Statistical inference for harmonizable symmetric $¥alpha$—stable processes

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We consider the class of stationary harmonizable stable processes $X=¥{ X(t): t¥in ¥mathbb{R} ¥}$ defined by
$$X(t)=Re¥left(¥int_{¥mathbb{R}} e^{itx} M_{¥alpha}(dx) ¥right), ¥quad t¥in¥mathbb{R},$$
where $M_{¥alpha}$ is an isotropic complex symmetric $¥alpha$-stable (S$¥alpha$S) random measure with finite control measure and the index of stability $¥alpha¥in(0,2)$. In this talk, we propose non—parametric estimates for the density f of this measure which we call a spectral density. Since X is known to be non—ergodic, and we have only one realisation of X at our disposal, empirical means cannot be used in this context. However, to is known that X admits a LePage type series representation consisting of sine and cosine functions with random amplitudes and frequencies, which can be estimated consistently using the periodogram. The estimated frequencies can be used to get a kernel density estimate (KDE) of the spectral density f.

In the second part of our talk, we focus on the statistical inference for stationary-increment harmonizable stable processes $Y=¥{ Y(t): t¥in ¥mathbb{R} ¥}$ defined by
$$Y(t)=Re¥left(¥int_{¥mathbb{R}} (e^{itx}−1) ¥Psi (x) N_{¥alpha}(dx) ¥right), ¥quad t¥in¥mathbb{R},$$
where $N_{¥alpha}$ is an isotropic complex S$¥alpha$S random measure with Lebesgue control measure. This class contains real harmonizable fractional stable motions, which are a generalization of the harmonizable representation of fractional Brownian motions to the stable regime, when $¥Psi(x)=|x|−H−1/¥alpha, x¥in¥mathbb{R}$. We show that the convolution of Y with a suitable measure yields a real stationary harmonizable S$¥alpha$S process with finite control measure. Then the results of the first part of the talk apply directly. Combined with KDE, this allows us to construct consistent estimators for the index of stability $¥alpha$ as well as the kernel function $¥Psi$ in the integral representation of Y (up to a constant factor). For real harmonizable fractional stable motions consistent estimators for the index of stability $¥alpha$ and its Hurst parameter H are given, which are computed directly from the periodogram frequency estimates.