Correlation Persistence in Financial Markets: A Network Theory Approach

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Abstract

I propose a persuasion model to show the correlation of stock price can substantially vary over time in a general network economy in which investors update their own information following by the Bayesian rules and evolving in the social network effect. To describe the mutual interconnections and model the financial network, an empirical strategy, Granger-Causality network, is proposed based on the multivariate vector autoregression (MVAR) models. The empirical study is conducted by examining the unique dataset that consists of all brokers’ daily trading information for a decade in Taiwan. The empirical results show the density of brokers’ financial network can be positively correlated to the market realized volatility. Moreover, the density can also be viewed as an indicator to the systemic risk of market. The empirical evidence shed new light not only on providing an explanation to the phenomenon of correlation persistence but also measuring the systemic risk of the market from the perspective of network.

Key words: Financial Network; Realized Volatility; Long Memory; Systemic Risk
JEL classification: C3; G2

1. Introduction

Rational choice theory is widely used for modeling individual behavior in making economic decisions that assumes a person will be able to fully understand the consequences and take all the time they need for every choice they make. But, in reality, an individual’s rationality is limited by the information he has, the cognitive limitations of their minds, and the finite amount of time they have to make a decision. Instead, it is argued that most speculative traders in stock market do not have any source of private information regarding asset payoffs, instead, they seem to make trading decisions based on media accounts, tips from friend, advice from analysts and experts, etc. Therefore, an individual who anticipates a future change in his expected believes should incorporate a wide variety of settings such as persuasion (DeMarzo, Vayanos and Zwiebel (2003)). Since these speculators believe their information are superior than
others, they tend to persuade other investors once the communication was taken place.

This paper mainly studies the dynamics of volatility that have long been documented in the literature. Return volatility of individual stocks and markets varies over time and leads to a heavy-tailed unconditional return distribution (Bollerslev and Jubinski (1999); Lobato and Velasco (2000)). Furthermore, the phenomenon of long memory is observed only after applying certain transformations such as square or absolute value to the series. A common explanation for such time-varying volatility is lumpy diffusion of information into the market (Clark (1973); Epps and Epps (1976); Andersen (1996)). In this study, I propose a persuasion model to study information communication on the effect of market volatility. In this economy, individuals can exchange their opinions once word-of-mouth (Hong, Kubik, and Stein (2005)) communication occurs. Investors dynamically update their beliefs evolving in a way that weight between their subjective opinions and information of individuals whom they listen to relative to other individuals. For example, investors $i$ and $j$ each have different information at time $t$, $x_t^i$ and $x_t^j$. Suppose that investors $i$ and $j$ meet. Investor $j$ tries to “persuade” investor $i$ by revealing his information. If investor $i$ “listens to” investor $j$, the updating rule for $i$ is:

$$x_{t+1}^i = (1 - \lambda_i)x_t^i + \lambda_i x_t^j$$ \hspace{1cm} (1)

where $\lambda_i \in [0, 1)$ represents the parameter of social isolation. The weight depends on the magnitude of investor’s social isolation. If investor is highly social isolated, i.e., $\lambda_i \to 0$, there would be no information persuasion with his neighbors so that the believes in determining his invest decisions remains unchanged.

The equilibrium analysis suggests that the information communication structure can affect volatility dynamics in the market. And, the price volatility would vary substantially over time in a general network economy. To implement the empirical study, I collect the dataset which contains all daily trading information from November 6, 2001 to March 2, 2011 in Taiwan stock market, covering the period of financial crisis in 2007 with The sample records the information of trading stocks, order types (buy or sell), trading amounts, and trading prices for each broker. During the sample period, there are total 95 brokers and 1330 common stocks that have complete trading records over the 2314 trading days.

This study focuses on studying the brokers trading data instead of using the account level. It is because of preventing tracking those infrequent trading behaviors in stock market. Since the majority of investors are rarely trade, identifying their mutual interaction could easily become irrelevant. The brokers trading dataset can also prevent constructing the sparse trading matrix.
and raising the estimation issues. Most importantly, my analysis of the size of the brokers financial network would be more stable over time.

The brokers trading data can be viewed as the trading records of a group of investors. To mimic each brokers’ short- and long-term realized trading gains and losses, I adopt the method developed in Barber, Lee, Liu, and Odean (2009) but focus on brokers’ trades rather than on investor groups. A statistical methodology I proposed is to identify the possible linkages based on the property of the brokers’ portfolio holdings. To capture the information diffusion process by a directed network structure, I adopt the multivariate vector autoregressive (MVAR) models to empirically measure their mutual interconnectedness from a system-wide perspective. The financial networks I constructed are in the spirit of Granger-Causality according to the parameter estimations of MVAR(1) model in Section 3.

The empirical results indicate that when the brokers network density increases, the market realized volatility would increase in the following. It makes the dynamic network density can be a systemic risk indicator, i.e., a probe to the market status. The remainder of the paper is organized as follows: Section 2 introduces the model. Section 3 describes the data and methodology. The main empirical results are illustrated in Section 4. Section 5 concludes.

2. Model

Assumes that there are $N$ investors, enumerated by $i \in \mathcal{N} = \{1, 2, ..., N\}$ in a $T + 1$-period economy, $t = 0, ..., T$. Each investor maximizes his expected utility of terminal wealth, and has constant absolute risk aversion (CARA) preferences with homogeneous risk aversion coefficient, $\gamma$,

$$U_i = E[-e^{-\gamma W_i, T}], \tag{2}$$

where $W_{i,T}$ is investor $i$’s terminal wealth.

The net supply (liquidity) of the risky asset traded in the economy is $L$. The terminal value, $v$, of the risky asset is assumed as

$$v = \bar{v} + \eta. \tag{3}$$

The first component of $v$ is a weighted sum of information coming from $H$ pub-
lic information sources, like financial statements, company reports, macroeconomic announcements, etc that are available publicly known by all investors. It is denoted by 
\[ \bar{v} = \sum_{h=1}^{H} w_h v_h. \] (4)

The second component, \( \eta \), is independent of the \( v_h \)'s and follows \( N(0, \sigma^2_v) \) which is unobservable. Therefore, the value of the risky asset is normally distributed with mean \( \bar{v} \) and variance \( \sigma^2_v \). The precision is denoted as \( \tau_v = \sigma_v^{-2} \).

At time 0, each investor receive a noise signal about the asset’s value from the \( H \) sources,
\[ s^0_i = \sum_{h=1}^{H} w_{i,h} v_h + \epsilon_i \] (5)
where \( w_{i,h} \) is denoted as investor \( i \)'s subjective weight on information source \( h \) and \( \epsilon_i \sim N(0, \sigma^2) \) is independent across information sources \( v_h \) and investors. Investors use their initial signals to form an estimate of the asset’s payoff. At time \( T \), the true value of the asset, \( v \), is revealed. The precision is denoted as \( \tau = \sigma^{-2} \).

In this study, I assumed that investors are connected in a simple directed graph\(^3\) and updated their information via a Bayesian rule which are different from the setting of Walden (2013).

**Investors’ Network**

Investors are assumed to be connected in a network, represented by a graph \( G = (N, E) \). The relation \( E \subset N \times N \) describes which investors are connected in the network. Investors gain information from other investors about the risky payoff via their connection within a network that can be modeled by vertices (nodes) representing the investors and the directed edges (links) representing the direction of learning. A convenient representation of the network is by the *adjacency matrix* \( A \in \{0, 1\}^{N \times N} \) with
\[ A_{ij} = \begin{cases} 
1 & \text{if investor } j \text{ is directly linked to } i, \\
0 & \text{otherwise.} 
\end{cases} \]
Here, I assume that investors are self-connected. That is, \( A_{ii} = 1, \forall i \). In the my model, investor gain information from other investors via persuasion. Whom an investor gains from is determined by a network which can be represented as following.

\(^3\) A graph is “simple” if multiple edges between the same pair of vertices are forbidden. A graph is called “directed” if edges exhibit inherent direction, implying every relationship presented is asymmetric.
Definition 1  For a given network $G \in \mathcal{G}$,

$$D(i) \equiv \{k \in \{1, \ldots, N\} \setminus \{i\} : k \to i\}$$  \hspace{1cm} (6)

is defined to capture investor $i$’s information structure.

Note that the vector $\mathbf{D} = (D(1), \ldots, D(N))'$ fully captures the interaction among investors impose by the network $G \in \mathcal{G}$. I refer $k \in D(i)$ as an information source for investor $i$. In addition, an investor is always an information source for himself.

**Information Updating Rules**

The graph $\mathcal{G}$ determines how investors share information with each other. Specifically, at time $t+1$, investor $i$ shares all signals he had received up until to time $t$ with all his neighbors. Let $\mathcal{S}^t_i$ denote the information set that investor $i$ has received up until $t$, either directly or via his network. The dynamics of $s^t_i \in \mathcal{S}^t_i$ are given the recursive formula

$$s^t_i = (1 - \lambda_i)s^{t-1}_i + \lambda_i \sum_{j \in D(i)} T_{ij} s^t_j \hspace{1cm} (7)$$

where $\lambda_i \in [0, 1)$ is a parameter that reflects the extent to which social network of friends is relevant to investor. It is denoted $\mathbf{\lambda} = (\lambda_1, \ldots, \lambda_N)'$. The term $(1 - \lambda_i)$ is the weight that investor $i$ places on his own public information at time $t - 1$. Here, the parameter $\lambda_i$ can be intuitively denoted as $i$’s degree of social interaction. Investor $i$ is said to be socially isolated if $i$’s time-$t$ information is not influenced by the social network of friends. Social isolation shuts down the social channel through which the provision of information affect $i$’s belief. $T_{ij}$ denotes the probability that investor $i$ listen to investor $j$ in $i$’s network, where $T_{ij} > 0$ if $j \in D(i)$ and $T_{ij} = 0$ otherwise. And

$$\sum_{j \in D(i)} T_{ij} = 1. \hspace{1cm} (8)$$

Definition 2  In most cases, I assume the investors are uniform listening, that is,

$$T_{ij} = \frac{1}{\#D(i)} \quad \text{for } j \in D(i). \hspace{1cm} (9)$$

**An Illustration**

Consider a network $G$ with $N = 5$ investors as shown in the directed graph of Figure 1. The information structures of investors are $D(1) = \{2, 3\}, D(2) = \emptyset, D(3) = \{1, 2, 5\}, D(4) = \{1, 2, 3, 5\}, D(5) = \{1\}$. The uniform listening
Fig. 1. Illustration of a social network (N=5)

matrix for Figure 1 is:

$$
T = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\frac{1}{3} & \frac{1}{3} & 0 & 0 \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\
1 & 0 & 0 & 0 
\end{bmatrix}
$$

(10)

3. Data and Methodology

**Taiwan Stock Market**

My dataset comes from the Taiwan stock market, which is the world’s 12th largest financial market. During the sample period, the Taiwan Stock Exchange (TSE) opens from 9:00 AM to 1:30 PM. Buy and sell orders interact to determine the executed price subject to applicable auto-matching rules every 90 seconds. Orders are executed in strict price and time priority. A daily price limit of 7% holds in each direction and a trade-by-trade intraday price limit of two ticks from the previous trade price. The commission fee of TSE is 0.1425% of the trading value and a transaction tax on stock sales of 0.3%. Capital gains are not taxed, whereas cash dividends are taxed at ordinary income tax rates for domestic investors and at 20% for foreign investors. Corporate income is taxed at a maximum rate of 25%, whereas personal income is taxed at a maximum rate of 40%. The accumulated investor account numbers at securities companies is approximately 16 million in 2011, which is almost 76% of the total population.

**Brokers Daily Trading Information**

I collect all daily trading information of all brokers in Taiwan over the period from November 6, 2001 to March 2, 2011 to carry out my empirical study.
I chose the brokers’ dataset as opposed to account-level data because tracking those infrequent investors’ trading behavior and identifying their mutual interaction could easily make noise and become irrelevant to study investors’ trading behavior via networks. Most importantly, it would also prevent to construct the sparse trading matrix. The dataset includes the stocks, order types (buy or sell), trading amounts, and trading prices that are traded by each broker every day. Therefore, I can investigate the trading behavior of a large amount of investors within a financial network in a market through the trading information of brokers. Further, these brokers can also be categorized into five types as banking (13 brokers), bills finance corporations (1 broker), specialized brokerage firms (32 brokers), integrated securities firms (31 brokers), and foreign financial institutions (18 brokers). During the sample period, there are total 95 brokers and 1330 stocks have complete trading information. The average the trading amount of these brokers is approximately 97% of the daily whole market’s total trading amount.

Short- and Long-term Trading Gains and Losses

The first step of my empirical work is to construct each broker’s daily buy and sell portfolios to mimic a group of investors’ daily stocks purchase and sale over a period of time. I use the method similar to Barber, Lee, Liu, and Odean (2009) but focus on brokers’ trades in the short- and long-term trading period to evaluate a group of investors’ trading gains and losses in each broker, which are denoted by two vectors \( G_t = (g_{1,t}, \ldots, g_{n,t})' \) and \( L_t = (\ell_{1,t}, \ldots, \ell_{n,t})' \) at time \( t \) respectively. Take the trading of broker Yuanta Securities on March 29, 2002 as an example, the mimicking portfolios can be constructed as follows: if Yuanta Securities buys 900 shares of HTC and sells 700 shares, I add 200 shares of HTC on the buy portfolio, whereas no HTC shares are added to the sell portfolio on that day. The purchase price will be recorded as the difference between the total value of buys and the total value of sells divided by the net shares. I consider the shares being included in the portfolios for a fixed horizon, \( z \), with the window length of 5 (short-term) and 20 (long-term) trading days.

The volume-weighted (VW) realized trading gains denoted by \( g_{z,i,t} \) are therefore calculated as:

\[
 g_{z,i,t} = \frac{\sum_{j=1}^{n_i} \#\text{net shares of stock } j \text{ purchased}}{\#\text{net total shares purchased by broker } i} \times r_{j,t+1},
\]

where \( r_{j,t+1} \) is the one-day return of the stock \( j \) after traded at time \( t \) and \( n_i \) denotes the number of stocks that the broker \( i \) holds considering \( z \)-days holding period. An analogous calculation occurs for the VW realized trading losses, \( \ell_{z,i,t} \), which is defined as

\[
 \ell_{z,i,t} = \sum_{j=1}^{n_i} \frac{\#\text{net shares of stock } j \text{ sold}}{\#\text{net total shares sold by broker } i} \times r_{j,t+1}.
\]
The net trading profit, \( p_{i,t} \), is defined as the difference of \( g_{i,t} \) and \( \ell_{i,t} \), that is,

\[
p_{i,t} = g_{i,t} - \ell_{i,t}.
\]  

(13)

In the following study, I use the net trading profit time series, \( p_{t} \), for each broker to fit the multivariate time series model as in the following Section and construct the brokers financial network to describe the mutual interconnectedness.

**Multivariate Vector Autoregression (MVAR)**

To construct the financial networks of 95 brokers, MVAR models can be used to empirically measure their mutual interconnectedness from a system-wide perspective. The network structure could therefore be inferred by using the Granger-Causality test for investigating whether one time series can provide forecasting power to others and therefore to determine the interconnectedness among these brokers. The causality can therefore reflect both statistic correlations and economic connections among the multivariate time series of a group of investors' net trading profit.

MVAR model is to consider \( Y_{t} = (y_{1t}, ..., y_{nt})^{\prime} \), which denotes a weakly stationary multivariate time series of dimension \( n \) defined on a probability space \( (\Omega, \mathcal{F}, \mathbb{P}) \). A \( p \)-th order vector autoregressive can be represented in a simple form as

\[
Y_{t} = c + \sum_{k=1}^{p} \Phi_{k}Y_{t-k} + \epsilon_{t},
\]  

(14)

where \( c \) denotes an \((n \times 1)\) vector of constants and \( \Phi_{k} \) denotes an \((n \times n)\) matrix of autoregressive coefficients for \( k = 1, 2, ..., p \). The \( \epsilon_{t} \) is a vector with \( \Sigma \) symmetric definite matrix. The stationary condition is satisfied if all roots of \(|\Phi(Z)| = 0|\) lie outside the unit circle.

For a large VAR model \((n > 2)\), the Granger-Causality test can be used to test whether one variable is influenced only by itself and not by other variables in the model system. \( Y_{t} \) can be arranged and partitioned in subgroups \( Y_{1t} \) and \( Y_{2t} \) with dimensions \( n_{1} \) and \( n_{2} \) \((n = n_{1} + n_{2})\), respectively. VAR model can be represented as a matrix form:

\[
\begin{bmatrix}
\Phi_{11}(\beta) & \Phi_{12}(\beta) \\
\Phi_{21}(\beta) & \Phi_{22}(\beta)
\end{bmatrix}
\begin{bmatrix}
Y_{1t} \\
Y_{2t}
\end{bmatrix} =
\begin{bmatrix}
C_{1} \\
C_{2}
\end{bmatrix} +
\begin{bmatrix}
\Sigma_{1} \\
\Sigma_{2}
\end{bmatrix}.
\]  

(15)

The Wald statistic to test \( H_{0} : C\beta = c \), where \( C \) is a \( s \times (n^{2}p + n) \) matrix of rank \( s \) and \( c \) is an \( s \)-dimensional vector can be obtained from

\[
\sqrt{T}(C\hat{\beta} - c)[C(\hat{\Gamma}^{-1} \otimes \Sigma)C^{\prime}]^{-1}(C\hat{\beta} - c) \overset{d}{\rightarrow} \chi^{2}(s).
\]  

(16)
In this study, I set the significance level of $\alpha = 0.01$.

**Granger-Causality Network**

Based on the MVAR model, the financial network I construct follows the similar concept of Grange causality as in Eichler (2007). I define a network $G = (V, E)$, where $V$ is a set of elements called nodes (95 brokers in this study) and $E$ is a set of directed edges which belong to the class

$$\{ j \rightarrow i | i, j \in V, i \neq j \} \notin E \iff \Phi_{ij}(k) = 0 \forall k,$$

(17)

where $\Phi$ is the autoregressive coefficients matrix in Equation (14). A network can be represented by an adjacency matrix $A \in \{0, 1\}^{n \times n}$, with

$$A_{ij} = \begin{cases} 1 & \text{if broker } j \text{ is directly linked to } i, \\ 0 & \text{otherwise}. \end{cases}$$

(18)

Moreover, I assume that a broker is connected with himself, that is, $A_{ii} = 1$ for all $i$. This class represents the directed edges corresponding to direct causal relations between the components of $Y_t$ which can be identified by Granger-Causality test as shown in Equation (16). For simplicity, I consider the case of $k = 1$ and use a 180-day rolling window to estimate $\Phi$ in Equation (14).

### 4. Empirical Results

All economic networks are heterogeneous with respect to both their agents and interaction strength which can also strongly vary in time. Previous studies of efficient and equilibrium networks assumed homogeneity. The most oblivious stylized fact of realized volatility has shown long range dependence (Andersen, Bollerslev, Diebold and Labys (2003)), which means the autocorrelation of the square and absolute returns shows very strong persistence that lasts for extended periods. Schennach (2013) shown that long memory can arise in a simple linear homogeneous economic subsystems with a short memory are interconnected to form a network.

**Realized Volatility (RV) and Density**

For calculating the market realized volatility, I follow the Andersen, Bollerslev, Diebold and Labys (2003) by using the one-minute data of Taiwan Capitalization Weighted Stock Index (TWSI) over the period from October 25, 2002 to March 2, 2011. The time series plot of TWSI RV and short-term density is in Figure 1. The time series plot of TWSI RV and long-term density is in Figure 2. It can find that when the density of brokers financial network becomes higher, the following market realized volatility almost rises as well.

**Regression Analysis**
Ozsoylev and Walden (2011) predicts that asset volatility increases as the network centralization increases. To test the prediction, we implement the regression of

\[ RV_t = \alpha z + \beta z DENS_t + \epsilon_t, \tag{19} \]

where \( RV \) is the market realized volatility, \( DENS \) is the density of brokers financial network in different time of \( z \). The main regression results are shown in Table 1.

From Table 1, it can find that the slopes are all very significant and positive. One standard deviation increase of brokers financial network density leads to 0.226% increase in market realized volatility for \( z = 5 \). For \( z = 20 \), 0.193% increase in market realized volatility.
Table 1
Regression of Realized Volatility on Network Density ($\times 10^{-3}$)

<table>
<thead>
<tr>
<th>$z$</th>
<th>$\alpha^z$</th>
<th>$\beta^z$</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-0.0683</td>
<td>2.26</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>(0.1229)</td>
<td>(0.9040)</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>-0.0203</td>
<td>1.93</td>
<td>***</td>
</tr>
<tr>
<td></td>
<td>(0.0991)</td>
<td>(0.7390)</td>
<td></td>
</tr>
</tbody>
</table>

Density and Systemic Risk

Moreover, I also plot the figures to illustrate the relationship between price and short- and long-term density in Figure 4 and Figure 5. From the both figures, we can find the time series plots of densities like reflections to the price pattern. When the density is at the relative low status, the price often reaches at the peak. The empirical results shows that the density can be an indicator to the systemic risk for the market.

Fig. 4. TWSI and Short-term Density

5. Conclusions

When explaining and understanding the social reality in which we exist, if the complexity of society rises, precise categorical statements will lose meaning and meaningful statements will cease to be precise and categorical. Fortunately, with the development of complex network analysis, networks become a natural tool to help obtain more fundamental insights in understanding both social and economic phenomena, such as contagion, globalization, information diffusion, neighborhood effects, and social learning.

Although networks have been appeared in almost every aspect of science
and technology, extracting insights from networks of trading behavior within a market remains a less undeveloped area. This study primarily investigates the relationship of the density of brokers network and the realized volatility of market. My contribution to the network literature is to use a persuasion model to explain the correlation persistence in the market and provide a multivariate time series model to measure the mutual interconnectedness among a group of investors. The empirical results indicate that network density is positively correlated with the realized volatility of market. When the network density increases, the following realized volatility of market also becomes large as well.

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