Abstract: Consider iid samples from a vector-valued Gaussian distribution are observed. When the covariance matrix of the Gaussian distribution has no structure, the MLE uniquely exists with probability one if and only if the sample size n is greater than or equal to the size of the covariance matrix. However, this is not the case where the covariance matrix has some structure and is described with a fewer number of parameters. We consider the case where the covariance matrix is the Kronecker product of two matrices (m1 x m1 and m2 x m2 matrices). We show that the existence and the uniqueness of the MLE are characterized by a rank of an m1 x m2 x n tensor. In particular, when one of m1, m2, or n is 2, the problem can be reduced by Kronecker's canonical form of two matrices. The tensor rank is given explicitly as the solution of an integer programming. The Groebner basis computation also gives the rank when m1, m2 and n are small.