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ABSTRACT

The authors propose an integrated model that links consumers' budget allocation decisions at two different stages. The first stage concerns consumers' decisions to allocate budget among product categories, and the second stage concerns allocating the assigned budget among brands. In each stage, the authors assume that consumers behave rationally by determining the optimal demand for the category as well as for the brand to maximize utility. The authors use scanner data from purchases of instant coffee, coffee cream, tea, and instant curry to investigate the role of consumer preference and price in a hierarchical budget allocation problem. They also discuss some implications of anticipating a change in quantity demanded because of a change in expenditure and price. Researchers can use the model to examine the marginal effects of expenditure not only on category but also on brand purchase quantity.

Keywords: expenditure allocation, multi-stage decisions, reference price, heterogeneity

Consumers' decision making is often considered a process of resource allocation among two or more options. For example, a decision hierarchy often begins with a problem of allocating time resources between leisure and labor, followed by a problem of allocating disposable income between consumption and savings. It continues with the allocation of the consumption budget among product categories. Ultimately, consumers make a decision to allocate the budget assigned to each category among brands. Undoubtedly, the outcome of a decision made at an earlier stage will influence decisions at subsequent stages. We argue that a better understanding of how the decisions at different stages relate with each other will provide useful insights for a marketer.

Therefore, we propose a framework for analyzing consumer decision making in multiple stages. Considering data availability, we focus our study on two stages: one in which the budget is allocated among product categories and another in which the budget assigned to each category is allocated among brands. In particular, we propose an integrated model that links consumer decisions in both stages. In the present research, we aim to determine how a change in consumer expenditure would affect demand for product categories and brands. We also address how a change in the price of a brand could influence demand allocation in both stages.

Modeling consumer demand allocation has been an area of interest for several decades. Researchers have developed an ample number of models to accommodate factors that are thought to be influential in consumer decision-making processes. A so-called ideal model should be consistent with economic theory and facilitate easy estimation and data fitting (Barten 1993). In general, it is difficult to generate a model that satisfies all of these conditions, as there is often a tradeoff among them. Theoretically consistent models are usually associated with computational difficulty, and the resulting demand functions are often unappealing. On the other hand, simpler models are usually easy to estimate, yet the derived demand function often falls short of theoretical consistency.

One stream of research begins with the specification of direct utility function and derives Marshallian demand function from the first-order condition for utility maximization (Stone 1954). Another approach utilizes a specified indirect utility function and applies Roy's identity to obtain estimable demand function (Christensen, Jorgenson, and Law 1975). The third stream, which induces demand systems that possess useful properties, starts from a specified expenditure function and applies Shephard's lemma to arrive at a demand function. The almost ideal demand system (AIDS) of Deaton and Muellbauer (1980) is the best-known example of this type of specification. Recently, Kim, Allenby, and Rossi (2002) employed a Bayesian inference approach that, to some extent, solves the computational problem arising in the estimation of a demand function derived from a direct utility function.

Most of the models proposed thus far have focused on one of the stages in the consumer decision-making process. One of the main objectives of these studies has been to investigate the role of price and income in the distribution of budget share among product categories. While single-stage frameworks have contributed a great deal to understanding consumer decision making, we expect to obtain richer findings by extending the scope of analysis to involve consumer decisions at several stages. For example, a marketer might want to know how a change in the proportion of category expenditures stemming from an increase in income would affect brand sales.

Marketing researchers have put considerable effort into analyzing consumer decisions at multiple stages simultaneously. For example, Bucklin and Lattin (1991) proposed a nested logit model of category purchase and brand choice to investigate planned and unplanned buying behavior. A model by Chib, Seetharaman, and Strijnev (2004) integrated category purchase and brand choice in a framework that involves no-purchase data. Models proposed by Mehta (2007) and Song and Chintagunta (2007) deal with consumer decisions of category purchase and brand choice. The latter also incorporated purchase quantity decisions into the model. However, all of these models assume that consumers purchase only one brand within a category (i.e., the corner solution) and, as such, are inapplicable in cases where consumers purchase multiple brands simultaneously.

We summarize some related studies in Table 1. References 1 through 3 are econometric models of demand systems dealing with budget allocation in one stage. The demand model of Hauser and Urban (1986) is a one-stage model that accommodates some findings in behavioral research. References 6 through 10 are multistage models, yet their focus is on discrete consumer decisions, such as category purchase and brand choice decision. These types of models have some limitations in explaining the effect of an expenditure change on category and brand spending. Lee et al. (2013) provided an inherently two-stage model that deals with the consumer problem of demand allocation. However, the model carries out allocation according to a certain sequence. That is, consumers first allocate expenditures between a category and the remainder, and then allocate the remaining amount between the next category and the remainder, and so on. While this modeling approach is useful for examining competition among categories, we argue that it is unsuitable for analyzing the marginal effect of expenditures in multiple stages. Our modeling approach contributes to the literature in the sense that it can provide a more comprehensive understanding of consumer decision making, as it integrates expenditure allocation problems at both the category and brand levels.

[Insert Table 1 about here.]

We have organized the remainder of this article as follows. The next section describes the model for budget allocation in two stages. We then introduce the data used for examining the proposed model. Subsequently, we provide the estimation results and discuss the implications of our model for the anticipation of demand changes. Finally, we offer our concluding remarks.

MODEL

Expenditure Allocation among Categories

Suppose that a consumer is facing a problem of allocating budget among m product categories and the outside good at purchase occasion t. We assume that the overall utility function of this consumer will increase with category expenditures at a decreasing rate, which is given by:

$$U(E_{1t}, \dots, E_{mt}, z_t) = \sum_{i=1}^{m} \psi_{it} \ln(E_{it} + 1) + \ln(z_t), \tag{1}$$

where E_{it} and z_t denote the expenditure allocated to category i and the outside good, respectively. We can represent category expenditure as $E_{it} = \tilde{p}_{it} x_{it}$, where $\tilde{p}_{it}$ is the unit price and x_{it} is the purchase quantity of category i. Here, we assume that product categories are infinitely divisible, and, therefore, x_{it} refers to the volume of category i purchased by a consumer at the time t[1].

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[1] In our empirical analysis, we define x_{it} as the purchased volume measured in grams. For most of packaged goods, it would be more realistic to consider x_{it} a discrete quantity, as in Lee and Allenby (2014). However, we treat it as a continuous variable in order to get a tractable link between the decisions in different stages.

Parameter ψ_{it} refers to category-specific attractiveness, whose value can vary across time. For U to be a valid utility function, it is necessary to impose a restriction such that $\psi_{it} \geq 0$. Without loss of generality, the attractiveness of the outside good is fixed to one. To accommodate market competition, we parameterize ψ_{it} as a function of baseline category preference and inventory effects.

$$\psi_{it} = \exp\left(\alpha_i + \sum_{j=1}^{m} \beta_{ji} s_{jt}\right) \quad (2)$$

Here, α_i represents baseline preference toward category i. s_{jt} denotes the inventory level of category j. Parameter β_{ji} captures the inventory effect of category j on category i's attractiveness. If β_{ji} takes a positive (negative) value, then we interpret that category j is a complement (substitute) for category i (Lee, Kim, and Allenby 2013). We let $\beta_{ij} \neq \beta_{ji}$ to allow for asymmetric substitution and complementary effects. The specification of inventory level is given by:

$$s_{jt} = \lambda_j^{dt}\left(s_{j,t-1} + 0.01x_{j,t-1}\right) \quad (3)$$

where dt denotes the number of days elapsed since the last purchase (Ailawadi et al. 2007). The initial value of inventory level s_{j0} is set to a hundredth of the average purchase quantity. λ_j is a parameter indicating the consumer consumption rate for category j. We fix $\lambda_j = 0.8$ to reduce the number of parameters. In this setting, consumers' inventory depletes in about two weeks, which is approximately the same as the average interpurchase time.

In the first stage, the consumer problem lies in determining how to allocate budget among product categories to maximize utility. We denote the total expenditure spent by a consumer at purchase occasion t by E_t. This utility maximization problem can be written as follows:

$$\max_{E_{1t},\dots,E_{mt},z_t} U(E_{1t},\dots,E_{mt},z_t) = \sum_{i=1}^{m} \psi_{it} \ln(E_{it}+1) + \ln(z_t)$$
$$\text{s.t.} \quad \sum_{i=1}^{m} E_{it} + z_t \le E_t \tag{4}$$

Defining the Lagrange function by:

$$V(E_{1t},\dots,E_{mt},z_t) = U(E_{1t},\dots,E_{mt},z_t) - \lambda\left(\sum_{i=1}^{m} E_{it} + z_t - E_t\right), \tag{5}$$

we obtain the Kuhn–Tucker condition for this utility maximization problem as follows:

$$\begin{aligned} \frac{\partial V}{\partial E_{it}} &= \frac{\psi_{it}}{E_{it}+1} - \lambda = 0, \ \text{if} \ E_{it} > 0 \\ \frac{\partial V}{\partial E_{it}} &= \frac{\psi_{it}}{E_{it}+1} - \lambda < 0, \ \text{if} \ E_{it} = 0 \\ \frac{\partial V}{\partial z_t} &= \frac{1}{z_t} - \lambda = 0 \end{aligned} \tag{6}$$

We can write the solutions for this problem as $E_{it}^* = \tilde{p}_{it} x_{it}^*$, where x_{it}^* is the optimum purchase quantity. In addition, $\tilde{p}_{it}$ is not the actual price but the unit price level perceived by consumers. While we can observe x_{it}^* from data, we cannot know the actual value of E_{it}^* because in general $\tilde{p}_{it}$ is unobservable. For this reason, we must determine how to appropriately assign the value of $\tilde{p}_{it}$. It is natural to posit that consumers form their beliefs about category unit price based on the price of various brands within the category. Accordingly, consumers use clues about the actual price of various brands encountered during a past or present shopping trip as a reference price representing the category. In particular, we use some specifications of category reference price and choose the one that produces the best fit.

Expenditure Allocation within Categories

In the second stage, consumers move on to allocating budget among the brands within each category. Conditional on the optimum category expenditure E_{it}^*, and brand prices $\{p_{ikt}\}$, this allocation problem reduces to a problem of determining which and how much of the brands to purchase to maximize category level utility, which is defined as:

$$U_i\left(y_{i1t}, \dots, y_{iK_it}\right) = \sum_{k=1}^{K_i} \frac{\phi_{ik}}{\gamma_{ik}} \ln(\gamma_{ik} y_{ikt} + 1)\,. \tag{7}$$

Here, y_{ikt} denotes the purchase quantity[2] of brand k in category i. ϕ_{ik} and γ_{ik} stand for the brand attractiveness and satiation parameter, respectively. In contrast to Equation 1, we assume brand attractiveness is invariant with respect to purchase occasion. Instead, we include γ_{ik} to accommodate consumers' preference for variety, as in Hasegawa, Terui, and Allenby (2012).

Remember that the budget for each category that maximizes overall utility U is $E_{it}^* = \tilde{p}_{it} x_{it}^*$. Consumers decide which and how much of the brands in category i to purchase as long as the total amount spent does not exceed $\tilde{p}_{it} x_{it}^*$. On the other hand, because x_{it}^* is the optimum category purchase quantity, the quantity of brands purchased must add up to x_{it}^*. This implies that consumers have to maximize U_i subject to budget and quantity constraints. This type of consumer problem is given as follows:

$$\begin{gathered}\max_{y_{i1t},\dots,y_{iK_it}} U_i\left(y_{i1t}, \dots, y_{iK_it}\right) = \sum_k \frac{\phi_{ik}}{\gamma_{ik}} \ln(\gamma_{ik} y_{ikt} + 1) \\ \text{s.t.} \quad \sum_k p_{ikt} y_{ikt} \le \tilde{p}_{it} x_{it}^* \quad \text{and} \quad \sum_k y_{ikt} = x_{it}^* \end{gathered} \tag{8}$$

In fact, utility maximization in Equation 8 can be subject to an awkward problem whereby the solution could not exist. This occurs because in most cases, budget and

[2] In line with the definition of x_{it} described previously, we define y_{ikt} as the volume of brand k purchased by a consumer at the time t.

quantity constraints cannot be simultaneously satisfied. Figure 1 illustrates this problem when there are two brands in category i. Suppose that the price of brand 2 is higher than the price of brand 1 and that the reference price lies somewhere in between. The solid and dashed lines in the figure represent quantity and budget constraints, respectively. If the indifference curve touches the quantity line at the point where the budget constraint coincides with the quantity constraint (point i), a consumer can choose the product bundle at this point because it satisfies both constraints. However, if it touches the quantity line at any point in area ii, this bundle would be unaffordable, as the amount needed to purchase it exceeds the budget constraint. On the other hand, if the indifference curve touches the quantity line at any point in area iii, the bundle is attainable; however, then the consumer would have some amount left over. The last two cases are the problems that must be handled. The problem in the second case is clear because the solution does not exist. The third case must be excluded because we assume that budget constraint is binding, provided that the marginal utility of the outside good is greater than zero.

[Insert Figure 1 about here.]

We assume that consumers will make an adjustment if the optimum bundle does not fall on the point where the budget line intercepts the quantity line. When the problem depicted in area ii arises, a consumer will draw from the proportion allocated to the outside good as much as $\sum_k p_{ikt}y_{ikt} - \tilde{p}_{it}x_{it}^*$ to make any bundle in this area affordable. Consequently, the consumption of the outside good decreases, and the utility obtained from consuming it becomes smaller. When consumers have more than enough money to buy any bundle in area iii, the amount left over will be allocated to the outside good. In this case, the budget for the outside good increases, and, therefore, consumers gain additional utility.

Here, we provide a formal representation of this budget adjustment. Let $c_{it} = \sum_k p_{ikt} y_{ikt} - \tilde{p}_{it} x_{it}^*$ and $z_{it} = |c_{it}|$. We define the two indicator variables I_ρ and I_ω as follows:

$$I_\rho = \begin{cases} 1 & \text{if } c_{it} > 0 \\ 0 & \text{if } c_{it} \leq 0 \end{cases}, \quad I_\omega = \begin{cases} 1 & \text{if } c_{it} < 0 \\ 0 & \text{if } c_{it} \geq 0 \end{cases} \tag{9}$$

Then, after implementing budget adjustment, a consumer utility maximization problem becomes:

$$\max_{y_{i1t}, \dots, y_{iK_i t}} U_i^*(y_{i1t}, \dots, y_{iK_i t}) = \sum_k \frac{\phi_{ik}}{\gamma_{ik}} \ln(\gamma_{ik} y_{ikt} + 1) + (I_\omega \omega_i - I_\rho \rho_i) \ln(z_{it} + 1)$$
$$\text{s.t.} \sum_k y_{ikt} = x_{it}^* \tag{10}$$

where U_i^* is the adjusted category utility function, and ω_i and ρ_i are parameters that capture the sensitivity of consumer utility to an increase or a reduction in the outside good expenditure. For this reason, we call them the gain and loss parameters.

The utility maximization in Equation 8 is similar to the problem of Satomura, Kim, and Allenby (2011), which imposes both budget and quantity constraints. However, in contrast to their problem, the quantity x_{it}^* is not the maximum quantity but the optimum quantity. This implies that the equality in quantity constraint must hold. Therefore, it is impossible to introduce a so-called outside good into the quantity constraint and to define a Langrage function that includes this term. The idea behind the budget adjustment assumption is to embed the budget constraint in the (adjusted) utility function and construct a utility maximization problem subject to only quantity constraint.

Defining the Lagrange function by:

$$V_i\left(y_{i1t},\dots,y_{iK_it}\right)=U_i^*\left(y_{i1t},\dots,y_{iK_it}\right)-\lambda\left(\sum_k y_{ikt}-x_{it}^*\right), \tag{11}$$

we determine the Kuhn–Tucker condition for Equation 10 as follows:

$$\begin{aligned}\frac{\partial V_i}{\partial y_{ikt}}&=\frac{\phi_{ik}}{(\gamma_{ik}y_{ikt}+1)}+\frac{(I_\omega\omega_i-I_\rho\rho_i)p_{ikt}}{(z_{it}+1)}-\lambda=0,\quad \text{if } y_{ikt}>0\\ \frac{\partial V_i}{\partial y_{ikt}}&=\frac{\phi_{ik}}{(\gamma_{ik}y_{ikt}+1)}+\frac{(I_\omega\omega_i-I_\rho\rho_i)p_{ikt}}{(z_{it}+1)}-\lambda<0,\quad \text{if } y_{ikt}=0\end{aligned} \tag{12}$$

For the budget adjustment assumption to hold, it is necessary that $U_i(\boldsymbol{y}'_{it})<U_i^*(\boldsymbol{y}''_{it})$, where $\boldsymbol{y}'_{it}=(y'_{i1t},\dots,y'_{iKt})$ and $\boldsymbol{y}''_{it}=(y''_{i1t},\dots,y''_{iKt})$ are the optimum bundles of the respective utility function. As already mentioned, consumers adjust their budget if either $\sum_k p_{ikt}y''_{ikt}>E_{it}^*$ or $\sum_k p_{ikt}y''_{ikt}<E_{it}^*$. Thus, we can formally express the condition for budget adjustment by:[3]

$$\begin{cases}\displaystyle\sum_k\frac{\phi_{ik}}{\gamma_{ik}}\{\ln(\gamma_{ik}y''_{ikt}+1)-\ln(\gamma_{ik}y'_{ikt}+1)\}>\rho_i\ln(z_{it}+1), & \text{if } \displaystyle\sum_k p_{ikt}y''_{ikt}>E_{it}^*\\ \displaystyle\sum_k\frac{\phi_{ik}}{\gamma_{ik}}\{\ln(\gamma_{ik}y'_{ikt}+1)-\ln(\gamma_{ik}y''_{ikt}+1)\}<\omega_i\ln(z_{it}+1), & \text{if } \displaystyle\sum_k p_{ikt}y''_{ikt}<E_{it}^*\end{cases} \tag{13}$$

The first inequality in Equation 13 implies that when the budget assigned to category i is not enough to afford the bundle $\boldsymbol{y}''_{it}$, consumers will draw some amount from the budget allocated to the outside good so as to make $\boldsymbol{y}''_{it}$ affordable if the difference between the utility of consuming $\boldsymbol{y}'_{it}$ and $\boldsymbol{y}''_{it}$ is greater than the loss of giving up some amount of the outside good.[4] The second inequality says that even when $\boldsymbol{y}'_{it}$ (which is greater than $\boldsymbol{y}''_{it}$) is affordable, consumers would instead prefer $\boldsymbol{y}''_{it}$ if the difference between the utility of consuming $\boldsymbol{y}'_{it}$ and $\boldsymbol{y}''_{it}$ is less than the gain from

[3] Although we assume that consumers implement budget adjustment, we do not restrict the parameters on the space bounded by Equation 13. However, we examine the plausibility of this assumption by using the parameter estimates and describe the analysis later under Discussion.

[4] Provided that the indifference curves are convex to the origin, we have $U_i(\boldsymbol{y}''_{it})>U_i(\boldsymbol{y}'_{it})$ if $\sum_k p_{ikt}y''_{ikt}>\tilde{p}_{it}x_{it}^*$ because $\boldsymbol{y}''_{it}>\boldsymbol{y}'_{it}$.

saving as much money as $E_{it}^{*} - \sum_k p_{ikt} y_{ikt}''$ to get more of the outside good.

Statistical Model

In this section, we aim to statistically represent the proposed model. We assume that uncertainty stems from the misspecification of the utility function. That is, the utility function specified in Equation 1 may be different from the actual one. To accommodate this type of error, we employ the approach proposed by Kim, Allenby, and Rossi (2002). They represented actual marginal utility as a function of model marginal utility and error term. We let u_{it}^{*} be the actual and $u_{it} = \partial U / \partial x_{it}$ be the model marginal utility. The relation between them is given by:

$$\ln(u_{it}^{*}) = \ln(u_{it}) + \varepsilon_{it}, \;\; \varepsilon_{it} \sim N(0,1) \tag{14}$$

where ε_{it} represents the specification error which is independent across categories and purchase occasions. Considering that budget constraint is binding and introducing the error term into Equation 6, we obtain the link between category expenditures and error term:

$$\begin{aligned} \varepsilon_{it} &= \ln\left(\frac{E_{it}+1}{\psi_{it}}\right) - \ln\left(E_t - \sum_{j=1}^{m} E_{jt}\right), \quad \text{if} \;\; E_{it} > 0 \\ \varepsilon_{it} &< \ln\left(\frac{E_{it}+1}{\psi_{it}}\right) - \ln\left(E_t - \sum_{j=1}^{m} E_{jt}\right), \quad \text{if} \;\; E_{it} = 0 \end{aligned} \tag{15}$$

As for the second stage model, we represent the true marginal utility u_{ikt}^{*} as a function of the marginal utility of the model and error term, as in the first stage.

$$\ln(u_{ikt}^{*}) = \ln(u_{ikt}) + \epsilon_{ikt}, \;\; \epsilon_{ikt} \sim N(0,1) \tag{16}$$

where $u_{ikt} = \partial U_i^{*} / \partial y_{ikt}$ is the model marginal utility of brand k in category i, and ϵ_{ikt} is an independent and identically distributed error term following a standard normal distribution. Thus, we substitute Equation 16 into Equation 12 to determine the relation between observed quantity and error term:

$$\begin{aligned} \epsilon_{ikt} &= \ln(\lambda) - \ln\left(\frac{\phi_{ik}}{(\gamma_{ik}y_{ikt}+1)} - \frac{(I_\rho\rho_i - I_\omega\omega_i)p_{ikt}}{(z_{it}+1)}\right) \quad \text{if } y_{ikt} > 0 \\ \epsilon_{ikt} &< \ln(\lambda) - \ln\left(\frac{\phi_{ik}}{(\gamma_{ik}y_{ikt}+1)} - \frac{(I_\rho\rho_i - I_\omega\omega_i)p_{ikt}}{(z_{it}+1)}\right) \quad \text{if } y_{ikt} = 0 \end{aligned} \tag{17}$$

We observe purchased quantity for each product category and brand. These quantities are considered as the optimum values that maximize consumers' utility at both levels (i.e., x^*_{it} and y^*_{ikt}). In addition, we also observe the actual price of each brand p_{ikt} and the total expenditure E_t. The likelihood function of the proposed model is a joint probability of E^*_{it} and y^*_{ikt}. However, it can be considered as a probability of observing x^*_{it} and y^*_{ikt} given $\tilde{p}_{it}$, p_{ikt}, E_t, and model parameters. Furthermore, this probability can be decomposed into the probability of observing x^*_{it} and the probability of observing y^*_{ikt} conditional on x^*_{it}.

$$\begin{aligned} \Pr(\{x^*_{it}\},\{y^*_{ikt}\}|\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\phi},\boldsymbol{\gamma},\boldsymbol{\rho},\boldsymbol{\omega}) &= \Pr(\{x^*_{it}\}|\boldsymbol{\alpha},\boldsymbol{\beta})\Pr(\{y^*_{ikt}\}|\{x^*_{it}\},\boldsymbol{\phi},\boldsymbol{\gamma},\boldsymbol{\rho},\boldsymbol{\omega}) \\ &= l_{1t}(\boldsymbol{\alpha},\boldsymbol{\beta}|\{x^*_{it}\})l_{2t}(\boldsymbol{\phi},\boldsymbol{\gamma},\boldsymbol{\rho},\boldsymbol{\omega}|\{y^*_{ikt}\},\{x^*_{it}\}) \end{aligned} \tag{18}$$

The first term in the last part of Equation 18 is the likelihood used to estimate parameters α and β. Suppose that, without loss of generality, the first n categories are purchased and the quantity of the remainder equals zero. Letting $r_{it} = \ln(\tilde{p}_{it}x^*_{it}+1) - \ln(\psi_{it}) - \ln\left(E_t - \sum_{j=1}^{m}\tilde{p}_{jt}x^*_{jt}\right)$, we can write the likelihood as follows:

$$\begin{aligned} &l_{1t}(\boldsymbol{\alpha},\boldsymbol{\beta}|\{x^*_{it}\}) \\ &= \boldsymbol{\Phi}(r_{1t},\cdots,r_{nt})|\mathbf{J}_1|\int_{-\infty}^{r_{(n+1)t}}\cdots\int_{-\infty}^{r_{mt}}\boldsymbol{\Phi}\left(\varepsilon_{(n+1)t},\cdots,\varepsilon_{mt}\right)\partial\varepsilon_{(n+1)t}\cdots\partial\varepsilon_{mt} \end{aligned} \tag{19}$$

where $\mathbf{J}_1$ is the Jacobian of change of variables from $\{\varepsilon_{it}\}$ to $\{x^*_{it}\}$ and $\boldsymbol{\Phi}$ denotes the multivariate normal density function.

The representation of the second likelihood is rather troublesome because we have λ in Equation 17. To eliminate λ, we define auxiliary variables as follows:

$$v_{ikt} = \epsilon_{ikt} - \epsilon_{ilt}\,, \quad s_{ikt} = \ln(u_{ilt}) - \ln(u_{ikt}) \tag{20}$$

where l is the index for any brand having purchase quantity greater than zero. Again, suppose that the first g brands are purchased and the others are not. Therefore, the second likelihood is given by:

$$l_{2t}(\boldsymbol{\phi}, \boldsymbol{\gamma}, \boldsymbol{\rho}, \boldsymbol{\omega} | \{y^*_{ikt}\}, \{x^*_{it}\})$$
$$= \boldsymbol{\Phi}(s_{i1t}, \cdots, s_{igt})|\mathbf{J}_2| \int_{-\infty}^{s_{i(g+1)t}} \cdots \int_{-\infty}^{s_{iK_it}} \boldsymbol{\Phi}(v_{i(g+1)t}, \cdots, v_{iK_it})\, \partial v_{i(g+1)t} \cdots \partial v_{iK_it} \tag{21}$$

where $\mathbf{J}_2$ is the Jacobian of change of variables from $\{\epsilon_{ikt}\}$ to $\{y^*_{ikt}\}$.

Reference Price Specifications

Past studies have proposed several surrogates for reference price based on various price clues that can be observed from data. Although most of the specifications proposed have been applied to brand-level reference price (Kalyanaram and Winer 1995), we argue that the same approach can be applied to category level. Accordingly, we utilize actual price data for approximating the true category reference prices. In particular, we estimate the model with various reference prices specified as follows.

RP1: $p_{it} = p_{ib,(t-1)}$

RP2: $p_{it} = \theta p_{i,(t-1)} + (1-\theta) p_{ib,(t-1)}$

RP3: $p_{it} = \frac{1}{K} \sum_{k=1}^{K} p_{ik,(t-1)}$

RP4: $p_{it} = \frac{1}{K} \sum_{k=1}^{K} p_{ikt}$

Here, b denotes the brand purchased at the time $t-1$. RP1 is the price of brand purchased in the previous purchase occasion. RP2 is the exponentially smoothed reference price. We determine the smoothing parameter θ by using a grid search. RP3 is the average price of focal brands in the previous purchase occasion. RP4 is the average price of focal brands on the day when the purchase is made. The first three reference price specifications are memory based, and the last is stimulus based.

Consumer Heterogeneity

To account for heterogeneity, we allow the parameters to vary across consumers. Thus, the estimation is conducted for all individual-level parameters. Let h be a suffix representing the individual consumer. We define the random effects specification for heterogeneous parameters as follows.

– $\boldsymbol{\alpha}_h \sim N(\bar{\boldsymbol{\alpha}}, \mathbf{D}_\alpha),\ \boldsymbol{\alpha}_h = (\alpha_{h1}, \alpha_{h2}, \ldots, \alpha_{hm})'$

– $\boldsymbol{\beta}_h \sim N(\bar{\boldsymbol{\beta}}, \mathbf{D}_\beta),\ \boldsymbol{\beta}_h = \text{vec}\{\beta_{hij}\}$

– $\boldsymbol{\phi}_{hi}^* \sim N(\bar{\boldsymbol{\phi}}_i, \mathbf{D}_{\phi_i}),\ \boldsymbol{\phi}_{hi}^* = (\phi_{hi1}^*, \phi_{hi2}^*, \ldots, \phi_{hiK}^*)',\ \phi_{hik} = \exp(\phi_{hik}^*)$

– $\boldsymbol{\gamma}_{hi}^* \sim N(\bar{\boldsymbol{\gamma}}_i, \mathbf{D}_{\gamma_i}),\ \boldsymbol{\gamma}_{hi}^* = (\gamma_{hi1}^*, \gamma_{hi2}^*, \ldots, \gamma_{hiK}^*)',\ \gamma_{hik} = \exp(\gamma_{hik}^*)$

– $\boldsymbol{\rho}_h^* \sim N(\bar{\boldsymbol{\rho}}, \mathbf{D}_\rho),\ \boldsymbol{\rho}_h^* = (\rho_{h1}, \rho_{h2}, \ldots, \rho_{hm})',\ \rho_{hi} = \exp(\rho_{hi}^*)$

– $\boldsymbol{\omega}_h^* \sim N(\bar{\boldsymbol{\omega}}, \mathbf{D}_\omega),\ \boldsymbol{\omega}_h^* = (\omega_{h1}, \omega_{h2}, \ldots, \omega_{hm})',\ \omega_{hi} = \exp(\omega_{hi}^*)$

Note that because parameters $\phi, \gamma, \rho,$ and ω are restricted to a non-negative value, they are reparameterized so that the restriction can be satisfied. In the appendix, we describe the procedure for estimating all of the hyper parameters.

DATA

We sourced the data from Customer Communications Co., Ltd, a marketing research company in Tokyo, Japan. The data comprise purchases record of customers who made shopping trips to a supermarket during one year from July 1, 2007, to June 30, 2008. We chose four product categories for the analysis: instant coffee, coffee cream, tea, and instant curry. We selected 79 customers who made at least 3 purchases in each category during the data period. This resulted in 3,821 observations in total. Table 4 shows a summary of category purchases. Among the four categories, coffee has the largest budget share, followed by tea, curry, and cream. In each category, interior solutions account for about one-third of purchase incidence.

[Insert Table 2 about here.]

Figure 2 shows monthly sales of the categories during the data period. The sales of coffee, cream, and tea appear to be correlated. In fact, the correlation coefficients are moderately high among the three products: 0.53 for instant coffee vs. coffee cream, 0.62 for instant coffee vs. tea, and 0.66 for coffee cream vs. tea. The correlation between each of these three products and instant curry is less than 0.29, revealing that instant curry is independent from the remainder.

[Insert Figure 2 about here.]

We display basic information for the brands in each category in Table 5. We masked actual brand names for confidentiality purposes. For the coffee, cream, and tea categories, we chose five brands. For curry, we chose seven brands. We chose brands based on the order of market share. In each category, the chosen brands account for about 40 to 78 percent of the total sales. The last brands in the table are an aggregation of the remainder.

Most of the brands are available in various sizes. We observed that within a brand, the unit price varies across sizes. In most cases, a bigger size has a lower unit price than a smaller size does. We derived the unit price of each brand by averaging the unit prices of different sizes. For brands in the curry category, most were priced in a similar range, except for brand 5, which is a premium brand. This reflects a high competition intensity in this category. In the coffee and tea category, the brands with the highest market share have relatively higher prices. This indicates that consumers base their choice largely on the quality preference rather than on price.

[Insert Table 3 about here.]

RESULT

Model Selection

The joint posterior density of the model was assessed by using Bayesian Markov chain Monte Carlo (MCMC) methods. In Appendix A, we describe the algorithm used for estimating model parameters in detail. For each parameter, we drew 20,000 iterations from a full conditional posterior distribution. We used the first 15,000 draws during a burn-in period, and we kept the remainder for calculating posterior means. We inspected chain convergence by splitting the samples kept for estimation into two subsamples and testing the difference between the posterior means derived from them (Geweke 1992). We confirmed that the difference was insignificant.

[Insert Table 4 about here.]

We estimated the proposed two-stage model with previously described reference price specifications along with the one-stage model. The one-stage model corresponds to the second stage in our proposed model, but has no any link to the first stage model. We compared the predictive ability of each model based on three criteria:[5] the log of marginal likelihood (LML), deviance information criterion (DIC: Spiegelhalter et al. 2002), and root-mean-square error (RMSE). We computed LML and DIC using

[5] We computed LML by using the overall likelihood from Equation 18 to compare the performance of the two-stage models. However, the computation of DIC is based only on the second term of Equation 18 to compare the performance of all models. RMSE is computed as the root of mean squared differences between holdout sample of y_{ikt} and their predictive values.

in-sample data to assess the goodness of fit. Furthermore, we computed RMSE using a holdout sample for validation purposes.

Table 4 presents the performances of the competing models. The values of DIC and RMSE revealed that the two-stage models fit the data better than do the one-stage models. The results indicate that involving consumer decisions in multiple stages leads to a significant improvement in model fit. In addition, the model with the RP3 specification (i.e., the average price of focal brands in the previous purchase occasion) outperforms the other two-stage models in terms of all criteria. However, the differences are moderate, and one may find that a model with a different RP specification performs better when applying it to different data. Next, we discuss the estimation results along with their implications for the best model.

First-Stage Model

We show the estimates of parameter α in Table 5. As for the outside good, we fixed product attractiveness at one for identification purposes, implying that the value of α for the category equals zero. Compared to all of the categories except the outside good, instant coffee has the greatest baseline attractiveness. With other things being equal, this indicates that coffee is the most preferred category among the four under consideration. The result is consistent with the fact that coffee accounted for a large portion of consumers' budget. However, the value of α is not necessarily proportional to budget share, as it also depends on category prices. A category with a relatively lower price can gain a high budget share despite having a small α. This is particularly true for the case of curry, whose unit prices were lower than the unit prices of other categories. While its α value is the smallest relative to the other categories, its budget share is slightly greater than that of cream. In Figure 3, we display the subutility functions of the categories when inventory levels are set to zero.

[Insert Table 5 about here.]

[Insert Figure 3 about here.]

Now we consider the estimates of this parameter at a disaggregated level. Figure 4 shows the distributions of parameter α over all customers. The figure shows that customers differ in their preferences toward certain categories, indicating the importance of accounting for consumer heterogeneity. For example, in the case of instant coffee, the estimates range from −3.67 to −0.09. This means that although coffee has the largest average value of α, some customers might have preferred tea to coffee. In particular, the results reveal that the extent to which parameter α varied over customers is relatively high for coffee, cream, and tea. On the other hand, the variation is smaller for curry than it is for the others.

[Insert Figure 4 about here.]

Table 6 shows the results for inventory effects β_{ji}. Self-inventory effects for instant coffee and coffee cream are significant and have the expected signs. However, this is not the case for the categories of tea and instant curry. The cross-inventory effect of instant coffee on coffee cream demand is significant and has a positive sign. This implies that consumers find coffee cream to be more attractive when they have enough coffee in storage. The effect, however, is asymmetric in the sense that the same effect does not apply to instant coffee when consumers have a higher inventory level of coffee cream. The cross-inventory effect of tea to instant coffee is also significant but with a negative sign. This means that instant coffee is less attractive when consumers have a high inventory of tea, but not vice versa.

Note that the cross-inventory effects of coffee cream on instant curry and of instant curry on tea are also observed to be significant. However, we conjecture that this result is a coincidence effect, as pointed out by Manchanda, Ansari, and Gupta (2007), because curry is a frequently purchased category.

[Insert Table 6 about here.]

Second Stage Model

Next, we provide the parameter estimates for the second stage of the demand allocation model. Table 7 presents the posterior means of brand attractiveness ϕ and satiation parameter γ. For the case of coffee, the magnitude of ϕ is roughly proportional to market share. Therefore, in this category consumer preference plays a significant role in the buying decision. In other words, consumers based their purchasing decision in the coffee category largely on their preference, not on brand prices. This might be because coffee is a hedonic good for which taste matters more than price (Khan, Dhar, and Wertenbroch 2005). On the other hand, the relations between brand attractiveness and market share for the other categories are not as strong as that observed for coffee. Consumers are more sensitive toward brand price for these other, more utilitarian categories.

The satiation parameter varied across brands within a category. This parameter governs the diminishing rate of marginal utility as quantity increases. The purchased quantity of a brand with higher γ will be less than that of a brand with lower γ, ceteris paribus. This is the case for both brand 1 and brand 2 in the tea category. Both brands have similar brand attractiveness, and the former was priced higher than the latter. However, the share of brand 1 is greater than that of brand 2 because the latter has a relatively higher satiation parameter.

[Insert Table 7 about here.]

Table 8 presents the estimates of parameter ρ and ω. The values of ω are greater than those of ρ for the coffee, cream, and tea categories. This means that the gain from saving an additional dollar is greater than the loss of reducing one dollar from the outside good expenditure. On the contrary, the result for curry is reversed. For this category, consumers are more resistant to reallocate the outside good budget for curry. We postulate that this is because curry was the least attractive category to consumers. As a result, drawing an additional dollar from the expenditure allocated to outside goods in order to satisfy the quantity constraint caused a significant loss in utility.

[Insert Table 8 about here.]

DISCUSSION

Expenditure Effect

From the results of the first-stage model, we can analyze the marginal effect of expenditure on category demands. If consumers increase their expenditure by one unit, how would this be distributed among product categories? The proportions can be computed using the Engel aggregation (Jehle and Reny 2011). Let $\mathbf{x}_{it}(E_t, \tilde{\mathbf{p}}_t)$ be consumers' Marshallian demand system for the category and $\tilde{\mathbf{p}}_t$ denotes a vector of the reference prices. At optimum value, the budget constraint requires $E_t = \tilde{\mathbf{p}}_t \cdot \mathbf{x}_{it}(E_t, \tilde{\mathbf{p}}_t)$ for all $\tilde{\mathbf{p}}_t$ and E_t. By differentiating both sides with respect to E_t, we get:

$$1 = \sum_i \tilde{p}_{it} \frac{\partial x_{it}}{\partial E_t} \tag{22}$$

Each element in the summation refers to the proportion that will be allocated to category

i from the additional one-unit expenditure. Denoting each element by ς_i, we can express it in terms of model parameters by using the category demand function given in Appendix B as follows:

$$\begin{aligned} \varsigma_i &= \tilde{p}_{it} \frac{\partial x_{it}(E_t, \tilde{\mathbf{p}}_t)}{\partial E_t} \\ &= \frac{\psi_{it}}{\sum_{j=1}^{m} \psi_{jt} + 1} \end{aligned} \tag{23}$$

Equation 23 indicates that the proportion of the additional one-unit expenditure to a category is equal to the relative attractiveness of the category. We show the average value ς_i of each category in Figure 5. The leftmost bar is the proportions averaged over all customers. As expected, a large portion of the expenditure is allocated to the outside good. The proportions of other categories are relatively small and correspond with average budget shares.

We next split customers into three segments based on their average expenditure per month in order to examine how the proportions relate to consumer spending. Segments 1, 2, and 3 are groups of customers with average expenditures of less than 4,000 yen, between 4,000 and 8,000 yen, and above 8,000 yen per month, respectively. As we can see, the segment with a greater average expenditure has a higher proportion of the outside good and a lower proportion of the others. This means that while the amounts spent on coffee, cream, tea, and curry increase as expenditure increases, the proportions become smaller for customers spending more money. One explanation for this result is that the consumption rates for these categories are relatively constant across customers, which implies that those with higher expenditures have lower budget shares of the categories. On the other hand, there is no significant correlation between expenditure elasticity and expenditure level. Another explanation is that “wealthier” customers do not necessarily buy high-priced brands compared to their counterparts with lower expenditure levels. Therefore, the amounts allocated to each category remain almost unchanged in terms of total expenditure.

[Insert Figure 5 about here.]

Figure 6 shows the scatter plots between the proportions allocated to all categories and the average expenditure per month of individual customers. The proportions allocated to coffee, cream, and tea appear to be negatively correlated with the amounts consumers spent per month. The correlation for curry is not as strong as that for the other categories, yet the proportion tends to decrease. On the other hand, the proportion for the outside good approximates to one as expenditure increases. We can expect that there is a point along the expenditure continuum beyond which the expenditure for the focal categories will remain unchanged. We can take this for granted because there would be a maximum quantity consumed by a consumer in a certain period.

[Insert Figure 6 about here.]

Next, we explore the effect of expenditure increase on brand purchases. We use the identity $E_t = \sum_i \sum_k p_{ikt} y_{ikt}(x_{it}, \tilde{p}_{it}, \mathbf{p}_{it}) + z_t$, where $y_{ikt}(x_{it}, \tilde{p}_{it}, \mathbf{p}_{it})$ denotes the demand function for brands given in Appendix B. We then compute the effect by differentiating the above identity with respect to E_t. Denoting the proportion allocated to brand k in category i from the additional one-unit expenditure by ς_{ik}, we can express it in terms of model parameters as follows:

$$\begin{aligned} \varsigma_{ik} &= \varsigma_i p_{ikt} \frac{\partial y_{ikt}(x_{it}, \tilde{p}_{it}, \mathbf{p}_{it})}{\partial \tilde{p}_{it} x_{it}} \\ &= \frac{\psi_{it}}{\sum_{j=1}^{m} \psi_{jt} + 1} \frac{1}{\left\{\sum_{l \neq k} \left(\frac{\phi_{il}\gamma_{ik}}{\phi_{ik}\gamma_{il}}\right) + 1\right\}} \end{aligned} \tag{24}$$

Now, suppose that all customers increased their expenditure by 1,000 yen on a certain purchase occasion. Thus, the total increase in expenditure of 97 customers is 97,000 yen. This induces an expenditure increase for each brand, which we can compute as $1000 \times \varsigma_{ik}$. Figure 7 shows how this amount would be allocated among product categories and then among brands. For example, out of the total increase in expenditure, 11,448 yen is allocated to the coffee category. This amount is further allocated to each brand within the category: 2,687 yen to brand 1, 2,224 yen to brand 2, and so on. The Figure also reveals that for coffee, the brand with the higher market share enjoys a larger portion of the incremental expenditure. The underlying reason for this is that brands with a higher market share have greater expenditure elasticity. However, this is not the case for the categories of cream and tea. For curry, the incremental increase in expenditure is shared about equally among all brands.

[Insert Figure 7 about here.]

Price Effect

While examining marginal effects of expenditure can provide insights into consumers' decisions concerning demand allocation, expenditure is an exogenous variable. It is almost impossible for a marketer to influence how much consumers will spend during a particular shopping trip. Accordingly, it would be more useful if we could understand the effect of a variable that is controllable to a marketer. Subsequently, we explore the impact of price changes on brand demand.

Note that a change in the price of any brand can impact not only demand for the brand but also the category reference price on the next purchase occasion. Consequently, it can lead to a change in the budget allocated to that category. This will ultimately induce a change in demand for the brand on the subsequent shopping trip. For this reason, we divide the price effect on demand into two periods; direct price effects occur at the same purchase occasion (period 1), and indirect price effects occur through the reference price at the next purchase occasion (period 2).

To obtain direct and indirect price effects, we begin by computing the demand of each brand evaluated at its average price. Next, we reduce the prices by 10 percent and again calculate demands under these new prices. We calculate the price effect in period 1 as a difference between the demand levels before and after the price reduction. We then calculate the changes in reference price and category expenditure due to price reduction to determine the quantity demanded on the next purchase occasion. We then reduced this quantity by the quantity before the price reduction to obtain the price effect in period 2. Table 9 presents price effects in two periods for each brand in four product categories. The numbers in the table represent an average price effect over all customers. The results reveal that the effect in period 1 overwhelmed the effect in period 2. This implies that a large portion of the effect occurs at the time the price changes, with almost no carry over effect.

[Insert Table 9 about here.]

Budget Adjustment Assumption

We imposed the assumption of budget adjustment to ensure that the utility maximization problems have some solutions. The condition for the assumption to hold is given in Equation 13. However, we do not restrict the parameters on the space bounded by this condition. Instead, we estimate the parameters on unbounded space and examine whether the results are consistent with the assumption. By doing so, we can inspect the plausibility of the assumption by assessing the probability that parameter estimates lie on the subspace bounded by the assumption. To compute this probability, we define φ_{it} as follows:

$$\begin{aligned}\varphi_{it} &= U_i^*(\boldsymbol{y}_{it}'')-U_i(\boldsymbol{y}_{it}') \\ &= \sum_k \frac{\phi_{ik}}{\gamma_{ik}} \ln\left(\frac{\gamma_{ik} y_{ikt}'' + 1}{\gamma_{ik} y_{ikt}' + 1}\right) + \{I_\omega \omega_i - I_\rho \rho_i\} \ln(z_{it} + 1)\end{aligned} \quad (25)$$

Equation 13 says that when either $\sum_k p_{ikt} y_{ikt}'' > \tilde{p}_{it} x_{it}^*$ or $\sum_k p_{ikt} y_{ikt}'' < \tilde{p}_{it} x_{it}^*$, consumers will adjust their budget if $\varphi_{it} > 0$. Therefore, to determine whether the assumption holds, it is sufficient to assess the probability that the parameters satisfy the condition requirement (i.e., $\Pr(\varphi_{it} > 0)$). Letting φ_{it}^c be the value of φ_{it} at the c-th iteration, we compute the probability as follows:

$$\Pr(\varphi_{it} > 0) \cong \frac{1}{CT} \sum_{c=1}^{C} \sum_{t=1}^{T} I(\varphi_{it}^c > 0) \quad (28)$$

We observed that out of 97 customers, more than 90 have probabilities greater than 0.8. For the tea category, all customers have probabilities greater than 0.8. The results reveal that even without restrictions imposed on the parameters, the estimates satisfy the assumption condition with high probability. Accordingly, we conclude that the assumption is highly plausible.

CONCLUDING REMARKS

In this paper, we proposed an integrated model of expenditure allocation among product categories and brands. We applied the model to purchase history data and examined its performance for different specifications of reference price. We also compared its performance with the one-stage model and found that the proposed model has a better predictive ability. The estimation results of category and brand attractiveness are consistent with our expectation about the roles of preference and price in consumer purchase decisions. We observed that baseline preference explained a large portion of variation in budget and market share. However, for some categories and brands, their prices also substantially influenced marginal utility per dollar, which ultimately affected their shares. In addition, we observed that the satiation parameter had a significant effect on the sales of some brands, indicating the importance of considering consumers' preference for variety.

Our framework provides a tool for exploring the marginal effect of expenditure in several decision levels. The empirical results demonstrated how an incremental increase in expenditure would be distributed among categories and brands. The advantage of the multi-stage demand model is that it can aid marketers in anticipating demand changes when expenditures increase due to a change in exogenous factors, such as economic conditions. We also examined how a change in brand price could lead to a change in the demand for both brand and category expenditures during the subsequent shopping trip. Our analysis indicated that the price effect on category expenditure was small compared to its effect on quantity demanded.

One of the major issues in modeling consumers' decision making in multiple stages is linking the decisions over different stages. An intuitive way to cope with this issue is to treat the solutions in the previous stage as additional constraints in the subsequent stage. However, with this treatment, in most cases the solutions in the subsequent stage that satisfy all constraints cannot be attained. We handled this problem by assuming that consumers flexibly adjust the expenditure that has been allocated to the outside good to arrive at the solutions that satisfy all constraints. We argued that this is a reasonable assumption because the outside good usually contains some low-priority products whose expenditure can be flexibly altered. Furthermore, there is a high probability that the parameters will take values that satisfy the condition required for the assumption to hold.

The proposed model can be extended to handle consumer decisions in more than two stages. However, data availability would be a major obstacle to doing so. Furthermore, our model is built on the assumption that consumers first decide to allocate expenditure among categories and then among brands, which is a type of planned buying behavior. Additional research is necessary to examine how the results would vary if we imposed the assumption of unplanned purchase behavior—that is, a two-stage model in which consumers decide which and how much of the brand to purchase first and then subsequently decide the budget allocated to each category. We leave these extensions to future research.

REFERENCES

Ailawadi, K. L., K. Gedenk, C. Lutzky, and S. A. Neslin (2007), "Decomposition of the Sales Impact of Promotion-Induced Stockpiling," *Journal of Marketing Research*, 44 (3), 450–467.

Barten, A. P. (1993), "Consumer Allocation Models: Choice of Functional Form," *Empirical Economics*, 18, 129–158.

Bucklin, R., and Lattin, J. M. (1991), "A Two-State Model of Purchase Incidence and Brand Choice," *Marketing Science*, 10 (Winter), 24–39.

Chib, S., P. B. Seetharaman, and A. Strijnev (2004), "Model of Brand Choice with a No-Purchase Option Calibrated to Scanner-Panel Data," *Journal of Marketing Research*, 41 (2), 184–196.

Christensen, L. R., D. W. Jorgenson, and L. J. Law (1975), "Transcendental Logarithmic Utility Functions," *American Economic Review*, 5, 367–382.

Deaton, A., and J. Muellbauer (1980), "An Almost Ideal Demand System," *American Economic Review*, 70, 312–326.

Geweke, J. (1992), "Evaluating the Accuracy of Sampling-Based Approaches to the Calculation of Posterior Moments," In *Bayesian Statistics 4*, ed. J. M. Bernardo, J. O. Berger, A. P. Dawid, and A. F. M. Smith, Oxford: Oxford University Press, 169–193.

Hasegawa, S., N. Terui, and G. M. Allenby (2012), "Dynamic Brand Satiation," *Journal of Marketing Research*, 48, 842–853.

Hauser, J. R., and G. L. Urban (1986), "The Value Priority Hypotheses for Consumer Budget Plans," *Journal of Consumer Research*, 12, 446–462.

Jehle, G. A., and P. J. Reny (2011), *Advanced Microeconomic Theory*, 3rd edition. New

York: Prentice Hall.

Kalyanaram, G., and R. S. Winer (1995), "Empirical Generalization from Reference Price Research," *Marketing Science,* 14 (3), 161–169.

Khan, U., R. Dhar, and K. Wertenbroch (2005), in *Inside Consumption: Frontiers of Research on Consumer Motives, Goals, and Desires*, ed. S. Ratneshwar and David Glen Mick, London: Routledge, 144-165.

Kim, J., G. M. Allenby, and P. E. Rossi (2002), "Modeling Consumer Demand for Variety," *Marketing Science*, 21 (3), 229–250.

Lee, S., and G. M. Allenby (2014), "Modeling Indivisible Demand," *Marketing Science,* Article in Advance, 1–18.

Lee, S., J. Kim, and G. M. Allenby (2013), "A Direct Utility Model for Asymmetric Complements," *Marketing Science,* 32 (3), 454–470.

Manchanda, P., A. Ansari, and S. Gupta (1999), "The Shopping Basket: A Model for Multicategory Purchase Incidence Decisions," *Marketing Science*, 18, 95–114.

Mehta, N. (2007), "Investigating Consumers' Purchase Incidence and Brand Choice Decisions Across Multiple Product Categories: A Theoretical and Empirical Analysis," *Marketing Science*, 26 (2), 196–217.

Satomura, T., J. Kim, and G. M. Allenby (2011), "Multiple Constraints Choice Model with Corner and Interior Solutions," *Marketing Science*, 30 (3), 481–490.

Song, I., and P. K. Chintagunta (2007), "A Discrete-Continuous Model for Multicategory Purchase Behavior of Customers," *Journal of Marketing Research*, 44, 595–612.

Spiegelhalter, D. J., N. G. Best, B. P. Carlin, and A. Van Der Linde (2002), "Bayesian Measures of Model Complexity and Fit," *Journal of the Royal Statistical Society:*

Series B, 64 (4), 583–616.

Stone, R. (1954), "Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand," *The Economics Journal*, 64, 511–527.

Table 1. Summary of the Selected Studies on Demand Model

No.	*Reference*	*Type of decision*	*Number of stages*	*Remarks*
1	Stone (1954)	Demand allocation	1 stage	
2	Christensen, Jorgenson, and Law (1975)	Demand allocation	1 stage	
3	Deaton and Muellbauer (1980)	Demand allocation	1 stage	
4	Hauser and Urban (1986)	Demand allocation	1 stage	
5	Kim, Allenby, and Rossi (2002)	Demand allocation	1 stage	
6	Bucklin and Lattin (1991)	Category purchase Brand choice	2 stages	Corner solutions
7	Bucklin and Gupta (1992)	Category purchase Brand choice	2 stages	Corner solutions
8	Chib, Seetharaman, and Strijnev (2004)	Category purchase Brand choice	2 stages	Corner solutions
9	Mehta (2007)	Category purchase Brand choice	2 stages	Corner solutions for brand purchase
10	Song and Chintagunta (2007)	Category purchase Brand choice Purchase quantity	2 stages	Corner solutions for brand purchase
11	Lee, Kim, and Allenby (2013)	Demand allocation	2 stages	Sequential allocation

Table 2. Purchase Summary

Product category	*Purchase incidence*	*Total expenditure*	*Interior solution*
Instant coffee	1,491	709,101	475
Coffee cream	1,172	250,242	377
Tea	916	325,813	284
Instant curry	978	251,816	253

Table 3. Basic Statistics of Brands

Brand	*Instant coffee*		*Coffee cream*		*Tea*		*Instant curry*	
	Share	*Unit price*	*Share*	*Unit price*	*Share*	*Unit price*	*Share*	*Unit price*
Brand 1	0.27	5.27	0.31	0.79	0.27	4.98	0.07	0.70
Brand 2	0.13	3.32	0.11	0.69	0.09	3.52	0.06	0.77
Brand 3	0.08	3.56	0.21	1.67	0.07	2.88	0.06	0.73
Brand 4	0.08	3.31	0.09	0.83	0.12	1.81	0.05	0.93
Brand 5	0.05	7.04	0.09	0.81	0.04	3.77	0.06	2.27
Brand 6	0.40	2.69	0.19	1.12	0.40	4.61	0.05	0.80
Brand 7							0.04	0.74
Brand 8							0.61	1.30

Table 4. Data Fit and Predictive Ability

Model	*LML (In-sample)*	*DIC (In-sample)*	*RMSE (Holdout sample)*
Two-stage model:			
with RP1	−121,332	84,121	980.64
with RP2	−122,125	84,063	979.57
with RP3	**−119,319**	**83,913**	**976.39**
with RP4	−120,806	84,142	981.20
One-stage model:		87,730	992.08

Table 5. Parameter Estimates of Category Attractiveness

Product category	*Post mean*	*Post STD*	$exp(\alpha)$
Instant coffee	−1.90	0.27	0.15
Coffee cream	−2.79	0.11	0.06
Tea	−2.43	0.10	0.09
Instant curry	−2.82	0.50	0.06
Outside good	0	fixed	1.00

Table 6. Parameter Estimates of Inventory Effect

		Instant coffee	*Coffee cream*	*Tea*	*Instant curry*
Inventory	*Instant coffee*	**−0.14 (0.05)**	**0.14 (0.06)**	−0.04 (0.07)	0.01 (0.06)
	Coffee cream	−0.09 (0.07)	**−0.21 (0.08)**	−0.11 (0.07)	**−0.27 (0.06)**
	Tea	**−0.21 (0.07)**	−0.08 (0.07)	−0.07 (0.08)	0.13 (0.07)
	Instant curry	−0.01 (0.06)	0.00 (0.05)	**−0.20 (0.08)**	0.07 (0.07)

Posterior standard deviations are in parentheses. Bold indicates significant parameter.

Table 7. Brand Attractiveness and Satiation Parameters

Brand	*Instant coffee*		*Coffee cream*		*Tea*		*Instant curry*	
	ϕ	γ	ϕ	γ	ϕ	γ	ϕ	γ
Brand 1	3.47	1.45	1.09	1.41	1.04	0.68	0.90	1.43
	(0.69)	(0.15)	(0.12)	(0.24)	(0.15)	(0.08)	(0.07)	(0.17)
Brand 2	2.02	0.68	2.12	1.10	1.18	2.03	1.24	1.42
	(0.41)	(0.09)	(0.22)	(0.11)	(0.10)	(0.31)	(0.10)	(0.13)
Brand 3	2.31	0.90	0.85	2.24	3.12	0.85	1.66	1.92
	(0.34)	(0.14)	(0.13)	(0.40)	(0.61)	(0.05)	(0.17)	(0.25)
Brand 4	1.59	1.07	0.98	1.65	1.44	0.78	0.95	1.08
	(0.27)	(0.23)	(0.11)	(0.33)	(0.16)	(0.11)	(0.07)	(0.11)
Brand 5	1.23	1.08	2.47	1.27	1.43	1.25	1.43	1.45
	(0.14)	(0.18)	(0.31)	(0.26)	(0.21)	(0.08)	(0.09)	(0.21)
Brand 6	1.20	1.45	2.36	3.19	2.74	1.74	1.53	1.84
	(0.01)	(0.27)	(0.27)	(0.67)	(0.39)	(0.26)	(0.15)	(0.25)
Brand 7							1.67	2.07
							(0.09)	(0.17)
Brand 8							2.78	1.04
							(0.27)	(0.12)

Posterior standard deviations are in parentheses.

Table 8. Estimates of Loss and Gain Parameters

	Loss parameter ρ		*Gain parameter* ω	
	Posterior mean	*Posterior STD*	*Posterior mean*	*Posterior STD*
Instant coffee	1.96	0.78	2.34	0.65
Coffee cream	0.04	0.03	0.89	0.23
Tea	0.70	0.14	0.94	0.30
Instant curry	1.58	0.36	0.43	0.17

Table 9. Price Effects

Brand	*Instant coffee*			*Coffee cream*			*Tea*			*Instant curry*		
	Period 1	*Period 2*	*Total*	*Period 1*	*Period 2*	*Total*	*Period 1*	*Period 2*	*Total*	*Period 1*	*Period 2*	*Total*
Brand 1	222	25	247	269	12	281	63	1	64	211	5	217
Brand 2	259	25	284	724	60	784	63	2	65	234	10	244
Brand 3	232	21	253	96	4	100	375	42	417	380	24	403
Brand 4	319	31	350	231	10	241	325	26	351	194	6	200
Brand 5	60	2	62	901	94	995	80	3	83	135	7	142
Brand 6	187	11	198	556	61	617	156	18	174	265	13	278
Brand 7										245	8	252
Brand 8										355	30	385

Figure 1. Budget and Quantity Constraint

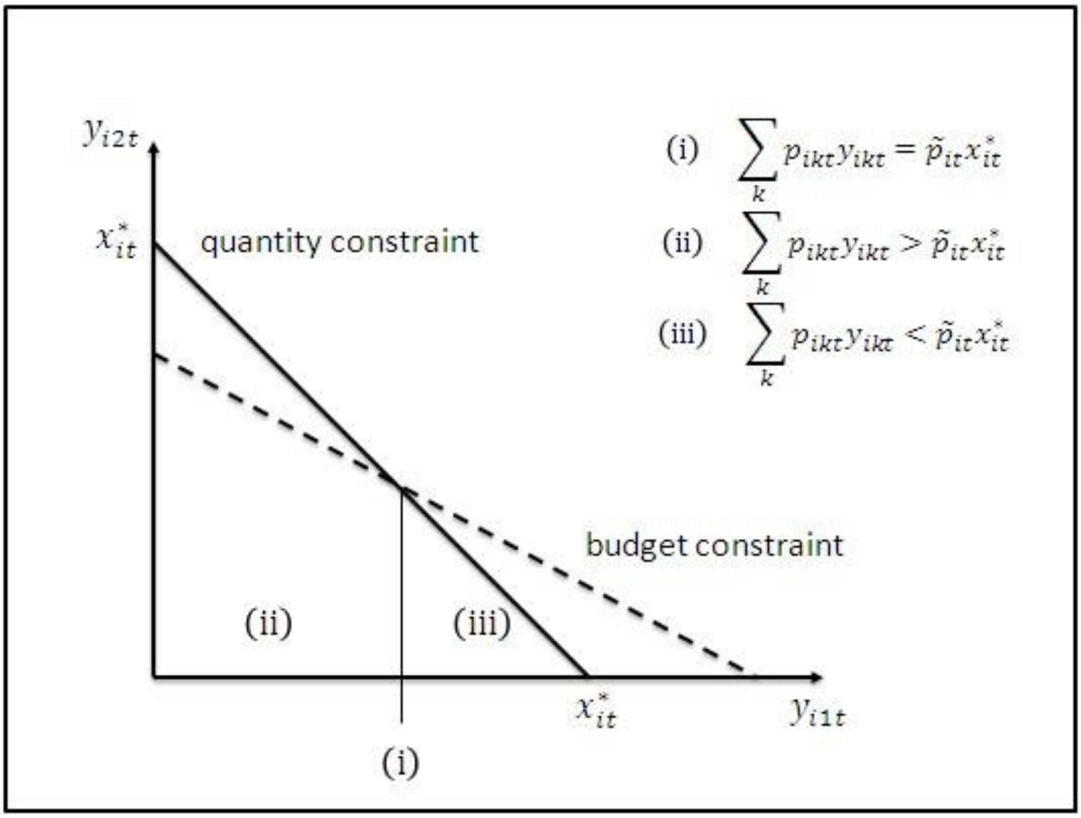


Figure 2. Monthly Sales

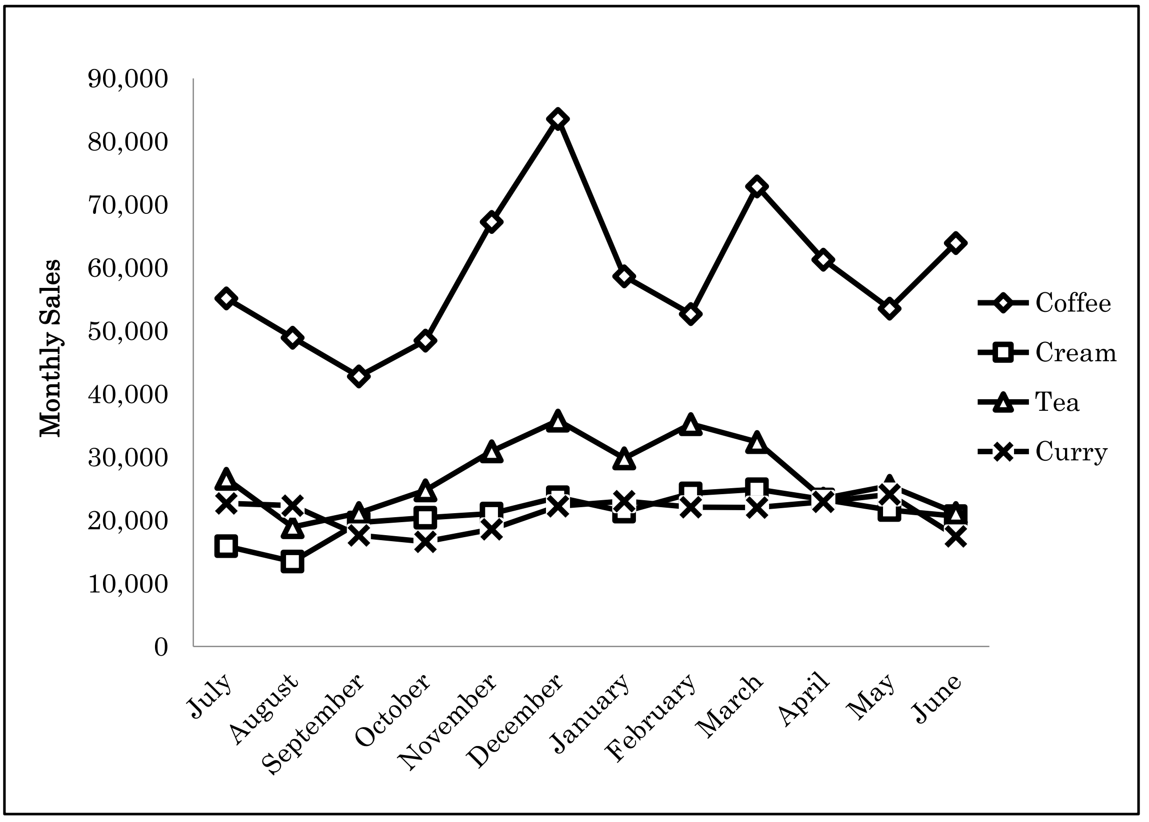


Figure 3. Subutility Functions

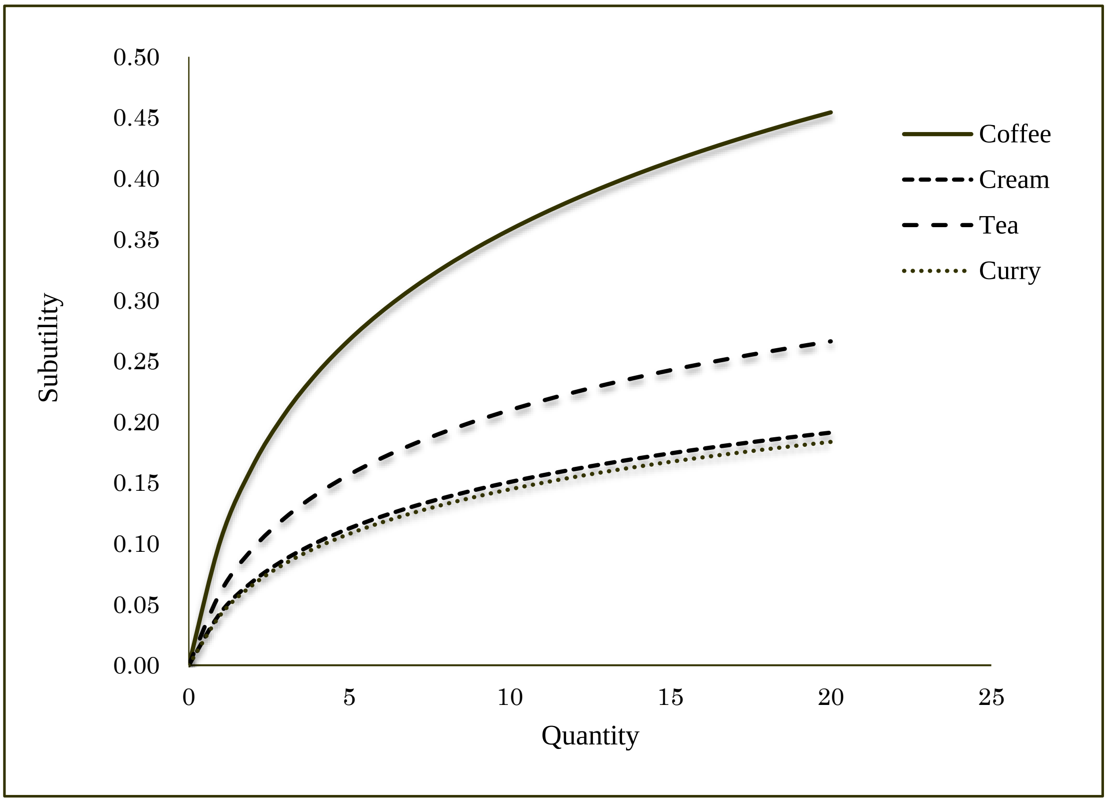


Figure 4. The Distribution of Baseline Preferences toward Category

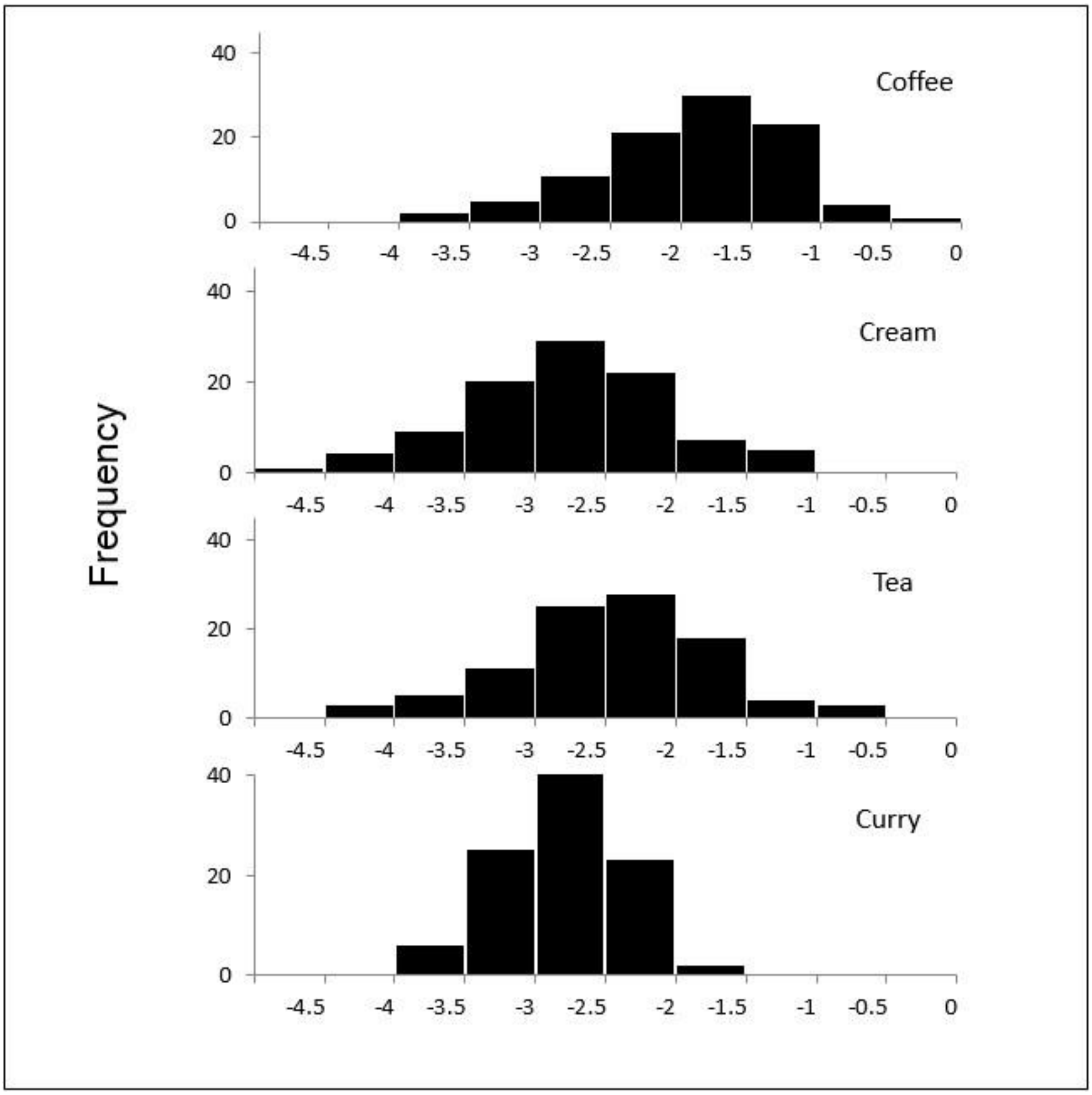


Figure 5. The Allocation of Additional One-Unit Expenditure

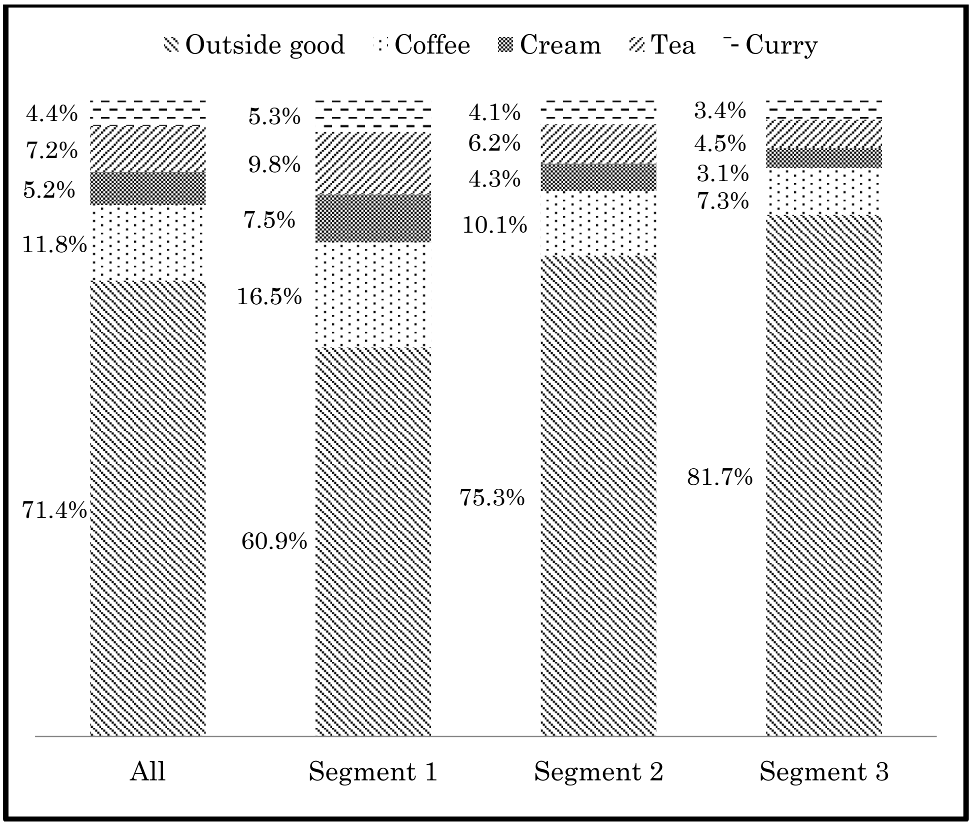


Figure 6. Proportions Allocated by Individual Customers

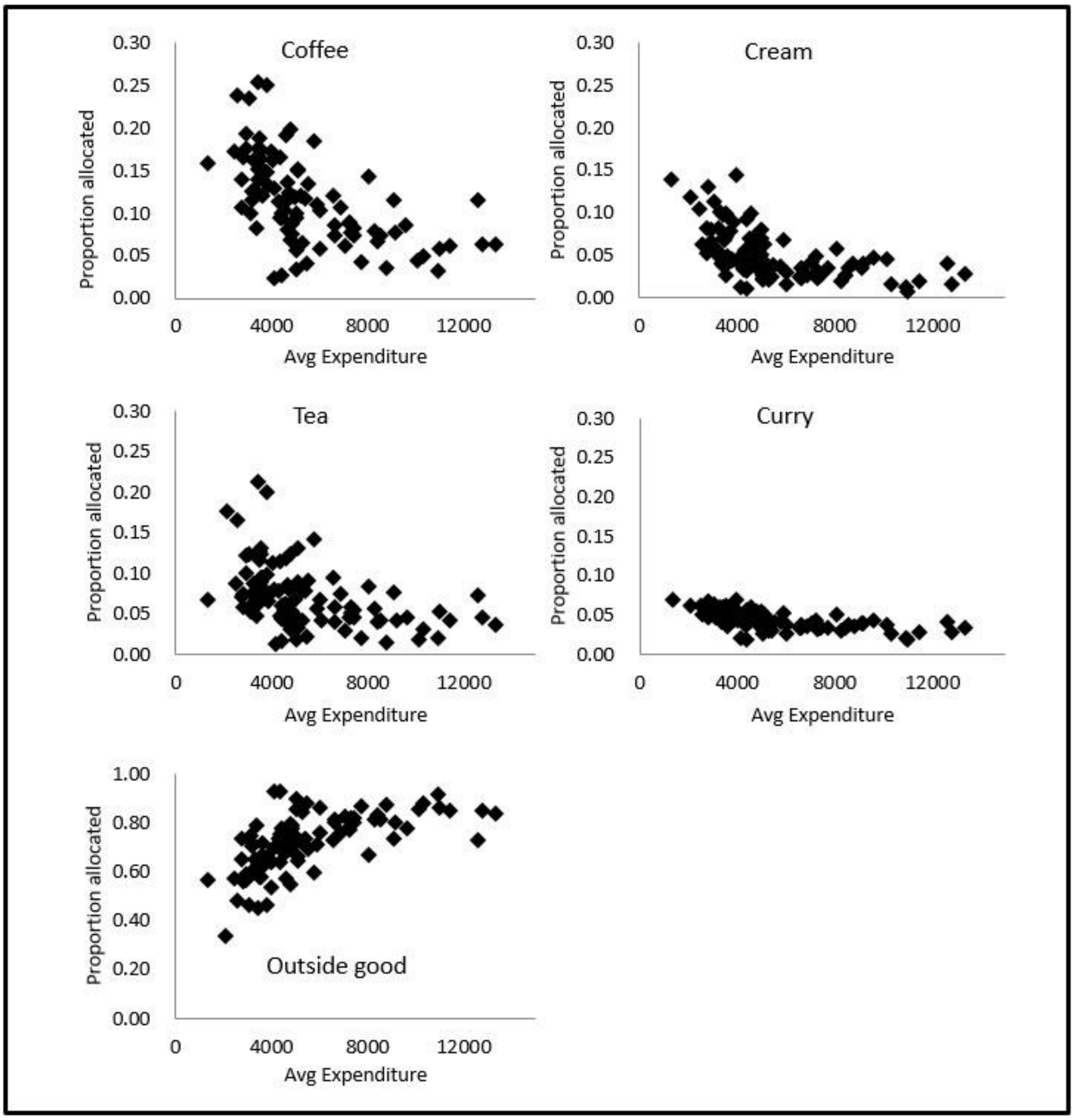
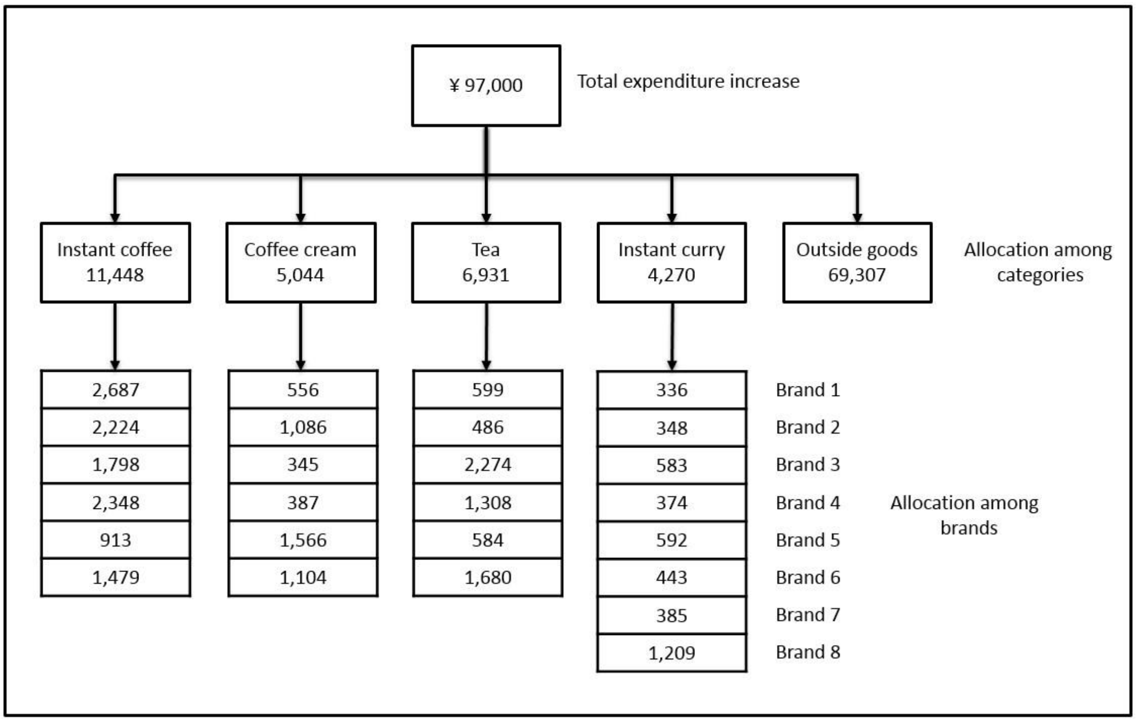


Figure 7. Allocation of Additional Expenditure among Categories and Brands



APPENDIX A: MCMC ALGORITHM

We estimated the proposed model by using the MCMC method. We defined the prior distribution for each parameter in the model description. We assume that the distributions are centered at the population mean. In particular, the hyper parameters are updated at each step within the chain by using the current value of individual parameters. Let $c = (1,2,\ldots,\mathrm{C})$ be the number of MCMC iterations and d and D be prior values. Thus, we can write the distributions of the hyper parameters at the c-th step as follows:

$$\pi\left(\bar{\boldsymbol{\alpha}}\middle|\left\{\boldsymbol{\alpha}_h^{(c)}\right\},\mathbf{D}_\alpha\right) = N\left(\frac{\sum_{h=1}^{H}\boldsymbol{\alpha}_h^{(c)}}{H},\frac{\mathbf{D}_\alpha}{H}\right) \tag{A.1}$$

$$\pi\left(\mathbf{D}_\alpha\middle|\left\{\boldsymbol{\alpha}_h^{(c)}\right\},\bar{\boldsymbol{\alpha}}\right) = IW\left(d_\alpha + H, D_\alpha + \sum_{h=1}^{H}\left(\boldsymbol{\alpha}_h^{(c)} - \bar{\boldsymbol{\alpha}}\right)\left(\boldsymbol{\alpha}_h^{(c)} - \bar{\boldsymbol{\alpha}}\right)'\right) \tag{A.2}$$

$$\pi\left(\bar{\boldsymbol{\beta}}\middle|\left\{\boldsymbol{\beta}_h^{(c)}\right\},\mathbf{D}_\alpha\right) = N\left(\frac{\sum_{h=1}^{H}\boldsymbol{\beta}_h^{(c)}}{H},\frac{\mathbf{D}_\beta}{H}\right) \tag{A.3}$$

$$\pi\left(\mathbf{D}_\beta\middle|\left\{\boldsymbol{\beta}_h^{(c)}\right\},\bar{\boldsymbol{\beta}}\right) = IW\left(d_\beta + H, D_\beta + \sum_{h=1}^{H}\left(\boldsymbol{\beta}_h^{(c)} - \bar{\boldsymbol{\beta}}\right)\left(\boldsymbol{\beta}_h^{(c)} - \bar{\boldsymbol{\beta}}\right)'\right) \tag{A.4}$$

$$\pi\left(\bar{\boldsymbol{\phi}}_i\middle|\left\{\boldsymbol{\phi}_{hi}^{*(c)}\right\},\mathbf{D}_{\phi_i}\right) = N\left(\frac{\sum_{h=1}^{H}\boldsymbol{\phi}_{hi}^{*(c)}}{H},\frac{\mathbf{D}_{\phi_i}}{H}\right) \tag{A.5}$$

$$\pi\left(\mathbf{D}_{\phi_i}\middle|\left\{\boldsymbol{\phi}_{hi}^{*(c)}\right\},\bar{\boldsymbol{\phi}}_i\right) = IW\left(d_{\phi_i} + H, D_{\phi_i} + \sum_{h=1}^{H}\left(\boldsymbol{\phi}_{hi}^{*(c)} - \bar{\boldsymbol{\phi}}_i\right)\left(\boldsymbol{\phi}_{hi}^{*(c)} - \bar{\boldsymbol{\phi}}_i\right)'\right) \tag{A.6}$$

$$\pi\left(\bar{\boldsymbol{\gamma}}_i\middle|\left\{\boldsymbol{\gamma}_{hi}^{*(c)}\right\},\mathbf{D}_{\gamma_i}\right) = N\left(\frac{\sum_{h=1}^{H}\boldsymbol{\gamma}_{hi}^{*(c)}}{H},\frac{\mathbf{D}_{\gamma_i}}{H}\right) \tag{A.7}$$

$$\pi\left(\mathbf{D}_{\gamma_i}\middle|\left\{\boldsymbol{\gamma}_{hi}^{*(c)}\right\},\bar{\boldsymbol{\gamma}}_{i_i}\right) = IW\left(d_{\gamma_i} + H, D_{\gamma_i} + \sum_{h=1}^{H}\left(\boldsymbol{\gamma}_{hi}^{*(c)} - \bar{\boldsymbol{\gamma}}_i\right)\left(\boldsymbol{\gamma}_{hi}^{*(c)} - \bar{\boldsymbol{\gamma}}_i\right)'\right) \tag{A.8}$$

$$\pi\left(\bar{\boldsymbol{\rho}}\middle|\left\{\boldsymbol{\rho}_h^{*(c)}\right\},\mathbf{D}_\rho\right) = N\left(\frac{\sum_{h=1}^{H}\boldsymbol{\rho}_h^{*(c)}}{H},\frac{\mathbf{D}_\rho}{H}\right) \tag{A.9}$$

$$\pi\left(\mathbf{D}_\rho \middle| \left\{\boldsymbol{\rho}_h^{*(c)}\right\}, \bar{\boldsymbol{\rho}}\right) = IW\left(d_\rho + H, D_\rho + \sum_{h=1}^{H}\left(\boldsymbol{\rho}_h^{*(c)} - \bar{\boldsymbol{\rho}}\right)\left(\boldsymbol{\rho}_h^{*(c)} - \bar{\boldsymbol{\rho}}\right)'\right) \quad (A.10)$$

$$\pi\left(\bar{\boldsymbol{\omega}} \middle| \left\{\boldsymbol{\omega}_h^{*(c)}\right\}, \mathbf{D}_\omega\right) = N\left(\frac{\sum_{h=1}^{H} \boldsymbol{\omega}_h^{*(c)}}{H}, \frac{\mathbf{D}_\omega}{H}\right) \quad (A.11)$$

$$\pi\left(\mathbf{D}_\omega \middle| \left\{\boldsymbol{\omega}_h^{*(c)}\right\}, \bar{\boldsymbol{\omega}}\right) = IW\left(d_\omega + H, D_\omega + \sum_{h=1}^{H}\left(\boldsymbol{\omega}_h^{*(c)} - \bar{\boldsymbol{\omega}}\right)\left(\boldsymbol{\omega}_h^{*(c)} - \bar{\boldsymbol{\omega}}\right)'\right) \quad (A.12)$$

We use the Metropolis–Hastings method to assess the posterior distributions. A new candidate is augmented by using the random walk algorithm, where the innovation variance was set as 0.9. The posterior means and standard deviations of the parameters were obtained from their posterior distributions we listed below.

1. $\boldsymbol{\alpha}_h | \{x_{hit}^*\}, \bar{\boldsymbol{\alpha}}, \mathbf{D}_\alpha, \boldsymbol{\beta}_h$
2. $\boldsymbol{\beta}_h | \{x_{hit}^*\}, \bar{\boldsymbol{\beta}}, \mathbf{D}_\beta, \boldsymbol{\alpha}_h$
3. $\boldsymbol{\phi}_{hi}^* | \{x_{hit}^*\}, \{y_{hit}^*\}, \bar{\boldsymbol{\phi}}_i, \mathbf{D}_{\phi_i}, \boldsymbol{\gamma}_{hi}^*, \boldsymbol{\rho}_h^*, \boldsymbol{\omega}_h^*$
4. $\boldsymbol{\gamma}_{hi}^* | \{x_{hit}^*\}, \{y_{hit}^*\}, \bar{\boldsymbol{\gamma}}_i, \mathbf{D}_{\gamma_i}, \boldsymbol{\phi}_{hi}^*, \boldsymbol{\rho}_h^*, \boldsymbol{\omega}_h^*$
5. $\boldsymbol{\rho}_h^* | \{x_{hit}^*\}, \{y_{hit}^*\}, \bar{\boldsymbol{\rho}}, \mathbf{D}_\rho, \boldsymbol{\phi}_{hi}^*, \boldsymbol{\gamma}_{hi}^*, \boldsymbol{\omega}_h^*$
6. $\boldsymbol{\omega}_h^* | \{x_{hit}^*\}, \{y_{hit}^*\}, \bar{\boldsymbol{\omega}}, \mathbf{D}_\omega, \boldsymbol{\phi}_{hi}^*, \boldsymbol{\gamma}_{hi}^*, \boldsymbol{\rho}_h^*$

APPENDIX B: DEMAND FUNCTION

We derived the Marshallian demand functions as the solutions of the direct utility maximization problems in Equations 4 and 10. The demand for category i is the optimum quantity that maximizes overall utility whose function is given by:

$$x_{it}(E_t, \tilde{p}_{it}) = \frac{\psi_{it}}{\sum_{j=1}^{m} \psi_{jt} + 1} \frac{E_t - \sum_{j \neq i} \left(\frac{\psi_{jt}}{\psi_{it}} - 1 \right)}{\tilde{p}_{it}}, \tag{B.1}$$

if its marginal utility per dollar is equal to λ, and $x_{it}(E_t, \tilde{p}_{it}) = 0$ if otherwise. We derive brand demand function when the difference between quantity and budget constraint is set to zero (i.e., $z_{it} = 0$). The demand function of brand k is given by:

$$y_{ikt}(x_{it}, \tilde{p}_{it}, \mathbf{p}_{it}) = \frac{\tilde{p}_{it} x_{it} + \sum_{l \neq k} \left(\frac{p_{ilt}}{\gamma_{il}} \right)}{p_{ikt} \left\{ \sum_{l \neq k} \left(\frac{\phi_{il} \gamma_{ik}}{\phi_{ik} \gamma_{il}} \right) + 1 \right\}} - \frac{\sum_{l \neq k} \left(\frac{\phi_{il}}{\gamma_{il}} \right)}{\left\{ \sum_{l \neq k} \left(\frac{\phi_{il} \gamma_{ik}}{\gamma_{il}} \right) + \frac{1}{\phi_{ik}} \right\}}, \tag{B.2}$$

if its marginal utility per dollar is equal to λ, and $y_{ikt}(x_{it}, \tilde{p}_{it}, \mathbf{p}_{it}) = 0$ if otherwise. $\mathbf{p}_{it} = (p_{i1t}, p_{i2t}, \ldots, p_{iKt})'$ is a vector of brand prices.