal tariffs with a variable trade elasticity.

Keywords: Optimal tariffs, variable trade elasticity, trade liberalization, country size.
JEL Classification Numbers: F12, F13, F16

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1 Introduction

A growing body of empirical evidence using aggregate and firm-level data has demonstrated that country size has a critical impact on the domestic trade share, one of the sufficient statistics for welfare along with the trade elasticity (Arkolakis et al., 2012). For example, using aggregate data on manufacturing for 25 countries, Eaton and Kortum (2002, 2011) show that larger countries tend to buy much more products from the domestic market than smaller countries. Using firm-level data on manufacturing, Bernard et al. (2007) and Mayer and Ottaviano (2008) show a similar trend in the United States and European countries respectively in the sense that larger countries tend to have a larger fraction of firms that sell their products for the domestic market than smaller countries. These pieces of evidence indicate that country size and trade liberalization can have an opposite effect on the domestic trade share (i.e., the share is higher, the larger and the *less* open are countries), even though both factors are associated with the major source of competitive pressures on operating firms in the domestic market.

In this paper, we explore the mechanism through which country size and trade liberalization work differently on firm selection into exporting and address quantitative relevance of these competitive pressures for optimal trade policy in recent trade models with imperfect competition and heterogeneous firms. As is standard in the literature, we employ an asymmetric-country version of the Melitz (2003) model with monopolistic competition and CES preferences. One of the well-known drawbacks in this framework is that firms' markups are constant which implies under firm heterogeneity that country size has no selection effects. To achieve our goal mentioned above, we make three key departures from the existing models. First, we develop a heterogeneous firm model without imposing specific parameterizations to a productivity distribution. Second, we exclude a freely traded outside good sector, which makes the factoral terms-of-trade (i.e., the wage) endogenous. Finally, we analyze not only iceberg trade costs but also import tariffs that raise government revenue. These distinctions jointly help understand the role of two competitive pressures in generating different effects in a single unified setting, allowing us to quantitatively measure the impact of these pressures on optimal trade policy.

Our starting point is to notice that not only does trade liberalization but also country size affects the wage. Figure 1 displays the transition in population and GDP per capita during 1960-2020, which are regarded as a measure of country size and the wage respectively.[1] Panel A illustrates the case of the United States, displaying a clear monotone relationship between population and GDP per capita. On the other hand, Panel B illustrates the case of Japan where population is gradually declining due mainly to the low birthrate. According to the Cabinet Office of Japan, population is expected to decrease from 124 million in 2020 to 97 million in 2050 and to 86 million in 2060. It is often said that gradient shrinking in its domestic market size together with heavy reliance on overseas demand could force Japan to see a steep decline in GDP per capita (Nikkei Asia, 2019). Further, since changes in the wage imply changes in production costs for firms, such changes could alter firms' behavior. For example, using micro-level datasets in Japanese manufacturing, Fukao et al. (2008) explore how productivity differences among establishments affect establishments' turnover during 1990-2003, the recession period known as the "lost decade." They find that the turnover rate is significantly higher for less productive establishments, but nearly a half of top 10 percent of most productive establishments simultaneously exit. This puzzling fact can be explained by the increasing wage in Japan in this period, which leads these productive establishments to seek for relatively cheaper labor in foreign markets such as China. In this way, country size gives rise to the shift in trade patterns that can be thought of as business destruction in an expanding country (Bertoletti and Etro, 2017).

[1] While county size is measured by population here, it is measured by labor force in the quantitative exercise in a later section. Either interpretation of country size is isomorphic for the main result of the paper.

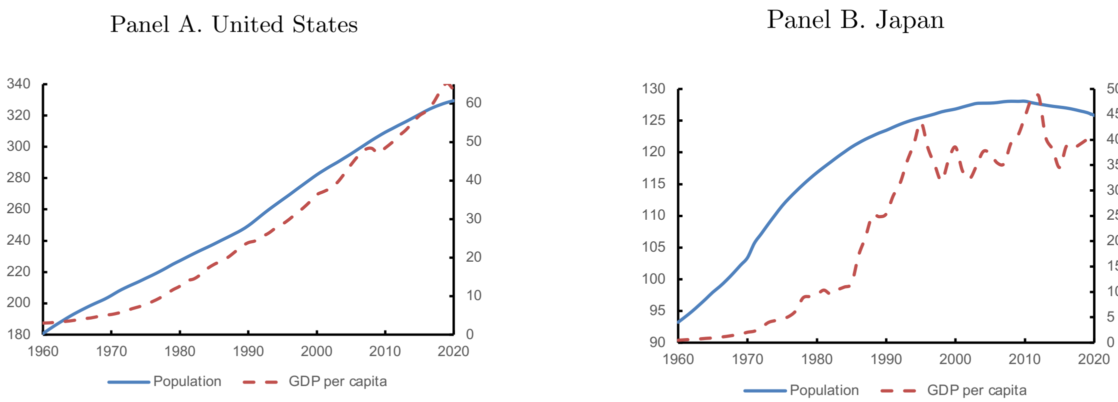

Figure 1 – Population and GDP per capita during 1960-2020

Source: World Bank Data.
Note: The left (right) scale measures population in units of million (GDP per capita in units of thousand US dollar).

Building on this observation, we first show that while unilateral trade liberalization entails selection effects, unilateral market expansion entails anti-selection effects in a country of origin. Unilateral reductions in trade costs lower firms' expected profits by inviting more intense competition, which directly induce less firms to enter a liberalizing country. With the endogenous wage, such reductions also worsen the factoral terms-of-trade (i.e., decrease the relative wage). This improves profitability by decreasing production costs and raises firms' expected profits, which indirectly induce more firms to enter a liberalizing country. In equilibrium, the indirect effect outweighs the direct effect, and hence unilateral trade liberalization entails selection effects in a country of origin, raising the domestic productivity cutoff at which the least inefficient firm can survive. In contrast, unilateral increases in country size do not directly affect firms' expected profits in an expanding country due to exogenously fixed firms' markups in CES preferences. Such increases only improve the factoral terms-of-trade (i.e., increase the relative wage) as demonstrated by Krugman (1980). This worsens profitability by increasing production costs, which makes it possible for inefficient firms to survive in an expanding country. Because of this endogenous impact of country size on the wage, unilateral market expansion entails anti-selection effects in a country of origin, decreasing the domestic productivity cutoff.

The effect of country size on selection contradicts that in Melitz and Ottaviano (2008). The reason stems from an outside good sector incorporated into their model in addition to a differentiated good sector, as shown by Demidova and Rodríguez-Clare (2013) for trade costs, and the same holds for country size. With an outside good, the difference in country size allows for a home market effect on trade patterns by muting the factoral terms-of-trade. This induces more firms to enter different sectors, so that a larger (smaller) country specializes in a differentiated (outside) good. Without an outside good like the present paper, in contrast, country size does not allow for the home market effect in trade patterns through endogenous changes in the wage. Hence it is not very surprising that our model gives a different effect of country size from Melitz and Ottaviano (2008), confirming that we have to be careful about under which conditions this popular assumption enables us to innocuously abstract from the wage channel.[2] While a larger country accommodates relatively more inefficient firms which negatively affects welfare, it can nonetheless enjoy greater welfare gains since a negative impact on declined productivity is dominated by a positive impact on increased product variety.

[2]Our setting also differs from Melitz and Ottaviano (2008) in consumers' preferences that generate constant/variable markups, but the absence of an outside good can reverse their result even with quadratic preferences; see Demidova (2017).

Given the different effect of unilateral trade liberalization and unilateral market expansion on firm selection, what can we say about their policy implications? In analyzing optimal trade policy, we show that the difference is important for the characterization of optimal tariffs, i.e., welfare-maximizing tariffs that each country would impose without fearing retaliation. In our model, optimal tariffs in a country are inversely related to its trading partner's export supply elasticity, which is composed of both the domestic trade share and the trade elasticity, as in the existing trade models. In contrast to previous work, however, trade liberalization and country size do not always lead to higher optimal tariffs in this paper. From a policy point of view, the effect of country size on optimal tariffs is of particular interest: a larger country does not always benefit from setting higher tariffs. Our model shows that although a larger country can enjoy a terms-of-trade gain from tariffs as in conventional optimal tariff theory, it accommodates more inefficient firms in the domestic market by anti-selection effects, which accelerates a welfare loss from protecting inefficient firms by tariffs. Then, whether the former benefit of tariffs dominates the latter cost depends critically on whether the "trade elasticity" (Arkolakis et al., 2012) is constant or variable. If the trade elasticity is variable and differs across markets as found by empirical work,[3] optimal tariffs can decrease with country size through the trade elasticity, identifying a potential importance to reconsider existing policy implications.

To help better appreciate the policy result, following Chaney (2008), let us decompose the trade elasticity into the intensive margin elasticity and the extensive margin elasticity where the former refers to the elasticity of each incumbent firm's shipment whereas the latter refers to the elasticity of new entrants' shipment. Since the intensive margin elasticity is constant under monopolistic competition and CES preferences, the variable nature of the trade elasticity should come from the extensive margin elasticity, which in turn depends on the micro structure of the model. In the homogeneous firm model where all firms export, there is no adjustment margin from entry of new firms (i.e., the extensive margin elasticity is zero) and the trade elasticity is the same as the intensive margin elasticity. In the heterogeneous firm model with an unbounded Pareto distribution, the extensive margin elasticity is constant (Chaney, 2008) and the trade elasticity is constant as well. In these special cases, country size affects optimal tariffs only through the domestic trade share, so that optimal tariffs increase with country size (Gros, 1987; Felbermayr et al., 2013). However, the result that the trade elasticity is constant does not generally hold. Even with a slight generalization of this distribution to a bounded Pareto distribution with a finite upper bound, the extensive margin elasticity is variable and so is the trade elasticity. In this more general case, country size affects optimal tariffs not only through the domestic trade share but also through the trade elasticity. Due to this additional channel that previous work has not taken into account, we find that optimal tariffs do not necessarily increase with country size.

We then quantitatively measure the discrepancies between optimal tariffs with a constant trade elasticity and those with a variable trade elasticity. Using a bounded Pareto distribution that makes the trade elasticity variable and standard parameter values obtained from the US data in Melitz and Redding (2015), we find that optimal tariffs are 16.6 percent in trade between two symmetric countries, which means that optimal tariffs with a variable trade elasticity are much lower than those with a constant trade elasticity in the literature.[4] In our calibration, the difference in this magnitude mainly stems from the bias that do not control for the difference in the extensive margin elasticities, rather than the difference in the calibrated parameters adopted in these papers and ours. Using the analytical solutions of the comparative statics outcomes with respect to trade liberalization and country size, we also quantitatively compare the difference in bilateral and unilateral impacts

[3]See Helpman et al. (2008), Novy (2013), Spearot (2013) and Bas et al. (2017) for empirical evidence on substantial variation in the trade elasticity across country pairs. As in the present paper, Helpman et al. (2008) and Bas et al. (2017) explain the variation focusing mainly on the extensive margin in monopolistic competition and CES preferences.

[4]Calibrating their model, Felbermayr et al. (2013) and Ossa (2014) report that optimal tariffs are 26.4 percent and 62 percent in the heterogeneous firm model under an unbounded Pareto distribution and the homogeneous firm model respectively.

of these competitive pressures on optimal tariffs with a variable trade elasticity. We find, for example, that bilateral reductions in variable trade costs increase optimal tariffs with a variable trade elasticity two times as big as optimal tariffs with a constant trade elasticity in the heterogeneous firm model because such reductions endogenously affect not only the domestic trade share but also the trade elasticity. Interestingly, even though optimal tariffs increase with country size under an bounded Pareto distribution as in the conventional optimal tariff theory, changes in optimal tariffs associated with country size are quantitatively very limited, which implies that large countries would not enjoy a large welfare gain from tariffs in trade war.

A number of papers have explored welfare and policy implications in the homogeneous and heterogeneous firm models. Regarding welfare implications, Arkolakis et al. (2012) derive a simple formula that can capture welfare gains only by the domestic trade share and the trade elasticity. As this insight applies to an important class of trade models, followup papers have examined extension/robustness of the welfare result. For example, Arkorakis et al. (2019) investigate demand functions that yield variable markups, Felbermayr et al. (2015) introduce tariffs that raise government revenue, and Melitz and Redding (2015) employ a general productivity distribution that makes the trade elasticity variable. We show that the welfare formula by Arkolakis et al. (2012) can be used to reconsider the conventional wisdom of optimal tariffs. In particular, conditional on the two sufficient statistics for welfare, the optimal level of import tariffs is the same across different trade models with a constant trade elasticity, but more generally it depends on the micro structure that makes the trade elasticity variable. We also find that firm heterogeneity distributed from outside unbounded Pareto affects a welfare measurement as in Melitz and Redding (2015), but the scope of this paper differs from theirs since we analytically show a new optimal tariff formula with a variable trade elasticity and quantitatively examine the bilateral and unilateral impacts of two major sources of competitive pressures on optimal tariffs.

As for policy implications, there is a large literature of optimal tariffs. Gros (1987) derives optimal tariffs in the homogeneous firm model which is inversely related to the trade elasticity and the trading partner's own trade share. Using Ossa's (2011) framework featured with new trade production relocation effects, Ossa (2014) provides a comprehensive analysis of optimal tariffs in a multi-sector, general equilibrium model which nests the traditional (terms-of-trade), new trade (profit-shifting) and political-economy motives in the homogeneous firm model. These analyses of optimal tariffs are extended to the heterogeneous firm model by Demidova and Rodríguez-Clare (2009) for a small economy and Felbermayr et al. (2013) for a large economy. In so doing, Felbermayr et al. (2013) also show that optimal tariffs are lower in the heterogeneous firm model than the homogeneous firm model, holding the domestic trade share equal.[5] While they find that the optimal level of import tariffs is strictly positive, one of the crucial limitations is that the trade elasticity is constant in either the homogeneous or heterogeneous firm model. As stressed by Melitz and Redding (2015), welfare changes are highly sensitive to the limitation, and even small deviations from this lead to different welfare implications by making the trade elasticity variable. In the context of trade policy, this implies that optimal tariffs that do not control for the difference in the extensive margin elasticities lead to a serious bias in policy evaluations. We highlight this caveat not only by analytically characterizing optimal tariffs with a variable trade elasticity, but also by quantitatively measuring their magnitude from our model calibrated to aggregate and firm-level US data, adopting the technique known as the exact hat algebra in the literature.[6]

[5] Recently, Costinot et al. (2020) provide a strict generalization of the optimal tariff results of Gros (1987) in the homogeneous firm model, and those of Demidova and Rodríguez-Clare (2009) and Felbermayr et al. (2013) in the heterogeneous firm model with an unbounded Pareto distribution. As in Felbermayr et al. (2013), they find that optimal tariffs are lower in the heterogeneous firm model holding *only* the domestic trade share equal, since self-selection of heterogeneous firms into exports makes the marginal rate of transformation between exports and local goods of a trading partner non-convex. In contrast, we show that optimal tariffs can be the same between the two models holding *both* the domestic trade share and the trade elasticity equal, depending on the difference in the extensive margin elasticities. If this conditioning is dropped, we find the similar result with theirs.

[6] See Ossa (2016) for a recent survey using this technique that applies for the analysis of optimal tariffs.

2 Model

2.1 Setup

Consider the Melitz (2003) model with two asymmetric countries indexed by i, j and one differentiated good sector. Country i is populated by a mass L_i of identical consumers whose preferences are

$$U_i = \left(\sum_{n=i,j} \int_{\omega \in \Omega_n} q_{ni}(\omega)^{\rho} d\omega \right)^{1/\rho}, \qquad 0 < \rho < 1,$$

where an elasticity of substitution between varieties is $\sigma = 1/(1-\rho) > 1$. Throughout this paper, we denote the exporting (importing) country by the first (second) subscript. Hence $q_{ji}(\omega)$ is an export quantity shipped from country j to country i of variety ω. As is well-known, the preferences yield the demand for $q_{ji}(\omega)$:

$$q_{ji}(\omega) = R_i P_i^{\sigma-1} (p_{ji}(\omega))^{-\sigma},$$

where R_i is aggregate expenditure of consumers and P_i is an associated price index in country i. Defining an aggregate good $Q_i \equiv U_i$, these satisfy $P_i Q_i = R_i$.

To produce varieties, upon paying fixed entry costs f_i^e (measured in country i's labor units with the wage w_i), a mass M_i^e of entrants randomly draw productivity φ from a distribution $G_i(\varphi)$ with support $(\varphi_{\min}, \varphi_{\max})$, where the upper bound is either finite ($\varphi_{\max} < \infty$) or infinite ($\varphi_{\max} = \infty$). If a firm from country j chooses to serve country i, it must incur variable trade costs $\theta_{ji} \geq 1$ (with $\theta_{jj} = 1$) and fixed trade costs f_{ji} (both measured in country j's labor units with the wage w_j). A government in each country imposes import tariffs on foreign varieties and the firm must also incur ad valorem tariffs $\tau_{ji} = 1 + t_{ji}$, where $\tau_{ji} \geq 1$ (with $\tau_{jj} = 1$). Tariffs are imposed before each firm sets markups, i.e., tariffs are modeled as cost shifters ignoring the aspect of demand shifters.[7] As a result, country i's government collects tariff revenue $(\tau_{ji} - 1) p_{ji}(\omega)/\tau_{ji}$ per unit, so that the firm receives only $p_{ji}(\omega)/\tau_{ji}$ per unit.

Following Helpman et al. (2008) and Melitz and Redding (2015), it is useful to define

$$J_i(\varphi^*) \equiv \int_{\varphi^*}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma-1} - 1 \right] dG_i(\varphi),$$
$$V_i(\varphi^*) \equiv \int_{\varphi^*}^{\varphi_{\max}} \varphi^{\sigma-1} dG_i(\varphi),$$

where $J_i(\varphi^*)$ and $V_i(\varphi^*)$ are strictly decreasing in φ^*.

2.2 Equilibrium Conditions

Under our preference assumption, a firm with productivity φ from country j to country i charges a constant markup $1/\rho$ over marginal cost $\theta_{ji} w_j/\varphi$ and tariffs τ_{ji}, and firm pricing rule satisfies $p_{ji}(\varphi) = \tau_{ji}\theta_{ji} w_j/(\rho\varphi)$. In the presence of tariffs, it is more convenient to define firm revenue *net of tariffs* $r_{ji}(\varphi) = p_{ji}(\varphi) q_{ji}(\varphi)/\tau_{ji}$. Combining the variety demand and firm pricing rule, this is given by

$$r_{ji}(\varphi) = \sigma B_i \tau_{ji}^{-\sigma} (\theta_{ji} w_j)^{1-\sigma} \varphi^{\sigma-1},$$

[7] See Felbermayr et al. (2015) for the differences between cost shifters and demand shifters of tariffs.

where

$$B_i = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} R_i P_i^{\sigma-1}$$

is the index of market demand. Since firm variable profit is $r_{ji}(\varphi)/\sigma$, the productivity cutoff that satisfies zero profit ($\frac{r_{ji}(\varphi^*_{ji})}{\sigma} = w_j f_{ji}$) is implicitly defined as

$$B_i \tau_{ji}^{-\sigma} (\theta_{ji} w_j)^{1-\sigma} (\varphi^*_{ji})^{\sigma-1} = w_j f_{ji}. \tag{1}$$

From (1), the ratio of the export productivity cutoff φ^*_{ji} to the domestic productivity cutoff φ^*_{jj} is given by

$$\left(\frac{\varphi^*_{ji}}{\varphi^*_{jj}}\right)^{\sigma-1} = \frac{\tau_{ji}^\sigma \theta_{ji}^{\sigma-1} f_{ji}}{f_{jj}} \frac{B_j}{B_i}.$$

To ensure selection into the export market ($\varphi^*_{ji} > \varphi^*_{jj}$), we assume not only that trade costs are large enough that $\tau_{ji}^\sigma \theta_{ji}^{\sigma-1} f_{ji}/f_{jj} > 1$ for i, j but also that relative market demand B_i/B_j – which is proportional to relative country size L_i/L_j – is not too different.

Free entry requires that the expected profits of entering the market in all operating countries should equal the fixed entry costs ($\sum_n \int_{\varphi^*_{in}}^{\varphi_{\max}} (\frac{r_{in}(\varphi)}{\sigma} - w_i f_{in}) dG_i(\varphi) = w_i f_i^e$). Using the definition of $J_i(\varphi^*)$ in Section 2.1, the free entry condition in country i is given by

$$\sum_{n=i,j} f_{in} J_i(\varphi^*_{in}) = f_i^e. \tag{2}$$

Since $J_i(\varphi^*)$ is strictly decreasing in φ^*, the free entry condition means that there is a one-to-one relationship between the domestic productivity cutoff φ^*_{ii} and the export productivity cutoff φ^*_{ij} .

Next, we look at the labor market clearing condition. Labor in the economy is used for entry and production ($L_i = L_i^e + L_i^p$). Using (1), (2) and the definition of $V_i(\varphi^*)$ in Section 2.1, the aggregate amount of labor used for the two activities in country i is expressed as (see Appendix A.1)

$$L_i = \frac{R_i - T_i}{w_i},$$

where $R_i = \sum_n \tau_{ni} R_{ni}$ is aggregate expenditure and $T_i = (\tau_{ji} - 1) R_{ji}$ is aggregate tariff revenue in country i.[8] Rewriting this equality as $R_i = w_i L_i + T_i$ helps us to better understand the labor market clearing condition. This means that country i's wage w_i is determined by the equality between aggregate expenditure R_i and aggregate labor income $w_i L_i$ plus aggregate tariff revenue T_i as in the usual general-equilibrium trade models. It is possible to show that the labor market clearing condition is equivalent with the trade balance condition ($R_{ij} = R_{ji}$) in that both conditions induce the same equality, $R_i = w_i L_i + T_i$.

To work on the general equilibrium, we need to rewrite the labor market clearing condition further. Let $\lambda_{ji} \equiv \tau_{ji} R_{ji} / \sum_n \tau_{ni} R_{ni}$ denote the foreign trade share from country j in country i, which is by definition inclusive of tariffs τ_{ji} paid by firms from country j to country i. Rewriting λ_{ji} as $R_{jj}/R_{ji} = \tau_{ji}(1-\lambda_{ji})/\lambda_{ji}$, it is possible to define the corresponding foreign trade share *net of tariffs*:

$$\tilde{\lambda}_{ji} \equiv \frac{R_{ji}}{\sum_n R_{ni}} = \frac{\lambda_{ji}}{\tau_{ji}(1-\lambda_{ji}) + \lambda_{ji}},$$

[8]Since r_{ji} is firm revenue net of tariffs, R_{ji} is aggregate revenue (or expenditure) from country j to country i net of tariffs.

Not surprisingly, we have $\tilde{\lambda}_{ji} = \lambda_{ji}$ if country i does not impose import tariffs ($\tau_{ji} = 1$). We also find it useful for our analysis to define a "tariff multiplier" (Felbermayr et al., 2015), i.e., the ratio of aggregate expenditure to aggregate labor income. Substituting λ_{ji} into $R_i = w_i L_i + (\tau_{ji} - 1) R_{ji}$,

$$\mu_i \equiv \frac{R_i}{w_i L_i} = \frac{\tau_{ji}}{\tau_{ji}(1 - \lambda_{ji}) + \lambda_{ji}},$$

where $\mu_i \geq 1$ as tariff revenue is redistributed to consumers and $\mu_i = 1$ in the absence of tariffs. Finally, using $w_i L_i = \sum_n R_{in}$ (labor income in country i consists of revenues earned by domestic firms and exporting firms from country i) and $R_{ij} = R_{ji}$ (trade is balanced between countries), the labor market clearing condition is expressed in terms of the foreign trade share net of tariffs:

$$w_i L_i = \sum_{n=i,j} \tilde{\lambda}_{in} w_n L_n. \tag{3}$$

Since $\tilde{\lambda}_{ij}$ depends on R_{ij}, (3) depends on the productivity cutoffs φ^*_{ij} and the market demand B_i determined by (1) and (2) respectively.

Now, we are ready for characterizing the key variables in levels. For given levels of exogenous variables, an equilibrium in levels can be defined as a set of $\{\varphi^*_{ii}, \varphi^*_{ij}, B_i, w_i\}$ which are jointly characterized by the system of eight equations in (1), (2), and (3) for i, j, where levels in country j's wage are normalized to unity by Walras's law, setting labor there as a numeraire. Once levels in these key variables are determined, the other endogenous variables can be written as a function of them. In particular, using the definition of B_i in (1), welfare per worker defined as the real wage is expressed as follows (see Appendix A.2):

$$W_i = \left(\frac{L_i}{\sigma f_{ii}}\right)^{\frac{1}{\sigma-1}} (\mu_i)^{\frac{1}{\rho}} \rho \varphi^*_{ii},$$

where the tariff multiplier μ_i enters the welfare expression in the present setting because tariff revenue is rebated back to consumers.

3 Trade Liberalization

The previous section has defined the equilibrium conditions and equilibrium variables in *levels*. This section defines the equilibrium conditions and equilibrium variables in *changes*. We first examine the impact of changes in trade barriers, holding all other exogenous variables constant. Demidova and Rodríguez-Clare (2013) study a welfare effect of asymmetric trade liberalization in the Melitz (2003) model, dispensing with the assumption of an outside good. They show that unilateral reductions in any trade barriers on either exports and imports always increase welfare in a liberalizing country, which stands in contrast to the presence of an outside good in the model with CES preferences (Demidova, 2008) and quadratic preferences (Melitz and Ottaviano, 2008). Here, with the help of the exact hat algebra, we analytically show their result. More importantly, we show that the endogenous wage can reverse the impact of country size on productivity, just as in the impact of trade liberalization on welfare in Section 4, and these analytical solutions are shown to be useful for quantifying the impact of bilateral changes in trade costs on optimal trade policy in Sections 5 and 6.[9]

[9]Though the result in this section is not entirely new, optimal tariffs cannot be characterized without the analytical solutions using the exact hat algebra. In addition, previous work has not computed with a general productivity distribution, which results in a crucial limitation for policy implications of optimal tariffs.

Suppose that country i unilaterally reduces trade costs of importing from country j. While we focus mainly on the impact of variable trade costs, the impacts of fixed trade costs and ad valorem tariffs are qualitatively similar. In contrast to variable and fixed trade costs, however, tariffs have a different effect on welfare through tariff revenue rebated back to consumers. Hence, the following analysis should be understood as the impact of exogenous changes in trade costs. We will characterize welfare-maximizing optimal tariffs after examining the impact of these exogenous changes.

Under the circumstance, let a "hat" denote proportional changes of variables ($\hat{x} \equiv dx/x$). Taking the log and differentiating the zero profit cutoff condition (1) with respect to θ_{ji}, we have

$$\hat{B}_i + (\sigma - 1)\hat{\varphi}^*_{ji} = \sigma \hat{w}_j + (\sigma - 1)\hat{\theta}_{ji}. \tag{4}$$

Similarly, differentiating the free entry condition (2) with respect to θ_{ji} yields

$$\sum_{n=i,j} f_{in} J_i'(\varphi^*_{in})\varphi^*_{in}\hat{\varphi}^*_{in} = 0. \tag{5}$$

Finally, taking the log and differentiating the labor market clearing condition (3) with respect to θ_{ji},

$$\hat{w}_i = \sum_{n=i,j} \delta_{in}(\hat{\tilde{\lambda}}_{in} + \hat{w}_n), \tag{6}$$

where

$$\delta_{ij} \equiv \frac{R_{ij}}{R_i} = \frac{\tilde{\lambda}_{ij} w_j L_j}{w_i L_i}.$$

As will described shortly, changes in $\tilde{\lambda}_{ij}$ in (6) are a function of changes in φ^*_{ii}.

Now, we are ready for characterizing the key variables in changes. Just like (1), (2) and (3) can be used to solve the model for the equilibrium in levels, (4), (5) and (6) can be used to solve for the equilibrium in changes. In the comparative statics considered here, for given changes in variable trade costs $\hat{\theta}_{ji}$, the equilibrium in changes is defined as a set of $\{\hat{\varphi}^*_{ii}, \hat{\varphi}^*_{ij}, \hat{B}_i, \hat{w}_i\}$ which are jointly characterized by the system of eight equations in (4), (5), and (6) for i, j, where changes in country j's wage are normalized to zero. Once changes in these key variables are determined, changes in the other endogenous variables can be written as a function of them. In particular, changes in welfare per worker are expressed as

$$\hat{W}_i = \frac{1}{\rho}\hat{\mu}_i + \hat{\varphi}^*_{ii}.$$

In what follows, we show that the system of equations (4), (5) and (6) can be solved for the equilibrium variables in changes. First, rearranging (5) reveals that changes in the export productivity cutoff are a constant multiple of changes in the domestic productivity cutoff as follows:

$$\hat{\varphi}^*_{ij} = -\alpha_i \hat{\varphi}^*_{ii}, \tag{7}$$

where

$$\alpha_i \equiv \frac{f_{ii} J_i'(\varphi^*_{ii})\varphi^*_{ii}}{f_{ij} J_i'(\varphi^*_{ij})\varphi^*_{ij}}.$$

Note that $\alpha_i > 0$ because $J_i(\varphi^*)$ is decreasing in φ^*. Hence (7) means that changes in θ_{ji} always shift φ^*_{ii} and φ^*_{ij} in the opposite directions. The next lemma records some important properties of α_i (see Appendix A.3):

Lemma 1

(*i*) *From the definitions of* $J_i(\varphi^*)$ *and* $V_i(\varphi^*)$ *in Section 2.1,*

$$\alpha_i = \frac{f_{ii}(\varphi^*_{ii})^{1-\sigma}V_i(\varphi^*_{ii})}{f_{ij}(\varphi^*_{ij})^{1-\sigma}V_i(\varphi^*_{ij})} = \frac{R_{ii}}{R_{ij}},$$

where $\alpha_i\alpha_j - 1 > 0$.

(*ii*) *From the definition of* α_i *and the trade balance condition,*

$$\lambda_{ji} = \frac{\tau_{ji}}{\alpha_i + \tau_{ji}}, \quad \tilde{\lambda}_{ji} = \frac{1}{\alpha_i + 1}, \quad \mu_i = \frac{\alpha_i + \tau_{ji}}{\alpha_i + 1}.$$

By definition, α_i is a function of φ^*_{ii} and φ^*_{ij}. Thus Lemma 1 means that once these productivity cutoffs are endogenously determined by (1), (2) and (3), α_i in turn pins down the foreign trade shares λ_{ji}, $\tilde{\lambda}_{ji}$ as well as the tariff multiplier μ_i.

Next, noting that (3) is rewritten as $\frac{w_iL_i}{\alpha_i+1} = \frac{w_jL_j}{\alpha_j+1}$ from Lemma 1 and using the relationship in (7), changes in the wage in (6) are related to changes in the domestic productivity cutoff as follows:

$$\hat{w}_i - \hat{w}_j = -\beta_i\hat{\varphi}^*_{ii} + \beta_j\hat{\varphi}^*_{jj}, \tag{8}$$

where

$$\beta_i \equiv \frac{\alpha_i}{\alpha_i + 1}[\sigma - 1 + \gamma_{ii} + (\sigma - 1 + \gamma_{ij})\alpha_i],$$
$$\gamma_{in} \equiv -\frac{d\ln V_i(\varphi^*_{in})}{d\ln\varphi^*_{in}}.$$

Note that β_i is a function of φ^*_{ii} and φ^*_{ij} as in α_i, while γ_{in} can be regarded as the extensive margin elasticity (Arkolakis et al., 2012) and thus β_i consists of the intensive margin elasticity $\sigma - 1$ and the extensive margin elasticity γ_{in}. The next lemma records some important properties of β_i (see Appendix A.4):

Lemma 2

(*i*) *From the definition of* β_i *and* $\tilde{\lambda}_{ji} = 1 - \tilde{\lambda}_{ii}$, β_i/α_i *is given by*

$$\frac{\beta_i}{\alpha_i} = \varepsilon_{ij} + (\gamma_{ii} - \gamma_{ij})(1 - \tilde{\lambda}_{ii}),$$

where $\varepsilon_{ij} \equiv \sigma - 1 + \gamma_{ij}$ *is the partial trade elasticity capturing only the direct effect of* θ_{ij} *on trade flows from country* i *to country* j.[10]

(*ii*) *From the definitions of* β_i *and* μ_i, *changes in* μ_i *are given by*

$$\hat{\mu}_i = (\tau_{ji} - 1)\lambda_{ii}\frac{\beta_i}{\alpha_i}\hat{\varphi}^*_{ii}.$$

[10]Since we only model cost-shifting tariffs, ε_{ij} is the partial trade elasticity with respect to not only variable trade costs θ_{ij} but also tariffs τ_{ij} (Felbermayr et al., 2015).

The first part of Lemma 2 says that β_i/α_i can be greater or smaller than the partial trade elasticity ε_{ij}, depending on the sign of $\gamma_{ii} - \gamma_{ij}$, i.e., the differential in the extensive margin elasticities between the domestic and export markets. From the definition of γ_{ij}, we know that the differential depends on $V_i(\varphi^*)$ which in turn depends on the distribution $G_i(\varphi)$. For example, if φ is drawn from an *unbounded* Pareto distribution with a shape parameter k and an infinite upper bound $\varphi_{\max} = \infty$, we have $\gamma_{ii} = \gamma_{ij} = k - (\sigma - 1)$ and hence β_i/α_i coincides with $\varepsilon_{ij}(= k)$. This lemma suggests that this property does not always hold under a general productivity distribution.

The second part says that the domestic productivity cutoff φ^*_{ii} is a single sufficient statistic for welfare even with tariff revenue.[11] Any changes in variable trade costs induce changes in the foreign trade share λ_{ji}, which affects redistribution of tariff revenue through the tariff multiplier μ_i. However, these changes are captured solely by changes in φ^*_{ii}, and changes in welfare per worker are expressed as

$$\hat{W}_i = \left(\frac{(\tau_{ji} - 1)\lambda_{ii}}{\rho} \frac{\beta_i}{\alpha_i} + 1 \right) \hat{\varphi}^*_{ii}. \tag{9}$$

Observe importantly that β_i/α_i enters the welfare expression in (9). This suggests that changes in welfare by variable trade costs depend crucially on the differential $\gamma_{ii} - \gamma_{ij}$. Using a *bounded* Pareto distribution with a finite upper bound $\varphi_{\max} < \infty$, Melitz and Redding (2015) show that $\gamma_{ii} - \gamma_{ij} < 0$ and missing this differential tends to understate the absolute changes in welfare. We show that the differential plays an important role in characterizing a country's optimal tariffs later.

We now can solve the system of eight equations ((4), (7), (8)) for eight unknowns ($\hat{\varphi}^*_{ii}, \hat{\varphi}^*_{ij}, \hat{B}_i, \hat{w}_i$ for i, j), where we have chosen $w_j = 1$ (hence $\hat{w}_j = 0$) as a numeraire. Solving (4), (7) and (8) simultaneously yields the following equilibrium relationships in changes:

$$\begin{aligned}
\hat{\varphi}^*_{ii} &= -\frac{\rho(\beta_j + \rho)}{\Xi} \hat{\theta}_{ji}, \\
\hat{\varphi}^*_{jj} &= -\frac{\rho(\beta_i - \rho\alpha_i)}{\Xi} \hat{\theta}_{ji}, \\
\hat{w}_i &= \frac{\rho^2(\beta_i + \alpha_i\beta_j)}{\Xi} \hat{\theta}_{ji},
\end{aligned} \tag{10}$$

where $\beta_i - \rho\alpha_i > 0$ (from the definitions of α_i and β_i) and

$$\Xi \equiv \sum_{n=i,j} (\beta_n + \rho) - \sum_{n=i,j} (\beta_n - \rho\alpha_n) > 0.$$

(10) shows that reductions in θ_{ji} increase $\varphi^*_{ii}, \varphi^*_{jj}$ and decrease w_i. From (9), these changes in turn mean that welfare rises not only in country j but also in country i because a decline in w_i is smaller than a decline in P_i (hence w_i/P_i rises). Tariff revenue rebated back to consumers also increases by raising μ_i (see Lemma 2(ii)), which additionally contributes to welfare gains.

The intuition behind the result is clearly seen by solving (4) and (7) first without (8):

$$\begin{aligned}
\hat{\varphi}^*_{ii} &= \frac{1}{\alpha_i\alpha_j - 1} \hat{\theta}_{ji} - \frac{\alpha_j + 1}{\rho(\alpha_i\alpha_j - 1)} \hat{w}_i, \\
\hat{\varphi}^*_{jj} &= -\frac{\alpha_j}{\alpha_i\alpha_j - 1} \hat{\theta}_{ji} + \frac{\alpha_i + 1}{\rho(\alpha_i\alpha_j - 1)} \hat{w}_i,
\end{aligned} \tag{11}$$

[11] This holds true for the case of variable trade costs that use real resources. In the case of tariffs that raise revenue, this revenue affects the welfare analysis (see Section 5).

where $\alpha_i\alpha_j - 1 > 0$ from Lemma 1. In (11), the first term represents the direct effect of reductions in θ_{ji}, while the second term represents the indirect effect of such reductions through changes in the terms of trade. The direct effect lowers (raises) expected profits and induces less (more) firms to enter in a liberalizing (non-liberalizing) country under free entry. Thus, reductions in θ_{ji} decrease φ_{ii}^* but increase φ_{jj}^*. Note that the direct effect exists even when the wage is exogenously fixed by a freely traded outside good.[12] In such a case, it follows from (9) that reductions in θ_{ji} decrease (increase) welfare in country i (country j) due to a rise (a fall) in P_i (P_j). The welfare effect is in line with previous work where unilateral trade liberalization reduces welfare in a liberalizing country (Demidova, 2008; Melitz and Ottaviano, 2008).

If the wage is endogenous, however, the indirect effect also changes expected profits. A decline in country i's relative wage improves (worsens) profitability in country i (country j) by changing production costs, which leads more (less) firms to enter the domestic market in the respective country under free entry. Hence, if the wage is endogenous, reductions in w_i (induced by reductions in θ_{ji}) increase φ_{ii}^* but decrease φ_{jj}^*, which works in the opposite direction to the direct effect. It follows from the equilibrium outcomes in (10) that the indirect effect outweighs the direct effect for φ_{ii}^* whereas the converse is true for φ_{jj}^*, and thus both cutoffs rise as a result of reductions in θ_{ji}.

The finding means that the endogenous wage has a critical impact on the home market effect on the trade patterns. Solving the price index P_i for the mass of entrants M_i^e yields

$$\frac{M_i^e}{M_j^e} = \left(\frac{w_i}{w_j}\right)^{\sigma-1} \frac{(P_i/P_j)^{1-\sigma} V_j(\varphi_{jj}^*) - \tau_{ji}^{-\sigma}\theta_{ji}^{1-\sigma} V_j(\varphi_{ji}^*)}{V_i(\varphi_{ii}^*) - \tau_{ij}^{-\sigma}\theta_{ij}^{1-\sigma}(P_i/P_j)^{1-\sigma} V_i(\varphi_{ij}^*)}.$$

If w_i is exogenous by an outside good, (7) and (11) reveal that M_i^e/M_j^e is decreasing in θ_{ji}, which means that trade liberalization in country i leads to redistribution of firms into the outside good (differentiated good) sector in country i (country j). Further, firm export revenue satisfies

$$\frac{r_{ij}(\varphi)}{r_{ji}(\varphi)} = \frac{B_j}{B_i}\left(\frac{\tau_{ij}}{\tau_{ji}}\right)^{-\sigma}\left(\frac{\theta_{ij}w_i}{\theta_{ji}w_j}\right)^{1-\sigma}.$$

From (4) and (11), $r_{ij}(\varphi)/r_{ji}(\varphi)$ is decreasing in θ_{ji}. This means that trade liberalization in country i changes the trade patterns in favor of country j, not only through firm entry (extensive margin) but also through firm revenue (intensive margin). If w_i is endogenous without an outside good, (10) shows that the two margins are not always decreasing in θ_{ji}, and hence the home market effect is not operative on the trade patterns. However, trade liberalization increases $\varphi_{ii}^*, \varphi_{jj}^*$ but decreases $\varphi_{ij}^*, \varphi_{ji}^*$. From Lemma 1, these changes increase the foreign trade share $\lambda_{ji}, \tilde{\lambda}_{ji}$, and hence reduce the domestic trade share $\lambda_{ii}, \tilde{\lambda}_{ii}$ for i, j, which ensures the welfare gains from unilateral trade liberalization in both countries (Arkolakis et al., 2012).

Though we have focused on the impact of variable trade costs on imports θ_{ji}, we can show that the impacts of *any* trade costs $(\theta_{ij}, \theta_{ji}, f_{ij}, f_{ji}, \tau_{ij}, \tau_{ji})$ on the productivity cutoffs are qualitatively similar (see Appendix A.5). In the case of variable trade costs on exports θ_{ij}, for example,

$$\begin{aligned}
\hat{\varphi}_{ii}^* &= -\frac{\rho(\beta_j - \rho\alpha_j)}{\Xi}\hat{\theta}_{ij}, \\
\hat{\varphi}_{jj}^* &= -\frac{\rho(\beta_i + \rho)}{\Xi}\hat{\theta}_{ij}, \\
\hat{w}_i &= -\frac{\rho^2(\beta_j + \alpha_j\beta_i)}{\Xi}\hat{\theta}_{ij}.
\end{aligned}$$

[12] To introduce an outside good, we require that L_i/L_j is not too different across countries to allow for complete specialization between countries (i.e., if L_i/L_j is too different, a small country can specialize in an outside good).

Thus, reductions in export costs θ_{ij} also increase the domestic productivity cutoffs in both countries as above. Only the difference is that reductions in *import* costs θ_{ji} *decrease* w_i, whereas reductions in *export* costs θ_{ij} *increase* w_i. The difference reflects the fact that the direct effect outweighs the indirect effect for φ^*_{ii} whereas the converse is true for φ^*_{jj}, because the signs of (11) are opposite in this trade liberalization. The same claim applies not only to variable trade costs, but also to fixed trade costs and tariffs.

Finally, we can say that, starting from a symmetric situation, the effect of trade liberalization is always greater in a liberalizing country than in a non-liberalizing country. Regarding variable trade costs on imports θ_{ji}, for example, evaluating (10) at $\alpha_i = \alpha_j$ and $\beta_i = \beta_j$ reveals that $\hat{\varphi}^*_{ii} > \hat{\varphi}^*_{jj}$. It follows immediately from (9) that welfare gains from trade liberalization are greater in country i than in country j. Clearly, a similar claim applies to variable trade costs on exports θ_{ij} in the sense that, starting from a symmetric situation, country j enjoys higher welfare gains from trade liberalization than country i.

Proposition 1 *Unilateral trade liberalization in variable and fixed trade costs on either exports or imports as well as tariffs has the following effects:*

(i) *The relative wage falls in a liberalizing country.*

(ii) *The domestic (export) productivity cutoff rises (falls), and the domestic (foreign) trade share falls (rises) in both countries.*

(iii) *Trade liberalization is unambiguously welfare-enhancing for both countries. Starting from a symmetric situation, the effect is always greater in a liberalizing country than in a non-liberalizing country.*

Proposition 1 is essentially the same as the finding in Demidova and Rodríguez-Clare (2013).[13] They show that the endogenous wage can reverse the impact of asymmetric trade liberalization on welfare in a liberalizing country due to a failure of the home market effect on the trade patterns without an outside good. While they graphically show the finding with a simple figure, we analytically show this with the exact hat algebra. More important, however, is our tractability to study the impact of another competitive measure, i.e., country size, which can be examined in a parallel manner with trade liberalization without imposing a specific productivity distribution. Further, the analytical solutions also allow us to quantitatively measure the impact of unilateral changes in trade costs as well as country size on optimal trade policy.

4 Country Size

Let us next investigate changes in country size, holding all the other exogenous variables constant, which has been extensively examined in the literature. Melitz and Ottaviano (2008) are the first to show that a country with larger size entails higher productivity and welfare through tougher competition in the domestic market, thereby reducing average markups across firms. Due to the existence of an outside good sector that gives rise to the home market effect on the trade patterns, however, they find that trade liberalization and country size have an opposite impact on welfare in a country of origin: a unilaterally liberalizing country can be worse off by relocating firm entry from a liberalizing country to a non-liberalizing country (sometimes referred to as "firm delocation" in the literature).

[13] The result also relates to Felbermayr and Jung (2012) and Felbermayr et al. (2013), although their analysis is less general than ours in that their model relies on an unbounded Pareto distribution, whereby variable and fixed trade costs are symmetric. These restrictions have crucial differences in trade policy implications.

We show that, in the absence of an outside good, the endogenous wage can reverse the impact of country size, just as in the impact of unilateral trade liberalization: the domestic productivity cutoff decreases with country size so that a country with larger size entails lower productivity, which stands in sharp contrast to Melitz and Ottaviano (2008) with an outside good. Although a larger country accommodates relatively more inefficient firms which negatively affects welfare, it can nonetheless enjoy greater welfare gains since a negative impact on declined productivity is dominated by a positive impact on increased product variety.[14]

Suppose that country i unilaterally expands market size L_i. Denoting proportional changes of variables by a "hat" once again, and taking the log and differentiating the zero cutoff condition (1) with respect to L_i,

$$\hat{B}_j + (\sigma - 1)\hat{\varphi}^*_{ij} = \sigma \hat{w}_i. \tag{12}$$

While changes in the free entry condition (5) is the same as before, taking the log and differentiating the labor market clearing condition (3) with respect to L_i,

$$\hat{w}_i + \hat{L}_i = \sum_{n=i,j} \delta_{in}(\hat{\tilde{\lambda}}_{in} + \hat{w}_n) + \delta_{ii}\hat{L}_i. \tag{13}$$

The definition of the equilibrium in changes with respect to L_i is similar to that with respect to θ_{ji} in the previous section: for given changes in country size $\hat{L}_i$, the equilibrium in changes can be defined as a set of $\{\hat{\varphi}^*_{ii}, \hat{\varphi}^*_{ij}, \hat{B}_i, \hat{w}_i\}$ which are jointly characterized by (5), (12), and (13) for i, j, where changes in country j's wage are normalized to zero. In the comparative statics here, however, changes in welfare per worker must be modified because changes in L_i have a direct influence on welfare in country i. It follows from Lemma 2 that such welfare changes that correspond to (9) are expressed as

$$\hat{W}_i = \left(\frac{(\tau_{ji} - 1)\lambda_{ii}}{\rho} \frac{\beta_i}{\alpha_i} + 1 \right) \hat{\varphi}^*_{ii} + \frac{\hat{L}_i}{\sigma - 1}, \tag{14}$$

which shows that, to know what happens to welfare as a result of unilateral market expansion, we need to take account of changes in L_i as well as changes in φ^*_{ii}.

As in unilateral trade liberalization, we can explicitly solve the system of equations in (5), (12), and (13). Since (5) remains the same, the one-to-one relationship between φ^*_{ii} and φ^*_{ij} in (7) also remains the same. In contrast, changes in the labor market clearing condition (13) are expressed as

$$\hat{w}_i - \hat{w}_j = -\beta_i \hat{\varphi}^*_{ii} + \beta_j \hat{\varphi}^*_{jj} - \hat{L}_i, \tag{15}$$

where the definitions of α_i and β_i appearing in the equilibrium in changes are exactly the same as those in the previous section. Noting that (5), (12), and (15) are eight equations with eight unknowns where $w_j = 1$, and solving these equations simultaneously yields

$$\begin{aligned} \hat{\varphi}^*_{ii} &= -\frac{\rho(\alpha_j + 1)}{\Xi}\hat{L}_i, \\ \hat{\varphi}^*_{jj} &= \frac{\rho(\alpha_i + 1)}{\Xi}\hat{L}_i, \\ \hat{w}_i &= \frac{\rho^2(\alpha_i\alpha_j - 1)}{\Xi}\hat{L}_i. \end{aligned} \tag{16}$$

[14] In the calibration exercise in Section 6, we show that a negative impact on declined productivity is quantitatively very small relative to a positive impact of increased product variety.

(16) shows that an increase in L_i decreases φ_{ii}^* but increases φ_{jj}^* and w_i. From (14), these changes mean that an increase in L_i always increase welfare in country j, whereas it can increase or decrease welfare in country i, depending upon the magnitudes of a decline in φ_{ii}^* (declined productivity) and a rise in L_i (increased product variety).

The intuition is again clearly explained by solving (7) and (12) first without (15):

$$\begin{aligned}\hat{\varphi}_{ii}^* &= -\frac{\alpha_j + 1}{\rho(\alpha_i\alpha_j - 1)}\hat{w}_i,\\ \hat{\varphi}_{jj}^* &= \frac{\alpha_i + 1}{\rho(\alpha_i\alpha_j - 1)}\hat{w}_i.\end{aligned} \tag{17}$$

Simple comparison between (11) and (17) immediately reveals that the direct effect of country size is absent in this case due to the peculiar and restrictive property of monopolistic competition and CES preferences, and there is only the indirect effect of these increases through changes in the terms of trade. Hence, if $\hat{w}_i = 0$ by a freely tradable outside good, (17) shows that country size has no impact on the domestic productivity cutoff ($\hat{\varphi}_{ii}^* = \hat{\varphi}_{jj}^* = 0$). Given this, (14) in turn shows that country size increases welfare in country i due solely to increased product variety, as in the standard heterogeneous firm model with CES preferences (Melitz, 2003), let alone the homogeneous firm model (Krugman, 1980).

If the wage is endogenous, however, county size indirectly changes expected profits. A rise in country i's relative wage worsens (improves) profitability in country i (country j) by changing production costs, which leads less (more) firms to enter the domestic market in the respective country under free entry. Hence, if the wage is endogenous, increases in w_i (induced by increases in L_i) decrease φ_{ii}^* but increase φ_{jj}^*. It is important to stress that the negative impact on φ_{ii}^* comes from the home market effect on w_i as in Krugman (1980). (The negative impact is absent in Krugman (1980) because firm productivity levels are exogenously given.) This causes higher marginal cost and lower profitability in an expanding country, which leads to less competitive pressures on firms and makes it possible for less productive firms to survive there. Note also that, in contrast to Melitz and Ottaviano (2008) in which country size has no impact on the productivity cutoffs of a trading partner, country size does affect these cutoffs in the present paper through the relative wage that changes competitiveness across trading countries.

As with unilateral trade liberalization, the home market effect on the trade patterns (induced by increases in country size) does not always work in the presence of the endogenous wage. From the labor market clearing condition, the mass of entrants is alternatively expressed as

$$\frac{M_i^e}{M_j^e} = \left(\frac{\sum_n f_{jn}(\varphi_{jn}^*)^{1-\sigma}V_j(\varphi_{jn}^*)}{\sum_n f_{in}(\varphi_{in}^*)^{1-\sigma}V_i(\varphi_{in}^*)}\right)\frac{L_i}{L_j}.$$

Furthermore, let $M_{ii} = [1 - G_i(\varphi_{ii}^*)]M_i^e$ and $M_{ij} = [1 - G_i(\varphi_{ij}^*)]M_i^e$ respectively denote the mass of domestic firms and that of exporting firms. Then the mass of these firms satisfies

$$\frac{M_{ii}}{M_{jj}} = \left(\frac{1 - G_i(\varphi_{ii}^*)}{1 - G_j(\varphi_{jj}^*)}\right)\frac{M_i^e}{M_j^e}, \quad \frac{M_{ij}}{M_{ji}} = \left(\frac{1 - G_i(\varphi_{ij}^*)}{1 - G_j(\varphi_{ji}^*)}\right)\frac{M_i^e}{M_j^e}.$$

If w_i is exogenous, country size has no impact on the productivity cutoffs and the values in the brackets above. Since the mass of entrants M_i^e/M_j^e increases proportionately to country size L_i/L_j in the present model with a single differentiated good sector, the mass of domestic firms M_{ii}/M_{jj} and exporting firms M_{ij}/M_{ji} also increases proportionately to country size. Therefore, market expansion in country i gives rise to the following pattern of firm entry across trading countries:

$$\frac{M_i^e}{M_j^e} = \frac{M_{ii}}{M_{jj}} = \frac{M_{ij}}{M_{ji}}.$$[15]

From (12) and (17), we have that firm revenue $r_{ij}(\varphi)/r_{ji}(\varphi)$ is not affected by country size. If w_i is endogenous, in contrast, country size has an impact on the productivity cutoffs and the values in the brackets above. While the mass of entrants does not necessarily increase more than proportionately to country size and thereby the home market effect is not operative on the trade patterns, the comparative statics in (16) show that the mass of domestic firms (exporting firms) always increases more (less) than proportionately to the mass of entrants. Therefore, market expansion in country i gives rise to the following pattern of firm entry:

$$\frac{M_{ij}}{M_{ji}} < \frac{M_i^e}{M_j^e} < \frac{M_{ii}}{M_{jj}}.$$

Further, $r_{ij}(\varphi)/r_{ji}(\varphi)$ is also decreasing in L_i. This means that market expansion in country i changes the trade patterns in favor of country j through both the extensive and intensive margins. Intuitively, country i with a relatively higher proportion of consumers has more incentive to save trade costs that are high enough to generate selection; thus firms find it less (more) profitable to export their goods to a smaller (larger) country. As a result, if country i is relatively larger than country j, the size difference allows relatively less (more) exporting firms to exist in country i (country j). Just like unilateral trade liberalization has a crucial impact on welfare by the shift in the trade patterns, unilateral market expansion also has a crucial impact on welfare by the shift from an expanding country with the high wage to a non-expanding country with the low wage. This finding is in line with recent research of monopolistic competition in which the wage (or per-capita income) plays a pivotal role. For example, in a model with additively separable indirect utilities, Bertoletti and Etro (2017) show that the shift in the trade pattern can be thought of as business destruction (creation) where a rich (poor) country with the high (low) wage is characterized by concentration (expansion) of exporting firms. In our model with CES preferences, the shift arises only when the wage is endogenous.

The fact that country size affects selection also yields empirically consistent predictions that larger (smaller) countries tend to be less (more) open. At the aggregate level, the domestic trade share in total expenditure in country i can be expressed from Lemma 1 as

$$\lambda_{ii} = \frac{\alpha_i}{\alpha_i + \tau_{ji}}, \quad \tilde{\lambda}_{ii} = \frac{\alpha_i}{\alpha_i + 1}.$$

Since φ_{ii}^* is decreasing in L_i and α_i is decreasing in φ_{ii}^*, the share is increasing in country size: the domestic spending share is higher, the larger is country size, as documented by aggregate data from various countries (Eaton and Kortum, 2002, 2011). While the large domestic trade share would encourage firms to export those products for which they have the large domestic market (known as the Linder hypothesis), this is not the case in our model due to the feedback from country size to selection. At the firm level, the ratio of exporting firms to domestic firms in country i (which is less than unity with export market selection) can be expressed as

$$\frac{M_{ij}}{M_{ii}} = \frac{1 - G_i(\varphi_{ij}^*)}{1 - G_i(\varphi_{ii}^*)}.$$

[15] In a multi-sector version of the model with an outside good, it can be easily shown that the mass of entrants rises more than proportionately to country size. This generates the home market effect on the trade pattens such that an increase in country size leads to disproportionately reallocations of labor to the differentiated good sectors, thereby allowing a larger country to enjoy higher welfare gains through increased product variety. Note, however, that the above entry pattern still holds even in this case.

It follows from (16) that the ratio is decreasing in country size, which implies that the share of exporting firms among operating firms is lower, the greater is country size. This is also in line with empirical evidence using firm-level data. Bernard et al. (2007) find that 18 percent of firms export in the United States, while Mayer and Ottaviano (2008) report that a much larger fraction of firms export in European countries.[16]

It remains to show the impact of country size on welfare in an expanding country. The impact depends on the magnitudes of a decline in φ_{ii}^* and a rise in L_i, where the former gives rise to a welfare loss by increasing the domestic trade share $\lambda_{ii}, \tilde{\lambda}_{ii}$ there. Applying (16) and rearranging, changes in welfare (14) can be expressed in terms of changes in φ_{ii}^* only (see Appendix A.6):

$$\hat{W}_i = \frac{1}{\sigma - 1}\left((\sigma-1)(\beta_i+\rho) - \frac{\sigma\beta_i}{\mu_i} - (\beta_j - \rho\alpha_j)\left(\frac{\alpha_i+1}{\alpha_j+1}\right)\right)\hat{\varphi}_{ii}^*.$$

Since unilateral market expansion in country i decreases φ_{ii}^*, such expansion leads to a welfare gain in that country if the value in the brackets is negative. Unfortunately this is not always the case, and we cannot say in general that the country gains from market expansion in the current model. It is possible to prove, however, that starting from a symmetric situation and free trade ($\mu_i = 1$), unilateral market expansion in country i unambiguously improves welfare in both countries.

Proposition 2 *Unilateral market expansion has the following effects:*

(*i*) *The relative wage rises in an expanding country.*

(*ii*) *The domestic (export) productivity cutoff falls (rises), and the domestic (foreign) trade share rises (falls) in an expanding country. The converse is true in a non-expanding country.*

(*iii*) *Starting from a symmetric situation and free trade, market expansion is unambiguously welfare-enhancing for both countries.*

The result in Proposition 2 has a noticeable difference from that in the existing literature. In an influential study on allocation efficiency with VES preferences, Dhingra and Morrow (2019) find that market expansion provides a welfare gain when consumers' preferences are "aligned," i.e., demand shifts alter private and social markups in the same directions. This means that market expansion increases welfare in CES preferences, but this is not true in our model. The reason is that unilateral market expansion entails anti-selection effects that work to decline productivity in a country of origin. As shown by Dhingra and Morrow (2019), one of sufficient conditions for welfare gains is that productivity does not decline after market expansion, which is not satisfied here. Hence, market expansion does not always lead to gains due to distortions from anti-selection effects in our setting, whereas distortions stem from variable markups in their setting.[17]

5 Trade Policy

So far, we have examined the impact of exogenous changes in the two competitive measures on key endogenous variables without specifying a productivity distribution function. In this section, we show that the generality is important for the characterization of a country's optimal trade policy.

[16]See also Amiti et al. (2014) who report that 24 percent of firms export in Belgium. Of course, this cannot be explained only by the difference in market size and it also reflects other economic factors such as the difference in trade costs and technology.

[17]Demidova and Rodríguez-Clare (2013) and Felbermayr and Jung (2018) also show similar results but the analysis of market size is confined to an unbounded Pareto distribution. We show that this limitation has serious policy implications in next sections.

5.1 Optimal Tariffs

Suppose that country i chooses a tariff rate on imports from country j to maximize welfare. For the moment, we focus on the effect of country i's tariffs τ_{ji} holding country j's tariffs τ_{ij} fixed, and derive optimal tariffs that maximize country i's welfare. In country j that faces tariffs by country i, the effect of τ_{ji} is essentially the same as that of θ_{ji}, and changes in welfare per worker with respect to τ_{ji} are expressed as

$$\hat{W}_j = \left(\frac{(\tau_{ij}-1)\lambda_{jj}}{\rho} \frac{\beta_j}{\alpha_j} + 1 \right) \hat{\varphi}^*_{jj}.$$

From Proposition 1, an increase in τ_{ji} decreases the domestic productivity cutoff φ^*_{jj} which lowers welfare in country j. In country i that imposes tariffs on country j, on the other hand, there is an additional effect of τ_{ji} on welfare through changes in redistribution of tariff revenue, which contributes to a welfare gain there. Noting that changes in the tariff multiplier μ_i with respect to τ_{ji} in Lemma 2 are given by

$$\hat{\mu}_i = (\tau_{ji}-1)\lambda_{ii}\frac{\beta_i}{\alpha_i}\hat{\varphi}^*_{ii} + \lambda_{ji}\hat{\tau}_{ji},$$

changes in welfare per worker in country i corresponding to (9) and (14) are expressed as

$$\hat{W}_i = \left(\frac{(\tau_{ji}-1)\lambda_{ii}}{\rho} \frac{\beta_i}{\alpha_i} + 1 \right) \hat{\varphi}^*_{ii} + \frac{\lambda_{ji}}{\rho}\hat{\tau}_{ji}.$$

In this expression of welfare changes, the first term is a welfare loss from tariffs (as inefficient firms are sheltered by tariffs), and the second term is a welfare gain from tariffs (as tariff revenue is rebated back to consumers). After rearranging, this can be written in terms of changes in φ^*_{ii} only (see Appendix A.7):

$$\hat{W}_i = \frac{\lambda_{ji}(\beta_i - \rho\alpha_i)}{\rho} \left(\frac{\beta_j - \rho\alpha_j}{\beta_j + \rho} - \frac{1}{\tau_{ji}} \right) \hat{\varphi}^*_{ii}. \tag{18}$$

Recall from Proposition 1 that an increase in τ_{ji} also decreases φ^*_{ii}. Setting $\tau_{ji} = 1$ in (18) then implies that a small import tariff τ_{ji} unambiguously improves country i's welfare (which comes at the expense of country j), and therefore the welfare-maximizing optimal tariffs are strictly positive for country i. We can also say that, starting from a symmetric situation, country i's gain from tariffs cannot compensate country j's loss and thus the effect of τ_{ji} on world welfare is always negative.

Before moving to characterizing country i's optimal tariffs, it is useful to relate the expression in (18) with that in the existing literature. Using λ_{ii} and μ_i in Lemma 1, (18) is alternatively written as

$$\hat{W}_i = -\frac{\alpha_i}{\beta_i}\hat{\lambda}_{ii} + \left(\frac{\beta_i - \rho\alpha_i}{\rho\beta_i} \right) \hat{\mu}_i. \tag{19}$$

Welfare changes in (19) encompass the results in Arkolakis et al. (2012) without tariff revenue (i.e., $\mu_i = 1$ and hence $\hat{\mu}_i = 0$) and those in Felbermayr et al. (2015) with tariff revenue for the Melitz (2003) model under an unbounded Pareto distribution with a shape parameter k and an infinite upper bound $\varphi_{\max} = \infty$. In fact, noting that the extensive margin elasticity is constant at $k - (\sigma - 1)$ and β_i/α_i coincides with the partial trade elasticity $\varepsilon_{ij} (= k)$ under the specific productivity distribution, (19) is expressed as

$$\hat{W}_i = -\frac{1}{\varepsilon_{ij}}\hat{\lambda}_{ii} + \left(1 + \frac{\eta}{\varepsilon_{ij}} \right) \hat{\mu}_i,$$

where $\eta \equiv \frac{k}{\sigma-1}(1+\frac{1-\sigma}{k}) > 0$. The above expression shows that welfare changes can be captured solely by the two sufficient statistics λ_{ii} and ε_{ij} without tariff revenue as indicated by the first term (Arkolakis et al., 2012), but their welfare formula requires qualification with tariff revenue if tariffs act as cost shifters as indicated by the second term (Felbermayr et al., 2015).

The results however depend critically on the assumption that the trade elasticity is constant, as stressed by Melitz and Redding (2015). To see this caveat in the current setting, using the general expression of β_i/α_i in Lemma 2 and rearranging, we can further express (19) as

$$\hat{W}_i = \frac{1}{\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij}} \left(\hat{M}_i^e - \hat{\lambda}_{ii} \right) + \left(\frac{1}{\rho} - \frac{1}{\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij}} \right) \hat{\mu}_i. \tag{20}$$

This expression is a counterpart to that in Melitz and Redding (2015, eq. (33)), albeit the difference that we derive welfare changes by tariffs that raise government revenue. Note that, beside the domestic trade share λ_{ii} and the trade elasticity ε_{ij}, welfare changes depend on the extensive margin elasticity differential between the domestic and export markets $\gamma_{ii} - \gamma_{ij}$, which they refer to as the "hazard differential." They argue that, when the differential exists, the micro structure matters for welfare beyond the domestic trade share and the trade elasticity, causing substantial discrepancies in welfare changes by variable trade costs between a constant and variable trade elasticity. The welfare expression in (20) indicates that the same critique applies to the analysis of tariffs.[18]

Under which productivity distributions does the extensive margin elasticity differential exist? Obviously, $\gamma_{ii} = \gamma_{ij} = k - (\sigma - 1)$ under an unbounded Pareto distribution. Consider a slight generalization from this to a bounded Pareto distribution with a shape parameter k and a finite upper bound $\varphi_{\max} < \infty$. In this more general case, the extensive margin elasticity is expressed as (Melitz and Redding, 2015)

$$\gamma_{in} = (k - (\sigma - 1)) \frac{\left(\frac{\varphi_{\min}}{\varphi_{in}^*}\right)^{k-(\sigma-1)}}{\left(\frac{\varphi_{\min}}{\varphi_{in}^*}\right)^{k-(\sigma-1)} - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^{k-(\sigma-1)}},$$

where unbounded Pareto can be treated as a limit case in which $\lim_{\varphi_{\max}\to\infty} \gamma_{in} = k - (\sigma - 1)$. It is easily seen that γ_{in} is strictly increasing in the productivity cutoff φ_{in}^*. From this, we have that (i) the trade elasticity $\varepsilon_{ij} = \sigma - 1 + \gamma_{ij}$ is variable and differs across markets; and (ii) $\gamma_{ii} - \gamma_{ij} < 0$ under selection into the export market. Using bounded Pareto, Melitz and Redding (2015) show that if the extensive margin is more elastic in the export market than in the domestic market ($\gamma_{ii} - \gamma_{ij} < 0$), welfare changes are under-estimated relative to those without this differential. We show that missing the differential leads to a serious bias in evaluations of optimal tariffs. While we will mainly consider bounded Pareto to deliver key policy implications, the fact that the trade elasticity is variable and differs across markets is not specific to this distribution. For example, noting that the gravity equation with a constant trade elasticity is mis-specified under any distribution other than unbounded Pareto, Head et al. (2014) explore the welfare gains from trade under a log-normal distribution. As in bounded-Pareto heterogeneity, log-normal heterogeneity induces a variable trade elasticity along with a negative differential, yielding important differences in the welfare gains from trade liberalization. Similarly, comparing unbounded Pareto and log-normal as potential distributions of firm heterogeneity, Bas et al. (2017) show that the latter distribution does a better job at matching theory and evidence.

[18] The differential also has a crucial impact on the mass of entrants. Under the unbounded Pareto distribution where $\gamma_{ii} - \gamma_{ij} = 0$, the mass does not respond to changes in exogenous variables and hence $\hat{M}_i^e = 0$. More generally, we can show that if $\gamma_{ii} - \gamma_{ij} \lesseqqgtr 0$, changes in the mass of entrants are given by $\hat{M}_i^e \gtreqqless 0$ (see Appendix A.4).

We now turn to characterizing the optimal tariffs for country i. Setting $\hat{W}_i = 0$ in (18) and solving for τ_{ji} yields the following expression for the optimal tariffs:[19]

$$\tau_{ji}^* = 1 + \underbrace{\frac{\rho}{\frac{\alpha_j}{\alpha_j+1}\left(\frac{\beta_j}{\alpha_j} - \rho\right)}}_{t_{ji}^*} = \frac{\beta_j + \rho}{\beta_j - \rho\alpha_j} > 1.$$

Moreover, using $\tilde{\lambda}_{jj} = \alpha_j/(\alpha_j + 1)$ from Lemma 1 and substituting β_j/α_j from Lemma 2, the optimal tariffs t_{ji}^* are implicitly characterized as

$$t_{ji}^* = \frac{\rho}{\tilde{\lambda}_{jj}\left(\varepsilon_{ji} + (\gamma_{jj} - \gamma_{ji})(1 - \tilde{\lambda}_{jj}) - \rho\right)}. \tag{21}$$

Hence, the optimal tariffs in country i are inversely related to country j's export supply elasticity, which is composed of the domestic trade share in country j ($\tilde{\lambda}_{jj}$) and the trade elasticity from country j to country i (ε_{ji}), as in the existing models in the literature. The crucial difference, however, is that the trade elasticity is not necessarily constant in this model.

It is worth stressing that the optimal tariff in (21) is a generalization of some of well-known optimal tariff formulas. If the underlying distribution is assumed to be unbounded Pareto with a shape parameter k and an infinite upper bound $\varphi_{\max} = \infty$, the extensive margin elasticities γ_{jj}, γ_{ji} are constant at $k - (\sigma - 1)$ and the trade elasticity ε_{ji} is constant at k. Thus, (21) reduces to

$$t_{ji}^* = \frac{\rho}{\tilde{\lambda}_{jj}(k - \rho)}. \tag{22}$$

This expression is exactly the same as the optimal tariffs shown by Felbermayr et al. (2013) in a heterogeneous firm model a la Melitz (2003) under the unbounded Pareto distribution. Further, it is also possible to consider a homogeneous firm model as a special case with a degenerated productivity distribution in which firms draw their productivity level of either zero or an exogenous productivity cutoff (see Melitz and Redding (2015) for details). If fixed and variable trade costs are sufficiently low that all homogeneous firms export in this model, we can easily show that the extensive margin elasticities γ_{jj}, γ_{ji} are constant at zero and the trade elasticity ε_{ji} is constant at $\sigma - 1$. Thus, (21) reduces to

$$t_{ji}^* = \frac{1}{\tilde{\lambda}_{jj}(\sigma - 1)}. \tag{23}$$

This expression is exactly the same as the optimal tariffs shown by Gros (1987) in a homogeneous firm model a la Krugman (1980).

At this standpoint, we need to mention two caveats for the optimal tariff formula in (21). First, we cannot say that the optimal tariffs are smaller in the heterogeneous firm model than in the homogeneous firm model. Just like the different trade models yield the different domestic trade shares $\tilde{\lambda}_{jj}$, these models also yield the different trade elasticities ε_{ji}. This means that the optimal tariffs in the different trade models are not directly comparable without taking account of the difference in the trade elasticity. The optimal tariff formula in (21) is useful to shed light on this point. Plugging (21) in $\gamma_{jj} - \gamma_{ji} = 0$ that holds in the heterogeneous model with

[19] Following Felbermayr et al. (2013), we use the F.O.C. of welfare maximization (18) assuming the sufficiency to be satisfied. Instead of using the F.O.C., Demidova (2017) looks at the direct impact of tariffs on aggregate quantity and finds the result that strongly resembles the one derived by Felbermayr et al. (2013).

unbounded Pareto and the homogeneous firm model, we find that conditional the two empirically observable moments above, the optimal tariffs are the same between the different trade models. Of course, the result is a direct implication of the insight by Arkolakis et al. (2012) for the optimal tariff setting: conditional on the two sufficient statistics for welfare $\tilde{\lambda}_{jj}, \varepsilon_{ji}$, changes in welfare induced by tariffs are the same and, consequently, levels of the optimal tariffs are also the same.

Second, the equivalence of the optimal tariffs between the different trade models holds only if the extensive margin elasticity differential is zero, i.e., $\gamma_{jj} - \gamma_{ji} = 0$. If this does not hold, the optimal tariffs are different even after controlling for the two sufficient statistics for welfare. For example, if the extensive margin is more elastic in the export market than in the domestic market ($\gamma_{ii} - \gamma_{ij} < 0$), welfare changes by tariffs tend to be under-estimated relative to those without this differential as seen in (20). Noting that the welfare-maximizing optimal tariffs are strictly positive in country i, this implies in our policy context that the government faces a smaller welfare loss from an increase in tariffs and therefore has more incentive to impose higher tariffs for $\gamma_{ii} - \gamma_{ij} < 0$ than for $\gamma_{ii} - \gamma_{ij} = 0$. In other words, the optimal tariffs that do not control for the differential $\gamma_{jj} - \gamma_{ji}$ tend to be under-estimated since $\varepsilon_{ji} > \varepsilon_{ji} + (\gamma_{jj} - \gamma_{ji})(1 - \tilde{\lambda}_{jj})$ in (21). Only in the special case of no differential, are the optimal tariffs the same across the different trade models.

Proposition 3 *Conditional on the domestic trade share and the trade elasticity, the optimal tariffs have the following properties:*

(i) *If the extensive margin elasticity is the same between domestic and export markets, levels of the optimal tariffs are the same across different trade models.*

(ii) *If the extensive margin is more (less) elastic in the export market than in the domestic market, levels of the optimal tariffs are greater (smaller) than those in the absence of this differential.*

In Proposition 3, we compare the optimal tariffs across the different trade models, holding *both* the domestic trade share and the trade elasticity equal that endogenously arise in the respective model. If the optimal tariffs are compared without such conditioning, the proposition no longer holds. The optimal tariffs in (21), (22) and (23) depend on the domestic trade share, which is a function of tariffs and hence is not always the same level. The fact that the optimal tariffs are implicitly characterized means that we cannot solve for the optimal tariffs in closed forms as in existing work (e.g., Gros, 1987; Felbermayr et al. 2013).[20] The optimal tariff formula in (21) makes it more difficult to compare the optimal tariffs across the different trade models since not only is the domestic trade share but also the trade elasticity and the hazard differential are also a function of tariffs. Hence, without conditioning above, it is potentially ambiguous to see which optimal tariffs are lower across the different trade models analytically. To figure out this comparison, we use numerical illustrations with the calibrated parameters in the next section.

Next, we examine the impacts of trade costs and country size on the optimal tariffs. Let us consider the optimal tariffs with a constant trade elasticity in (22) and (23), where changes in the exogenous variables affect the optimal tariffs only through the domestic trade share. Proposition 1 says that reductions in *any* trade costs increase the domestic productivity cutoff φ^*_{jj}, which decrease the domestic trade share $\tilde{\lambda}_{jj}$. Proposition 2 says that increases in country i's market size (or decreases in country j's size) increase φ^*_{jj}, which also decrease $\tilde{\lambda}_{jj}$. These comparative statics mean that the optimal tariffs for country i are higher, the lower are any trade costs between countries or the larger is country i's relative size. Further noting that reductions in country j's tariffs

[20] From this reason, Felbermayr et al. (2013) compare the optimal tariffs in the heterogeneous firm model and the homogeneous firm model, holding *only* the domestic trade share equal. Costinot et al. (2020) also compare the optimal tariffs in a similar way.

increase country i's optimal tariffs, the best response function is downward-sloping so that tariffs are strategic substitutes. These properties of the optimal tariffs hold true for those with a constant trade elasticity in the literature; see Gros (1987) for the homogeneous firm model and Felbermayr et al. (2013) for the heterogeneous firm model. If we consider the optimal tariffs with a variable trade elasticity in (21), however, these properties are not necessarily satisfied since changes in the exogenous variables affect the optimal tariffs not only through the domestic trade share but also through the trade elasticity.

This additional channel for the optimal tariffs can be shown more formally by making clear the relationship between the extensive margin elasticity differential and the partial trade elasticity in a general productivity distribution. Applying the comparative statics in Propositions 1 and 2 to Lemma 2, the following lemma is immediately obtained (see Appendix A.8).[21]

Lemma 3

(i) *If the extensive margin is more (less) elastic in the export market than in the domestic market, reductions in trade costs between countries decrease (increase) the trade elasticity.*

(ii) *If the extensive margin is more (less) elastic in the export market than in the domestic market, increases in country i's relative market size decrease (increase) the trade elasticity.*

Lemma 3 implies that, if $\gamma_{jj} - \gamma_{ji} \neq 0$, the trade elasticity is not constant and hence differs across markets. In the case of variable trade costs, for example, we have

$$\gamma_{jj} - \gamma_{ji} \lesseqgtr 0 \quad \Longrightarrow \quad \frac{d\varepsilon_{ji}}{d\theta_{ji}} \gtreqless 0, \ \frac{d\varepsilon_{ji}}{d\theta_{ij}} \gtreqless 0.$$

If the extensive margin is more elastic in the export market than in the domestic market where the differential is negative, reductions in variable trade costs decrease the trade elasticity. This accords with empirical evidence that the trade elasticity is smaller when trade liberalization concerns country pairs where the trade volume is already large (Bas et al., 2017). Only in the special case of no differential, is the trade elasticity invariant to any trade costs and the same across markets.

It is easily shown that changes in the exogenous variables have an additional effect on the optimal tariffs. Consider reductions in variable trade costs θ_{ji}. If the differential is negative ($\gamma_{jj} - \gamma_{ji} < 0$), such reductions decrease the partial trade elasticity ($\frac{d\varepsilon_{ji}}{d\theta_{ji}} > 0$) as well as the domestic trade share in country j ($\frac{d\tilde{\lambda}_{jj}}{d\theta_{ji}} > 0$). It follows immediately from (21) that, due to an extra adjustment through ε_{ji} that is absent in (22) and (23), the impact on the optimal tariffs is reinforced. If the differential is positive, the converse is true in that the impact on the optimal tariffs is attenuated. Only when there is no differential, is the partial trade elasticity constant and reductions in θ_{ji} affect the optimal tariffs only through decreases in the domestic trade share. These highlight a potential bias in evaluating the optimal tariffs without allowing for a variable trade elasticity that differs across markets. In other words, the optimal tariffs that do not control for the differential tend to be under/over-estimated not only in terms of levels but also in terms of changes induced by exogenous shocks (see Appendix A.9).

[21] Strictly speaking, we require that the extensive margin elasticity γ_{ji} is a monotonic function in the productivity cutoff φ^*_{ji} for this lemma. With this restriction, the sign of $\gamma_{jj} - \gamma_{ji}$ is the same for a given productivity distribution and does not change with the key exogenous variables so long as export market selection is ensured. As seen above, the property holds for a bounded Pareto distribution (Head et al. (2014) show this for a log-normal distribution), in which case the sign of the differential does not switch in the comparative statics exercises here. We are not sure if this always holds for other popular productivity distributions and the discussion below simply bypasses the question.

Proposition 4 *Reductions in trade costs between countries or increases in country i's relative market size have the following effects on the optimal tariffs:*

(*i*) *If the extensive margin elasticity is the same between the domestic and export markets, they increase the optimal tariffs only through decreases in the domestic trade share.*

(*ii*) *If the extensive margin is more (less) elastic in the export market than in the domestic market, they reinforce (attenuate) the impact on the optimal tariffs through decreases (increases) in the trade elasticity.*

One of interesting results in this proposition arises when $\gamma_{jj} - \gamma_{ji} > 0$ and increases in the trade elasticity (induced by exogenous shocks) are greater than decreases in the domestic trade share. In this case, the model predicts that the optimal tariffs for country i are lower, the lower are trade costs between the two countries and the larger is relative size in country i. The impact of country size on the optimal tariffs accords with recent research. For example, Naito (2019) finds a significant negative relationship between GDP and tariffs across countries, meaning that larger countries tend to set lower tariffs. To account for this fact that is inconsistent with the existing optimal tariff theory, Naito (2019) develops a dynamic Ricardian model in which long-run welfare effects of tariffs on revenue and economic growth jointly characterize the optimal tariffs, which is shown to be decreasing in a country's absolute advantage parameter.

While the present model yields a similar prediction, the mechanism behind the result is different.[22] Our model predicts that a larger country accommodates relatively more inefficient firms in the domestic market by lowering the domestic productivity cutoff which negatively affects welfare. If a larger country is allowed to choose tariffs to maximize welfare, it can enjoy a terms-of-trade gain by setting higher tariffs in our model as in the conventional optimal tariff theory. At the same time, however, such tariffs also accelerate a welfare loss from protecting inefficient firms because a larger country suffers from anti-selection effects in the domestic market. Taking these effects together, the optimal tariffs are decreasing in country size only if a welfare loss from protecting inefficient firms is stronger than a welfare gain from improving the terms-of-trade, which can occur under the condition that $\gamma_{jj} - \gamma_{ji} > 0$ in this model. If this is the case, a larger country does not always benefit from higher tariffs due to the feedback from country size to firm election. In the next section, we also show that, even if $\gamma_{jj} - \gamma_{ji} < 0$ so that the optimal tariffs are increasing in country size as in the conventional optimal tariff theory, the impact of country size on the optimal tariffs is quantitatively very limited relative to what one would expect from the findings in the existing literature.

5.2 Nash Tariffs

We have characterized the optimal tariffs in country i, taking tariffs in country j as given without fearing any retaliation. Now consider the situation in which both countries set tariffs so as to maximize respective welfare but country i sets tariffs taking account of the possibility to retaliate against country j's tariffs and vice versa. From Lemma 3, we know that if $\gamma_{jj} - \gamma_{ji} \leq 0$, country i's optimal tariffs are decreasing in country j's tariffs, since an increase in τ_{ij} leads to decreases in $\tilde{\lambda}_{jj}$ and ε_{ji}. As indicated earlier, the best response functions are downward-sloping and the optimal tariffs are strategic substitutes for one another. If $\gamma_{jj} - \gamma_{ji} > 0$ and increases in ε_{ji} are greater than decreases in $\tilde{\lambda}_{jj}$, country i's optimal tariffs are increasing in country j's tariffs. In this case, the best response functions are upward-sloping and the optimal tariffs are strategic complements for one another. As usual, the optimal tariffs in Nash equilibrium are determined at which the best response

[22] From the finding in Section 4, the larger country i's size L_i, the larger its national income $w_i L_i + (\tau_{ji} - 1)R_{ji}$ for given τ_{ji}. Hence, a larger country entails larger GDP as well as GDP per capita in this model.

functions intersect in the (τ_{ji}, τ_{ij}) space, but the variable nature of the trade elasticity alters the equilibrium properties of such tariffs. Further, the optimal tariffs are bounded from above and below in Nash equilibrium. On the one hand, if tariffs are sufficiently high that no firm exports from country i, the domestic trade share in country j approaches to unity in (21). Thus, the lower bound of the optimal tariffs are

$$\underline{\tau}^*_{ji} = 1 + \frac{\rho\alpha_j}{\beta_j - \rho\alpha_j} = \frac{\beta_j}{\beta_j - \rho\alpha_j}.$$[23]

On the other hand, if trade costs are sufficiently low that all operating firms export from country j ($\varphi^*_{jj} = \varphi^*_{ji}$), we have $\alpha_j = f_{jj}/f_{ji}$ (from the definition of α_j) and $\gamma_{jj} = \gamma_{ji}$ (from the definition of γ_{ji}) giving rise to $\beta_j/\alpha_j = \varepsilon_{ji}$. Using these and $\tilde{\lambda}_{jj} = \alpha_j/(\alpha_j + 1)$ in (21), the upper bound of the optimal tariffs are

$$\overline{\tau}^*_{ji} = 1 + \frac{\rho\left(1 + \frac{f_{jj}}{f_{ji}}\right)}{\frac{f_{jj}}{f_{ji}}\left(\varepsilon_{ji} - \rho\right)} = \frac{\varepsilon_{ji} + \rho\frac{f_{ji}}{f_{jj}}}{\varepsilon_{ji} - \rho}.$$

Note from Lemma 2 and Proposition 4 that, unless $\gamma_{jj} - \gamma_{ji} = 0$, both $\underline{\tau}^*_{ji}$ and $\overline{\tau}^*_{ji}$ are variable in the present model with a general productivity distribution. Therefore, these bounds vary endogenously with changes in the exogenous variables.

To better appreciate the equilibrium properties of the Nash tariffs, we follow Felbermayr et al. (2013) in assuming that the two countries are symmetric and choose their tariff non-cooperatively. In Nash equilibrium, they impose the same optimal tariffs $\tau^*_{ij} = \tau^*_{ji} \equiv \tau^*$ with the equalized wage across countries $w_i = w_j \equiv w = 1$. Exploiting the symmetry, let us also define

$$\begin{aligned}
&\theta_{ij} = \theta_{ji} \equiv \theta, \quad f_{ii} = f_{jj} \equiv f_d, \quad f_{ij} = f_{ji} \equiv f_x, \quad L_i = L_j \equiv L, \quad \varphi^*_{ii} = \varphi^*_{jj} \equiv \varphi^*_d, \quad \varphi^*_{ij} = \varphi^*_{ji} \equiv \varphi^*_x, \\
&\tilde{\lambda}_{ii} = \tilde{\lambda}_{jj} \equiv \tilde{\lambda}, \quad \varepsilon_{ij} = \varepsilon_{ji} \equiv \varepsilon, \quad \gamma_{ii} = \gamma_{jj} \equiv \gamma_d, \quad \gamma_{ij} = \gamma_{ji} \equiv \gamma_x, \quad \alpha_i = \alpha_j \equiv \alpha, \quad \beta_i = \beta_j \equiv \beta.
\end{aligned}$$

Then, finding the Nash tariffs is equivalent to finding a solution to the fixed point problem $\tau = f(\tau)$ in (21) where the dependence of $f(\tau)$ on θ, f_x and L is understood:

$$f(\tau) = 1 + \frac{\rho}{\tilde{\lambda}\left(\varepsilon - (\gamma_d - \gamma_x)(1 - \tilde{\lambda}) - \rho\right)}.$$

Since all of the key endogenous variables (i.e., $\tilde{\lambda}$, ε, $\gamma_d - \gamma_x$) are a function of tariffs, the fixed point problem only implicitly characterizes the Nash tariffs as in the optimal tariffs in the previous subsection. Nevertheless, we can discuss the following equilibrium properties of the Nash tariffs.

From Propositions 1 and 4, if $\gamma_d - \gamma_x < 0$, we know that not only is the domestic trade share $\tilde{\lambda}$ but also the trade elasticity ε is increasing in τ. In this case, $f(\tau)$ is strictly decreasing in τ, reflecting that tariffs are strategic substitutes. In contrast, if $\gamma_d - \gamma_x > 0$, $\tilde{\lambda}$ is increasing in τ but ε is decreasing in τ. In this case, $f(\tau)$ is strictly increasing in τ under the condition that reductions in τ lead to increases in ε relatively more than decreases in $\tilde{\lambda}$, reflecting that tariffs are strategic complements. Figure 2 depicts a 45-degree line plus a $f(\tau)$ curve for two possible cases: tariffs are strategic substitutes in Panel A and tariffs are strategic complements in Panel B. In either panel, the Nash tariffs are found at which a 45-degree line and a $f(\tau)$ curve intersect. Such tariffs lie within the shaded area in the figure where the lower and upper bounds are respectively denoted by $\underline{\tau}^*_{ij} = \underline{\tau}^*_{ji} \equiv \underline{\tau}^*$ and $\overline{\tau}^*_{ij} = \overline{\tau}^*_{ji} \equiv \overline{\tau}^*$.

[23]This bound also represents country i's optimal tariffs when country i is treated as a limit case of a small economy ($\tilde{\lambda}_{jj} = 1$). From Lemma 2, $\beta_j/\alpha_j = k$ with unbounded Pareto and hence $\underline{\tau}^*_{ji} = k/(k - \rho)$ as in Demidova and Rodríguez-Clare (2009).

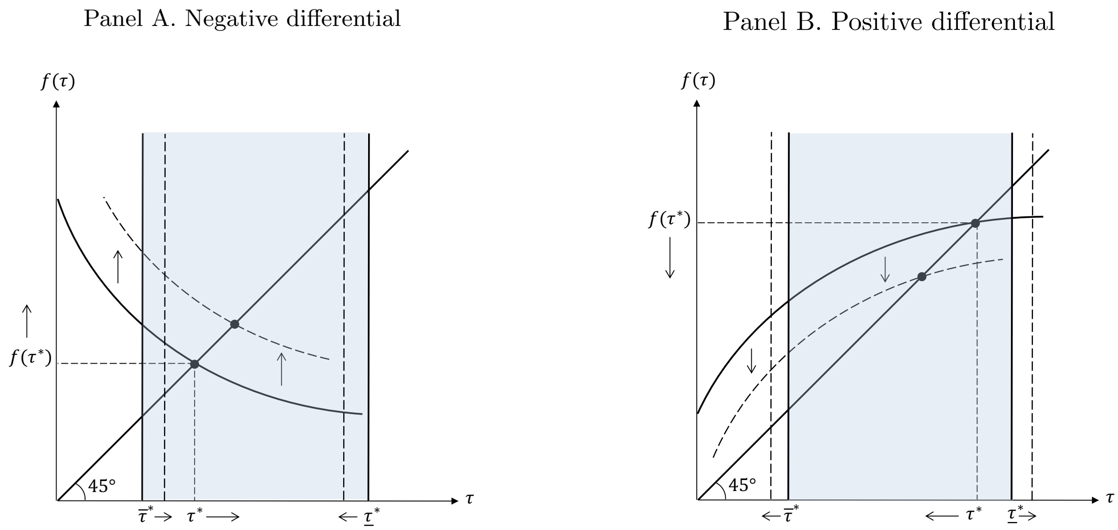

Figure 2 – Effect of trade liberalization on Nash tariffs

Consider the impact of trade liberalization at the symmetric situation. *Bilateral* reductions in trade costs (both variable θ and fixed f_x) decrease the domestic trade share $\tilde{\lambda}$ as in *unilateral* reductions in these costs. We should note however that the magnitude of a decline in $\tilde{\lambda}$ is different between them, which can be seen from the impact on the productivity cutoffs. In the case of variable trade costs θ, for example, solving the system of three equations ((4), (5)) at the symmetric situation for three unknowns $(\hat{\varphi}_d^*, \hat{\varphi}_x^*, \hat{B})$, we get

$$\hat{\varphi}_d^* = -\frac{1}{\alpha+1}\hat{\theta}, \quad \hat{\varphi}_x^* = \frac{\alpha}{\alpha+1}\hat{\theta}. \tag{24}$$

Comparing (10) and (24) reveals that reductions in variable trade costs have different impacts on the cutoffs. Since $\tilde{\lambda} = \alpha/(\alpha+1)$ from Lemma 1, reductions in θ also have different impacts on the domestic trade share. On top of that, if the differential is negative ($\gamma_d - \gamma_x < 0$), such reductions also decrease the trade elasticity ε. In this case, the $f(\tau)$ curve shifts up and the Nash tariffs τ^* become higher, thereby narrowing the gap between the upper and lower bounds (i.e., tariffs tend to converge) as a result of such reductions in Panel A. If the differential is positive ($\gamma_d - \gamma_x > 0$), the converse is true in Panel B. Finally, if the differential is zero ($\gamma_d - \gamma_x = 0$), such reductions have no impact on ε and the Nash tariffs τ^* become higher only through a decline in $\tilde{\lambda}$, thereby leaving the two bounds unaffected.

Consider next the impact of market size at the symmetric situation. Bilateral increases in market size (L) have no impact on the domestic trade share $\tilde{\lambda}$, because the indirect effect through the terms-of-trade is not operative at the symmetric situation with the equalized wage $\hat{w}_i = \hat{w}_j = 0$ (see (17)).[24] As a result of this, bilateral increases in market size have no effect on the productivity cutoffs at all. Noting that the extensive margin elasticities γ_d, γ_x are a function of these cutoffs, this also means that market size has no effect on the partial trade elasticity $\varepsilon = \sigma - 1 + \gamma_x$ and the extensive margin elasticity differential $\gamma_d - \gamma_x$. Consequently, the $f(\tau)$ curve does not shift at all, so that the Nash tariffs τ^* and the two bounds $\overline{\tau}^*, \underline{\tau}^*$ also remain unchanged. In contrast to trade liberalization above, this impact of market size necessarily holds irrespective of the sign of the differential $\gamma_d - \gamma_x$ (see Appendix A.10).

[24] As in trade costs, unilateral changes in market size have different impacts on the cutoffs from bilateral changes; see (16).

The important upshot of our argument is that policy evaluations of the Nash tariffs that do not control for the differential $\gamma_d - \gamma_x$ can lead to a serious bias even in an environment in which countries choose tariffs non-cooperatively. This is of particular importance for assessment of trade policy in globalization where reductions in transportation or communication costs are significant across countries. Our model reveals that, whenever $\gamma_d - \gamma_x \neq 0$, there is an additional channel through which trade costs endogenously affect the Nash tariffs, i.e., a variable trade elasticity. In fact, recent work using firm-level data has identified empirical relevance of this aspect, paying particular attention to the role of the extensive margin. For example, estimating trade flows in their generalized gravity equation featured with firm heterogeneity under the bounded Pareto distribution, Helpman et al. (2008) find substantial variation in the trade elasticity with respect to observable trade costs (such as distance) between country pairs, which means that $\gamma_d - \gamma_x \neq 0$.[25] Calibrating their heterogeneous firm model under the bounded Pareto distribution into US firm-level data, Melitz and Redding (2015) also show that missing the variable nature of the trade elasticity can give rise to a quantitatively large discrepancy from the true welfare gains from trade liberalization. In the context of trade policy, these insights suggest that the micro structure that makes the trade elasticity variable matters for evaluating the Nash tariffs correctly, since globalization has a crucial impact on the characterization of the Nash tariffs not only through the domestic trade share but also through the trade elasticity. As emphasized by Melitz and Redding (2015), these economic factors are empirically observable moments, and therefore we need to take fully into account these objects for trade policy evaluations. In the next section, we calibrate the optimal tariffs with a variable trade elasticity (21) and compare them with those with a constant trade elasticity in (22) and (23), in order to quantitatively assess a potential bias that arises from omitting the important aspect of $\gamma_d - \gamma_x \neq 0$.

Proposition 5 *Evaluating at a symmetric situation, the Nash tariffs have the following equilibrium properties:*

(i) *If the extensive margin elasticity is the same between the domestic and export markets, reductions in trade costs increase the Nash tariffs only through decreases in the domestic trade share.*

(ii) *If the extensive margin is more (less) elastic in the export market than in the domestic market, they reinforce (attenuate) the impact on the Nash tariffs through decreases (increases) in the trade elasticity.*

(iii) *Regardless of the sign of the extensive margin elasticity differential, market size has no impact on the Nash tariffs.*

We conclude this section by mentioning the welfare effect of the exogenous variables in Nash equilibrium evaluated at the symmetric situation. First, welfare changes with respect to variable trade costs in (9) similarly hold in Nash equilibrium, and bilateral reductions in variable trade costs improve welfare due to a rise in φ_d^*. Next, welfare changes with respect to market size in (14) are simply given by $\hat{W} = \hat{L}/(\sigma - 1)$, since a fall in the productivity cutoff does not enter stems from that φ_d^* is invariant to market size with the equalized wage. Thus bilateral increases in market size improve welfare due solely to increased product variety even with tariff revenue. Finally, welfare changes with respect to tariffs in (18) are expressed as

$$\hat{W} = \left(\frac{(\tau - 1)(\beta - \rho\alpha)}{\rho(\alpha + \tau)} \right) \hat{\varphi}_d^*, \tag{25}$$

and bilateral reductions in tariffs (from $\tau \geq 1$) improve welfare by increasing in φ_d^*.

[25] See Head et al. (2014) and Bas et al. (2017) for similar findings under the log-normal distribution. In explaining the variation, they also focus on the role of the extensive margin.

6 Quantitative Relevance

This section shows the quantitative relevance of the theoretical results. To calibrate the model, we mainly use a bounded Pareto distribution that makes the trade elasticity variable. We treat country i as the rest of the world (ROW) and country j as the United States, and choose the standard parameter values obtained from the US data. We first examine the impact of *bilateral* changes in trade costs on the optimal tariffs, and then examine the impact of *unilateral* changes in trade costs and market size on the optimal tariffs. In both cases, the optimal tariffs are evaluated at a symmetric situation in an initial equilibrium.

6.1 Calibration

We choose the elasticity of substitution between varieties $\sigma = 4$ and hence $\rho = (\sigma - 1)/\sigma = 0.75$. We also set the number of workers equal to the US labor force and $L_j = 148$ million in 2002, and choose US labor as a numeraire $w_j = 1$. In the initial equilibrium, the two countries have the same labor force and wage. From the finding in Section 5, this suggests that the impact of *bilateral* changes in market size (in terms of labor force) has no impact on the optimal tariffs. In contrast, from the finding in Section 4, the the impact of *unilateral* changes in market size has an impact on the optimal tariffs by changing the wage.

Following Melitz and Redding (2015), we calibrate θ_{ji} and τ_{ji} to match the average fraction of exports in firm sales in US manufacturing, which is 0.14 (Bernard et al., 2007). In contrast to their study that matches this number to variable trade costs only, we also consider tariffs and $\frac{\tau_{ji}^{-\sigma}\theta_{ji}^{1-\sigma}}{1+\tau_{ji}^{-\sigma}\theta_{ji}^{1-\sigma}} = 0.14$ in the current study. We set τ_{ji} equal to 1.045 which matches the world applies tariff rate (weighted mean, all products) in 2002.[26] Together with $\sigma = 4$, we get $\theta_{ji} = 1.7$. We also set f_{ji} equal to 0.535 and normalize $f_{jj} = f_j^e = 1$ as those in Melitz and Redding (2015).

The bounded Pareto distribution with a finite upper support $\varphi_{\max}$ is given by (Feenstra, 2017)

$$G(\varphi) = \frac{1 - \left(\frac{\varphi_{\min}}{\varphi}\right)^k}{1 - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^k}.$$

For simplicity, the distribution is assumed to be the same across two countries with the same parameter values. We set the shape parameter $k = 4.25$ and normalize the scale parameter $\varphi_{\min} = 1$. We set the upper bound of the bounded Pareto distribution $\varphi_{\max}$ equal to 2.85 as in Melitz and Redding (2015). Given the distribution, the share of firms that export from country j to country i, $\chi_{ji} \equiv [1 - G(\varphi_{ji}^*)]/[1 - G(\varphi_{jj}^*)]$, is

$$\chi_{ji} = \frac{\left(\frac{\varphi_{\min}}{\varphi_{ji}^*}\right)^k - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^k}{\left(\frac{\varphi_{\min}}{\varphi_{jj}^*}\right)^k - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^k}.$$

The average share of firms that export in US manufacturing is 0.18 (Bernard et al., 2007). In order to match the number, we need to know the values of two unknowns $\varphi_{jj}^*, \varphi_{ji}^*$ in the initial equilibrium. Another system of equations for the unknowns is selection into the export market, which is at the symmetric situation ($B_i = B_j$)

$$\left(\frac{\varphi_{ji}^*}{\varphi_{jj}^*}\right)^{\sigma-1} = \frac{\tau_{ji}^{\sigma}\theta_{ji}^{\sigma-1}f_{ji}}{f_{jj}}.$$

[26] The world tariff rate and US labor force are obtained from the World Bank Data. Since Bernard et al. (2007) use the data in year 2002, we also choose the data in the same year for these variables.

Solving these two relationships for the two unknowns, φ_{jj}^* is expressed in the initial equilibrium as

$$(\varphi_{jj}^*)^{-k} = \frac{\varphi_{\max}^k (1-\chi_{ji})}{\tau_{ji}^{-\frac{k\sigma}{\sigma-1}} \theta_{ji}^{-k} \left(\frac{f_{ji}}{f_{jj}}\right)^{-\frac{k}{\sigma-1}} - \chi_{ji}}.$$

Plugging the calibrated parameters yields $\varphi_{jj}^* = 1.16$ and $\varphi_{ji}^* = 1.70$ under the bounded Pareto distribution. Note that these cutoffs are not uniquely determined under the unbounded Pareto distribution with $\varphi_{\max} = \infty$. Once the values of these cutoffs are determined, the other key endogenous variables are automatically pinned down, as shown in Lemma 1.

An implicit solution of the optimal tariffs for country i is given in (21). This denotes the optimal tariffs set by country i on exports from country j (i.e., optimal tariffs for ROW). We do not consider t_{ij}^* (i.e., optimal tariffs for US) here, because we use the calibrated values from the US data for the three endogenous variables, ε_{ji}, $\gamma_{jj} - \gamma_{ji}$, $\tilde{\lambda}_{jj}$ in (21), all of which are those of country j. If we try to quantify the optimal tariffs for US, we instead need information of the above three observable moments of the ROW, which are harder to find in the empirical literature than those of US. In the quantitative exercise that follows, therefore, we quantify the optimal tariffs *faced* by US firms.

The key endogenous variables in (21) are given by

$$\varepsilon_{ji} = \sigma - 1 + \gamma_{ji},$$

$$\gamma_{jn} = (k-(\sigma-1)) \frac{\left(\frac{\varphi_{\min}}{\varphi_{jn}^*}\right)^{k-(\sigma-1)}}{\left(\frac{\varphi_{\min}}{\varphi_{jn}^*}\right)^{k-(\sigma-1)} - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^{k-(\sigma-1)}},$$

$$\tilde{\lambda}_{jj} = \frac{\alpha_j}{\alpha_j + 1},$$

where $n = i, j$ and

$$\alpha_j = \tau_{ji}^{\sigma} \theta_{ji}^{\sigma-1} \frac{V(\varphi_{jj}^*)}{V(\varphi_{ji}^*)}$$ [27]

evaluated at the symmetric situation. Using the productivity cutoffs obtained from the calibrated parameters, we can quantify the three empirically observable moments and thus the optimal tariffs in the initial equilibrium. Further, the comparative statics outcomes in the previous sections allow us to address the quantitative impact of unilateral changes in trade costs and market size as well as the impact of bilateral changes in trade costs on the optimal tariffs from the initial equilibrium.

We are also interested in addressing how the optimal tariffs with a variable trade elasticity quantitatively differs from the optimal tariffs with a constant trade elasticity for given levels of trade costs and market size. As discussed in Section 5, an implicit solution of the latter type of the optimal tariffs is given in (22) in the heterogeneous firm model with the unbounded Pareto distribution, and (23) in the homogeneous firm model with the degenerated distribution. While all of the optimal tariffs t_{ji}^* depend on the domestic trade share $\tilde{\lambda}_{jj}$, this share itself is a function of tariffs, which makes it difficult to solve the optimal tariffs in (21), (22) and (23) in closed forms, as found the existing literature (e.g., Gros, 1987; Felbermayr et al. 2013). In other words, we cannot undertake an analytical comparison of the optimal tariffs across the different trade models without

[27] It can be easily shown that under the bounded Pareto distribution,

$$V(\varphi_{jn}^*) = \frac{k\varphi_{\min}^k}{k-(\sigma-1)} \left((\varphi_{jn}^*)^{-(k-(\sigma-1))} - \varphi_{\max}^{-(k-(\sigma-1))}\right).$$

conditioning on the two sufficient statistics for welfare by Arkolakis et al. (2012). The numerical illustrations with the calibrated parameters help us figure out this comparison without such conditioning.

In what follows, we first examine the quantitative impact of bilateral changes in trade costs on the optimal tariffs at the symmetric situation in the initial equilibrium. To do this, we apply the analytical solutions of the comparative statics outcomes (24) to the optimal tariffs across the different trade models (21), (22) and (23). The impact of bilateral changes in market size on the optimal tariffs is omitted due to the reason detailed in Section 5. We then examine the quantitative impact of unilateral changes in trade costs and market size on the optimal tariffs, applying the analytical solutions of the comparative statics outcomes (10) and (16) to the optimal tariffs above. In contrast to the first exercise, not only do trade costs but also market size has a crucial impact on the optimal tariffs through changes in the wage that alter the terms-of-trade between countries. As a result, changes in the optimal tariffs with respect to trade costs are also quantitatively different between the first and second exercises.

6.2 Bilateral Effect on Optimal Tariffs

In this subsection, we use the short-hand notations of the key variables at the symmetric situation introduced in Section 5. In the next subsection where a unilateral effect on the optimal tariffs is examined, we re-attach the country subscripts to the relevant variables.

Substituting the calibrated parameters into γ_d, γ_x yields the extensive margin elasticity for the domestic market $\gamma_d = 1.85$ and for the export market $\gamma_x = 2.63$, which in turn yields the hazard differential $\gamma_d - \gamma_x = -0.77$ as well as the partial trade elasticity $\varepsilon = 5.63$. Further, substituting the calibrated parameters yields $\alpha = 13.38$, which in turn yields the domestic trade share $\tilde{\lambda} = 0.93$ that is the value reported by Arkolakis et al. (2012). Finally, plugging the calibrated values in (21) evaluated at the symmetric situation, we get optimal tariffs $t^* = 0.166$. Therefore, the optimal tariffs are 16.6 percent in trade between two symmetric countries. This means that the optimal tariffs with a variable trade elasticity are much lower than those with a constant trade elasticity found in the existing literature. For example, Felbermayr et al. (2013) report that the optimal tariffs are 26.4 percent in the heterogeneous firm model under the unbounded Pareto distribution, while Ossa (2014) report that the optimal tariffs are 62 percent in the homogeneous firm model. The result shows that the variable nature of the trade elasticity lowers the magnitude of the optimal tariffs substantially. Of course, the difference in the optimal tariff values also comes from the difference in the calibrated parameters adopted in these papers and ours, but we find that the difference in this magnitude mainly stems from the bias that do not control for the difference in the extensive margin elasticities $\gamma_d - \gamma_x$.[28]

Next, we examine the quantitative impact of bilateral changes in trade costs on the optimal tariffs. While we study changes in variable trade costs from the initial equilibrium $\theta = 1.7$ holding the other parameter values constant here, it is possible to examine changes in fixed trade costs from the initial equilibrium $f_x = 0.535$ (see Appendix B.1). Consider then unilateral effects of θ on the productivity cutoffs, φ_d^*, φ_x^*. From the comparative statics outcomes in (24), it follows that 1 percent reductions in θ leads to $\frac{1}{\alpha+1}$ percent increases in φ_d^* and $\frac{\alpha}{\alpha+1}$ percent decreases in φ_x^* respectively. Substituting $\alpha = 13.38$ obtained from the calibrated values in the initial equilibrium, we can compute changes in φ_d^* and φ_x^* induced by changes in variable trade costs from the initial equilibrium. Using these changes in the productivity cutoffs, we can also compute changes in the key endogenous moments ε, $\gamma_d - \gamma_x$, $\tilde{\lambda}$ and hence changes in the optimal tariffs t^*.

[28]For example, even if the calibrated parameters by Felbermayr et al. (2013) are substituted into (21), we find that the value of the optimal tariffs is of a comparable magnitude in the baseline version. In our quantitative exercise, we mainly use the calibrated parameters by Melitz and Redding (2015) since the domestic trade share by theirs fits well with the one observed in the US data relative to that by Felbermayr et al. (2013).

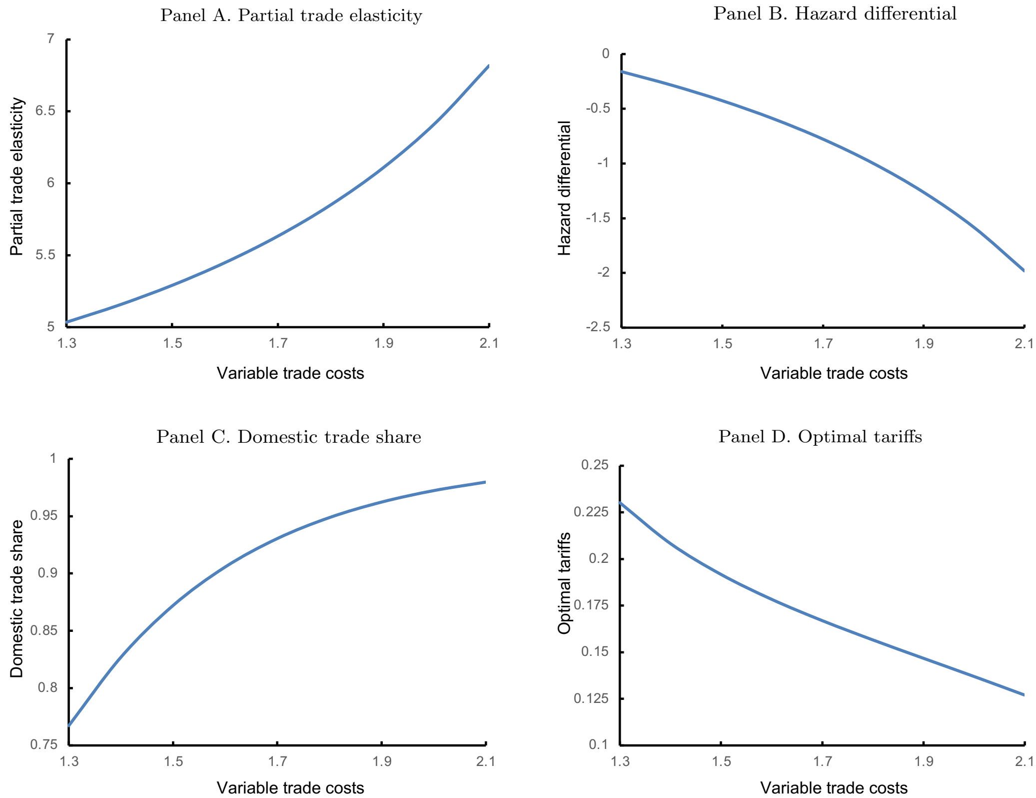

Figure 3 – Unilateral effect of variable trade costs

Figure 3 shows changes in the key endogenous variables induced by changes in $\theta \in [1.3, 2.1]$ where $\theta = 1.7$ in the initial equilibrium. Panels A, B, C and D respectively display changes in the partial trade elasticity ε, the hazard differential $\gamma_d - \gamma_x$, the domestic trade share $\tilde{\lambda}$, and the optimal tariffs t^*.

Panel A shows that the partial trade elasticity is increasing in variable trade costs under the bounded Pareto distribution. This elasticity is higher under the bounded Pareto distribution than under the unbounded Pareto distribution ($\varepsilon = 4.25$) or degenerated distribution ($\varepsilon = 3$) in any level of variable trade costs. Panel B shows that the hazard differential is decreasing in variable trade costs, as the extensive margin elasticity γ_d (γ_x) is increasing in φ_d^* (φ_x^*) under the bounded Pareto distribution. This stands in sharp contrast to the unbounded Pareto or degenerated distribution in which the hazard differential is always zero. The domestic trade share is increasing in variable trade costs as in Panel C, but it is lower under the bounded Pareto distribution than under the unbounded Pareto or degenerated distribution due to the higher trade elasticity as in Panel A.[29] Finally, Panel D shows that the optimal tariffs are decreasing in variable trade costs. Although the property is well-known in the literature, this operates not only through the higher domestic trade share in Panel C but also through the higher trade elasticity in Panel A in the current paper. A simple inspection of (21) reveals that the negative effect from the lower hazard differential in Panel B works to increase the optimal tariffs, but this is dominated by the positive effect from the higher trade elasticity and larger domestic trade share here. Consequently, changes in variable trade costs have the greater effect on the optimal tariffs under the bounded Pareto distribution than under the unbounded Pareto or degenerated distribution.

[29] These quantitative impacts on the three observable moments are basically the same as those in Melitz and Redding (2015), but the difference in a variable or constant trade elasticity has a crucial impact on evaluations of optimal trade policy.

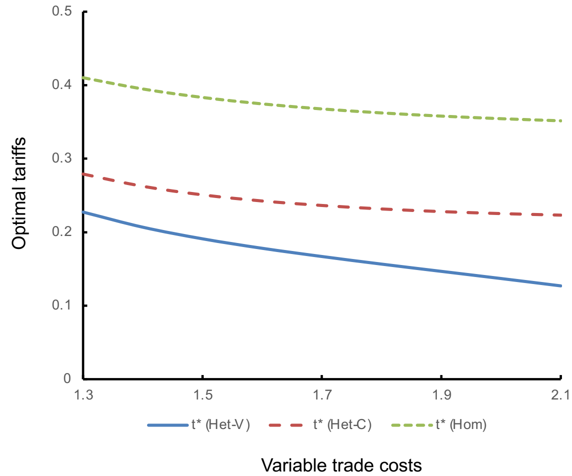

Figure 4 – Optimal tariffs across different trade models

Figure 4 compares the optimal tariffs across the different trade models. This comparison is made without conditioning on the two sufficient statistics for welfare by Arkolakis et al. (2012), as Proposition 3 shows that the optimal tariffs are always higher under the bounded Pareto distribution than under the bounded Pareto or degenerated distribution with such conditioning. From this reason, we compare the optimal tariffs conditioning on the domestic productivity cutoff (instead of the two sufficient statistics) in the initial equilibrium across the different trade models.[30] In Figure 3, the solid, dashed and dotted curves represent the optimal tariffs in (21), (22) and (23) respectively. In the initial equilibrium with $\theta = 1.7$, the optimal tariffs are 16.6 percent, 23.6 percent and 36.7 percent respectively. Thus, a variable trade elasticity leads to the lower optimal tariffs. In our quantitive exercise, levels of the optimal tariffs with a variable trade elasticity are around two-thirds (smaller than a half) of those with a constant trade elasticity in the heterogeneous (homogeneous) firm model. Moreover, in the initial equilibrium, the difference in *levels* in the optimal tariffs can be mainly explained by the difference in the trade elasticity relative to the difference in the domestic trade share.

We also compare the difference in *changes* in the optimal tariffs with respect to variable trade costs. From Lemma 3, it follows that so long as there is a hazard differential ($\gamma_d - \gamma_x \neq 0$), changes in variable trade costs lead to changes not only in the domestic trade share but also in the trade elasticity. This additional channel on the optimal tariffs need to be taken into account for assessment of trade policy in globalization, as argued in the end of Section 5. Consider, for example, the impact of bilateral reductions in variable trade costs from $\theta = 1.7$ to $\theta = 1.3$. With a variable trade elasticity, such reductions increase the optimal tariffs by 36.2 percent (from $t^* = 0.166$ to $t^* = 0.227$). With a constant trade elasticity, in contrast, they increase in the optimal tariffs by only 18.0 percent (from $t^* = 0.236$ to $t^* = 0.279$) in the heterogeneous firm model or 11.5 percent (from $t^* = 0.367$ to $t^* = 0.410$) in the homogeneous firm model. Hence, changes in the optimal tariffs with a variable trade elasticity are up to a factor of two (three) relative to those with a constant trade elasticity in the heterogeneous (homogeneous) firm model. This difference reflects that changes in variable trade costs affect the optimal tariffs through endogenous changes in the trade elasticity in (21), which is captured by that the gap between the solid line and the other two lines is narrower, the smaller is θ in Figure 3.

[30]We choose f_x to match the average share of firms that export in US manufacturing ($\chi = 0.18$) under the bounded/unbounded Pareto distributions. This yields a slightly higher value of f_x under the unbounded Pareto distribution, which is needed to adjust the difference in the upper bound of the Pareto distribution (Melitz and Redding, 2015). From the calibrated parameters, we set $\varphi_x^* = 1.70$ for (21) but $\varphi_x^* = 1.74$ for (22) and (23), while keeping $\varphi_d^* = 1.16$ for all cases.

It is worth emphasizing that the difference in changes in the optimal tariffs also gives rise to the difference in a welfare gain. While an increase in the optimal tariffs (induced by reductions in θ) negatively affects each country's welfare, (9) suggests that this effect is dominated by the positive effect of an aggregate productivity gain. Hence trade liberalization always generates a welfare gain, even though it increases the optimal tariffs. Nonetheless, the welfare gain from trade liberalization differs across the different trade models, since changes in the optimal tariffs with respect to bilateral changes in variable trade costs are different across these models. To see this, consider again the impact of bilateral reductions in variable trade costs from $\theta = 1.7$ to $\theta = 1.3$. Plugging the calibrated values into (9), such reductions increase each country's welfare by 3.52 percent (4.86 percent) in the heterogeneous firm model with a variable (constant) trade elasticity. This difference in welfare changes reflects that the optimal tariffs rise by trade liberalization relatively more for a variable trade elasticity, which reduces relative magnitude of the welfare gain from trade liberalization.[31]

Although we have focused on the impact of variable trade costs, the impact of tariffs is similarly analyzed. Applying the comparative statics outcome in Proposition 1, we can examine the quantitative impact of tariffs on the empirically observable moments in Figure 3. Applying welfare changes in (25), we can also address the quantitative effect of tariffs on welfare.

6.3 Unilateral Effect on Optimal Tariffs

So far, we have examined the impact of bilateral changes in trade costs. In reality, however, trade liberalization is asymmetric and often takes place in a unilateral way. As shown by Demidova and Rodríguez-Clare (2013), unilateral trade liberalization also gives rise to different welfare implications from bilateral trade liberalization. Using the analytical solutions of the comparative statics in Sections 3 and 4, we readily address the quantitative difference between these two kinds of trade liberalization. Moreover, since unilateral increases in market size alter the relative wage, market size also has a crucial impact on the equilibrium variables. From these reasons, we investigate the impact of unilateral changes in trade costs and market size on the optimal tariffs.

We re-attach the country subscripts to all relevant variables in the analysis below. Suppose that country i unilaterally changes variable trade costs of importing from country j (θ_{ji}). Evaluating (10) at the symmetric situation ($\alpha_i = \alpha_j, \beta_i = \beta_j$) and using (7), changes in the productivity cutoffs in country j are

$$\hat{\varphi}^*_{jj} = -\frac{\rho(\beta_j - \rho\alpha_j)}{\Xi}\hat{\theta}_{ji},$$

$$\hat{\varphi}^*_{ji} = \frac{\rho\alpha_j(\beta_j - \rho\alpha_j)}{\Xi}\hat{\theta}_{ji}.$$

Just as in bilateral effects of θ, 1 percent reductions in θ_{ji} leads to $\frac{\rho(\beta_j - \rho\alpha_j)}{\Xi}$ percent increases in φ^*_{jj} and $\frac{\rho\alpha_j(\beta_j - \rho\alpha_j)}{\Xi}$ percent decreases in φ^*_{ji} respectively. It is important to note, however, that unilateral effects of θ_{ji} have smaller impacts on the productivity cutoffs than bilateral effects of θ, because only country i changes variable trade costs on imports θ_{ji}, keeping country j's variable trade costs of imports θ_{ij} constant. Using the calibrated values in the initial equilibrium, we obtain $\alpha_i = \alpha_j = 13.38$, $\beta_i = \beta_j = 74.6$, and $\Xi = 1510$ at the symmetric situation. Substituting these values, we can compute changes in φ^*_{jj} and φ^*_{ji} from the initial equilibrium, which are in turn used to compute changes in the key endogenous moments ε_{ji}, $\gamma_{jj} - \gamma_{ji}$, $\tilde{\lambda}_{jj}$ and therefore changes in the optimal tariffs t^*_{ji} in (21). All of these results are qualitatively similar with (but quantitatively different from) Figure 3 and we relegate the corresponding figure to Appendix B.2.

[31] So long as the welfare gains from trade are measured by changes in the domestic productivity cutoff (9), we cannot directly compare these gains between the heterogeneous and homogeneous firm models because that productivity cutoff is exogenous in the latter model. Thus, we need to use changes in the domestic trade share as in (19) and (20).

Table 1: Unilateral effect of trade costs and market size

Panel A. Variable trade costs

θ_{ji}	1.3	1.4	1.5	1.6	**1.7**	1.8	1.9	2.0	2.1
ε_{ji}	5.313	5.385	5.461	5.544	**5.632**	5.728	5.831	5.944	6.066
$\gamma_{jj} - \gamma_{ji}$	−0.449	−0.523	−0.602	−0.686	**−0.777**	−0.875	−0.980	−1.094	−1.219
$\tilde{\lambda}_{jj}$	0.819	0.858	0.888	0.912	**0.930**	0.944	0.955	0.964	0.971
t^*_{ji}	0.204	0.191	0.181	0.173	**0.166**	0.161	0.155	0.150	0.146
ΔW_j	0.0162	0.0122	0.0081	0.0040	**0**	−0.0040	−0.0081	−0.0122	−0.0162
ΔW_i	0.0189	0.0142	0.0094	0.0047	**0**	−0.0047	−0.0094	−0.0142	−0.0189

Panel B. Fixed trade costs

f_{ji}	0.35	0.40	0.45	0.5	**0.535**	0.55	0.60	0.65	0.70
ε_{ji}	5.465	5.550	5.552	5.599	**5.632**	5.647	5.697	5.750	5.804
$\gamma_{jj} - \gamma_{ji}$	−0.605	−0.649	−0.695	−0.742	**−0.777**	−0.792	−0.843	−0.897	−0.953
$\tilde{\lambda}_{jj}$	0.921	0.923	0.926	0.928	**0.930**	0.931	0.933	0.935	0.938
t^*_{ji}	0.174	0.172	0.170	0.168	**0.166**	0.166	0.164	0.162	0.160
ΔW_j	0.0079	0.0058	0.0036	0.0015	**0**	−0.0006	−0.0028	−0.0049	−0.0071
ΔW_i	0.0093	0.0067	0.0042	0.0017	**0**	−0.0007	−0.0032	−0.0057	−0.0082

Panel C. Market size

L_j	110	120	130	140	**148**	150	160	170	180
ε_{ji}	5.546	5.568	5.591	5.614	**5.632**	5.637	5.661	5.685	5.710
$\gamma_{jj} - \gamma_{ji}$	−0.689	−0.711	−0.734	−0.758	**−0.777**	−0.782	−0.806	−0.831	−0.856
$\tilde{\lambda}_{jj}$	0.925	0.927	0.928	0.929	**0.930**	0.930	0.931	0.932	0.934
t^*_{ji}	0.170	0.169	0.168	0.167	**0.166**	0.166	0.165	0.164	0.163
ΔW_j	−0.0356	−0.0263	−0.0169	−0.0075	**0**	0.0018	0.0112	0.0206	0.0300
ΔW_i	−0.0039	−0.0029	−0.0018	−0.0008	**0**	0.0002	0.0012	0.0022	0.0033

Notes: The numbers with bold letters indicate the values in the initial equilibrium with the calibrated parameters. Market size (L_j) is measured in units of million.

It is possible to examine unilateral effects of fixed trade costs f_{ji} or market size L_j holding other parameters fixed. As for market size, evaluating (16) at the symmetric situation and substituting the calibrated values, we can compute changes in the optimal tariffs in (21) by unilateral changes in L_j from the initial equilibrium. If country j's market size unilaterally increases from the initial equilibrium, φ^*_{jj} falls and φ^*_{ji} rises. The fact that an increase in L_j shift these cutoffs in the same direction as an increase in θ_{ji} implies that country i's optimal tariffs t^*_{ji} are decreasing in country j's market size L_j (or increasing in country i's relative size L_i/L_j), as is well-known in the literature of the optimal tariff theory. In this sense, we can say that the *qualitative* impact on the optimal tariffs is similar between θ_{ji} and L_j. Interestingly, the *quantitative* impact is different between these exogenous variables drastically. The same claim also applies to the qualitative/quantitative difference between θ_{ji} and f_{ji}.

Table 1 illustrates the quantitative difference in the impacts of variable trade costs θ_{ji}, fixed trade costs f_{ji} and market size L_j on the key endogenous moments of the model. Compared to unilateral changes in variable trade costs θ_{ji} in Panel A, unilateral changes in fixed trade costs f_{ji} in Panel B or those in market size L_j in

Panel C have much limited effects on the optimal tariffs. Consider, for example, the difference in unilateral changes in θ_{ji} and L_j, which have qualitatively similar impacts on the optimal tariffs as observed above. From the initial equilibrium, we find that 23 percent increases in θ_{ji} (from $\theta_{ji} = 1.7$ to $\theta_{ji} = 2.1$) decrease country i's optimal tariffs by 12.3 percent (from $t^*_{ji} = 0.166$ to $t^*_{ji} = 0.146$), whereas 21 percent increases in L_j (from $L_j = 148$ to $L_j = 180$) decrease country i's optimal tariffs by only 1 percent (from $t^*_{ji} = 0.166$ to $t^*_{ji} = 0.163$). This implies that a country with relatively larger size could benefit from setting higher tariffs on its imports (as in the conventional optimal tariff theory), this quantitative impact is weaker than what one would expect. Intuitively, the difference reflects that market size has no direct effect on the productivity cutoffs under CES preferences. Comparing (11) and (17) reveals that the indirect effect through the terms-of-trade is of the same magnitude evaluated at the symmetric situation. From this analytical result, it follows that unilateral changes in variable trade costs have a greater impact on the equilibrium variables than those in market size, due to the direct effect that is missing in unilateral changes in market size. Hence, the quantitative effect of market size on the optimal tariffs is much smaller than that of variable trade costs in the current setting.

We also compare changes in welfare induced by unilateral changes in the key exogenous variables. The last two rows in Panels A and B show that, starting from a symmetric situation, the effect of trade liberalization is always greater in a liberalizing country than in a non-liberalizing country (Proposition 1). The quantitative effect on welfare is quite different between variable and fixed trade costs due to the difference outlined above. For example, 23 percent reductions in variable trade costs (from $\theta_{ji} = 1.7$ to $\theta_{ji} = 1.3$) generate a 1.6 (1.8) percent welfare gain for a non-liberalizing (liberalizing) country;[32] however, 25 percent reductions in fixed trade costs (from $f_{ji} = 0.535$ to $f_{ji} = 0.40$) generate only a 0.5 (0.6) percent welfare gain for a non-liberalizing (liberalizing) country. As for unilateral changes in market size, on the other hand, the last two rows in Panel C show that, starting from a symmetric situation, the effect of market expansion is welfare-enhancing not only for a non-expanding country but also for an expanding country, because the negative effect of market size on declined productivity in Proposition 2 is much smaller than the positive effect of increased product variety. For example, we find that a 21 percent increase in country j's market size (from $L_j = 148$ million to $L_j = 180$ million) generate 3 (0.3) percent welfare gains for an expanding (non-expanding) country. In country j where its size unilaterally increases, the model predicts two opposing effects on welfare (see (14)). In the quantitative exercise, the negative anti-selection effect is only -0.3 percent, while the positive variety effect is 3.3 percent. Therefore, even though market size can lead to an aggregate productivity loss, this does not predominate the well-established benefit from increased product variety in the new trade theory literature.

The key policy implication arising from this quantitative exercise is that a country with large size would not necessarily enjoy a large welfare gain from setting high tariffs in trade war. If this country is allowed to choose tariffs to maximize welfare, it can benefit from tariffs by improving the terms-of-trade. At the same time, such tariffs accelerate a welfare loss from protecting inefficient firms since a large country suffers from anti-selection effects in the domestic market. Under the bounded Pareto distribution where the extensive margin elasticity differential is negative, the former benefit of tariffs dominates the latter cost and hence country i's optimal tariffs are decreasing in country j's market size (or increasing country i's relative market size), as is known in the optimal tariff theory. However, changes associated with market size have quantitatively limited impacts on the endogenous variables, thereby leading to small changes in the optimal tariffs. The result, of course, is partly accounted for by constant markups in CES preferences but it is also accounted by anti-selection effects which hold in more general preferences with the endogenous wage. Thus, we hope that the finding highlights a potential importance to reconsider existing policy implications.

[32] This welfare gain in bilateral trade liberalization is smaller than that in unilateral trade liberalization (3.52 percent) as seen in the last subsection.

7 Conclusion

This paper presents a heterogeneous firm model of trade to study optimal tariffs with a variable trade elasticity. To provide a better understanding of the impact of trade liberalization and country size on optimal trade policy, we drop the assumption of an outside good sector and employ a general productivity distribution that makes the trade elasticity variable. Our key contribution to the trade policy literature can be broadly summarized as follows. The optimal level of import tariffs is inversely related to the two empirically observable moments – the domestic trade share and the trade elasticity – where the second integrant is either constant or variable depending on the micro structure of the model. If the trade elasticity is constant and the same across markets as assumed in previous work, the optimal level of import tariffs is the same between different trade models, holding both the domestic trade share and trade elasticity equal. However, if the trade elasticity is variable and differs across markets as found by empirical work, the optimal level of import tariffs tends to be under/over-estimated due mainly to an additional channel through a variable trade elasticity. The same applies to changes in optimal tariffs by changes in exogenous variables, and the bilateral/unilateral impacts of trade liberalization and country size on the optimal tariffs depend critically on the micro structure that makes the trade elasticity variable. These insights basically go through for Nash trade policies.

We also investigate the quantitative relevance of our theoretical results to help better appreciate a new role played by a variable trade elasticity in characterizing the optimal tariffs. Using a bounded Pareto distribution that makes the trade elasticity variable and standard parameter values from the US data in the literature, we find that the optimal tariffs with a variable trade elasticity are much lower than those with a constant trade elasticity found in the existing literature, which suggests that the variable nature of the trade elasticity lowers the magnitude of the optimal tariffs substantially. In our calibration, the difference in this magnitude mainly stems from the bias that do not control for the difference in the extensive margin elasticities, rather than the difference in the calibrated parameters adopted in the previous literature and ours. Using the analytical solutions of the comparative statics outcomes with respect to trade liberalization and country size, we also quantitatively compare the difference in the bilateral and unilateral impacts of these competitive pressures on the optimal tariffs with a variable trade elasticity. One of the most important policy implications arising from this quantitative exercise is that a large country would not always enjoy a quantitatively large welfare gain from setting high tariffs in trade war, because country size has a very limited impact on the terms-of-trade and hence the firm-level variables under CES preferences.

Nevertheless, much remains to be done. In the theory side, the variable nature of the trade elasticity comes only from the extensive margin which is made possible by departing from an unbounded Pareto distribution in this paper. However, it would come also from the intensive margin that is related to the firm-level elasticity in reality. To correctly examine the variability of the trade elasticity in trade policy evaluations, it would be necessary to drop CES preferences with constant markups and instead employ VES preferences with variable markups that differ across firms, in which case a variable trade elasticity would play a more important role in optimal trade policy. There is also a gap between the theoretical and quantitative analyses. The model shows that strategic relationships across countries' optimal tariffs depend on the difference in the extensive margin elasticities. To study trade policy in which tariffs are strategic complements, we need to replace a (bounded or unbounded) Pareto distribution by another one, as this distribution induces a non-positive differential and tariffs are always strategic substitutes. The strategic complement case can be more relevant to some real-world problem, but we are not certain about which firm productivity distributions yield a positive differential and whether the resulting quantitation is able to provide a good fit for aggregate and firm-level data. We leave these theoretical and quantitative extensions and their implications for trade policy to future work.

A Proofs

A.1 Labor Market Clearing Condition

We first show that the labor market clearing condition is given by

$$L_i = \frac{R_i - T_i}{w_i}.$$

Aggregate labor in country i's economy is given by $L_i = L_i^e + L_i^p$, where L_i^e and L_i^p denote aggregate labor used for entry and production respectively. On the one hand, the labor market clearing condition for entry requires $L_i^e = M_i^e f_i^e$. Recalling that firm revenue is defined as that net of tariffs $r_{ij}(\varphi) = \frac{p_{ij}(\varphi)q_{ij}(\varphi)}{\tau_{ji}}$ and using (1), (2) and the definition of $J_i(\varphi^*)$ in Section 2.1, aggregate labor used for entry is expressed as

$$L_i^e = \frac{M_i^e}{w_i} \sum_{n=i,j} \left\{ \frac{1}{\sigma} \int_{\varphi_{in}^*}^{\varphi_{\max}} r_{in}(\varphi) dG_i(\varphi) - \left[1 - G_i(\varphi_{in}^*)\right] w_i f_{in} \right\}.$$

On the other hand, with a linear cost function, the labor market clearing condition for production requires

$$L_i^p = M_i^e \sum_{n=i,j} \int_{\varphi_{in}^*}^{\varphi_{\max}} \left(f_{in} + \frac{\theta_{in} q_{in}(\varphi)}{\varphi} \right) dG_i(\varphi).$$

Noting that firm pricing rule generates the relationship $q_{ij}(\varphi) = \frac{\tau_{ij} r_{ij}(\varphi)}{p_{ij}(\varphi)} = \frac{\rho \varphi r_{ij}(\varphi)}{\theta_{ij} w_i}$, aggregate labor used for production is expressed as

$$L_i^p = \frac{M_i^e}{w_i} \sum_{n=i,j} \left\{ \left[1 - G_i(\varphi_{in}^*)\right] w_i f_{in} + \frac{\sigma - 1}{\sigma} \int_{\varphi_{in}^*}^{\varphi_{\max}} r_{in}(\varphi) dG_i(\varphi) \right\}.$$

Summing up aggregate labor used for entry and production, we get

$$\begin{aligned} L_i &= \frac{M_i^e}{w_i} \sum_{n=i,j} \int_{\varphi_{in}^*}^{\varphi_{\max}} r_{in}(\varphi) dG_i(\varphi) \\ &= \frac{\sum_n R_{in}}{w_i}, \end{aligned}$$

where $R_{in} = M_i^e \int_{\varphi_{in}^*}^{\varphi_{\max}} r_{in}(\varphi) dG_i(\varphi)$ is aggregate revenue (or expenditure) of goods from country i to country $n = i, j$ net of tariffs. The result follows from $R_i = \sum_n \tau_{ni} R_{ni}$ (aggregate expenditure in country i consists of expenditure on domestic goods in country i and imported goods from country j) and $R_{ij} = R_{ji}$ (trade is balanced between countries).

Next, we show that the labor market clearing condition is equivalent with the trade balance condition. On the one hand, aggregate labor income in country i consists of revenues by domestic firms and exporting firms of country i net of tariffs, $w_i L_i = \sum_n R_{in}$. On the other hand, aggregate expenditure in country i satisfies $R_i = \sum_n \tau_{ni} R_{ni}$ as seen above. From these, the trade balance condition, $R_{ij} = R_{ji}$, is rearranged as

$$\underbrace{R_{ii} + R_{ij}}_{w_i L_i} = \underbrace{R_{ii} + \tau_{ji} R_{ji}}_{R_i} - \underbrace{(\tau_{ji} - 1) R_{ji}}_{T_i}.$$

Hence, both conditions are equivalent in the sense that they induce the same equality, $R_i = w_i L_i + T_i$.

A.2 Welfare Expression

Welfare per worker is given by

$$\begin{aligned} W_i &\equiv \frac{U_i}{L_i} \\ &= \frac{R_i}{L_i P_i} \\ &= \frac{\mu_i w_i}{P_i} \end{aligned}$$

where the second equality follows from defining $U_i \equiv Q_i$ and $P_i Q_i \equiv R_i$, and the third equality follows from noting that $R_i = \mu_i w_i L_i$ (from the definition of the tariff multiplier μ_i). Further, substituting $R_i = \mu_i w_i L_i$, aggregate market demand is expressed as

$$B_i = \frac{(\sigma - 1)^{\sigma - 1}}{\sigma^{\sigma}} \mu_i w_i L_i P_i^{\sigma - 1}.$$

Substituting this into (1) that pins down φ_{ii}^* and rearranging, the real wage is

$$\frac{w_i}{P_i} = \left(\frac{\mu_i L_i}{\sigma f_{ii}} \right)^{\frac{1}{\sigma - 1}} \rho \varphi_{ii}^*,$$

which shows that, to know what happens to welfare as a result of unilateral trade liberalization, we just need to see what happens to φ_{ii}^*. Note that this real wage becomes the same as that in the standard Melitz model without tariff revenue ($\mu_i = 1$); see for example Demidova and Rodríguez-Clare (2013). Finally, substituting w_i/P_i into above W_i establishes the result.

A.3 Proof of Lemma 1

We first show that

$$\begin{aligned} \alpha_i &\equiv \frac{f_{ii} J_i'(\varphi_{ii}^*) \varphi_{ii}^*}{f_{ij} J_i'(\varphi_{ij}^*) \varphi_{ij}^*} \\ &= \frac{f_{ii} (\varphi_{ii}^*)^{1-\sigma} V_i(\varphi_{ii}^*)}{f_{ij} (\varphi_{ij}^*)^{1-\sigma} V_i(\varphi_{ij}^*)}. \end{aligned} \tag{A.1}$$

The definition of α_i in (A.1) follows from solving (5) for $\hat{\varphi}_{ij}^*$ as in (7), whereas the equality in (A.1) follows from differentiating $J_i(\varphi^*)$ with respect to φ^*:

$$\begin{aligned} J_i'(\varphi^*) &= -\left(\frac{\sigma - 1}{\varphi^*} \right) [J_i(\varphi^*) + 1 - G_i(\varphi^*)] \\ &= -(\sigma - 1)(\varphi^*)^{-\sigma} V_i(\varphi^*), \end{aligned}$$

where the second equality comes from the definitions of $J_i(\varphi^*)$ and $V_i(\varphi^*)$ that satisfy

$$J_i(\varphi^*) + 1 - G_i(\varphi^*) = (\varphi^*)^{1-\sigma} V_i(\varphi^*).$$

Substituting this equality into the definition of α_i gives us the result.

Next, we show several properties of α_i.

- The first property is that $\alpha_i\alpha_j - 1 > 0$. To show this, from (1), we have that

$$\left(\frac{\varphi_{ij}^*}{\varphi_{ii}^*}\right)^{\sigma-1} = \frac{\tau_{ij}^{\sigma}\theta_{ij}^{\sigma-1}f_{ij}}{f_{ii}}\frac{B_i}{B_j}. \tag{A.2}$$

Substituting this equality into $\alpha_i\alpha_j$ that satisfies (A.1),

$$\alpha_i\alpha_j = (\tau_{ij}\tau_{ji})^{\sigma}(\theta_{ij}\theta_{ji})^{\sigma-1}\left(\frac{V_i(\varphi_{ii}^*)V_j(\varphi_{jj}^*)}{V_i(\varphi_{ij}^*)V_j(\varphi_{ji}^*)}\right) > 1.$$

The inequality follows from $\varphi_{ij}^* > \varphi_{ii}^*$ and noting that $V_i(\varphi^*)$ is strictly decreasing in φ^*.

- The second property is that $\alpha_i = R_{ii}/R_{ij}$. Using (1), $R_{ij} = M_i^e \int_{\varphi_{ij}^*}^{\varphi_{\max}} r_{ij}(\varphi)dG_i(\varphi)$ is given by

$$R_{ij} = M_i^e \sigma w_i f_{ij}(\varphi_{ij}^*)^{1-\sigma}V_i(\varphi_{ij}^*). \tag{A.3}$$

The result follows from substituting (A.3) into the equality of (A.1).

- The third property is that $\lambda_{ji}, \tilde{\lambda}_{ji}$ and μ_i are written in terms of α_i. By definition,

$$\begin{aligned}
\lambda_{ji} &= \frac{\tau_{ji}R_{ji}}{R_{ii}+\tau_{ji}R_{ji}} = \frac{\tau_{ji}R_{ij}}{R_{ii}+\tau_{ji}R_{ij}} = \frac{\tau_{ji}}{\alpha_i+\tau_{ji}},\\
\tilde{\lambda}_{ji} &= \frac{\lambda_{ji}}{\tau_{ji}(1-\lambda_{ji})+\lambda_{ji}} = \frac{1}{\alpha_i+1},\\
\mu_i &= \frac{\tau_{ji}}{\tau_{ji}(1-\lambda_{ji})+\lambda_{ji}} = \frac{\alpha_i+\tau_{ji}}{\alpha_i+1}.
\end{aligned} \tag{A.4}$$

This follows from the second property and the trade balance condition.

A.4 Proof of Lemma 2

We first show that $\varepsilon_{ij} = \sigma - 1 + \gamma_{ij}$. Following Melitz and Redding (2015) and using the domestic trade share $\lambda_{jj} = \alpha_j/(\alpha_j + \tau_{ij})$ from (A.4), this elasticity is defined as

$$\varepsilon_{ij} = -\frac{\partial \ln\left(\frac{1-\lambda_{jj}}{\lambda_{jj}}\right)}{\partial \ln \theta_{ij}} = \frac{\partial \ln\left(\frac{\alpha_j}{\tau_{ij}}\right)}{\partial \ln \theta_{ij}}.$$

Note, in our asymmetric-country setting, that ε_{ij} is defined as the elasticity of the import share relative to the domestic share in country j. Moreover, $\alpha_j = R_{jj}/R_{ji} = R_{jj}/R_{ij}$ and ε_{ij} is also defined as the elasticity of import demand relative to domestic demand net of tariffs in country j, as in Arkolakis et al. (2012) without tariffs and Felbermayr et al. (2015) with tariffs. Using (1) and (A.3), $\alpha_j/\tau_{ij} = R_{jj}/\tau_{ij}R_{ij}$ is expressed as

$$\frac{\alpha_j}{\tau_{ij}} = \frac{M_j^e}{M_i^e}\left(\frac{\tau_{ij}\theta_{ij}w_i}{w_j}\right)^{\sigma-1}\frac{V_j(\varphi_{jj}^*)}{V_i(\varphi_{ij}^*)}.$$

Since the partial trade elasticity is estimated from a gravity equation, holding national income and price index constant (Arkolakis et al., 2012), reductions in θ_{ij} have no impact on M_i^e, M_j^e and w_i, w_j appearing in α_j/τ_{ij}. To show this in our model, we follow Melitz and Redding (2015) in holding the domestic productivity cutoffs $(\varphi_{ii}^*, \varphi_{jj}^*)$ constant. It then follows from (8) that the wage effects are muted as $\hat{w}_j = 0$. Further applying (A.3) to the labor market clearing condition,

$$L_i = M_i^e \sigma \sum_{n=i,j} f_{in} (\varphi_{in}^*)^{1-\sigma} V_i(\varphi_{in}^*).$$

Taking the log and differentiating this equality with respect to θ_{ij} and using (7),

$$\hat{M}_i^e = \frac{\alpha_i}{\alpha_i + 1} (\gamma_{ii} - \gamma_{ij}) \hat{\varphi}_{ii}^*, \tag{A.5}$$

and the entry effects are muted so long as φ_{ii}^* is held constant. Taking the partial derivative of α_j/τ_{ij} with respect to θ_{ij} holding φ_{jj}^* constant,

$$\varepsilon_{ij} = \frac{\partial \ln(\alpha_j/\tau_{ij})}{\partial \ln \theta_{ij}} = (\sigma - 1) - \frac{\partial \ln V_i(\varphi_{ij}^*)}{\partial \ln \varphi_{ij}^*} \frac{\partial \varphi_{ij}^*}{\partial \theta_{ij}},$$

where $\partial \ln V_i(\varphi_{ij}^*)/\partial \ln \varphi_{ij}^* = d \ln V_i(\varphi_{ij}^*)/d \ln \varphi_{ij}^*$ from the definition of $V_i(\varphi^*)$ and $\partial \ln \varphi_{ij}^*/\partial \ln \theta_{ij} = 1$ from (A.2). In a similar vein, we can show that ε_{ij} is the partial trade elasticity of τ_{ij}.

Next, we show that φ_{ii}^* is a single sufficient statistic for welfare even with tariff revenue. Taking the log and differentiating μ_i in Lemma 1 with respect to θ_{ji},

$$\hat{\mu}_i = -\frac{(\tau_{ji} - 1)\alpha_i}{(\alpha_i + \tau_{ji})(\alpha_i + 1)} \hat{\alpha}_i.$$

Substituting $\hat{\alpha}_i = -[\sigma - 1 + \gamma_{ii} + (\sigma - 1 + \gamma_{ij})\alpha_i]\hat{\varphi}_{ii}^*$ and the definitions of β_i and λ_{ii} in Lemmas 1 and 2 into (9) gives us the result.

A.5 Proof of Proposition 1

We first show (10). From (4), (7), and (8), it follows that

$$\hat{B}_i + (\sigma - 1)\hat{\varphi}_{ii}^* = \sigma \hat{w}_i, \tag{A.6}$$

$$\hat{B}_j + (\sigma - 1)\hat{\varphi}_{jj}^* = \sigma \hat{w}_j, \tag{A.7}$$

$$\hat{B}_j + (\sigma - 1)\hat{\varphi}_{ij}^* = \sigma \hat{w}_i, \tag{A.8}$$

$$\hat{B}_i + (\sigma - 1)\hat{\varphi}_{ji}^* = \sigma \hat{w}_j + (\sigma - 1)\hat{\theta}_{ji}, \tag{A.9}$$

$$\hat{\varphi}_{ij}^* = -\alpha_i \hat{\varphi}_{ii}^*, \tag{A.10}$$

$$\hat{\varphi}_{ji}^* = -\alpha_j \hat{\varphi}_{jj}^*, \tag{A.11}$$

$$\hat{w}_i - \hat{w}_j = -\beta_i \hat{\varphi}_{ii}^* + \beta_j \hat{\varphi}_{jj}^*. \tag{A.12}$$

Note that (A.6)-(A.12) are the system of seven equations with seven unknowns where we can normalize $w_j = 1$ and $\hat{w}_j = 0$ by Walras's law. From (A.6), (A.9), (A.10), (A.12) and (A.7), (A.8), (A.11), (A.12) respectively,

$$\begin{aligned} (\rho + \beta_i)\hat{\varphi}_{ii}^* - (\beta_j - \rho\alpha_j)\hat{\varphi}_{jj}^* &= -\rho\hat{\theta}_{ji}, \\ -(\beta_i - \rho\alpha_i)\hat{\varphi}_{ii}^* + (\beta_j + \rho)\hat{\varphi}_{jj}^* &= 0, \end{aligned}$$

where

$$\beta_i - \rho\alpha_i = \frac{\alpha_i}{\alpha_i + 1}[\sigma - 1 - \rho + \gamma_{ii} + (\sigma - 1 - \rho + \gamma_{ij})\alpha_i] > 0.$$

Solving for $\hat{\varphi}^*_{ii}$ and $\hat{\varphi}^*_{jj}$ and subsequently substituting them into (A.12) yields (10). Then,

$$\frac{d\varphi^*_{ii}}{d\theta_{ji}} < 0,\ \frac{d\varphi^*_{jj}}{d\theta_{ji}} < 0,\ \frac{d\varphi^*_{ij}}{d\theta_{ji}} > 0,\ \frac{d\varphi^*_{ji}}{d\theta_{ji}} > 0,\ \frac{dB_i}{d\theta_{ji}} > 0,\ \frac{dB_j}{d\theta_{ji}} > 0,\ \frac{dw_i}{d\theta_{ji}} > 0.$$

Further, from (9), we have that $dP_i/d\theta_{ji} > 0$ and $dP_j/d\theta_{ji} > 0$. In contrast, if w_i is exogenous,

$$\frac{d\varphi^*_{ii}}{d\theta_{ji}} > 0,\ \frac{d\varphi^*_{jj}}{d\theta_{ji}} < 0,\ \frac{d\varphi^*_{ij}}{d\theta_{ji}} < 0,\ \frac{d\varphi^*_{ji}}{d\theta_{ji}} > 0,\ \frac{dB_i}{d\theta_{ji}} < 0,\ \frac{dB_j}{d\theta_{ji}} > 0,\ \frac{dw_i}{d\theta_{ji}} = 0,$$

and, from (9), we have that $dP_i/d\theta_{ji} < 0$ and $dP_j/d\theta_{ji} > 0$. These differences imply that variable trade costs have different impacts on the trade patterns through the extensive and intensive margins.

Next, we show that the impacts of fixed trade costs and tariffs are similar to those of variable trade costs. Following similar steps, we can derive the equilibrium in changes for f_{ji}:

$$\begin{aligned}
\hat{\varphi}^*_{ii} &= -\frac{\beta_j + \rho}{\sigma\Xi}\hat{f}_{ji}, \\
\hat{\varphi}^*_{jj} &= -\frac{\beta_i - \rho\alpha_i}{\sigma\Xi}\hat{f}_{ji}, \\
\hat{w}_i &= \frac{\rho(\beta_i + \alpha_i\beta_j)}{\sigma\Xi}\hat{f}_{ji},
\end{aligned} \tag{A.13}$$

and those for f_{ij}:

$$\begin{aligned}
\hat{\varphi}^*_{ii} &= -\frac{\beta_j - \rho\alpha_j}{\sigma\Xi}\hat{f}_{ij}, \\
\hat{\varphi}^*_{jj} &= -\frac{\beta_i + \rho}{\sigma\Xi}\hat{f}_{ij}, \\
\hat{w}_i &= -\frac{\rho(\beta_j + \alpha_j\beta_i)}{\sigma\Xi}\hat{f}_{ij}.
\end{aligned}$$

and those for τ_{ji}:

$$\begin{aligned}
\hat{\varphi}^*_{ii} &= -\frac{\beta_j + \rho}{\Xi}\hat{\tau}_{ji}, \\
\hat{\varphi}^*_{jj} &= -\frac{\beta_i - \rho\alpha_i}{\Xi}\hat{\tau}_{ji}, \\
\hat{w}_i &= \frac{\rho(\beta_i + \alpha_i\beta_j)}{\Xi}\hat{\tau}_{ji},
\end{aligned} \tag{A.14}$$

and those for τ_{ij}:

$$\begin{aligned}
\hat{\varphi}^*_{ii} &= -\frac{\beta_j - \rho\alpha_j}{\Xi}\hat{\tau}_{ij}, \\
\hat{\varphi}^*_{jj} &= -\frac{\beta_i + \rho}{\Xi}\hat{\tau}_{ij}, \\
\hat{w}_i &= -\frac{\rho(\beta_j + \alpha_j\beta_i)}{\Xi}\hat{\tau}_{ij}.
\end{aligned}$$

Hence, reductions in any trade costs on exports and imports raise φ^*_{ii} and φ^*_{jj}, but starting from a symmetric situation (i.e., $\alpha_i = \alpha_j$ and $\beta_i = \beta_j$), the effect of trade liberalization is always greater in a liberalizing country than in a non-liberalizing country. Only the difference is that reductions in *import* costs $\theta_{ji}, f_{ji}, \tau_{ji}$ *reduce* w_i, whereas reductions in *export* costs $\theta_{ij}, f_{ij}, \tau_{ij}$ *raise* w_i.

A.6 Proof of Proposition 2

We first show (16). From (12) and (15),

$$\hat{B}_i + (\sigma - 1)\hat{\varphi}^*_{ii} = \sigma \hat{w}_i, \tag{A.15}$$

$$\hat{B}_j + (\sigma - 1)\hat{\varphi}^*_{jj} = \sigma \hat{w}_j, \tag{A.16}$$

$$\hat{B}_j + (\sigma - 1)\hat{\varphi}^*_{ij} = \sigma \hat{w}_i, \tag{A.17}$$

$$\hat{B}_i + (\sigma - 1)\hat{\varphi}^*_{ji} = \sigma \hat{w}_j, \tag{A.18}$$

$$\hat{w}_i - \hat{w}_j = -\beta_i \hat{\varphi}^*_{ii} + \beta_j \hat{\varphi}^*_{jj} - \hat{L}_i. \tag{A.19}$$

From (A.10), (A.15), (A.18), (A.19) and (A.11), (A.16), (A.17), (A.19) respectively,

$$(\beta_i + \rho)\hat{\varphi}^*_{ii} - (\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj} = -\hat{L}_i,$$
$$-(\beta_i - \rho\alpha_i)\hat{\varphi}^*_{ii} + (\beta_j + \rho)\hat{\varphi}^*_{jj} = \hat{L}_i.$$

Solving for $\hat{\varphi}^*_{ii}$ and $\hat{\varphi}^*_{jj}$ and substituting them into (A.19) yields (16).

Next, we show that (14) can be expressed in terms of the domestic productivity cutoff φ^*_{ii} only. Substituting $\hat{L}_i = -(\beta_i + \rho)\hat{\varphi}^*_{ii} + (\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj}$ derived above into (14),

$$\begin{aligned}
\hat{W}_i &= \left(\frac{(\tau_{ji} - 1)\lambda_{ii}}{\rho}\frac{\beta_i}{\alpha_i} + 1\right)\hat{\varphi}^*_{ii} + \frac{1}{\sigma - 1}(-(\beta_i + \rho)\hat{\varphi}^*_{ii} + (\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj}) \\
&= \frac{1}{\rho}\left((1 - \lambda_{ii})\beta_i - \lambda_{ii}\frac{\beta_i}{\alpha_i} + \rho - \frac{\beta_i + \rho}{\sigma}\right)\hat{\varphi}^*_{ii} + \frac{1}{\sigma - 1}(\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj} \\
&= \frac{1}{\sigma - 1}\left((\sigma - 1)(\beta_i + \rho) - \sigma\beta_i\left(\frac{\alpha_i + 1}{\alpha_i + \tau_{ji}}\right) - (\beta_j - \rho\alpha_j)\left(\frac{\alpha_i + 1}{\alpha_j + 1}\right)\right)\hat{\varphi}^*_{ii},
\end{aligned}$$

where the second equality comes from rewriting $\lambda_{ii} = \alpha_i/(\alpha_i + \tau_{ji})$ in (A.4) as $\tau_{ji}\lambda_{ii} = \alpha_i(1 - \lambda_{ii})$ and the third equality comes from rewriting the first two relationships in (16) as

$$\hat{\varphi}^*_{jj} = -\left(\frac{\alpha_i + 1}{\alpha_j + 1}\right)\hat{\varphi}^*_{ii}.$$

Finally, we show that starting from a symmetric situation and free trade, market expansion unambiguously improves welfare for country i. Evaluating at $\alpha_i = \alpha_j, \beta_i = \beta_j$ and $\mu_i = 1$,

$$\hat{W}_i = -\frac{1}{\sigma - 1}\left(\beta_i - (\sigma - 1)\rho + (\beta_i - \rho\alpha_i)\right)\hat{\varphi}^*_{ii},$$

where $\beta_i - (\sigma - 1)\rho > 0$. The desired result follows from $\hat{\varphi}^*_{ii} < 0$. Together with (7) and (12),

$$\frac{d\varphi^*_{ii}}{dL_i} < 0,\ \frac{d\varphi^*_{jj}}{dL_i} > 0,\ \frac{d\varphi^*_{ij}}{dL_i} > 0,\ \frac{d\varphi^*_{ji}}{dL_i} < 0,\ \frac{dB_i}{dL_i} > 0,\ \frac{dB_j}{dL_i} < 0,\ \frac{dw_i}{dL_i} > 0.$$

Further, from (14), we have that $dP_i/dL_i < 0$ and $dP_j/dL_i < 0$. In contrast, if w_i is exogenous,

$$\frac{d\varphi^*_{ii}}{dL_i} = 0,\ \frac{d\varphi^*_{jj}}{dL_i} = 0,\ \frac{d\varphi^*_{ij}}{dL_i} = 0,\ \frac{d\varphi^*_{ji}}{dL_i} = 0,\ \frac{dB_i}{dL_i} = 0,\ \frac{dB_j}{dL_i} = 0,\ \frac{dw_i}{dL_i} = 0,$$

and, from (14), $dP_i/dL_i < 0$ and $dP_j/dL_i = 0$.

A.7 Proof of Proposition 3

We first show the derivation of (18). Taking the log and differentiating W_i with respect to τ_{ji},

$$\begin{aligned}\hat{W}_i &= \frac{1}{\rho}(\tau_{ji}-1)\left(\frac{\alpha_i}{\alpha_i+\tau_{ji}}\right)\frac{\beta_i}{\alpha_i}\hat{\varphi}^*_{ii} + \frac{1}{\rho}\left(\frac{\tau_{ji}}{\alpha_i+\tau_{ji}}\right)\hat{\tau}_{ji} + \hat{\varphi}^*_{ii} \\ &= \left(\frac{(\tau_{ji}-1)\lambda_{ii}}{\rho}\frac{\beta_i}{\alpha_i}+1\right)\hat{\varphi}^*_{ii} + \frac{1}{\rho}\lambda_{ji}\hat{\tau}_{ji},\end{aligned}$$

where the second equality follows from $\lambda_{ii} = \alpha_i/(\alpha_i+\tau_{ji})$ and $\lambda_{ji} = \tau_{ji}/(\alpha_i+\tau_{ji})$ from Lemma 1. Compared to (9), there is an additional term that captures changes in tariff revenue raised by changes in τ_{ji}. Taking the log and differentiating (1) with respect to τ_{ji} gives the counterparts to (A.6) and (A.9). Cancelling $\hat{B}_i$ out from these and using (7) and (8) that hold for changes in τ_{ji},

$$\hat{\tau}_{ji} = -(\beta_i+\rho)\hat{\varphi}^*_{ii} + (\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj}.$$

Further, noting that $\lambda_{ji} = 1-\lambda_{ii}$ and substituting $\hat{\tau}_{ji}$ derived above,

$$\hat{W}_i = -\frac{1}{\rho}\frac{\lambda_{ii}}{\alpha_i}(\beta_i-\rho\alpha_i)\hat{\varphi}^*_{ii} + \frac{1}{\rho}\lambda_{ji}(\beta_j-\rho\alpha_j)\hat{\varphi}^*_{jj}. \tag{A.20}$$

Since an increase in tariffs decreases φ^*_{ii} and φ^*_{jj}, (A.20) shows that tariffs in country i have a positive (negative) impact on welfare in country i by increasing (decreasing) the consumption of domestic (imported) varieties. In fact, $\hat{\varphi}^*_{ii}$ and $\hat{\varphi}^*_{jj}$ have the following relationship from (A.14):

$$\hat{\varphi}^*_{jj} = \left(\frac{\beta_i-\rho\alpha_i}{\beta_j+\rho}\right)\hat{\varphi}^*_{ii}.$$

Substituting this into (A.20) and rearranging,

$$\hat{W}_i = \frac{\beta_i-\rho\alpha_i}{\rho}\left(-\frac{\lambda_{ii}}{\alpha_i} + \frac{\lambda_{ji}(\beta_j-\rho\alpha_j)}{\beta_j+\rho}\right)\hat{\varphi}^*_{ii}.$$

Further, substituting $\lambda_{ii}/\alpha_i = \lambda_{ji}/\tau_{ji}$ from (A.4) into the above, we obtain the expression in (18).

Next, we show that starting from a symmetric situation, country i's gain from tariffs cannot compensate country j's loss. Adding $\hat{W}_i$ in (A.20) and $\hat{W}_j$ in the main text,

$$\begin{aligned}\hat{W}_i + \hat{W}_j &= -\frac{1}{\rho}\frac{\lambda_{ii}}{\alpha_i}(\beta_i-\rho\alpha_i)\hat{\varphi}^*_{ii} + \left(\frac{(\tau_{ji}-1)\lambda_{jj}}{\rho}\frac{\beta_j}{\alpha_j} + 1 + \frac{\lambda_{ji}}{\rho}(\beta_j-\rho\alpha_j)\right)\hat{\varphi}^*_{jj} \\ &= \frac{\beta_i-\rho\alpha_i}{\rho\Xi}\left(\frac{\beta_j+\rho}{\alpha_i+\tau_{ji}} - \frac{(\tau_{ji}-1)\beta_j}{\alpha_j+\tau_{ij}} - \rho - \frac{\tau_{ji}(\beta_j-\rho\alpha_j)}{\alpha_i+\tau_{ji}}\right)\hat{\tau}_{ji},\end{aligned}$$

where the second equality follows from using (A.4) and (A.14). Notice that the first term is positive and the others are negative in the brackets, and thus changes in total welfare are in general ambiguous, as in changes in country i's welfare. However, evaluating at a symmetric situation where $\alpha_i = \alpha_j$, $\beta_i = \beta_j$ and $\tau_{ij} = \tau_{ji}$,

$$\hat{W}_i + \hat{W}_j = -\frac{\beta_i-\rho\alpha_i}{\rho\Xi}\left(\frac{(\tau_{ji}-1)(\beta_i+\rho+\beta_i-\rho\alpha_i)}{\alpha_i+\tau_{ji}}\right)\hat{\tau}_{ji},$$

where the value in the brackets is positive from observing that $\tau_{ji}-1 \geq 0$. This establishes the desired result.

Finally, we show the derivation of (19) and (20). Taking the log and differentiating W_i with respect to τ_{ji}, welfare changes can be simply expressed as

$$\hat{W}_i = \frac{\hat{\mu}_i}{\rho} + \hat{\varphi}^*_{ii}.$$

To show that changes can be expressed in terms of changes in λ_{ii} and μ_i, we use the fact that $\lambda_{ii} \times \mu_i = \alpha_i/(\alpha_i + 1)$ from (A.4). Taking the log and differentiating this with respect to τ_{ji},

$$\hat{\lambda}_{ii} + \hat{\mu}_i = -\frac{\beta_i}{\alpha_i}\hat{\varphi}^*_{ii}. \tag{A.21}$$

Solving for $\hat{\varphi}^*_{ii}$ and substituting it into the above welfare changes gives us the expression in (19). Note that these changes in $\hat{W}_i$ and $\hat{\lambda}_{ii} + \hat{\mu}_i$ hold with respect to θ_{ji} and f_{ji}, and (19) also applies to variable and fixed trade costs. Regarding (20), using the general expression of β_i/α_i in Lemma 2, let us further express (19) as

$$\hat{W}_i = -\left(\frac{\alpha_i + 1}{\varepsilon_{ij}(\alpha_i + 1) + \gamma_{ii} - \gamma_{ij}}\right)\hat{\lambda}_{ii} + \left(\frac{1}{\rho} - \frac{\alpha_i + 1}{\varepsilon_{ij}(\alpha_i + 1) + \gamma_{ii} - \gamma_{ij}}\right)\hat{\mu}_i.$$

After rearranging, this can be rewritten as

$$\begin{aligned}\hat{W}_i = -\left(\frac{1}{\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij}}\right)\hat{\lambda}_{ii} - \left(\frac{\alpha_i(\gamma_{ii} - \gamma_{ij})}{(\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij})((\alpha_i + 1)\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij})}\right)\hat{\lambda}_{ii} \\ + \left(\frac{1}{\rho} - \frac{1}{\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij}} - \frac{\alpha_i(\gamma_{ii} - \gamma_{ij})}{(\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij})((\alpha_i + 1)\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij})}\right)\hat{\mu}_i.\end{aligned}$$

Solving (A.5) for $\hat{\varphi}^*_{ii}$ that holds for changes in τ_{ji} and substituting this and β_i/α_i into (A.21),

$$\hat{\lambda}_{ii} = -\left(\frac{(\alpha_j + 1)\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij}}{\alpha_i(\gamma_{ii} - \gamma_{ij})}\right)\hat{M}^e_i - \hat{\mu}_i.$$

Substituting this into the second $\hat{\lambda}_{ii}$ above yields the expression $\hat{W}_i$ in (20), which becomes the same as that in Melitz and Redding (2015) without tariff revenue ($\hat{\mu}_i = 0$).

A.8 Proof of Lemma 3

We first show that, if the hazard differential $\gamma_{jj} - \gamma_{ji}$ is negative (positive), the trade elasticity ε_{ji} is increasing (decreasing) in trade costs. Let $\phi \in \{\theta_{ij}, \theta_{ji}, f_{ij}, f_{ji}, \tau_{ij}, \tau_{ji}\}$ denote a set of trade costs between countries. From the definition of γ_{jn}, let us re-express this as a function of the productivity cutoff φ^*_{jn} for $n = i, j$:

$$\gamma_j(\varphi^*_{jn}) \equiv -\frac{d\ln V_j(\varphi^*_{jn})}{d\ln \varphi^*_{jn}}.$$

If $\gamma_j(\varphi^*_{jn})$ is strictly increasing (decreasing) in the productivity cutoff φ^*_{jn}, the differential is negative (positive) so long as selection into the export market is satisfied:

$$\gamma'_j(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \gamma_{jj} - \gamma_{ji} \lesseqgtr 0.$$

Thus, if the extensive margin elasticity $\gamma^*_{jn} = \gamma_j(\varphi^*_{jn})$ is a monotonic function in the productivity cutoff φ^*_{jn}, the sign of the differential is the same for a given productivity distribution $G_j(\varphi)$. Exploiting the fact that

φ^*_{jj} is held constant to derive the partial trade elasticity $\varepsilon_{ji} = \sigma - 1 + \gamma_{ji}$ (see Appendix A.4), we get

$$\frac{d\varepsilon_{ji}}{d\phi} = \gamma'_j(\varphi^*_{ji})\frac{d\varphi^*_{ji}}{d\phi}.$$

Since $\frac{d\varphi^*_{ji}}{d\phi} > 0$ from Proposition 1, so long as $\gamma_j(\varphi^*_{ji})$ is a monotonic function of φ^*_{jn},

$$\gamma'_j(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \frac{d\varepsilon_{ji}}{d\phi} \gtreqless 0. \tag{A.22}$$

Next, we show that, if the differential is negative (positive), the trade elasticity is decreasing (increasing) in country i's market size, while the converse is true for country j's market size. Differentiating ε_{ji} with respect to L_i and L_j respectively and noting that $\frac{d\varphi^*_{ji}}{dL_i} < 0$ and $\frac{d\varphi^*_{ji}}{dL_j} > 0$ from Proposition 2, we get

$$\gamma'_j(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \frac{d\varepsilon_{ji}}{dL_i} \lesseqgtr 0,$$
$$\gamma'_j(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \frac{d\varepsilon_{ji}}{dL_j} \gtreqless 0.$$

A.9 Proof of Proposition 4

We first show that, if the differential is negative (positive), reductions in trade costs have the impact on the optimal tariffs t^*_{ji} not only by decreasing the domestic trade share $\tilde{\lambda}_{jj}$ but also by decreasing (increasing) the trade elasticity ε_{ji}. The optimal tariff in (21) is rewritten as

$$t^*_{ji} = \frac{\rho}{\tilde{\lambda}_{jj}\left(\frac{\beta_j}{\alpha_j} - \rho\right)},$$

where reductions in trade costs always decrease $\tilde{\lambda}_{jj}$ irrespective of the sign of $\gamma_{jj} - \gamma_{ji}$ from Proposition 1. Thus, it suffices to show that, if $\gamma_{jj} - \gamma_{ji}$ is negative (positive), β_j/α_j decreases (increases) with ϕ. From Lemmas 1 and 2, $\beta_j/\alpha_j = \varepsilon_{ji} + (\gamma_{jj} - \gamma_{ji})/(\alpha_j + 1)$ and differentiating this with respect to ϕ,

$$\begin{aligned}\frac{d(\beta_j/\alpha_j)}{d\phi} &= \gamma'_j(\varphi^*_{ji})\frac{d\varphi^*_{ji}}{d\phi} + \frac{-\gamma'_j(\varphi^*_{ji})\frac{d\varphi^*_{ji}}{d\phi}(\alpha_j+1) - (\gamma_{ji} - \gamma_{jj})\frac{d\alpha_j}{d\phi}}{(\alpha_j+1)^2} \\ &= \frac{\alpha_j}{\alpha_j+1}\left(\frac{d\varepsilon_{ji}}{d\phi} - \left(\frac{\gamma_{jj} - \gamma_{ji}}{\alpha_j(\alpha_j+1)}\right)\frac{d\alpha_j}{d\phi}\right).\end{aligned}$$

Using the impact of ϕ on ε_{ji} in Lemma 3 and $\frac{d\alpha_j}{d\phi} > 0$ from Proposition 1,

$$\gamma'_j(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \frac{d(\beta_j/\alpha_j)}{d\phi} \gtreqless 0. \tag{A.23}$$

Next, we show that market size has a similar impact on t^*_{ji}. From the impact of market size on $\tilde{\lambda}_{jj}$ from Proposition 2, it suffices to see the impact of L_i, L_j on β_j/α_j:

$$\gamma'_j(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \frac{d(\beta_j/\alpha_j)}{dL_i} \lesseqgtr 0,$$
$$\gamma'_j(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \frac{d(\beta_j/\alpha_j)}{dL_j} \gtreqless 0.$$

A.10 Proof of Proposition 5

We first show the impact of the key exogenous variables evaluated at the symmetric situation.

- Impact of variable trade costs: Evaluating (A.6)-(A.11) at the symmetric situation where $B_i = B_j \equiv B$, $\varphi_{ii}^* = \varphi_{jj}^* \equiv \varphi_d^*$, $\varphi_{ij}^* = \varphi_{ji}^* \equiv \varphi_x^*$ (and ignoring (A.12) by setting $w_i = w_j \equiv w = 0$), we get

$$\hat{B} + (\sigma - 1)\hat{\varphi}_d^* = 0,$$
$$\hat{B} + (\sigma - 1)\hat{\varphi}_x^* = (\sigma - 1)\hat{\theta},$$
$$\hat{\varphi}_x^* = -\alpha\hat{\varphi}_d^*.$$

  Noting that these are the system of three equations with three unknowns $(\hat{\varphi}_d^*, \hat{\varphi}_x^*, \hat{B})$, we can solve for

$$\hat{\varphi}_d^* = -\frac{1}{\alpha + 1}\hat{\theta}, \quad \hat{\varphi}_x^* = \frac{\alpha}{\alpha + 1}\hat{\theta}, \quad \hat{B} = \frac{\sigma - 1}{\alpha + 1}\hat{\theta}. \tag{A.24}$$

  Further, noting from (A.4) that $\tilde{\lambda} = \alpha/(\alpha + 1)$, changes in $\tilde{\lambda}$ are given by

$$\hat{\tilde{\lambda}} = -\left(\frac{\sigma - 1 + \gamma_d + (\sigma - 1 + \gamma_x)\alpha}{\alpha + 1}\right)\hat{\varphi}_d^* = -\frac{\beta}{\alpha}\hat{\varphi}_d^*.$$

  Since φ_d^* is decreasing in θ, reductions in θ decrease $\tilde{\lambda}$ irrespective of the sign of $\gamma_d - \gamma_x$. As for the partial trade elasticity, differentiating $\varepsilon = \sigma - 1 + \gamma(\varphi_x^*)$ with respect to θ,

$$\frac{d\varepsilon}{d\theta} = \gamma'(\varphi_x^*)\frac{d\varphi_x^*}{d\theta}.$$

  As φ_x^* is increasing in θ, if $\gamma(\varphi_h^*)$ is increasing (decreasing) in φ_h^* for $h = d, x$ so that $\gamma_d - \gamma_x < (>)0$, reductions in θ decrease (increase) ε. Together with the impact on $\tilde{\lambda}$ above, if the differential is negative (positive and thereby increases in ε are greater than decreases in $\tilde{\lambda}$), reductions in θ shift up (down) the $f(\tau)$ curve in Figure 2, which in turn increase (decrease) the Nash tariffs τ^*.

- Impact of fixed trade costs: The similar proof applies to reductions in fixed trade costs f_x. In particular, noting that changes in φ_x^* are given by $\hat{B} + (\sigma - 1)\hat{\varphi}_x^* = \hat{f}_x$ while changes in φ_d^*, B are similar to those induced by θ, changes in the three unknowns satisfy

$$\hat{\varphi}_d^* = -\frac{1}{(\sigma - 1)(\alpha + 1)}\hat{f}_x, \quad \hat{\varphi}_x^* = \frac{\alpha}{(\sigma - 1)(\alpha + 1)}\hat{f}_x, \quad \hat{B} = \frac{1}{\alpha + 1}\hat{f}_x. \tag{A.25}$$

  Since reductions in f_x shift φ_d^* and φ_x^* in the same direction as those in θ, such reductions also shift $\tilde{\lambda}$, ε and τ^* in the same direction.

- Impact of market size: Evaluating (A.10)-(A.11), (A.15)-(A.18) at the symmetric situation (and ignoring (A.19) by setting $w_i = w_j \equiv w = 0$), we obtain the following solutions for the three unknowns:

$$\hat{\varphi}_d^* = \hat{\varphi}_x^* = \hat{B} = 0.$$

  Thus, irrespective of the sign of $\gamma_d - \gamma_x$, market size have no impact on the firm-level variables φ_d^*, φ_x^* at the symmetric situation due to firms' constant markups. Since φ_d^* and φ_x^* are invariant to market size in the symmetric situation, neither $\tilde{\lambda}$, ε or τ^* is affected by market size as well.

Next, we show that, if the differential is negative (positive), reductions in trade costs narrow (widen) the gap between $\bar{\tau}^*$ and $\underline{\tau}^*$, while increases in market size have no impact on these bounds irrespective of the sign of the differential in Nash equilibrium. On the one hand, evaluating $\bar{\tau}^*$ at the symmetric situation

$$\bar{\tau}^* = \frac{\varepsilon + \rho\frac{f_x}{f_d}}{\epsilon - \rho},$$

and differentiating it with respect to θ, we get

$$\frac{d\bar{\tau}^*}{d\theta} = \frac{\rho\left(1 - \frac{f_x}{f_d}\right)}{(\varepsilon - \rho)^2}\frac{d\varepsilon}{d\theta},$$

where $1 - \frac{f_x}{f_d} < 0$ for selection into the export market. Then it follows from (A.22) that

$$\gamma'(\varphi_h^*) \gtreqless 0 \quad \Longrightarrow \quad \frac{d\bar{\tau}^*}{d\theta} \gtreqless 0,$$

Thus, if $\gamma(\varphi_h^*)$ is strictly increasing (decreasing) in φ_h^*, the upper bound $\bar{\tau}^*$ is stritly increasing (decreasing) in θ. Regarding $\underline{\tau}^*$, on the other hand, let us rewrite it as

$$\underline{\tau}^* = \frac{\frac{\beta}{\alpha}}{\frac{\beta}{\alpha} - \rho}.$$

Differentiating this expression with respect to θ,

$$\frac{d\underline{\tau}^*}{d\theta} = -\frac{\rho}{(\frac{\beta}{\alpha} - \rho)^2}\frac{d(\beta/\alpha)}{d\theta}.$$

Then it follows from (A.23) that

$$\gamma'(\varphi_h^*) \gtreqless 0 \quad \Longrightarrow \quad \frac{d\underline{\tau}^*}{d\theta} \lesseqgtr 0.$$

Thus, if $\gamma(\varphi_h^*)$ is strictly increasing (decreasing) in φ_h^*, the lower bound $\underline{\tau}^*$ is strictly decreasing (increasing) in θ; however, if $\gamma(\varphi_h^*)$ is constant, so is β/α and the lower bound is not affected. While f_x has a similar impact on the two bounds, L has no impact on φ_d^*, φ_x^* and hence $\tilde{\lambda}$ as well as ε as shown above, which means that increases in L have no impact on the two bounds.

Finally, we show that bilateral reductions in tariffs always improves welfare in Nash equilibrium, as shown in (25). Evaluating (A.20) at a symmetric situation,

$$\begin{aligned}
\hat{W} &= -\frac{1}{\rho}\frac{\lambda}{\alpha}(\beta - \rho\alpha)\hat{\varphi}_d^* + \frac{1}{\rho}(1-\lambda)(\beta - \rho\alpha)\hat{\varphi}_d^* \\
&= -\frac{1}{\rho}(\beta - \rho\alpha)\left(\frac{\lambda}{\alpha} - (1-\lambda)\right)\hat{\varphi}_d^* \\
&= -\frac{1}{\rho}(\beta - \rho\alpha)\left(\frac{1}{\alpha+\tau} - \frac{\tau}{\alpha+\tau}\right)\hat{\varphi}_d^* \\
&= \left(\frac{(\tau-1)(\beta-\rho\alpha)}{\rho(\alpha+\tau)}\right)\hat{\varphi}_d^*,
\end{aligned}$$

where the third equality follows from (A.4). The result follows from noting that φ_d^* is decreasing in any trade costs and $\tau - 1 \geq 0$ in Nash equilibrium.

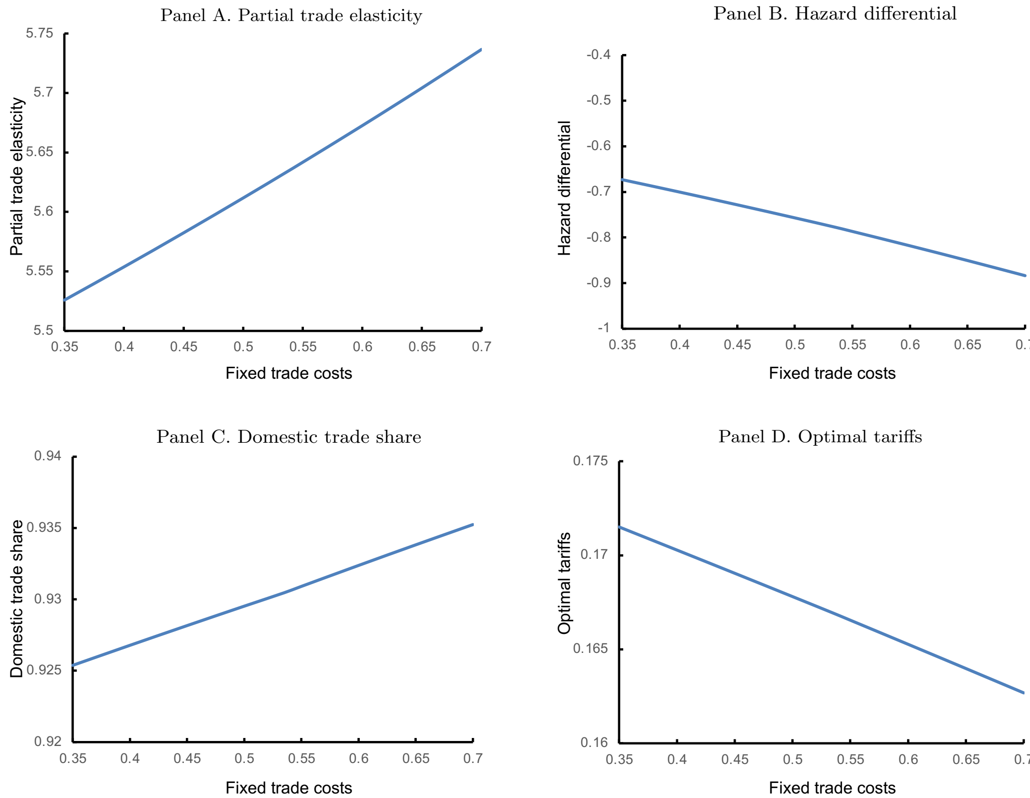

Figure B.1 – Bilateral effect of fixed trade costs

B Additional Quantitative Results

B.1 Bilateral Effect on Optimal Tariffs

To compute bilateral effects of variable trade costs θ in Figure 3, we rely on the analytical solutions in (A.24). To compute bilateral effects of fixed trade costs f_x, we rely on the analytical solutions in (A.25). Substituting the calibrated parameters, it is possible to examine changes in fixed trade costs from the initial equilibrium, holding the other parameter values constant.

Figure B.1 displays changes in the key endogenous variables induced by changes in $f_x \in [0.35, 0.7]$ where $f_x = 0.535$ in the initial equilibrium. It is obvious that this figure is a counterpart to Figure 3. The comparison between Figures 3 and B.1 reveals that fixed trade costs have qualitatively similar patterns. However, we find that the impact of such trade costs on the optimal tariffs is quantitatively much smaller than that of variable trade costs. For example, 25 percent reductions in fixed trade costs (from $f_x = 0.535$ to $f_x = 0.4$) increase the optimal tariffs only 2 percent. In contrast, 23 percent reductions in variable trade costs (from $\theta = 1.7$ to $\theta = 1.3$) increase the optimal tariffs by 36.2 percent as seen in the main text. The intuition is clearly seen by comparing (A.24) and (A.25): using the hat notation, bilateral changes in variable trade costs are given by $(\sigma - 1)\hat{\theta}$ while bilateral changes in fixed trade costs are given by $\hat{f}_x$. Thus the impact of fixed trade costs on the productivity cutoffs is $(\sigma - 1)$ times as low as that of variable trade costs.

Next, we compare changes in the optimal tariffs with respect to bilateral changes in fixed trade costs across the different trade models. Figure B.2 represents the result. As in Figure 4, the optimal tariffs with a variable

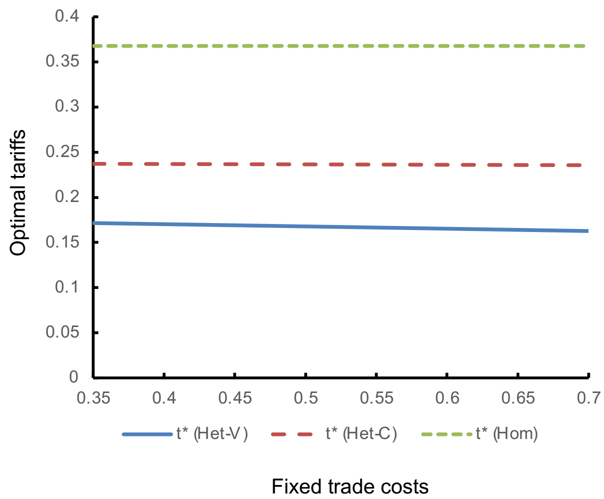

Figure B.2 – Optimal tariffs across different trade models

trade elasticity (21) are lower than the optimal tariffs with a constant trade elasticity (22) and (23) for any range of fixed trade costs $f_x \in [0.35, 0.7]$. However, the impact of fixed trade costs on the optimal tariffs are quantitatively much smaller than that of variable trade costs in the heterogeneous firm model with a variable or constant trade elasticity. In the homogeneous firm model, fixed trade costs have no effect on the optimal tariffs because productivity is exogenously given.

B.2 Unilateral Effect on Optimal Tariffs

Consider unilateral effects of the key exogenous variables on the productivity cutoffs. Suppose that country i unilaterally changes variable trade costs of imports from country j. Evaluating (10) at the symmetric situation and using (7), unilateral effects of variable trade costs on the productivity cutoffs in country j are

$$\begin{aligned} \hat{\varphi}^*_{jj} &= -\frac{\rho(\beta_j - \rho\alpha_j)}{\Xi}\hat{\theta}_{ji}, \\ \hat{\varphi}^*_{ji} &= \frac{\rho\alpha_j(\beta_j - \rho\alpha_j)}{\Xi}\hat{\theta}_{ji}. \end{aligned} \tag{B.1}$$

Similarly, evaluating the analytical solution in (A.13) at the symmetric situation and using (7), unilateral effects of fixed trade costs of imports from country j in country i have the following effects on these cutoffs:

$$\begin{aligned} \hat{\varphi}^*_{jj} &= -\frac{\beta_j - \rho\alpha_j}{\sigma\Xi}\hat{f}_{ji}, \\ \hat{\varphi}^*_{ji} &= \frac{\alpha_j(\beta_j - \rho\alpha_j)}{\sigma\Xi}\hat{f}_{ji}. \end{aligned} \tag{B.2}$$

Regarding market size, suppose that country j unilaterally expands its size. Evaluating the analytical solution in (16) at the symmetric situation and using (7), unilateral effects of market size on the cutoffs are

$$\begin{aligned} \hat{\varphi}^*_{jj} &= -\frac{\rho(\alpha_j + 1)}{\Xi}\hat{L}_j, \\ \hat{\varphi}^*_{ji} &= \frac{\rho\alpha_j(\alpha_j + 1)}{\Xi}\hat{L}_j. \end{aligned}$$

Plugging the calibrated parameters into the system of the equations, we can compute changes in the three endogenous variables $\varepsilon_{jj}, \gamma_{jj} - \gamma_{ji}, \tilde{\lambda}_{jj}$ and hence the optimal tariffs t_{ji}^* induced by each exogenous variable.

Figure B.3 displays unilateral effects of variable trade costs in country i (θ_{ji}). We can easily see that these results are qualitatively similar with those in Figure 3, but quantitatively different from that figure because we hold variable trade costs in country j (θ_{ij}) constant. As a result, the unilateral impact of variable trade costs on each endogenous variables is relatively smaller than the bilateral impact of such trade costs. Analytically, comparing (A.24) and (B.1) at the symmetric situation, bilateral changes in the domestic productivity cutoff ($\hat{\varphi}_d^*$) are greater than unilateral changes in that cutoff ($\hat{\varphi}_{jj}^*$) if and only if

$$\rho(\alpha_j + 1)(\beta_j - \rho\alpha_j) < \Xi. \tag{B.3}$$

Substituting Ξ defined in the main text, this inequality is always satisfied and hence the impacts are greater for bilateral changes than unilateral changes for variable trade costs.

The important difference from Figure 3 is changes in the relative wage in Panel E. Since we evaluate the initial equilibrium at the symmetric situation, *bilateral* reductions in variable trade costs have no impact on the terms-of-trade (i.e., $w_i = w_j \equiv w = 1$ and hence $\hat{w}_i = \hat{w}_j = \hat{w} = 0$). In contrast, *unilateral* reductions in variable trade costs do have a crucial impact on the terms-of-trade even evaluated at the symmetric situation as indicated by (10). This is a quantitative illustration of the finding by Demidova and Rodríguez-Clare (2013), and the analytical solutions of (10) in the present paper help to quantify the difference between bilateral and unilateral changes in variable trade costs, which operates through changes in the relative wage. We find that 23 percent unilateral reductions in variable trade costs in country i (from $\theta_{ji} = 1.7$ to $\theta_{ji} = 1.3$) decrease the relative wage of country i by 9.4 percent (from $w_i/w_j = 1$ to $w_i/w_j = 0.906$).

Figure B.4 displays unilateral effects of fixed trade costs in country i (f_{ji}). While the qualitative impacts of unilateral changes are similar between variable and fixed trade costs as shown in (B.1) and (B.2), fixed trade costs have much weaker quantitative impacts on the endogenous variables than variable trade costs. Although this is not surprising from the results in Figure B.1, we also find the *unilateral* effects of fixed trade costs in (B.2) are greater than *bilateral* effects of fixed trade costs in (A.25), which stands in sharp contrast to variable trade costs. Comparing (A.25) and (B.2) at the symmetric situation, unilateral changes in the domestic productivity cutoff ($\hat{\varphi}_{jj}^*$) are greater than bilateral changes in that cutoff ($\hat{\varphi}_d^*$) if and only if (B.3) holds. Reflecting this, 25 percent reductions in fixed costs (from $f_{ji} = 0.535$ to $f_{ji} = 0.4$) leads to 3.2 percent increases in the optimal tariffs, which is greater than the same amount of reductions in f_x (2 percent) reviewed in Section B.1. The intuition behind this result is elusive and we find it hard to explain why unilateral effects are greater than bilateral effects for fixed trade costs.

Finally, Figure B.5 displays unilateral effects of market size in country j (L_j). Contrary to bilateral changes, unilateral changes in market size have a crucial impact on all the endogenous variables through changes in the terms-of-trade in Panel E.[33] However, the quantitative impact of market size is very smaller than that of variable trade costs in Figure B.3 and is approximately similar to that of fixed trade costs in Figure B.4. We do recognize that the limited impact of market size largely relies on the restrictive feature of monopolistic competition and CES preferences, and the same result would not necessarily hold in monopolistic competition and VES preferences since market size works to reduce firms' average markups (Melitz and Ottaviano, 2008). We expect however that the key policy result would qualitatively continue to hold in such a setting in that a variable trade elasticity would play a more important role in optimal trade policy.

[33] From (16) and noting $w_j = 1$, changes in the relative wage are given by $\hat{w}_i = -\frac{\rho^2(\alpha_i\alpha_j - 1)}{\Xi}\hat{L}_j$ and the curve in Panel E is decreasing in L_j.

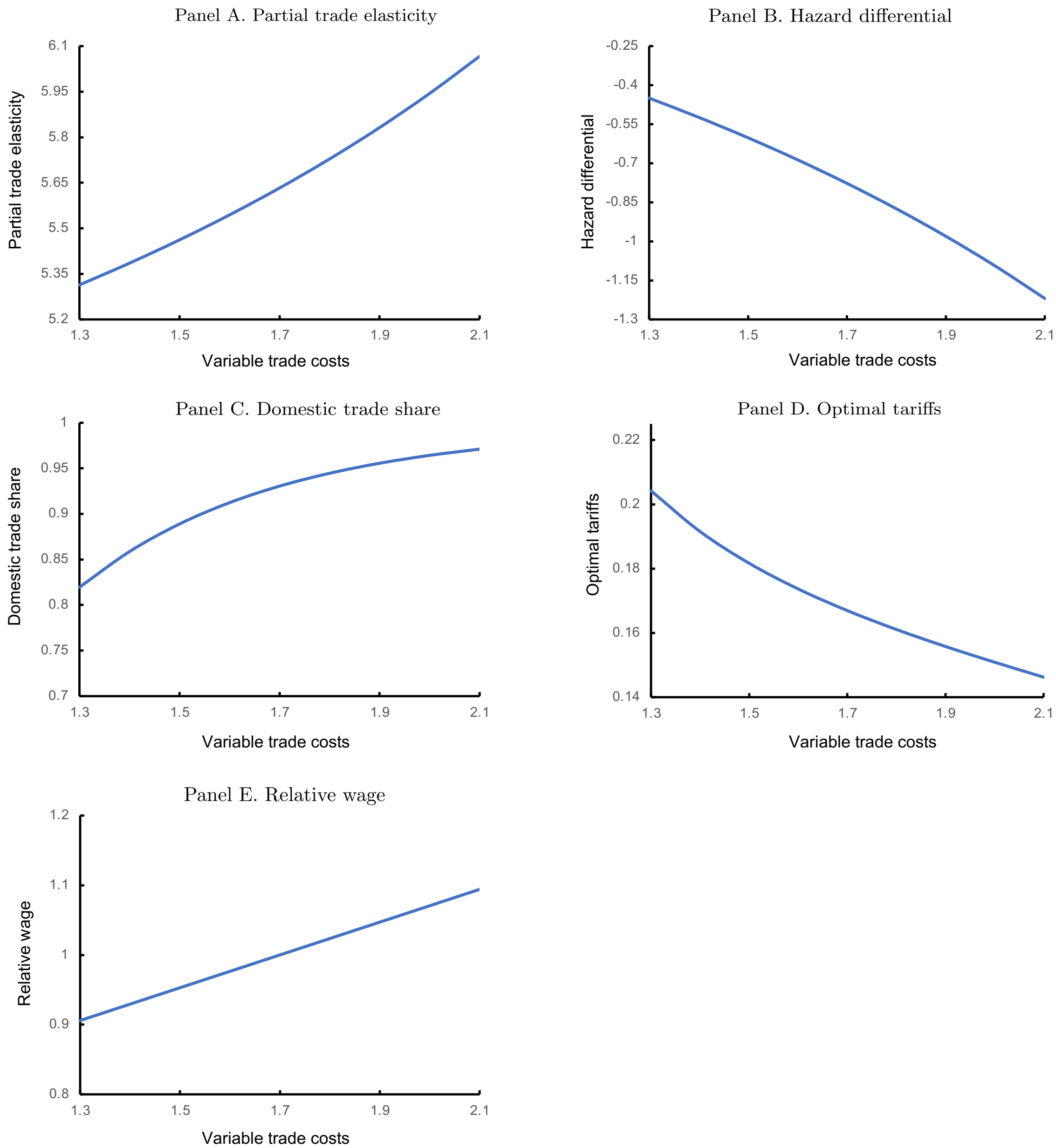

Figure B.3 – Unilateral effect of variable trade costs

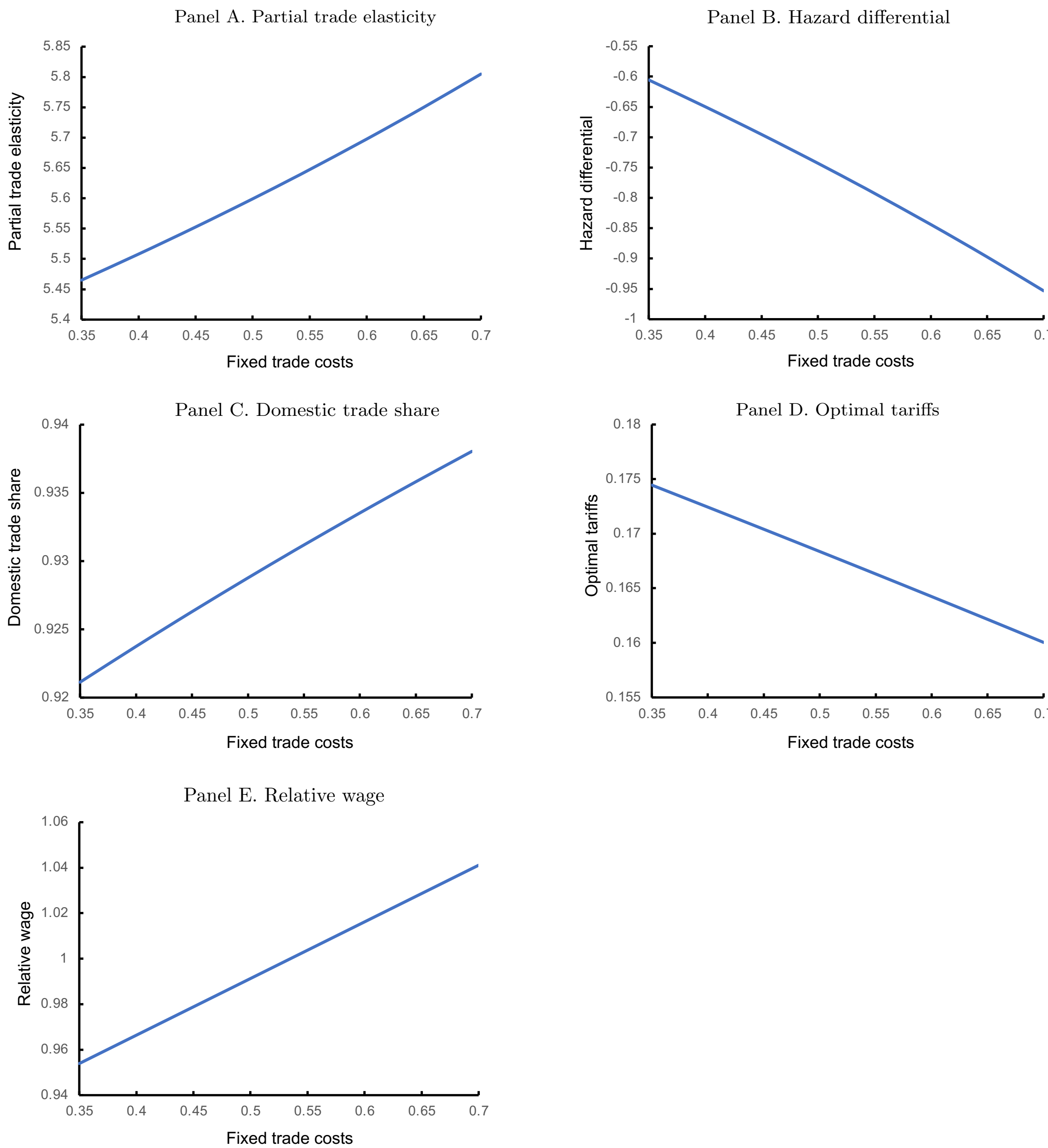

Figure B.4 – Unilateral effect of fixed trade costs

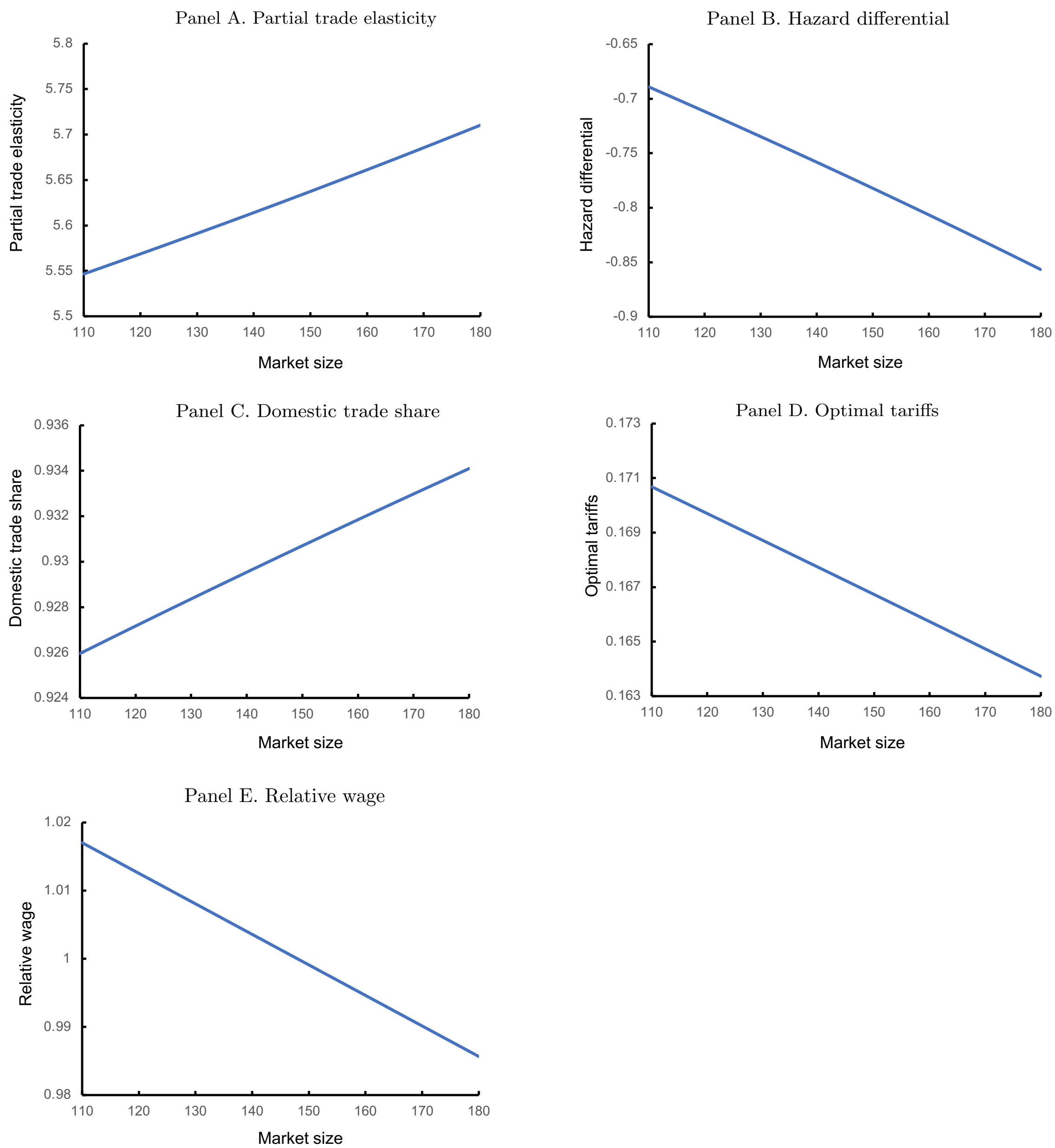

Figure B.5 – Unilateral effect of market size

References

Amiti M, Itskhoki O, Konings J. 2014. Importers, Exporters, and Exchange Rate Disconnect. *American Economic Review* 104, 1942-1978.

Arkolakis C, Costinot A, Donaldson D, Rodríguez-Clare A. 2019. The Elusive Pro-Competitive Effects of Trade. *Review of Economic Studies* 86, 46-80.

Arkolakis C, Costinot A, Rodríguez-Clare A. 2012. New Trade Models, Same Old Gains? *American Economic Review* 102, 94-130.

Bas M, Mayer T, Thoenig M. 2017. From Micro to Macro: Demand, Supply, and Heterogeneity in the Trade Elasticity. *Journal of International Economics* 108, 1-19.

Bernard AB, Jensen JB, Redding SJ, Schott SK. 2007. Firms in International Trade. *Journal of Economic Perspectives* 21, 105-130.

Bertoletti P, Etro F. 2017. Monopolistic Competition when Income Matters. *Economic Journal* 127, 1217-1243.

Chaney T. 2008. Distorted Gravity: The Intensive and Extensive Margins of International Trade. *American Economic Review* 98, 1707-1721.

Costinot A, Rodríguez-Clare A, Werbubg I. 2020. Micro to Macro: Optimal Trade Policy with Firm Heterogeneity. *Econometrica* 88, 2739-2776.

Demidova S. 2008. Productivity Improvements and Falling Trade Costs: Boon or Bane? *International Economic Review* 49, 1437-1462.

Demidova S. 2017. Trade Policies, Firm Heterogeneity, and Variable Markups. *Journal of International Economics* 108, 260-273.

Demidova S, Rodríguez-Clare A. 2009. Trade Policy under Firm-Level Heterogeneity in a Small Economy. *Journal of International Economics* 78, 100-112.

Demidova S, Rodríguez-Clare A. 2013. The Simple Analytics of the Melitz Model in a Small Economy. *Journal of International Economics* 90, 266-272.

Dhingra S, Morrow J. 2019. Monopolistic Competition and Optimum Product Diversity under Firm Heterogeneity. *Journal of Political Economy* 127, 196-232.

Eaton J, Kortum S. 2002. Technology, Geography, and Trade. *Econometrica* 70, 1741-1779.

Eaton J, Kortum S. 2011. Putting Ricardo to Work. *Journal of Economics Perspectives* 26, 65-89.

Feenstra RC. 2017. Restoring the Product Variety and Pro-competitive Gains from Trade with Heterogeneous Firms and Bounded Productivity. *Journal of International Economics* 110, 16-27.

Felbermayr G, Jung B. 2012. Unilateral Trade Liberalization in the Melitz Model: A Note. *Economics Bulletin* 32, 1724-1730.

Felbermayr G, Jung B. 2018. Market Size and TFP in the Melitz Model. *Review of International Economics* 24, 869-891.

Felbermayr G, Jung B, Larch M. 2013. Optimal Tariffs, Retaliation, and the Welfare Loss from Tariff Wars in the Melitz Model. *Journal of International Economics* 89, 13-25.

Felbermayr G, Jung B, Larch M. 2015. The Welfare Consequences of Import Tariffs: A Quantitative Perspective. *Journal of International Economics* 97, 295-309.

Fukao K, Kim YG, Kwon HU. 2008. Plant Turnover and TFP Dynamics in Japanese Manufacturing. In *Micro-Evidence for the Dynamics of Industrial Evolution: The Case of the Manufacturing Industry in Japan and Korea*. ed. Lee JD, Heshmati A. Nova Science Publication, 23-59.

Gros D. 1987. A Note on the Optimal Tariff, Retaliation and the Welfare Loss from Tariff Wars in a Framework with Intra-Industry Trade. *Journal of International Economics* 23, 357-367.

Head K, Mayer T, Thoenig M. 2014. Welfare and Trade without Pareto. *American Economic Review* 104, 310-316.

Helpman E, Melitz MJ, Rubinstein Y. 2008. Estimating Trade Flows: Trading Partners and Trading Volumes. *Quarterly Journal of Economics* 123, 441-487.

Krugman P. 1980. Scale Economies, Production Differentiation, and the Pattern of Trade. *American Economic Review* 70, 950-959.

Mayer T, Ottaviano GIP. 2008. The Happy Few: The Internationalisation of European Firms. *Intereconomics* 43, 135-148.

Melitz MJ. 2003. The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. *Econometrica* 71, 1695-1725.

Melitz MJ, Ottaviano GIP. 2008. Market Size, Trade, and Productivity. *Review of Economic Studies* 75, 295-316.

Melitz MJ, Redding SJ. 2015. New Trade Models, New Welfare Implications. *American Economic Review* 105, 1105-1146.

Naito T. 2019. A Larger Country Sets a Lower Optimal Tariff. *Review of International Economics* 27, 643-665.

Nikkei Asia. 2019. US and China to fight for top GDP in 2060 while Japan dips to 5th. *Nikkei Asia*. https://asia.nikkei.com/Economy/US-and-China-to-fight-for-top-GDP-in-2060-while-Japan-dips-to-5th

Novy D. 2013. International Trade without CES: Estimating Translog Gravity. *Journal of International Economics* 89, 271-282.

Ossa R. 2011. A "New Trade" Theory of GATT/WTO Negotiations. *Journal of Political Economy* 119, 122-152.

Ossa R. 2014. Trade Wars and Trade Talks with Data. *American Economic Review* 104, 4104-4146.

Ossa R. 2016. Quantitative Models of Commercial Policy. *Handbook of Commercial Policy* Volume 1a, Elsevier: North Holland, 207-259.

Spearot AC. 2013. Variable Demand Elasticities and Trade Liberalization. *Journal of International Economics* 89, 26-41.