Spatial and temporal modeling of heteroscedastic volatility behaviors in social science

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Abstract

This study is concerned with spatial and spatio-temporal extensions of time series volatility models. Volatility which is a variance or conditional variance in a model is one of the most important concepts in financial econometrics. The seminal work of Engle (1982) proposes autoregressive conditional heteroscedasticity (ARCH) models and the most important extension of the model is generalized ARCH (GARCH) models proposed by Bollerslev (1986). Univariate volatility models are generalized to multivariate cases in many ways for modeling the dynamic relationships between volatility processes of multiple assets returns. The curse of dimensionality becomes a major obstacle for generalization because there are $\frac{n(n+1)}{2}$ quantities in a conditional covariance matrix for a n-dimensional time series. We introduce not only geographical distances but also financial distances among components of multivariate series and apply spatial weight matrices into multivariate volatility models to overcome the difficulty. We propose spatial ARCH models, spatial GARCH models and spatial autoregressive moving average models with generalized autoregressive conditional heteroskedasticity processes as spatial and spatio-temporal extensions of volatility models. Parameters in the models are estiamted by the quasi-maximum likelihood estimation method and Asymptotic properties of these estimators are derived. Simulation studies and applications to land price data in Tokyo areas and Japanese and U.S. sock market data are conducted to demonstrate empirical properties of the models.

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