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Abstract

The dynamic Nelson-Siegel (DNS) model and even the Svensson generalization of the model have trouble in fitting the short maturity yields and fail to grasp the characteristics of the Japanese government bonds (JGBs) yield curve, which is flat at the short end and have multiple inflection points. Therefore, a closely related generalized Nelson-Siegel model (GDNS) with two slopes and curvatures is considered and compared empirically to the traditional DNS in terms of in-sample fit as well as out-of-sample forecasts. Furthermore, the GDNS with time-varying volatility component, modelled as standard EGARCH process, is also considered to evaluate its performance in relation to the GDNS. The GDNS models unanimously outperforms the DNS in terms of in-sample fit as well as out-of-sample forecasts. Moreover, the extended model that accounts for time-varying volatility outpace the other models for fitting the yield curve and produce relatively more accurate 6- and 12-month ahead forecasts, while the GDNS model comes with more precise forecasts for very short forecast horizons.

Keywords: Term structure of interest rates, Latent factors model, State-space model, Kalman filter, EGARCH, Forecasting, Bond market

JEL Classification: C32, C53, C51, E43, G12, G17

1. Introduction

The term structure of interest rates is a static function that relates the time-to-maturity to the zero rates at a given point in time. The conventional way of measuring the term structure is by means of the spot rate curve, or yield curve, on zero-coupon bonds. However, the entire term structure is not directly observable, which gives rise to the need to estimate it using some approximation technique. There are a wide variety of diverse yield models, with objective to accurately model and describe the future yield curve structure as much possible. However, the modelling of a yield curve is more complicated than any other asset pricing. In recent years, the Nelson-Siegel (1987) model and its extended versions have been credited for its high efficacy in the in-sample fitting and out-of-sample forecasting of the term structures of interest rates. Many existing studies (as well as some major central banks around the globe) have been employing the class of Nelson-Siegel (NS) models including the Svensson (1995), Bliss (1997), and Bjork and Christensen (1999) models to estimate and construct zero-coupon yield curves. The Nelson-Siegel type models, because of its parsimonious structure and efficiency in capturing the general shapes of the yield curves, rank them very popular among the term structure models and, therefore, are widely used by market practitioners and central banks.[1]

However, when we estimate the Japanese government bonds (JGBs) yield curve, selecting a method without careful consideration might result in the estimation of a curve that does not grasp the characteristics of the JGBs yield curve and the use of such a zero curve could lead to wrong conclusions. For JGBs since 1999, yield curves under the zero interest rate policy (ZIRP) and the quantitative easing monetary policy (QEMP) have distinctive features. During this periods, the yield curve has a flat shape near zero at the short-term maturities. The second feature frequently seen in the JGBs interest rate term structure is that it has a complex shape with multiple inflection points.[2] Moreover, at some dates the curve is initially falling and then gradually rising (Ullah *et al.* 2013a, b). Some models and estimation methods may not grasp this kind of curve features and shape. Using the Nelson-Siegel functional form, Ullah *et al.* (2014b) has shown that both the dynamic Nelson-Siegel (DNS) as well as affine Nelson-Siegel (AFNS) cannot fit attractively the short maturities if the estimate of decay parameter λ is constrained to be smaller than 0.025 (which fits well long maturities).[3] On the other hand, leaving the λ to be unconstrained implies to fit short maturities very well (λ takes the value around 0.2751). In addition, Christensen *et al.* (2009) show that the main in-sample problem with the regular Nelson-Siegel model is that, for reasonable choices of λ (which are empirically in the range from 0.5 to 1.0 for U.S. Treasury yield data), the

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[1] However, the original Nelson-Siegel type of models are not on par with the dynamic term structure models such as affine or quadratic term structure models, or the Heath *et al.* (1992) arbitrage-free models.

[2] For example, on February 17, 2009, the seven-year interest rate becomes relatively low compared with the six-year and eight-year rates (Kikuchi and Shintani, 2012). Detail description of these features of JGBs yield curve is given in Ullah *et al.* (2013a, 2014b), Kim and Singleton, (2012), and Kikuchi and Shintani, (2012).

[3] The parameter λ in Nelson-Siegel spot rate function specifies the location of the hump or the U-shape on the yield curve. Therefore, the range of shapes the curve can take is dependent on λ. The small values of λ, which have rapid decay in regressors, tend to fit low maturities interest rates quite well and larger values of λ lead to more appropriate fit of longer maturities spot rates.

factor loading for the slope and the curvature factors decay rapidly to zero as a function of maturity. Thus, the model only has the level factor to fit yields with maturities of ten years or longer. In empirical estimation this limitation shows up as a lack of fit either at the short end of curve or of the long-term yields (Christensen *et al.* 2009). This implies that the regular Nelson-Siegel model in both forms, i.e., affine and non-affine versions, cannot replicates the stylized facts and features of the Japanese bond market yield curve, particularly during the ZIRP and QEMP periods.

The more carefully and thorough investigation of JGBs yield curve shapes and descriptive features and of the Nelson-Siegel yield curve functional form, suggest that the single decay parameter λ in the function is at the heart of this problem. The parameter λ determines the exponential decay rate and there is a trade-off between fitting the curvature at short maturities and at long maturities.

In order to avoid such difficulties and select a better candidate model to accurately grasp the characteristics of the JGBs yield curve, in this paper, we consider the generalized version of the Nelson-Siegel model that is discussed in Svensson (1995) and Christensen *et al.* (2009). Foremost among these is the Svensson (1995) extension to the Nelson-Siegel curve.[4] The Svensson extension adds a second curvature term, which allows for a better fit at long maturities. Following Svensson (1995) and Christensen *et al.* (2009), we add second slope and also second curvature to the standard Nelson-Siegel model and introduce a dynamic version of this model, called as generalized dynamic Nelson-Siegel (GDNS) model, which corresponds to a modern five-factor term structure model.[5] The inclusion of second slope will be helpful to fit the very short maturities attractively, as we restrict the role of the newly added slope and curvature factors to the short end of the curve by assuming $\lambda_1 < \lambda_2$.[6] Moreover, prior studies (e.g., Koopman *et al.* 2010; Ullah *et al.* 2014b) show that the inclusion of common volatility component in the DNS improves not only improves the in-sample fitting performance but also the forecasts at the longer horizons. Therefore, we generalize this approach to the GDNS (Nelson-Siegel model with two slopes and curvature factors).

More specifically, to avoid lack of fitting at the short end of JGBs yield curve, this paper extend the standard DNS three-factor model to the five-factor model and also consider its extended version that accounts for time-varying volatility. We estimate the three-factor (DNS), five-factor (GDNS) and five-factor with time-varying volatility (GDNS-EGARCH) models and compare the results in terms of in-sample fitting and out-of-sample forecasting using the JGBs yield data. We find remarkably good in-sample fit and out-of-sample for the GDNS model. We show that the

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[4] It is used at the Federal Reserve Board (see Gurkaynak *et al*. 2007), the European Central Bank (see Coroneo *et al*. 2008), and many other central banks (see Bank for International Settlements, 2005). Detailed discussion is given in De Pooter (2007).

[5] Assuming a second-order differential equation, to describe the movements of the yield curve, with the assumption of real and un-equal roots, the solution will be the instantaneous implied forward rate function. The solution for the yield function can be found by integrating the forward rate function. The resulting yield curve function will consists of five factors (one level, two slopes and two curvatures factors) and two decay parameters, i.e., λ_1 and λ_2, which corresponds to two different roots of the second-order differential equation.

[6] Besides its wide use, the Svensson (1995) model in its dynamic form cannot be derived in the standard finance arbitrage-free affine term structure representation (Christensen *et al.* 2009). However, the model with two slopes and curvature can easily be derived in the arbitrage-free affine framework, by generalizing the method adopted in Christensen *et al.* (2011).

five-factor GDNS model of the yield curve not only outperforms the standard DNS in terms of in-sample fit but also at all forecast horizons. The inclusion of EGARCH effect is helpful only for the long horizon forecasts, but worsens the short horizon forecasts. Moreover, we relate the five factors in the GDNS models to the relevant macroeconomic variables that is helpful to interpret the factors in the macroeconomic scenario, which is one of the widely debated issue that Svensson (1995) fails to offer insight into the economic nature of the underlying forces that drive movements in interest rates. This issue has been addressed by a burgeoning macro-finance literature, which is described in Rudebusch and Wu (2007, 2008).

The remainder of the paper is structured as follows. Section 2 briefly describes the dynamic Nelson-Siegel model and generalized dynamic Nelson-Siegel with and without the time-varying common volatility component (we call the former GDNS-EGARCH and the latter as GDNS). Estimation method is also discussed in the same section. Section 3 presents the data structure and estimation results, while Section 4 describes the out-of-sample forecast performance of the models. Finally, section 5 concludes the paper, and appendices contains some additional details (helpful to understand the models in detail).

2. The term structure models

In this section, we briefly review the three term structure models that are used to estimate and forecast the yield curve, namely the dynamic Nelson-Siegel (DNS) three-factor model, the extended five-factor generalized dynamic Nelson-Siegel (GDNS) model and the GDNS with time-varying volatility (GDNS-EGARCH). In section 2.4, the models are presented in generalized state-space framework along with a brief reference to the estimation method employed in this study to estimate and forecast the yield curve. Lastly, the procedure of implementing the maximization routine and selection of initial values (seeds) for parameters are discussed in section 2.5.

2.1. The dynamic Nelson–Siegel model (DNS)

The Nelson-Siegel model is able to provide a good fit to the cross section of yields at a given point in time, and this is a key reason for its popularity among the financial market practitioners. However, to understand the evolution of the bond market over time, a dynamic representation is required. Diebold and Li (2006) have shown such representation by replacing the parameters with time-varying factors- the so called DNS model. The DNS model fits the yield curve with the simple functional form as:

$$R_t(m) = \beta_{1t} + \beta_{2t}\left[\frac{1-\exp(-\lambda m)}{\lambda m}\right] + \beta_{3t}\left[\frac{1-\exp(-\lambda m)}{\lambda m} - \exp(-\lambda m)\right] + \varepsilon_t(m) \qquad (1)$$

where $R_t(m)$ is the zero-coupon yield for maturity m at time t, $m = 1,2,\dots,N;\ t = 1,2,\dots,T$. Given the Nelson-Siegel factor loadings, Diebold and Li (2006) show that β_{1t}, β_{2t} and β_{3t} can be interpreted as level, slope, and curvature factors. Therefore, we consider $\beta_t = (\beta_{1t}, \beta_{2t}, \beta_{3t})'$ as the unobservable vector of three latent factors of level, slope and curvature respectively. The constant parameter λ is the decay parameter of the factor loading of the yield curve slope. In addition, it is easy to show that the parameter λ determines the exponential decay rate and there is a trade-off between fit curvature at short maturities and at long maturities.

Furthermore, once the model is viewed as a factor model, a dynamic structure can be postulated for the three factors, which yields a fully dynamic version of the original Nelson-Siegel model. For modelling the entire yield curve consistently and simultaneously, we need a state-space formulation of the model.[7] Therefore, we assume that the yield curve latent factors vector β_t follow a vector autoregressive process of first order, which allows to formulate the yield curve latent factors model in the state-space form, with observation and transition equations (2 and 3 respectively) as:

$$R_t = \Lambda(\lambda)\beta_t + \varepsilon_t \tag{2}$$

$$\beta_{t+1} = (I_3 - A)\mu + A\beta_t + v_{t+1} \tag{3}$$

$$\begin{bmatrix} \varepsilon_t \\ v_{t+1} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Omega & 0 \\ 0 & \Sigma_v \end{bmatrix} \right) \tag{4}$$

where R_t is (N×1) vector of zero-coupon yields, $\Lambda(\lambda)$ is (N×3) matrix of loadings, $\beta_t = (\beta_{1t}, \beta_{2t}, \beta_{3t})'$ is the (3×1) vector of latent factors of the yield curve, μ is (3×1) vectors of factors mean, and A is (3×3) full-matrix of parameters. The ε_t and v_t are (N×1) and (3×1) innovations vectors of the observation and state equations respectively, Ω is (N×N) covariance matrix of the measurement equation innovations, and Σ_v is (3×3) covariance matrix of the state innovations.

The main reasons why the Nelson-Siegel model is popular among market practitioners are its parsimonious functional form (in contrast to spline models) and its effectiveness in capturing the general shapes of the yield curve (including increasing, decreasing, humped, inverted-humped, and even S-type). Inspired by these advantages, several extensions have been proposed to increase the flexibility of model to fit optimally more irregular and complex shapes such as twists.[8]

2.2. The generalized dynamic Nelson-Siegel model (GDNS)

Despite the good empirical performance and parsimonious structure, the DNS model suffer from the lack of fitting the yield curve when the curve has multiple humps and inflection points. The single decay parameter λ implies either to fit attractively the short rate or the long rates. This difficulty can

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[7] Moreover, Diebold *et al.* (2006) find that the time series of estimated factors of Nelson-Siegel model are highly persistent, which implies that these can easily be modeled as AR(1) or VAR(1). Using the Japanese market data Ullah *et al.* (2013a), and Ullah *et al.* (2013b) find that the three latent factors of yield curve are highly persistent and VAR(1) specification is more appropriate than the AR(1) and random walk specifications.

[8] Two typical examples in this line are Bjork and Christensen (1999) and Svensson (1995). Bjork and Christensen (1999) make progress in this direction by adding a fourth factor to the NS model. The fourth factor also affects short-term maturities as the second component and can be interpreted as a second slope factor. The model only introduces one more parameter β_{4t} in that two slope factors are governed by the same parameter but with different decay rates. Svensson (1995) suggests another kind of four-factor extended NS specification by adding a second curvature factor.

be overcome by introducing the second slope and curvature factors in the standard DNS model.[9]

In the JGBs market, for the estimated λ (which are empirically in the range from 0.025 to 0.019), the factor loading for the curvature factor does not increase sharply to play its due role at the short end, while the slope factor loading is close to one (does not decay rapidly), and thus, the model only has the level and slope factors to fit yields with maturities of 3-month to 36-month (for these maturities the yield curve is flat and also has some humps). Therefore, sometimes the estimated yield becomes negative if much weighted is assigned to slope factor than the level factor because of having negative estimate of β_{2t} (due to the fact of observing the upward sloped curve almost for all t during the ZIRP and QEMP periods). Furthermore, if λ is allowed freely to vary, the model suffers ruthlessly from the lack of fit at the long end of curve.

To overcome this limitation in fitting the cross section of yields, we employ the extended version of the Nelson-Siegel yield curve with an additional slope as well as curvature factors to the JGBs market data, defined as:

$$\begin{aligned} R_t(m) = \beta_{1t} &+ \beta_{2t}\left[\frac{1-\exp(-\lambda_1 m)}{\lambda_1 m}\right] + \beta_{3t}\left[\frac{1-\exp(-\lambda_2 m)}{\lambda_2 m}\right] \\ &+ \beta_{4t}\left[\frac{1-\exp(-\lambda_1 m)}{\lambda_1 m} - \exp(-\lambda_1 m)\right] \\ &+ \beta_{5t}\left[\frac{1-\exp(-\lambda_2 m)}{\lambda_2 m} - \exp(-\lambda_2 m)\right] + \varepsilon_t(m) \end{aligned} \quad (5)$$

This generalized dynamic Nelson-Siegel model, which we denote as the GDNS model, is a five-factor model with one level, two slopes, and two curvatures factors. Here β_{1t} is the asymptotic value of the spot rate function, which can be seen as the long-term interest rate and is assumed to be positive ($\beta_{1t} > 0$). Furthermore, β_{2t} and β_{3t} determines the rate of convergence with which the spot rate function approaches its long-term trend. Furthermore, the factors β_{4t} and β_{5t} determines the size and the form of the humps. The two slopes and curvatures factors are governed by the two different decay rates, i.e., λ_1 and λ_2. In this framework, β_{2t} and β_{3t} refers to the first slope and slope factors, while β_{4t} and β_{5t} can be termed as first and second curvature factors, respectively.

If $\lambda_1 < \lambda_2$, then the value of λ_1 will serve to fit the long rates attractively (the first slope and curvature factors, i.e., β_{2t} and β_{4t} loadings will approach comparatively slowly to its asymptotic values, apparent in figure 1), while λ_2 will be helpful to fit the short end of cure more reasonably (the second slope and curvature, i.e., β_{3t} and β_{5t} loadings will decay more rapidly, shown in figure 1).

[9] The model with two slope factors (as in Bjork and Christensen, 1999) or two curvatures (such as in Svensson, 1995) may also serve the purpose of fitting curves with special shapes, such as twists, but Christensen *et al.* (2009) shows that the model, which accounts for two slope and curvature factors simultaneously, outperforms the standard Bjork and Christensen, (1999) and Svensson, (1995) models. Secondly, the models with either two slopes or two curvatures cannot be derived in the affine framework (for detail see Christensen *et al.* 2009).

This extension increases the flexibility of the model, by allowing for two different slope and curvature factors that are governed by two different parameters λ_1 and λ_2, to fit curves with special shapes and multiple humps, such as twists. However, there is a trade-off between better fitting and parameters estimation. The functional form imposed on the forward rates to derive the spot rate function as in (5) leads to a flexible, smooth parametric function of the term structure that is capable of capturing many of the typically observed shapes that the of JGBs yield curve assumes over time and captures most of its empirical properties.

It is worthwhile to mention that we impose the restriction of $\lambda_1 < \lambda_2$, which is non-binding due to symmetry. The factor loadings of the two slopes and curvatures in the yield function of the GDNS model are illustrated in figure 1 with λ_1 and λ_2 set equal to our estimates in section 3.

<<Figure 1>>

The GDNS is a five-factor model, the time dimension of the five factors can be modeled in terms of dynamics as an AR or VAR. However, we assume that the five time-varying factors follows the VAR(1) process because of the consistency with the DNS model presented in the previous section. The model in state-space framework can be written as:

$$R_t = \Lambda(\lambda_1, \lambda_2)\beta_t + \varepsilon_t \tag{6}$$

$$\beta_{t+1} = (I_5 - A)\mu + A\beta_t + v_{t+1} \tag{7}$$

$$\begin{bmatrix} \varepsilon_t \\ v_{t+1} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Omega & 0 \\ 0 & \Sigma_v \end{bmatrix}\right) \tag{8}$$

The state-space specification for the GDNS is exactly similar to the one specified for DNS in (2-4), but the dimension of matrices and vectors is different. Here, $\Lambda(\lambda_1, \lambda_2)$ is (N×5) matrix of loadings, $\beta_t = (\beta_{1t}, \beta_{2t}, \beta_{3t}, \beta_{4t}, \beta_{5t})'$ is the (5×1) vector of latent factors of the yield curve, μ is (5×1) vectors of factors mean, A is (5×5) full-matrix of parameters, v_t is (5×1) innovations vector of the state equation, and Σ_v is (5×5) covariance matrix of the state innovations. The dimensions of R_t, ε_t and Ω are similar to the one defined for the DNS model.

2.3. The generalized dynamic Nelson-Siegel model with time-varying volatility (GDNS-EGARCH)

The standard DNS as well as GDNS models assumes that the volatility in interest rates is constant over time. However, the interest rates are the result of trading at bond market and the volatility in the series may be time-varying. Therefore, incorporating the concept of common volatility component of Harvey *et al.* (1992), we decompose the error term, ε_t, in the GDNS model as:

$$\varepsilon_t = \Gamma_\varepsilon \varepsilon_t^* + \varepsilon_t^+ , \qquad \varepsilon_t^+ \sim N(0, \Omega), \qquad \varepsilon_t^* | \zeta_{t-1} \sim N(0, h_t) \tag{9}$$

where Γ_ε and ε_t^+ are (N×1) vectors of loadings and noise component respectively, and ε_t^* is a scalar representing the common disturbance term that has the time-varying variance denoted as h_t.

The h_t follows the EGARCH(1,1) specification, which is given by:[10]

$$\log(h_t) = \gamma_0 + \gamma_1 \frac{\varepsilon_{t-1}^*}{\sqrt{h_{t-1}}} + \gamma_2 \log(h_{t-1}) + \psi\left(\left|\frac{\varepsilon_{t-1}^*}{\sqrt{h_{t-1}}}\right| - \mathbb{E}\left[\left|\frac{\varepsilon_{t-1}^*}{\sqrt{h_{t-1}}}\right|\right]\right) \quad (10)$$

where $\mathbb{E}\left(\left|\varepsilon_{t-1}^*/\sqrt{h_{t-1}}\right|\right)$ is the expectation of the absolute value of a standard normally distributed random variable, which is equal to $\sqrt{2/\pi}$. This specification for variance dynamics enable the common volatility component in the GDNS model to account for asymmetric response to positive and negative shocks.

Adding a common component allows the model to capture latent exogenous shocks that affect the entire yield curve and are not captured by the five-factor structure of the level, slopes and curvatures factors. This extension increases the flexibility of the term structure model and enables it to better fit more complex shapes of the yield curve.

In the state-space representation the complete model (GDNS-EGARCH) can be written as:

$$R_t = [\Lambda(\lambda_1, \lambda_2) \quad \Gamma_\varepsilon] \begin{bmatrix} \beta_t \\ \varepsilon_t^* \end{bmatrix} + \varepsilon_t^+ \quad (11)$$

$$\alpha_{t+1} = \begin{bmatrix} (I_5 - A)\mu \\ 0 \end{bmatrix} + \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \alpha_t + \begin{bmatrix} v_{t+1} \\ \varepsilon_{t+1}^* \end{bmatrix} \quad (12)$$

$$\begin{bmatrix} \varepsilon_t^+ \\ v_{t+1} \\ \varepsilon_{t+1}^* \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Omega & 0 & 0 \\ 0 & \Sigma_v & 0 \\ 0 & 0 & h_{t+1} \end{bmatrix}\right) \quad (13)$$

where $\alpha_t = (\beta_{1t}, \beta_{2t}, \beta_{3t}, \beta_{4t}, \beta_{5t}, \varepsilon_t^*)'$ is (6×1) latent vector and Γ_ε is (N×1) vector showing the sensitivity of various yields to a common shock component. The definitions and dimensions of all remaining matrices and vectors is same as discussed in the GDNS model specification. Furthermore, we assume that the innovations, ε_t^+ and v_t, as well as common volatility component, ε_t^*, have Gaussian distribution. The model in equations (11 – 13) provides a flexible framework to fit the yield curve, while simultaneously accounts for the time-varying stochastic volatility in yields for all maturities.

2.4. Statistical formulation of the models and estimation method

In this subsection, the models are presented in the general state-space framework along with the estimation procedure. The estimation is based on the Kalman filter. For convenience, we introduce some new notations and rewrite the signal and state equations to obtain the generalized form for models. The state-space form of the models is given as:

[10] Koopman *et al.* (2010) has used the GARCH specification to model the variance h_t, but financial markets respond in different ways to positive and negative shocks and it is a common knowledge that volatility tends to increase quickly when negative news reaches to traders and investors, whereas, positive news usually has a much less pronounced effect (Ullah *et al.* 2014a).

$$R_t = HX_t + w_t , \quad \forall\, t = 1,2, \dots , T \tag{14}$$
$$X_t = C + FX_{t-1} + u_t \tag{15}$$
$$\begin{bmatrix} w_t \\ u_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} G & 0 \\ 0 & Q_t \end{bmatrix}\right) \tag{16}$$

where the expressions of $F, X_t, C, H, G, Q_t, w_t$ and u_t in case of DNS, GDNS and GDFNS-EGARCH are given in Appendix-I. In all three models, the matrix G is assumed to be diagonal for computational traceability, while the covariance matrix Q_t is non-diagonal. Moreover, the transition and the measurement errors are assumed to be orthogonal to the initial state.

The Kalman filter algorithm is implemented along the lines of Hamilton (1994) to evaluate the Gaussian likelihood function and obtain the latent factor as well as estimates of the hyper-parameters. Denoting the optimal estimate of latent factors X_t given the information until time $t-1$ or t, as $\hat{X}_{t|t-1}$ and $\hat{X}_{t|t}$ respectively, the recursive prediction step is calculated as:

$$\hat{X}_{t|t-1} = C + F\hat{X}_{t-1} \tag{17}$$
$$P_{t|t-1} = FP_{t-1}F' + Q_t \tag{18}$$

where $P_{t|t-1}$ is the mean square error (MSE) matrix at the prediction step. Using the measurement equation, these estimates are improved by observing R_t , thus in the update step:

$$\hat{X}_{t|t} = \hat{X}_{t|t-1} + P_{t|t-1}H'K_t^{-1}\eta_t \tag{19}$$
$$P_{t|t} = P_{t|t-1} - P_{t|t-1}H'K_t^{-1}HP_{t|t-1} \tag{20}$$

where $\eta_t = R_t - H\hat{X}_{t|t-1}$ (the forecast error vector) and $K_t = HP_{t|t-1}H' + G$ (the MSE matrix of η_t). The Kalman filter iterative process begins with X_0 and P_0 being set at the μ and unconditional covariance matrix respectively as discussed in Hamilton (1994). The last diagonal element of P_0 in EGARCH based model is set equal to h_1, which is the unconditional expectation of the log variance and computed as $\mathbb{E}[\log(h_t)] = \gamma_0(1-\gamma_2)^{-1}$.

Furthermore, in the GDNS-EGARCH model matrix Q_t contains h_{t+1} that is modeled by EGARCH process and relies on latent shocks at time t, which are unobservable. The h_{t+1} is computed by taking the conditional expectation at $t-1$of the latent variables in (10). For detail description on the computation of EGARCH process in the Nelson-Siegel framework, see Ullah *et al.* (2014a).[11]

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[11] The conditional expectation at $t-1$ of the latent variables in (10) gives:

$$\log(h_t) = \gamma_0 + \gamma_1\mathbb{E}_{t-1}\left(\frac{\varepsilon_{t-1}^*}{\sqrt{h_{t-1}}}\right) + \gamma_2\log(h_{t-1}) + \psi\mathbb{E}_{t-1}\left(\left|\frac{\varepsilon_{t-1}^*}{\sqrt{h_{t-1}}}\right| - \mathbb{E}_{t-1}\left[\left|\frac{\varepsilon_{t-1}^*}{\sqrt{h_{t-1}}}\right|\right]\right)$$

The model parameters vector $\xi = (\lambda_1, \lambda_2, H, C, F, G, Q_t)$ are estimated by maximizing the log-likelihood function, assuming that the forecasting errors η_t are Gaussian. The numerical optimization routine of Nelder-Mead, being popular as simplex algorithm, is used to maximize the log likelihood function and obtain the MLE estimates of the parameters.

For inferences, the covariance matrix of the estimates is calculated by inverting the negative of the Hessian evaluated at the optimum, where the Hessian itself was approximated by finite differences after reverting back to the original parameterization, as suggested by Hamilton (1994).

2.5. Models analysis

Estimating the parameters of the Nelson-Siegel family of models in the state-space framework is done by finding parameter values that optimize the likelihood function. Due to the large number of parameters in the models presented, particularly the GDNS and GDNS-EGARCH, the optimization problems are of high dimensional and the likelihood surface may be very noisy (have multiple local maxima). In order to start the optimization procedure, we choose certain initial values for the model parameters that were expected to be most likely that lead to the global optimum. However, the sensitivity of the optimization outcome to the initial values will be large if certain dynamics are not present in the data and, hence, the algorithm can encounter difficulties in finding the global maximum of the likelihood function.

This problem is avoided by choosing the more appropriate initial values for parameters vector denoted as $\xi^{(0)}$ in order to obtain the optimal MLE estimates of the hyper parameters. The process of choosing the initial values is carried out in multiple steps. We start with the most simple three-factor DNS model and subsequently extend the analysis to the highly parameterized GDNS and GDNS-EGARCH models.

Initially few parameters are allowed to vary freely (i.e., mean vector μ), while all other parameters are fixed to some constant values (fixed values are taken from Ullah *et al.* 2013b) and the log likelihood function is optimized over a randomly drawn sample from the entire data-set to obtain the optimal estimates for freely varying parameters. In the second stage, the parameters vector is expanded (decay parameter λ is included) and the log likelihood function is optimized once again. This information is incorporated into the next run. During the second run, μ is seeded with the previous run optimal values. In this way the parameter vector is steadily expanded until it includes the full set of parameters in the model and the parameter vector is fully specified. The full specification includes 39 parameters in the parameters vector. In this incremental progressing, we identify the starting values for almost all the parameters in the fully specified model. These identified values are then used as initial values, i.e., the parameters vector $\xi^{(0)}$, for optimizing the full model on the entire sample data-set. This process is discussed in detail in Ullah *et al.* (2014c).

This whole process for the DNS model involves running the model 10 times, each time with an

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where the estimate of $\mathbb{E}_{t-1}(\varepsilon^*_{t-1})$ is the last element of $X_{t|t}$ from the update step.

expanded parameters vector. Similar process is carried out to obtain the optimal seeds for the GDNS and GDNS-EGARCH models. The incremental steps for the GDNS model involves 14 runs, while 20 runs for the GDNS-EGARCH models. These simulated estimates are then used as initial values for the parameters vector to optimize the full models.[12]

3. Empirical application

In this section we provide empirical evidences on the in-sample fitting performance of the three models, i.e., DNS, GDNS and GDNS-EGARCH. In doing so we answer two principle questions: (i) what is the role of second slope and curvature factors in terms of in-sample fitting the yield curve? (ii) does incorporating the time-varying volatility in the yield curve model improve the performance of underlying model and what is the implication of implied time-varying volatility? We employ the Kalman filter algorithm to the panel of zero-coupon yields for various maturities derived from the bond pricing data in the Japanese bond market to obtain optimal estimates of the latent factors and the MLE estimates of the unknown parameters. The details of the data-set are provided in section 3.1. The estimation results of models of the yield curve are presented in section 3.2. Furthermore, some performance features of the models in terms of in-sample fitting are also presented in the same section.

3.1. Data description

The empirical results are based on the Japanese interest rates that are constructed by employing the Fama-Bliss (1987) methodology of calculating the unsmoothed Fama-Bliss zero-coupon yields from bond pricing data. These have been constructed from average bid-ask price quotes, retrieved from the Japan Securities Dealers Association (JSDA) and the Tokyo Stock Exchange (TSE) bonds files. The bonds with maturity of less than two months and inflation indexed bonds are excluded from the sample.[13] The remaining quotes are used to construct forward rates using the Fama and Bliss (1987) methodology. The forward rates are then averaged to construct constant maturity spot rates. These unsmoothed yields exactly price the underlying bonds.

The resulting balanced panel data-set consists of 20 maturities over the period January 1996 to December 2013 with maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, 120, 180, 240 and 300 months (20 maturities).

The summary statistics of the yields for various maturities along with a three-dimensional plot of the data-set are discussed in detail in appendix II. The descriptive statistics and three-dimensional plot of the data show that the typical yield curve have been upward sloping and the short rates are

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[12] See Ullah *et al.* (2014c) for detail on drawing the random sample from the data-set, convergence criterion and steps of expanding the parameters vector.

[13] The bonds of maturity less than two months have almost same prices because of the very low interest during the sampled period and it implies to some strange estimates of the zero-coupon rates, such as the rate for one-month maturity is higher than of the one and half month maturity bonds. Moreover, the inflation indexed bonds have floating rates (coupon is not fixed) that change in each period. Therefore, the bonds with maturity of less than two months and floating rates are omitted from the sample at the stage of calculating the zero-coupon rates.

almost zero during the prolonged period except with a little rise in late 2006 and early 2007. During the period of a rise in very low maturity interest rates, the fall in the slope is also apparent. Moreover, short maturities are less volatile than long rates.

3.2. Estimation results

For given values of the system matrices, we use the Kalman filter to evaluate minimum mean square linear estimates (MMSLE) of the state vector at time t given the observation. The Kalman filter is also used to evaluate the log likelihood function via the prediction error decomposition. The maximum likelihood estimates of the unknown parameters are obtained via the numerical optimization of the log likelihood function. To generate the results in this paper, we used the Simplex algorithm to perform the optimization.

The Kalman filter is initialized using the $X_0 = \mu$ and unconditional covariance matrix of the state vector P_0, which is derived from the Gaussian distribution, given that the innovations of both signal and state equations are normally distributed. The Kalman filter algorithm is sensitive to the initializing values of parameters, we use the estimates of the parameters of the simulation exercise, discussed in section 2.5, as the initial values.

Table 1 presents the estimated mean-reversion matrix A and the estimated vector of mean parameters μ in panel 1, along with the estimated parameters of the EGARCH equation obtained for the GDNS-EGARCH model in panel 2. The results reveal that all the diagonal elements of the mean revision matrix A are highly significant in all three setups. The level factor is most persistent in the DNS model, followed by the GDNS-EGARCH model. Considering the slope and curvature (first slope and curvature in GDNS and GDNS-EGARCH frameworks), the results indicate that the slope factor is more persistent in DNS and GDNS model than the curvature, while the latter is more persistent in the GDNS-EGARCH model. Regarding the second slope and curvature factors in GDNS and GDNS-EGARCH models, the persistency of β_{3t} and β_{5t} is reasonably lower as compared to the other factors, however, the second curvature is more persistent than the second slope factor in both setups. The results also show that the cross factors dynamics play a significant role in explaining the variation in all factors. In the DNS framework, the lagged curvature has positive significant impact on the level as well as the slope factors. The level factor has also significant negative impact on the curvature factor. In the generalized (extended) models, i.e., GDNS and GDNS-EGARCH, the cross-factor effects are almost similar in terms of statistical significance and direction of impact, except the impact of $\beta_{2,t-1}$ on β_{1t} and $\beta_{1,t-1}$ on β_{3t} (these impacts are significant in the GDNS while statistically insignificant in the GDNS-EGARCH specification). These two terms may be become insignificant because of the inclusion of the common volatility component in the model. Since, 13 out of 20 cross-factor interaction terms are significant statistically at 5 % level in the GDNS model, while 11 out of 20 are significant in the EGARCH based framework. The results show that the level, first slope and two curvatures play more prominent role than the rest of factors. More interestingly, by including the second slope and curvature factors, the persistency of level factor falls remarkably (comparing the DNS with the GDNS and GDNS-EGARCH level factor persistency). The first slope and curvature in the extended models are more persistent than the second slope and curvature factors.

<<Table 1>>

The results indicate that the estimates of mean vector μ are highly significant in all three setups. For the estimated mean parameters, we find little change after adding the second slope and curvature factor to the model. It seems like the uncertainty about these parameters has declined notably. This ties in well with the fact that the factors have become less persistent, which allows the estimation to determine their means more precisely. Moreover, the elements of the estimated mean vector are a bit larger in the EGARCH based model as compared to the mean vector of the GDNS model, however, the uncertainty about the parameters falls significant in the GDNS-EGARCH model.

The estimate of decay parameter in the DNS framework is almost similar to the estimated λ_1 for the GDNS and GDNS-EGARCH models. These estimates suggest that curvature factor loading reaches its maximum at about 72 months maturity in the DNS model, while the first curvature loading at about 63 and 66 months maturity in the GDNS and GDNS-EGARCH frameworks, respectively. This confers that the estimated λ in DNS and λ_1 in GDNS and GDNS-EGARCH serves to fit long rates attractively, because of very low decay rate of the slope and curvature factors loadings. Therefore, the first slope and curvature factors will affect the important intermediate range of maturities from 5 to 20 years of maturity.

The estimated λ_2 in the two extended models suggest that the second curvature loading peaks at about 21 and 15 months maturity in the GDNS and GDNS-EGARCH models respectively. Therefore, the second slope and curvature factors take on very different roles in the fit of the model as compared to the standard DNS model. The rapid decay rate of the second slope loading will be helpful to fit the short rates more accurately and precisely, as it was one of the problem with the standard DNS model in fitting the JGBs yield curve during the zero interest rate policy (ZIRP) period. Furthermore, to clearly illustrate the role of second slope and curvature, in figure 1 we plot the loadings of slope and curvature factors for all the three models. Figure 1 shows that the path of loadings for slope and curvature in DNS and first slope and curvature in both extended models is almost similar because of having almost the same estimate for λ (in DNS) and λ_1 (in GDNS and GDNS-EGARCH). The second slope and curvature loadings in GDNS and GDNS-EGARCH corresponds to the estimate of λ_2, which have very high decay rate and are helpful to fit attractively the short rates. Thus, these two factors have a limited impact on yields beyond the five-year maturity. The inclusion of the new slope and curvature factors to the DNS model are also helpful to refrain the very short rates from becoming negative during the ZIRP period.

Panel 2 of table 1 presents the estimates of the EGARCH equation parameters for the GDNS-EGARCH model. All the four estimates are statistically significant and the significance of fourth parameter, i.e., ψ supports the hypothesis of asymmetric volatility in the common shock component. Moreover, the high and significant estimate of the γ_1 indicates that much weight is put on recent shocks. The lag volatility coefficient γ_2 in the EGARCH equation is low but statistically different from zero. Therefore, the volatility of the common component is highly sensitive to the latest innovations; it increases quickly with large shocks and reverts soon thereafter. In order to obtain a better insight, in figure 2, common volatility h_t is plotted over time. Some historical events regarding the Japanese monetary policy and world financial market are clearly illustrated in the graph.

<<Figure 2>>

During early 1990s, the Japanese economy slowed down considerably because of stock market bubble burst of 1990.[14] As a consequence, the discount rate was lowered to stimulate the economy and the monetary policy was also relaxed in response to weakening of the economy. The official discount rate (ODR) was gradually lowered from 5.5% to 1% in April 1995, and finally to 0.5% in September 1995. Due to easy monetary policy and fall in the discount rate, we observe that level factor of yield curve as well as the slope factor fall (apparent in figure 3 and 4). This phenomena is reflected by a higher volatility in bond prices and consequently in yields on bonds, as we observe a higher volatility in 1996. The impacts of Asian currency crisis of 1997 and bad debt crisis in 1997-98 are also clearly highlighted by a rise in the estimated conditional volatility. The big spike in late 1999 and early 2000 corresponds to the adoption of zero interest rate policy (ZIRP) by the Bank of Japan (BOJ). The next two hikes, i.e., in May 2002 and mid-2004 are also relevant to the monetary policy regimes of the Japanese economy, as discussed in Ullah *et al*. (2014a). During the first period, the BOJ launched quantitative easy monetary policy (QEMP) to affect long-term interest rates in order to stimulate the economy, while in mid-2004 the forward rates jumped up sharply because of higher expectation of an exit from deflation and ZIRP in the near future. Furthermore, the rise in 2008 and 2010 corresponds to the global financial crisis of 2008 and Euro-zone crisis of 2010 respectively. The last spike that occurs in early 2013 matches to the recent momentous move of the BOJ about the quantitative easing of monetary policy.

In figure 3, we compare the level, the first slope, and the first curvature factors in the GDNS, GDNS-EGARCH and DNS models to their corresponding empirical proxies. Moreover, the second curvature of GDNS is also plotted with the ten-year maturity yield. The empirical level factor (L_t) is defined as the 25-year yield, slope (S_t) as the difference between the 25-year and 3-month yields and curvature (C_t) as two times the 2-year yield minus the sum of the 25-years and 3- month zero-coupon yields. The pairwise correlation of estimated level factor in all three setups with empirically defined level factor is $\rho\left(\hat{\beta}_{1t}^{DNS}, L_t\right) = 0.9681, \rho\left(\hat{\beta}_{1t}^{GDNS}, L_t\right) = 0.8274$ and $\rho\left(\hat{\beta}_{1t}^{GDNS-EGARCH}, L_t\right) = 0.8421$, whereas the correlation of the estimated first slope (i.e., $\hat{\beta}_{2t}$) with the empirical slope is $\rho\left(\hat{\beta}_{2t}^{DNS}, S_t\right) = -0.9576, \rho\left(\hat{\beta}_{2t}^{GDNS}, S_t\right) = -0.9029$ and$\rho(\beta_{2t}^{GDNS-EGARCH}, S_t) = -0.6522$. The correlation for first estimated curvature and empirical

[14] Stock prices plummeted in the summer of 1992, to the level of 15,000 in Nikke 225 index, losing more than 60% of the peak value in two and half years. The quarter-to-quarter GDP growth rate became negative in the spring-summer of 1992.

curvature is $\rho(\hat{\beta}_{3t}^{DNS}, C_t) = 0.7106, \rho(\hat{\beta}_{4t}^{GDNS}, C_t) = 0.6121$ and $\rho(\hat{\beta}_{4t}^{GDNS-EGARCH}, C_t) = 0.9079$, while the correlation of second curvature $\hat{\beta}_{5t}$ with the ten-year maturity yields is -0.9436 and -0.9369 in the GDNS and GDNS-EGARCH models respectively.[15] It indicates that the level factor is affected by the addition of a second slope and curvature factors in both GDNS and GDNS-EGARCH models. Also, the first slope and curvature factors have very similar sample paths across all three models, but their role become more limited to the long end in GDNS and GDNS-EGARCH as compared to its role in the DNS model. More importantly, there is a clear correlation between the second curvature factor and the ten-year yield (also evident from the right-bottom graph in figure 3), in both GDNS and GDNS-EGARCH models, implying that the impact of second slope is influential at the short end of the curve (until 10-year maturity).

<<Figure 3>>

Given the fairly large estimated correlation across factors in three models and also with their empirical proxies, and closely following pattern of empirically defined factors, it is legitimated to term the estimated latent variables as level, slope and curvature factors.

Furthermore, the estimated factors of the extended models, i.e., GDNS and GDNS-EGARCH are closely related to the macroeconomic fundamental and will be helpful for the policy related issues (i.e., monetary policy) and forecasting the future state of economy. The time series of the yield curve factors' estimates of GDNS model with potentially related macroeconomic variables are plotted in figure 4. The figure show the time series path of the estimated level $\hat{\beta}_{1t}$ and first slope $\hat{\beta}_{2t}$ factors against money supply (MS_t), the first curvature $\hat{\beta}_{4t}$ against the inflation rate (INF_t) and second slope $\hat{\beta}_{3t}$ factor against the growth rate in industrial production (IP_t).[16] The pattern of level factor along with the first slope factor is closely related to annual growth of money supply as depicted in the top pane of figure 4. It confers that shocks to monetary policy are important sources of variation in long end of the yield curve and pricing the long-term maturity bonds. The figure shows that fall in money growth is accompanied by a rise in the level factors, i.e., $\hat{\beta}_{1t}$ and fall in the slope of yield curve ($\hat{\beta}_{2t}$ rises), while an increase in money supply is reflected by a fall in $\hat{\beta}_{1t}$ and $\hat{\beta}_{2t}$ (yield curve becomes steeper). It means that the monetary policy signal transmits through the yield curve level and spread factors to the real sector. It suggests that the shift of long end and hence, the slope of yield curve have important information about the state of economy. This mechanism is more obviously illustrated by the second slope factor, i.e., $\hat{\beta}_{3t}$ (pane

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[15] The pairwise correlation of estimated factors across three models is as follows. The correlation of estimated level factor is $\rho(\hat{\beta}_{1t}^{DNS}, \hat{\beta}_{1t}^{GDNS}) = 0.9217, \rho(\hat{\beta}_{1t}^{DNS}, \hat{\beta}_{1t}^{GDNS-EGARCH}) = 0.9805$ and $\rho(\hat{\beta}_{1t}^{GDNS}, \hat{\beta}_{1t}^{GDNS-EGARCH}) = 0.8750$, for the estimated first slope factor is $\rho(\hat{\beta}_{2t}^{DNS}, \hat{\beta}_{2t}^{GDNS}) = 0.9343, \rho(\hat{\beta}_{2t}^{DNS}, \hat{\beta}_{2t}^{GDNS-EGARCH}) = 0.5523$ and $\rho(\hat{\beta}_{2t}^{GDNS}, \hat{\beta}_{2t}^{GDNS-EGARCH}) = 0.5317$, whereas for the first curvature across three models is $\rho(\hat{\beta}_{3t}^{DNS}, \hat{\beta}_{4t}^{GDNS}) = 0.5352, \rho(\hat{\beta}_{3t}^{DNS}, \hat{\beta}_{4t}^{GDNS-EGARCH}) = 0.5671$and $\rho(\hat{\beta}_{4t}^{GDNS}, \hat{\beta}_{4t}^{GDNS-EGARCH}) = 0.6281$. The correlation of second slope and second curvature factors in the GDNS and GDNS-EGARCH models is 0.9193 and 0.9435 respectively. It shows that the estimated factors follows almost the same pattern and are closely correlated across models.

[16] The data for the macroeconomic variables, the annualized growth of industrial production (IP_t), the growth rate of $M2$ money supply (MS_t) as an indicator of monetary policy; and inflation rate (INF_t), measured as annualized monthly changes in the consumer price index is obtained from the International Financial Statistics (IFS).

2 of figure 4), which has a very clear relation with the industrial production. The figure shows that decrease in second slope ($\hat{\beta}_{3t}$ rises) is followed by a fall in the real activity with one period lag and vice-versa, suggesting that decline in the slope of yield curve (becoming flat or more negatively sloped) can be considered as a signal of economic slowdown.[17]

<<Figure 4>>

Moreover, the variation in inflation is closely explained by the first curvature factor of the yield curve. The CPI based inflation rate closely follows the pattern of curvature factor of yield curve as depicted in the bottom panel of figure 4. The correlation between inflation rate and curvature factor is $\rho(\hat{\beta}_{4t}, INF_{t-1}) = -0.4366$, $\rho(\hat{\beta}_{4,t-1}, INF_t) = -0.3108$ and $\rho(\hat{\beta}_{4t}, INF_t) = -0.3473$. In addition, the curvature factor is also closely related to the growth rate in money supply.

Overall figure 4 suggests that during the initial period of adopting the ZIRP and QEMP and world financial crisis, we observe a decline in the yields of long-term bonds and slope of yield curve and during the period of recovery, the yield curve long end as well as slope are on the increasing trend. In particular, the curvature reflects the cyclical fluctuations of the economy too. Like the yield curve spread, a decrease in curvature is signaling towards economic slowdown and vice versa. It is worth noting that the fall in curvature appears to complement the transition from an upward sloping yield curve to a flat one. Furthermore, the curvature factor seems either to anticipate the future inflation or complemented by inflation rate, suggesting that the curvature factor is the main driving force of the inflation rate, and transmits the stance of monetary policy in yield curve shape and hence the economy.

<<Table 2>>

To compare the transition errors of the three models, the estimates of covariance matrices of the state innovations as depicted by Σ_v are reported in table 2 (panel 1). The results indicate that all diagonal elements of the matrix Σ_v that correspond to the variance of the state innovations are statistically significant in all three setups. Regarding the off-diagonal elements, the results show that two out of three covariance terms for DNS, four out of ten covariance terms for both GDNS and GDNS-EGARCH are statistically different from zero. Relatively, most of the variance and covariance terms for the GDNS-EGARCH are smaller than the corresponding term in GDNS and DNS, whereas that of GDNS are smaller than in the DNS setup. As most of the covariance terms in GDNS and GDNS-EGARCH are statistically not different from zero, we employ the Wald and Likelihood ratio (LR) tests for the joint significance of the off-diagonal elements of the matrix Σ_v, (i.e., considering the diagonal Σ_v). The results of Wald and LR tests are reported in the second panel of table 2 for all three models. Both the test statistics are highly significant and reject the null-hypothesis of the diagonality of the Σ_v matrix for all three setups. The result is consistent with our prior expectation that the innovations of transition system are cross correlated and Σ_v cannot be

[17] One should be aware of two big fall in industrial production during late 2008 and early 2011. These two falls do not correspond to the domestic policy shocks, therefore, are not reflected by the yield curve slope. The former corresponds to the global financial crisis, during this period the Japanese exports fall sharply and hence the real activity slows down. The second fall refers to the impact of the great East Japan earthquake and tsunami of 2011.

reduced to a diagonal matrix.

To get more deep insight regarding the in-sample fit performance of the three model, in table 3 summary statistics for the fitted errors of three models are reported. With its additional flexibility, the GDNS model does show reasonable improvement in fit for all maturities over the DNS model, especially in the short maturities range both in terms of MAE and RMSE. The improvement for the short maturities is consistent with the premise that the second slope and curvature factors operating at short maturities. There is also a slightly better GDNS model fit with long-term yields. Regarding the comparison of the two extended models, the results indicate that GDNS-EGARCH outperforms the GDNS for 11 out of 20 maturities, while the latter have good performance for long maturities (i.e., 180, 240 and 300 months) and maturities range from 18-month to 3-year. Moreover, it is evident that the residuals autocorrelations across time for all maturities is considerably smaller of the EGARCH based model (GDNS-EGARCH) as compared to both DNS and GDNS models. If we focus on the fit of the GDNS and GDNS-EGARCH models in table 3, we see fairly uniform improvement in the fit in the maturity range from 36- month to ten years and a dramatic improvement in the fit of the 3- to 36-month maturities yields. The improved fit for the short-maturity yield in the GDNS based models relative to the DNS model reflects the important role of second slope and curvature factors.

<<Table 3>>

Overall, the results in table 3 suggest that there seems to be need for the more flexible and complex models to fit the yield curve attractively. Furthermore, the residual autocorrelation, which is a common phenomenon in most of the statistical class of models can be removed by considering the time-varying volatility in the yield curve models, as the common volatility component swift out the time-series effect from the residuals.

A more visual inspection can be performed by looking at average observed (empirical) and fitted curves, as shown in figure 5.

<<Figure 5>>

All three models are almost coinciding and capture yield dynamics well, however, the DNS model suffer from over-estimating the average yield curve for very short and long maturities. The GDNS seems to be more attractive to fit the curve as it coincide at all point with the empirical observed average yields. However, the improvement over GDNS-EGARCH is very small. It appears that the DNS model does not fit the very short end very well, because the factors loading of curvature factor is very flat at the short end and play more significant role at the long end. Therefore, the inclusion of another slope and curvature factors can serve at the short end of curve.

The overall conclusion from in-sample fit is consistent with the findings in Christensen *et al.* (2009) as the inclusion of another slope and curvature factors considerably improve the in-sample fit of yield curve model, especially at the short maturities. Moreover, the improvement of EGARCH based model over the GDNS is very minor in terms of residuals MAE and RMSE, however, in terms of residuals persistency across time the model that accounts for time-varying volatility outperforms the rest of two models. The great success of the extended models may be due to two slopes and curvatures factors that play a different role at the two ends of the curve.

4. Out-of-sample forecasting

Up to now, we have illustrated the in-sample fit performance of three Nelson-Siegel type models. However, a good yield curve model should also come with more accurate and precise forecasts of the future yields as emphasized by Duffee (2002). Furthermore, there is no guarantee that the more flexible models which achieve a better in-sample fit will also perform well in out-of-sample forecasting because of potential over-fitting. In this section, we investigate whether the in-sample superiority of the more flexible models, i.e., GDNS and GDNS-EGARCH models carry over to out-of-sample forecasts.

The yield forecasting is done by constructing factor predictions using the state equations and subsequently substituting these predictions (predicted state variables) in the measurement equations to obtain interest rate forecasts. In the forecasting procedure, we first estimate the parameters of the different models over a subsample period for the state space framework as in (14-16). From the Kalman filter we obtain the filtered latent factors and subsequently predict the h-month ahead forecast at every point in the out-of-sample period by iterating forward the state equation h-periods. The h-month ahead forecast of the state vector is given by:

$$\hat{X}_{t+h|t} = \left[I_d - \left(\sum_{i=0}^{h-1} \hat{F}^i\right)\right]\hat{C} + \hat{F}^h \hat{X}_{t|t} \tag{21}$$

where I_d is the ($d \times d$) identity matrix ($d = 3, 5$ and 6 for DNS, GDNS and GDNS-EGARCH models respectively), $\hat{C}$ and $\hat{F}$ are the state equation parameters estimates and $\hat{X}_{t|t}$ is the last available factor estimates in the update step. Three forecast horizons, $h = 1,6$ and 12 months ahead, are considered. We estimate and forecast recursively, using data from January 1996 to the time that the forecast is made, beginning in January 2007 and extending through December 2013.[18] After the state vector is forecasted, the h-month ahead yield forecasts follow from:

$$\hat{R}_{t+h|t} = \hat{H}\hat{X}_{t+h|t} \tag{22}$$

where $\hat{R}_{t+h|t}$ is the (N×1) vector define as $\hat{R}_{t,t+h}(m)$, the forecasted yield in period t for $t+h$ period, and the other parameters are as defined before. The forecast errors for the h-step ahead forecasts are calculated as: $e_{t,t+h} = R_{t+h} - \hat{R}_{t,t+h}$, where R_{t+h} is the actual observed yield vector at $t+h$ and the $\hat{R}_{t,t+h}$ is the (N×1) vector of the h-month ahead forecasted yields in period t.

Moreover, in the EGARCH based model the common shock component is zero in expectation and it

[18] It means that we estimate and forecast recursively. For the first forecast, the models are estimated by using data from 1996:01 to 2006:12. Then, one month data is added to re-estimate the models, and another forecast is constructed.

does not play a direct role in the forward iterations of the state equation to forecast the yield curve. Therefore, the predictions only depend on the five factors representing level, two slopes and two curvatures, because only the first five elements of C and the upper (5×5) block of F contain non-zero values. However, the time-varying volatility component is accounted for in the filtering steps in the Kalman filter and, therefore, affects the estimates of the factors. Hence, the common shock does have an indirect influence on the predictions through $\hat{X}_{t|t}$.

As a benchmark, we can include the AR(1) or VAR(1) model of yield to forecast the term structure, but it is clearly evident from the relevant literature that the standard DNS model outperforms all the naive time series forecasts for short as well as long forecast horizons (Diebold and Li, 2006; De Pooter 2007; Ullah *et al.* 2013b). Therefore, we do not report the results of the comparison of all three models (considered in this study) with the forecasts of AR(1) model.[19]

4.1. Term structure forecast results

Tables 4, 5 and 6 show the results of the forecasts of the DNS, GDNS and GDNS-EGARCH models for maturities of 3, 6, 12, 24, 36, 60, 120, 180, 240 and 300 months of the $h = 1$,6 and 12 months, respectively. The tables report various descriptive features of the computed errors, including mean, standard deviation, MAE, RMSE and autocorrelation at various displacements. Three main aspects, MAE, RMSE and errors persistency, are focused to evaluate the performance of each of the three models.

For a 1-month forecast horizon, in table 4, there seems that the GDNS model dominates the scene in terms of MAE and RMSE, whereas GDNS-EGARCH outpace the other two models in terms of errors persistency. Furthermore, the GDNS model clearly outperforms its counterpart DNS model for all maturities in terms of all the considered properties, i.e., MAE, RMSE and errors autocorrelations. As compared to the EGARCH based specification, the GDNS has lower forecast errors until 10 years maturities, whereas the former is better for maturities beyond 120 months. Between the DNS and GDNS-EGARCH, the latter model has worse forecast power than the former as the RMSE is higher for all maturities except the last three maturities (very long maturities). The performance of the GDNS-EGARCH at the long end may be attributed to the second slope and curvature factors. Even with very small forecast errors, the forecasts of GDNS and DNS do not seem optimal as the forecast errors are highly persistent.

<<Table 4>>

Predictions results at the medium and long horizons (i.e., 6- and 12-month ahead) show a better performance of the GDNS-EGARCH than the rest of the two models in terms of all the considered attributes. For a 6-month ahead forecast the model with time-varying volatility component outperforms the other two models in terms of mean forecast errors, MAE, RMSE and lag

[19] We, in fact, ran the random walk and AR(1) model of yield, but all three models, i.e., DNS, GDNS and GDNS-EGARCH models outpace the benchmark AR(1) and random walk specification of yield forecasts, therefore the results are not reported.

autocorrelation for all maturities, whereas the GDNS has lower MAE and RMSE than the standard DNS for all maturities. The 6-month ahead forecasts results seem not as good as the one-month ahead forecasts in terms of RMSE for the DNS and GDNS models.

<<Table 5>>

<<Table 6>>

For 12-month ahead, the GDNS-EGARCH have better performance than the rest of the two other models in terms of RMSE and errors persistency for all maturities. Moreover, the GDNS model has more accurate and precise forecasts than the DNS for most of the maturities, whereas the lag autocorrelation of the forecast errors is same for both models.

In summary, the out-of-sample forecasts results indicate that for very short horizon forecasts, i.e., one month, the GDNS model outperforms the rest of two models and even the simple DNS comes with more accurate forecasts than the complex GDNS-EGARCH model. While for the medium and long forecast horizons, such as 6- and 12-month ahead, the EGARCH based model have more accurate forecasts than their counterpart GNDS and simple DNS model. Moreover, the GDNS have better performance than the DNS at both 6- and 12-month ahead forecast horizons.

4.2. Out-of-sample forecast accuracy comparisons

We use two standard forecast error evaluation criteria to assess the quality of the out-of-sample forecasts. In particular, we report the trace root mean squared prediction error (TRMSPE) as well as compute the Diebold and Mariano (1995) test statistic for loss differential quadratic errors. The former combines the forecast errors of all maturities and summarizes the performance of each model, thereby allowing for a direct comparison between models. The Diebold and Mariano (DM) test is a standard statistical test that compares the squared forecast errors of two competing models and is most commonly applied test for comparing the forecast accuracy.

4.2.1. Trace root mean squared prediction error

For evaluating the forecast performance, the full sample of forecast errors, i.e., for all 20 maturities and forecast periods, is considered to compute the trace root mean squared prediction error (TRMSPE). Given a sample of T out-of-sample forecasts of N distinct maturities with h−month ahead forecast horizon, we compute the TRMSPE as follows:

$$TRMSPE = \sqrt{\frac{1}{NT}\sum_{m=1}^{N}\sum_{t=1}^{T}\left[R_{t+h}(m) - \hat{R}_{t,t+h}(m)\right]^2} \tag{23}$$

where $\hat{R}_{t,t+h}(m)$ is the forecasted yield in period t for $t+h$ period, $[R_{t+h}(m) - \hat{R}_{t,t+h}(m)]$ is the forecast errors at $t+h$ for yield and t starts from 2007:01 for $h = 1$, 2007:06 for $h = 6$, and 2007:12 for $h = 12$. The results of TRMSPE for all three models are given in table 7. At first sight the results shows that the forecasts becomes worse as the forecast horizon becomes longer, and there is no single model that dominates for all the forecast horizons. However, the performance of GDNS-EGARCH based specification is better at the six-month and one-year ahead forecasts. For the one-month ahead forecast, the GDNS beats the rest of the two models.

<<Table 7>>

Furthermore, the GDNS outperforms the DNS for all three forecast horizons. More importantly, the simple DNS outperforms the GDNS-EGARCH model for one-month ahead forecast horizon.

4.2.2. Diebold–Mariano test

We employ the Diebold and Mariano (1995) test to assess the forecast performance for each maturity for the different pairs of models. The comparison is made in three pairs between the three models, i.e., (i) the GDNS against the simple DNS model, (ii) GDNS-EGARCH against the DNS model, and (iii) GDNS-EGARCH against the GDNS model. Given a pair of two competing forecasting models, i.e., 1 and 2, the difference between the two quadratic loss functions is computed as $d_t = e_{1t}^2 - e_{2t}^2$, where e_{1t}^2 and e_{2t}^2 are the quadratic loss functions of the two competing models, the DM test statistic is computed as:

$$DM = \frac{\bar{d}}{\sqrt{2\pi \hat{f}_d(0)/T}} \sim N(0,1) \tag{24}$$

where $\hat{f}_d(.)$ is the consistent estimate of the spectral density of d_t and $\bar{d}$ is the sample mean of d_t for $t = 1,2,\dots,T$. The null-hypothesis is: $H_0: E(d_t) = 0$ (meaning that both models have same squared forecasts errors) against the alternative hypothesis $H_1: E(d_t) \neq 0$. We apply the Diebold and Mariano (1995) test to forecast errors of three pairs of models and the results are presented in table 8 for all the three forecasts horizons.

The results in table 8 for the first pair point towards the universal significant difference of the RMSE for all three horizons and all maturities forecasts of the GDNS and DNS model except the 10-, 15- and 20-year maturities for $h = 12$. All the *DM-statistics* are significantly different from zero (except the 3 maturities for the 12-month forecast horizon) .The negative values show that the GDNS model outperforms all the competing forecasts of the DNS specification (in first pair e_{1t} and e_{2t} are the forecast errors of the GDNS and DNS models respectively).

Concerning the second pair of comparison, the result indicate that the GDNS-EGARCH outperforms the DNS for all maturities for $h = 6$ and 12, as all *DM-stat* are negative and significantly different from zero. For $h = 1$, the DNS has lower forecast errors until 10-year maturity than the GDNS (most test stat are significant and positive), whereas GDNS has more accurate forecasts for very long maturities (15-, 20- and 25-year maturities). In this pair e_{1t} and e_{2t} correspond to the forecasts errors of GDNS-EGARCH and DNS respectively.

<<Table 8>>

The *DM-Stat* reported in table 8 for the third pair of models (GDNS-EGARCH against the GDNS), indicates a significant difference of the RMSE for all the three forecasts horizons of the two competing models. It is worthwhile to mention that the negative values indicate superiority of GDNS-EGARCH model forecasts as e_{1t}^2 and e_{2t}^2 refer to the quadratic loss functions of GDNS-EGARCH model and GDNS model respectively. The results show that the GDNS-EGARCH unanimously outperforms the GDNS for $h = 6$ and 12. For $h = 1$, the GDNS has more accurate forecasts than the EGARCH based model for maturities until 10-year, whereas the latter outperforms the former for maturities beyond 120-month.

The results of Diebold and Mariano (1995) test suggest the resilient predictive power of the two slope and curvature based extended model at all three horizons. Moreover, the Diebold and Mariano (1995) test validates the results obtained from the TRMSPE criterion of evaluating the forecast performance of the models.

The results of the aforementioned two tests in this study unanimously suggest that the generalized dynamic Nelson-Siegel specifications (model with two slopes and curvatures) outperform the competing benchmark DNS model. Moreover, the inclusion of time-varying volatility component implies to worsen the 1-month ahead forecasts, whereas improves the predictive power of the model at longer horizons forecasting.

5. Conclusion

Estimating the JGBs zero curve with the dynamic Nelson-Siegel model has trouble in fitting the short maturity yields and fails to grasp the characteristics of the JGBs yield curve. For JGBs, since 1999, yield curves under the zero interest rate policy and the quantitative easing monetary policy are distinctive. During these periods, the yield curve has a flat shape near zero at the short-term maturities and often has a complex shape with multiple inflection points. In this study, we address these issues and consider the generalized version of the Nelson-Siegel model with two slopes and curvatures, the so called generalized dynamic Nelson-Siegel (GDNS) model, which corresponds to a modern five-factor term structure model. The inclusion of second slope and curvature is helpful to fit the very short maturities, by restricting the role of the newly added slope and curvature factors to the short end of the curve. We argue that in addition to second curvature as in Svensson (1995), the second slope improves the model performance in terms of in-sample fit as well as out-of-sample forecasts. Finally, we show that introducing the common volatility component improve the in-sample fit of the model.

Moreover, the volatility in bond market is found to be asymmetrically affected by the positive and negative shock and recent shocks play more prominent role in explaining the current volatility rather than the lag volatility.

Regarding the out-of-sample forecasts, the results indicate that the model with two slopes and curvatures (GDNS) model outperforms its counterpart DNS model for all forecasts horizons.

Allowing for time-varying volatility in the model (GDNS-EGARCH) enables it to better capture dynamics in the most volatile yields and produce relatively more accurate forecast at 6- and 12-month ahead horizons. However, the GDNS and even the simple DNS model outperform the GDNS-EGARCH at the short one-month forecast horizon. It seems that the GDNS model has higher forecasting capability for the short forecast horizons, i.e., one month, while the GDNS-EGARCH model has excellent performance for the medium and longer forecast horizons.

Summarizing, it turns out that the richer parameterization of model leads to a better in-sample fit and out-of-sample performance. Since, the GDNS based term structure models are characterized by having a more rich specification, which usually improves in-sample as well as out-of-sample forecasts, implying that the simplicity of the model comes at the cost of poor fit and future forecast.

Appendix-I

Coefficients and latent variable in the general state-space form

In the statistical formulation of the models in section 2.4, the matrices and vectors for the state and observations equations should be considered as follows. The matrices and vectors in state-space system in (14-16) for the simple DNS model should be defined as:

$H = \Lambda(\lambda)$: (N×3)	$X_t = [\beta_{1t}, \beta_{2t}, \beta_{3t}]'$: (3×1)	$w_t = \varepsilon_t$: (N×1)
$C = [I_3 - A]\mu$: (3×1)	$F = A$: (3×3)	$u_t = v_t$: (3×1)
$G = \Omega$: (N×N)	$Q_t = \Sigma_v$: (3×3)	

where $\Lambda(\lambda)$ is (N×3) matrix of loadings, $\beta_t = (\beta_{1t}, \beta_{2t}, \beta_{3t})'$ is the (3×1) vector of latent factors of the yield curve, μ is (3×1) vectors of factors mean, and A is (3×3) full-matrix of parameters. ε_t and v_t are (N×1) and (3×1) innovations vectors of the observation and state equations respectively, Ω is (N×N) covariance matrix of the measurement equation innovations, and Σ_v is (3×3) covariance matrix of the state innovations (assumed to be a full-matrix rather than a diagonal one).

While, for the GDNS model in the state-space system can be written as:

$H = \Lambda(\lambda_1, \lambda_2)$: (N×5)	$X_t = [\beta_{1t}, \beta_{2t}, \beta_{3t}, \beta_{4t}, \beta_{5t}]'$: (5×1)	$w_t = \varepsilon_t$: (N×1)
$C = [I_5 - A]\mu$: (5×1)	$F = A$: (5×5)	$u_t = v_t$: (5×1)
$G = \Omega$: (N×N)	$Q_t = \Sigma_v$: (5×5)	

The state-space specification for the GDNS is exactly similar to the one for DNS, but here the dimension of matrices and vectors is different. $\Lambda(\lambda_1, \lambda_2)$ is (N×5) matrix of loadings, $\beta_t = (\beta_{1t}, \beta_{2t}, \beta_{3t}, \beta_{4t}, \beta_{5t})'$ is the (5×1) vector of latent factors of the yield curve, μ is (5×1) vectors of factors mean, A is (5×5) full-matrix of parameters, v_t is (5×1) innovations vectors of the state equation, and Σ_v is (5×5) covariance matrix of the state innovations (assumed to be a full-matrix). The dimensions of ε_t and Ω are similar to the one defined for the DNS model.

Furthermore, for the GDNS-EGARCH model the system should be defined as:

$H = [\Lambda(\lambda_1, \lambda_2) \quad \Gamma_\varepsilon]$: (N×6) $\quad X_t = \alpha_t = [\beta_{1t}, \beta_{2t}, \beta_{3t}, \beta_{4t}, \beta_{5t}, \varepsilon_t^*]'$: (6×1) $\quad w_t = \varepsilon_t^+$: (N×1)

$C = \begin{bmatrix} (I_5 - A)\mu \\ 0 \end{bmatrix}$ (6×1) $\quad F = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}$: (6×6) $\quad u_t = \begin{bmatrix} v_{t+1} \\ \varepsilon_{t+1}^* \end{bmatrix}$: (6×1)

$G = \Omega$: (N×N) $\quad Q_t = \begin{bmatrix} \Sigma_v & 0 \\ 0 & h_{t+1} \end{bmatrix}$: (6×6)

where $\alpha_t = (\beta_{1t}, \beta_{2t}, \beta_{3t}, \beta_{4t}, \beta_{5t}, \varepsilon_t^*)'$ is (6×1) latent vector and Γ_ε is (N×1) vector showing the sensitivity of various yields to a common shock component. The definitions and dimensions of all remaining matrices and vectors is same as discussed in the GDNS model specification. In all three model the matrix Ω is assumed to be diagonal for computational traceability, while the covariance matrix Σ_v is non-diagonal.

Appendix-II

Data description

We consider JGB yields with maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, 120, 180, 240 and 300 months. The yields are derived from bid/ask average price quotes, from January 1996 through December 2013, using the Fama and Bliss (1987) methodology.

Table A1 provides summary statistics for the data-set. For each maturity, we report mean, standard deviation, minimum, maximum, skewness, kurtosis, and autocorrelation coefficients at various displacements. The summary statistics reveal that the average yield curve is upward sloping. Unconditional volatility increases by maturity and yields for all maturities are persistent, however, relatively short rates persistency is higher than those of the long rates.

<<Table A1>>

<<Figure A1>>

In addition to the findings in table A1, we see few interesting characteristics in figure A1, which plots cross-section of yields over time. The first noticeable fact is that long term yields vary significantly over time. Second, the short rates are almost zero during the prolonged period except with a little rise in late 2006 and early 2007 that causes a fall in the slope of the curves (apparent in figure). Moreover, when short rates are stuck at zero, the long end seem more volatile than the short end of the curves.

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<<Tables>>

Table 1. Latent factors and EGARCH models parameters estimates of the GDNS-EGARCH, GDNS and DNS models

Panel 1: Estimates of matrix A and vector μ						
	μ	$A(.,1)$	$A(.,2)$	$A(.,3)$	$A(.,4)$	$A(.,5)$
GDNS-EGARCH model						
$A(1,.)$	**3.3001** (0.0439)	**0.7289** (0.0157)	0.0596 (0.3172)	**0.1502** (0.0950)	**-0.1193** (0.0323)	**-0.1588** (0.0289)
$A(2,.)$	**-2.5629** (0.0160)	-0.0277 (0.0276)	**0.8100** (0.0213)	**-0.4039** (0.0774)	-0.0618 (0.1248)	**-0.1296** (0.0239)
$A(3,.)$	**-0.7042** (0.0560)	0.0636 (0.0498)	**-0.1367** (0.0580)	**0.4217** (0.0191)	**0.3232** (0.1988)	0.0179 (0.0272)
$A(4,.)$	**-1.6677** (0.0460)	-0.0567 (0.1711)	0.0571 (0.1997)	**-0.3406** (0.1355)	**0.9869** (0.0451)	0.0283 (0.1348)
$A(5,.)$	**-0.4235** (0.1470)	**-0.4420** (0.0243)	0.0079 (0.6519)	**-0.1083** (0.0523)	**0.2030** (0.0858)	**0.4586** (0.2418)
λ_1	**0.0271**	(0.0001)				
λ_2	**0.1098**	(0.0002)				
GDNS model						
$A(1,.)$	**2.7494** (0.1423)	**0.6786** (0.0158)	**0.0606** (0.0115)	**0.0505** (0.0021)	**-0.0920** (0.0038)	**-0.0504** (0.0045)
$A(2,.)$	**-2.5450** (0.0711)	-0.0195 (0.0133)	**0.9268** (0.0072)	**-0.2227** (0.0085)	-0.0699 (0.2190)	**-0.1675** (0.0044)
$A(3,.)$	**-0.0640** (0.0583)	**0.1473** (0.0020)	**-0.1217** (0.0053)	**0.3908** (0.0063)	**0.2583** (0.0049)	0.0232 (0.0440)
$A(4,.)$	**-1.6820** (0.4497)	-0.0995 (0.0645)	0.0717 (0.0532)	**-0.2098** (0.0059)	**0.9200** (0.0012)	0.0215 (0.0267)
$A(5,.)$	**-0.6042** (0.0983)	**-0.1310** (0.0083)	0.0361 (0.0029)	**-0.0976** (0.0036)	**0.0903** (0.0066)	**0.6199** (0.0065)
λ_1	**0.0287**	(0.0005)				
λ_2	**0.0919**	(0.0022)				
DNS model						
$A(1,.)$	**2.0355** (0.0127)	**0.8685** (0.0179)	0.0380 (0.0411)	**0.1960** (0.0620)		
$A(2,.)$	**-1.7975** (0.1116)	0.1287 (0.1059)	**0.8665** (0.0052)	**0.3180** (0.0058)		
$A(3,.)$	**-1.9433** (0.0639)	**-0.1081** (0.0154)	0.0333 (0.0397)	**0.6375** (0.0483)		
λ	**0.0254**	(0.0004)				

Panel 2: EGARCH model parameter estimates in the GDNS-EGARCH model				
GDNS-EGARCH model	γ_0	γ_1	γ_2	ψ
	0.1112 (0.0491)	**0.7583** (0.0909)	**0.6456** (0.0409)	**0.2832** (0.1079)

Note: The table reports the estimates for the parameters of the transition equation of GDNS- EGARCH (five-factor model with time-varying volatility), GDNS (five-factor model) and DNS (three-factor model) models and of EGARCH specification. Panel 1 presents the estimates for the vector μ and matrix A along with the decay parameters λ estimates, while panel 2 shows the parameters' estimates of the volatility processes (EGARCH) of the common component in the five-factor yield curve model. The standard errors are in parenthesis. Bold entries denote that parameter estimates are significant at the 5% level.

Table 2. Estimates of covariance matrix $\boldsymbol{\Sigma}_v$ and its diagonality test

Panel 1: Estimates of covariance matrix $\boldsymbol{\Sigma}_v$					
	$\Sigma_v(.,1)$	$\Sigma_v(.,2)$	$\Sigma_v(.,3)$	$\Sigma_v(.,4)$	$\Sigma_v(.,5)$
GDNS-EGARCH model					
$\Sigma_v(1,.)$	**0.5459** (0.0378)				
$\Sigma_v(2,.)$	-0.0009 (0.0101)	**0.1377** (0.0093)			
$\Sigma_v(3,.)$	0.0029 (0.065)	**0.1564** (0.0125)	**0.0002** (0.0001)		
$\Sigma_v(4,.)$	**0.0569** (0.0107)	**0.7973** (0.0518)	0.0181 (0.0555)	**0.0097** (0.0018)	
$\Sigma_v(5,.)$	0.0025 (0.0383)	**0.3279** (0.0193)	0.0038 (0.0311)	0.0189 (0.0808)	**0.0028** (0.0012)
GDNS model					
$\Sigma_v(1,.)$	**0.6088** (0.0038)				
$\Sigma_v(2,.)$	0.0041 (0.0090)	**0.1656** (0.0030)			
$\Sigma_v(3,.)$	0.0024 (0.0017)	**0.3065** (0.0096)	**0.0002** (0.0001)		
$\Sigma_v(4,.)$	**0.0306** (0.0015)	**0.9634** (0.0029)	0.0126 (0.0120)	**0.0076** (0.0027)	
$\Sigma_v(5,.)$	0.0026 (0.0161)	**0.4343** (0.0058)	0.0031 (0.0050)	0.0110 (0.0169)	**0.0021** (0.0004)
DNS model					
$\Sigma_v(1,.)$	**0.5911** (0.0374)				
$\Sigma_v(2,.)$	0.0025 (0.0034)	**0.3041** (0.0152)			
$\Sigma_v(3,.)$	**0.6097** (0.0512)	**0.7967** (0.0196)	**0.0098** (0.0036)		

Panel 2: Tests for diagonality of covariance matrix $\boldsymbol{\Sigma}_v$									
Model	GDNS-EGARCH model			GDNS model			DNS model		
Test	Test statistic	df	P-value	Test statistic	df	P-value	Test statistic	df	P-value
Wald Test	**65.0753**	10	0.000	**57.6382**	10	0.000	**51.4872**	3	0.000
LR Test	**73.5302**	10	0.000	**69.2951**	10	0.000	**41.05394**	3	0.000

Note: The table shows the estimates of the covariance matrices Σ_v of the innovations in the state equations for the GDNS- EGARCH (five-factor model with time-varying volatility), GDNS (five-factor model) and DNS (three-factor model) yields models in panel 1. The lower panel (panel 2) presents the results of the Wald test and likelihood ratio (LR) test for the null hypothesis that the covariance matrix Σ_v is diagonal. Both test statistics are chi-square with their respective degrees of freedom (df). The P-value is the probability value of the test statistic. The standard errors are in parenthesis. Bold entries denote that parameter estimates are significant at the 5% level.

Table 3. Descriptive statistics of the yield curve residuals

	GDNS-EGARCH model						GDNS model						DNS model					
Maturity	Mean	SD	MAE	RMSE	$\hat{\rho}$ (1)	$\hat{\rho}$ (6)	Mean	SD	MAE	RMSE	$\hat{\rho}$ (1)	$\hat{\rho}$ (6)	Mean	SD	MAE	RMSE	$\hat{\rho}$ (1)	$\hat{\rho}$ (6)
3	-0.035	0.192	0.145	0.135	0.616	0.518	-0.037	0.196	0.148	0.199	0.831	0.642	-0.069	0.298	0.252	0.306	0.919	0.660
6	0.019	0.200	0.150	0.215	0.625	0.548	-0.078	0.204	0.156	0.218	0.846	0.651	-0.107	0.304	0.256	0.321	0.924	0.670
9	0.045	0.252	0.148	0.226	0.601	0.502	-0.118	0.218	0.174	0.247	0.860	0.655	-0.141	0.311	0.262	0.340	0.926	0.676
12	0.065	0.240	0.197	0.255	0.608	0.546	-0.169	0.236	0.204	0.290	0.874	0.658	-0.185	0.318	0.271	0.368	0.926	0.681
15	0.051	0.265	0.185	0.241	0.761	0.547	-0.196	0.257	0.225	0.323	0.886	0.662	-0.207	0.327	0.280	0.386	0.926	0.684
18	0.023	0.289	0.365	0.426	0.676	0.546	-0.108	0.281	0.178	0.232	0.897	0.667	-0.242	0.335	0.291	0.413	0.924	0.686
21	-0.014	0.312	0.358	0.418	0.545	0.547	-0.170	0.307	0.180	0.229	0.905	0.671	-0.174	0.345	0.196	0.239	0.923	0.687
24	-0.057	0.332	0.354	0.412	0.416	0.541	-0.124	0.332	0.191	0.234	0.912	0.675	-0.125	0.354	0.217	0.280	0.921	0.687
30	-0.143	0.385	0.342	0.396	0.412	0.537	-0.188	0.382	0.232	0.269	0.922	0.681	-0.139	0.273	0.293	0.278	0.919	0.687
36	-0.226	0.410	0.333	0.386	-0.562	0.541	-0.250	0.427	0.287	0.327	0.928	0.686	-0.353	0.392	0.355	0.398	0.918	0.686
48	-0.338	0.459	0.309	0.363	0.517	0.535	-0.261	0.497	0.376	0.428	0.934	0.689	-0.372	0.427	0.382	0.473	0.917	0.684
60	-0.238	0.518	0.301	0.360	0.518	0.515	-0.375	0.543	0.440	0.500	0.936	0.690	-0.491	0.459	0.491	0.829	0.919	0.682
72	-0.233	0.549	0.299	0.360	0.610	0.507	-0.263	0.572	0.464	0.528	0.936	0.689	-0.480	0.487	0.480	0.529	0.922	0.680
84	-0.214	0.574	0.287	0.349	0.457	0.499	-0.296	0.587	0.448	0.512	0.936	0.686	-0.310	0.512	0.481	0. 570	0.925	0.679
96	-0.215	0.605	0.267	0.333	0.525	0.489	-0.332	0.595	0.407	0.469	0.934	0.683	-0.439	0.534	0.439	0.494	0.928	0.677
108	-0.213	0.616	0.253	0.320	0.514	0.486	-0.259	0.598	0.364	0.423	0.933	0.680	-0.177	0.554	0.378	0.437	0.930	0.676
120	-0.201	0.601	0.244	0.315	0.430	0.494	-0.230	0.597	0.322	0.380	0.931	0.676	-0.218	0.571	0.338	0.380	0.932	0.674
180	-0.091	0.644	0.255	0.319	0.705	0.481	-0.261	0.581	0.210	0.264	0.922	0.656	-0.200	0.633	0.207	0.299	0.937	0.666
240	0.049	0.643	0.271	0.334	0.628	0.483	-0.119	0.568	0.188	0.244	0.915	0.639	-0.157	0.670	0.195	0.165	0.937	0.661
300	0.161	0.667	0.262	0.317	0.691	0.475	-0.175	0.560	0.221	0.283	0.910	0.626	-0.163	0.693	0.230	0.290	0.937	0.657

Note: The table presents summary statistic of the residuals for different maturity times of the measurement equation of GDNS- EGARCH (five-factor model with time-varying volatility), GDNS (five-factor model) and DNS (three-factor model) yield curve models, using monthly data 1996:01–2013:12. RMSE and MAE is the root mean squared errors and mean absolute error respectively. $\hat{\rho}\,(i)$ denotes the sample autocorrelations at displacements of 1 and 6 months. The number of observations is 216.

Table 4. Out-of-sample 1-month ahead forecasting results

Maturity	Mean	SD	MAE	RMSE	$\hat{\rho}$ (1)	$\hat{\rho}$ (6)	$\hat{\rho}$ (12)
Forecast summary for GDNS-EGARCH model							
3	0.1124	0.2875	0.2246	0.3069	0.1401	0.2163	-0.0804
6	0.1468	0.2402	0.1977	0.2801	0.0875	0.1945	-0.1099
12	0.1411	0.1927	0.1680	0.2377	0.0053	0.1608	-0.1006
24	-0.0249	0.1834	0.1447	0.1838	-0.3976	0.0175	-0.1496
36	-0.2383	0.1974	0.2811	0.3086	-0.3108	0.0162	-0.1378
60	-0.5107	0.2215	0.5240	0.5561	0.0387	0.1045	0.0137
120	-0.3856	0.2022	0.3934	0.4348	0.2545	0.1693	0.0879
180	-0.0752	0.1917	0.1608	0.2046	0.1281	0.0937	-0.0732
240	0.1409	0.1717	0.1724	0.2212	0.0786	0.0846	-0.1291
300	0.2184	0.1790	0.2258	0.2816	0.2290	0.0877	-0.0782
Forecast summary for GDNS model							
3	0.0307	0.0960	0.0860	0.1002	0.5801	0.1091	0.1623
6	0.0454	0.0866	0.0834	0.0972	0.5203	0.0760	0.1102
12	0.0716	0.0744	0.0859	0.1029	0.4371	0.0074	0.0163
24	0.1029	0.0863	0.1085	0.1339	0.1741	-0.0373	0.0032
36	0.1029	0.1052	0.1077	0.1467	0.1359	0.0485	-0.0072
60	0.0782	0.1362	0.1036	0.1562	0.2535	0.1224	0.0705
120	0.1217	0.1479	0.1445	0.1507	0.1454	0.0578	0.0798
180	0.1677	0.1650	0.1900	0.2344	0.1608	0.0722	-0.0452
240	0.1825	0.1707	0.2043	0.2491	0.2565	0.1066	-0.0115
300	0.1225	0.1927	0.1703	0.2273	0.4403	0.1482	0.0438
Forecast summary for DNS model							
3	0.2470	0.1334	0.2585	0.2803	0.8340	0.4373	0.0321
6	0.2330	0.1308	0.2452	0.2667	0.8317	0.4223	0.0245
12	0.2087	0.1197	0.2194	0.2401	0.8390	0.4158	0.0184
24	0.1636	0.1142	0.1739	0.1991	0.7952	0.3863	0.0182
36	0.1150	0.1261	0.1272	0.1700	0.7165	0.3795	0.0297
60	0.0514	0.1513	0.1103	0.1588	0.6355	0.3870	0.1031
120	0.1495	0.1682	0.1844	0.2241	0.3930	0.3293	0.1488
180	0.2846	0.1566	0.2927	0.3243	0.2292	0.2195	0.0046
240	0.3667	0.1357	0.3679	0.3907	0.1333	0.1582	-0.0506
300	0.3528	0.1371	0.3550	0.3781	0.2822	0.1788	-0.0349

Note: The table reports the results of out-of-sample 1-month ahead forecasting using state-space specification for the GDNS- EGARCH (five-factor model with time-varying volatility), GDNS (five-factor model) and DNS (three-factor model) yield curve models for various maturities. We estimate the models recursively from 1996:01 to the time that the forecast is made, beginning in 2007:01 and extending through 2013:12. We define forecast errors at $t+1$ as $R_{t+1}(m) - \hat{R}_{t,t+1}(m)$, where $\hat{R}_{t,t+1}(m)$ is the $t+1$ month ahead forecasted yield at period t, and we report the mean, standard deviation, mean absolute errors (MAE) and root mean squared errors (RMSE) of the forecast errors, as well as their first, 6^{th} and 12^{th} sample autocorrelation coefficients.

Table 5. Out-of-sample 6-month ahead forecasting results

Maturity	Mean	SD	MAE	RMSE	$\hat{\rho}$ (1)	$\hat{\rho}$ (6)	$\hat{\rho}$ (12)
Forecast summary for GDNS-EGARCH model							
3	-0.9588	0.5117	0.2570	0.2950	0.1821	-0.0106	-0.0985
6	-0.6973	0.4417	0.2239	0.3002	0.1808	-0.0050	-0.0912
12	-0.5438	0.3579	0.3060	0.3211	0.1805	-0.0036	-0.0900
24	-0.8725	0.3331	0.2885	0.3242	0.1683	-0.0345	-0.0470
36	-1.3757	0.3657	0.1629	0.2910	0.2496	0.0428	0.0148
60	-2.0853	0.4391	0.2740	0.3137	0.4366	0.2368	0.1561
120	-2.3906	0.5012	0.2037	0.3725	0.6701	0.4541	0.3049
180	-2.2280	0.4917	0.3280	0.3809	0.6364	0.3771	0.2537
240	-2.0882	0.4581	0.3882	0.4371	0.6251	0.2969	0.1646
300	-2.0602	0.4577	0.3602	0.4097	0.6641	0.2276	0.1136
Forecast summary for GDNS model							
3	0.2451	0.2030	0.2621	0.3173	0.8425	0.3861	0.2194
6	0.2714	0.1920	0.2784	0.3316	0.8564	0.4099	0.2331
12	0.3077	0.1710	0.3077	0.3514	0.8593	0.3705	0.2161
24	0.3462	0.1596	0.3462	0.3808	0.8263	0.3268	0.2079
36	0.3437	0.1770	0.3437	0.3859	0.8087	0.3536	0.2007
60	0.2978	0.2389	0.3004	0.3807	0.8026	0.3635	0.2045
120	0.2807	0.3376	0.3375	0.4371	0.7659	0.1924	0.2055
180	0.2774	0.3955	0.3814	0.4807	0.7669	0.1141	0.1233
240	0.2569	0.4192	0.3785	0.4890	0.7725	0.0843	0.1040
300	0.1699	0.4440	0.3655	0.4723	0.7777	0.1150	0.0721
Forecast summary for DNS model							
3	0.5424	0.2453	0.5424	0.5945	0.8815	0.5642	0.1015
6	0.5268	0.2423	0.5268	0.5791	0.8829	0.5694	0.1106
12	0.4920	0.2309	0.4921	0.5427	0.8777	0.5681	0.1182
24	0.4318	0.2186	0.4323	0.4832	0.8894	0.5760	0.1427
36	0.3756	0.2197	0.3792	0.4343	0.8952	0.5925	0.1836
60	0.3105	0.2385	0.3218	0.3904	0.8961	0.6307	0.2620
120	0.4433	0.2765	0.4569	0.5214	0.8922	0.6375	0.3581
180	0.6078	0.2539	0.6078	0.6580	0.8901	0.6107	0.3372
240	0.7065	0.2171	0.7065	0.7386	0.8968	0.6193	0.3466
300	0.6998	0.2112	0.6998	0.7305	0.9037	0.6152	0.3215

Note: The table reports the results of out-of-sample 6-month ahead forecasting using state-space specification for the GDNS-EGARCH (five-factor model with time-varying volatility), GDNS (five-factor model) and DNS (three-factor model) yield curve models for various maturities. We estimate the models recursively from 1996:01 to the time that the forecast is made, beginning in 2007:01 and extending through 2013:12. We define forecast errors at $t+6$ as $R_{t+6}(m) - \hat{R}_{t,t+6}(m)$, where $\hat{R}_{t,t+6}(m)$ is the $t+6$ months ahead forecasted yield at period t, and we report the mean, standard deviation, mean absolute errors (MAE) and root mean squared errors (RMSE) of the forecast errors, as well as their first, 6th and 12th sample autocorrelation coefficients.

Table 6. Out-of-sample 12-month ahead forecasting results

Maturity	Mean	SD	MAE	RMSE	$\hat{\rho}$ (1)	$\hat{\rho}$ (6)	$\hat{\rho}$ (12)
Forecast summary for GDNS-EGARCH model							
3	-0.1960	0.5088	0.2066	0.3981	0.2957	0.0970	-0.1071
6	-0.1680	0.4396	0.2822	0.3616	0.3130	0.1076	-0.1013
12	-0.2060	0.3397	0.3107	0.3736	0.3258	0.1151	-0.0949
24	-0.0022	0.2699	0.0022	0.3030	0.3216	0.1113	-0.0648
36	-0.3549	0.2743	0.2549	0.3019	0.3891	0.1451	0.0546
60	0.3464	0.3355	0.2464	0.2161	0.5607	0.2989	0.2601
120	0.4136	0.4193	0.3136	0.3583	0.7600	0.4765	0.4050
180	0.1674	0.4094	0.5674	0.6162	0.7410	0.4185	0.3605
240	-0.0844	0.3794	0.4844	0.5314	0.7287	0.3795	0.3385
300	0.1341	0.3768	0.4341	0.6820	0.7546	0.3684	0.2883
Forecast summary for GDNS model							
3	0.4019	0.1866	0.4019	0.4125	0.8832	0.5361	0.0605
6	0.3906	0.1821	0.3906	0.4303	0.8799	0.5484	0.0765
12	0.3608	0.1776	0.3608	0.4015	0.8786	0.5589	0.0973
24	0.3034	0.1673	0.3034	0.3458	0.8764	0.5633	0.1256
36	0.2525	0.1695	0.2539	0.3033	0.8749	0.5740	0.1393
60	0.2059	0.1985	0.2208	0.2848	0.8760	0.5865	0.1622
120	0.3925	0.2789	0.4125	0.4802	0.8867	0.6151	0.2408
180	0.5850	0.2658	0.5850	0.6416	0.8730	0.5781	0.1868
240	0.7013	0.2270	0.7013	0.7365	0.8884	0.5792	0.1526
300	0.7021	0.2190	0.7021	0.7349	0.9030	0.5480	0.1163
Forecast summary for DNS model							
3	0.2902	0.2948	0.3225	0.4120	0.9262	0.3144	0.0612
6	0.3258	0.2804	0.3392	0.4283	0.9283	0.3200	0.0597
12	0.3706	0.2456	0.3709	0.4435	0.9262	0.3136	0.0480
24	0.4004	0.1639	0.4004	0.4322	0.8933	0.2748	0.0803
36	0.3767	0.1237	0.3767	0.3961	0.7938	0.3861	0.2213
60	0.2785	0.2602	0.2807	0.3797	0.8236	0.4638	0.0340
120	0.1523	0.5694	0.4651	0.5849	0.8487	0.3862	-0.0246
180	0.0741	0.7030	0.5824	0.7012	0.8372	0.3318	-0.0667
240	0.0092	0.7675	0.6367	0.7612	0.8417	0.3063	-0.0748
300	-0.1096	0.8198	0.7026	0.8205	0.8454	0.2749	-0.0790

Note: The table reports the results of out-of-sample 12-month ahead forecasting using state-space specification for the GDNS- EGARCH (five-factor model with time-varying volatility), GDNS (five-factor model) and DNS (three-factor model) yield curve models for various maturities. We estimate the models recursively from 1996:01 to the time that the forecast is made, beginning in 2007:01 and extending through 2013:12. We define forecast errors at $t + 12$ as $R_{t+12}(m) - \hat{R}_{t,t+12}(m)$, where $\hat{R}_{t,t+12}(m)$ is the $t + 12$ months ahead forecasted yield at period t, and we report the mean, standard deviation, mean absolute errors (MAE) and root mean squared errors (RMSE) of the forecast errors, as well as their first, 6th and 12th sample autocorrelation coefficients.

Table 7. TRMSPE results for out-of-sample forecasts accuracy comparisons

Models	1-Month Forecasts	6-Month Forecasts	12-Months Forecast
GDNS-EGARCH	0.3426	0.2805	0.5420
GDNS	0.1523	0.3906	0.6896
DNS	0.2285	0.5154	0.8177

Note: The table reports the Trace Root Mean Squared Prediction Error (TRMSPE) results of out-of-sample forecasts accuracy comparison for horizons of one, 6 and 12 months for the GDNS-EGARCH (five-factor model with time-varying volatility), GDNS (five-factor model) and DNS (three-factor model) yield curve models. In computing the TRMSPE, the full sample of forecast errors, i.e., all 20 maturities are considered.

Table 8. Diebold-Mariano test-statistic

Maturity	GDNS against the DNS			GDNS -EGARCH against the DNS			GDNS -EGARCH against the GDNS		
	$h = 1$	$h = 6$	$h = 12$	$h = 1$	$h = 6$	$h = 12$	$h = 1$	$h = 6$	$h = 12$
3	-16.1197	-5.6590	-4.5744	6.2309	-5.7338	-4.3965	4.6225	-4.0934	-3.6948
6	-17.3014	-5.7459	-4.6164	7.5572	-6.3357	-5.0498	4.3847	-3.6128	-3.3839
12	-18.6461	-5.9578	-4.8574	4.7051	-6.8170	-5.5443	4.0098	-2.8836	-2.9928
24	-15.4306	-5.7198	-5.1394	**-0.9358**	-7.4021	-5.8402	3.4245	-3.6946	-3.9072
36	-12.1140	-5.1633	-4.9224	9.9702	-3.0128	-2.9585	7.7791	-5.1795	-4.8942
60	-9.0910	-4.0592	-3.5434	11.2967	-8.0174	-6.9923	11.663	-6.4979	-5.1444
120	-4.3118	-1.7738	**-0.5361**	11.2739	-7.6368	-5.6484	7.1096	-6.2767	-3.6512
180	-1.4093	-2.5706	**0.8075**	-5.1324	-7.2303	-5.0971	-1.2871	-5.8346	-2.8920
240	-1.1294	-2.0717	**0.7601**	-1.8927	-7.1725	-4.8872	**-0.4498**	-5.7670	-2.4532
300	-2.5528	-3.0951	-2.5554	-2.6976	-7.0722	-4.8844	-3.9152	-5.7728	-2.3494

Note: The table presents Diebold–Mariano forecast accuracy comparison test results of 1, 6 and 12 months ahead forecast horizons for the GDNS- EGARCH (five-factor model with time-varying volatility), GDNS (five-factor model) and DNS (three-factor model) yield curve models. Three different pairs of comparison are considered, i.e., the GDNS against the DNS, GDNS -EGARCH against the DNS and GDNS -EGARCH against the GDNS forecasts. The null hypothesis is that the two forecasts have the same mean squared errors. The negative sign show the superiority of GDNS and GDNS -EGARCH over DNS model in the first and second pairs of comparisons respectively, while superiority of GDNS –EGARCH over GDNS (positive sign indicates preference of GDNS over GDNS –EGARCH model) in the third pair of comparison. Bold entries denote that the test statistic is insignificant at the 10% level.

Table A1. Descriptive statistics of yields data across maturities

Maturity	Mean	SD	Max	Min	Skewness	Kurtosis	$\hat{\rho}$ (1)	$\hat{\rho}$ (6)	$\hat{\rho}$ (12)
3	0.2094	0.2108	0.6956	0.0004	0.8717	2.2229	0.8812	0.7738	0.5673
6	0.2121	0.2136	0.7330	0.0041	0.8636	2.2869	0.8674	0.7508	0.5809
9	0.2285	0.2265	0.7699	0.0017	0.8518	2.2889	0.8614	0.7210	0.5818
12	0.2702	0.2408	0.8720	0.0041	0.6956	2.1064	0.8459	0.6773	0.5324
15	0.3000	0.2616	0.9910	0.0002	0.6922	2.1563	0.8491	0.6674	0.5444
18	0.3314	0.2845	1.1136	0.0130	0.7287	2.3042	0.8513	0.6620	0.5465
21	0.3638	0.3063	1.2377	0.0265	0.7444	2.4077	0.8535	0.6612	0.5498
24	0.3952	0.3276	1.3614	0.0192	0.7776	2.5691	0.8563	0.6649	0.5473
30	0.4673	0.3683	1.6030	0.0266	0.8743	2.9792	0.8598	0.6660	0.5425
36	0.5397	0.4097	1.8466	0.0505	0.9548	3.3446	0.8616	0.6693	0.5414
48	0.7044	0.4839	2.3083	0.0886	1.0271	3.7517	0.8659	0.6819	0.5348
60	0.8559	0.5458	2.6796	0.1139	1.0691	3.9729	0.8714	0.6938	0.5287
72	1.0030	0.5869	2.9704	0.1537	1.1181	4.1919	0.8722	0.6970	0.5206
84	1.1554	0.6057	3.1963	0.2364	1.1785	4.4380	0.8700	0.6925	0.5053
96	1.3080	0.6120	3.3739	0.3470	1.2049	4.6344	0.8663	0.6864	0.4891
108	1.4432	0.6108	3.5155	0.4468	1.2537	4.8852	0.8665	0.6860	0.4809
120	1.5618	0.6095	3.6304	0.5283	1.2698	5.0693	0.8663	0.6844	0.4708
180	1.9339	0.5410	3.7790	0.7577	1.2324	5.2687	0.8435	0.6319	0.4063
240	2.1755	0.5113	3.9802	0.9341	1.3751	6.2681	0.8358	0.6020	0.3286
300	2.3484	0.4743	3.9030	1.0703	0.8599	5.1924	0.8273	0.5621	0.2757

Note: The table shows descriptive statistics for monthly yields at different maturities. The last three columns contain sample autocorrelations at displacements of 1, 6 and 12 months. The sample period is 1996:01–2013:12. The number of observations is 216.

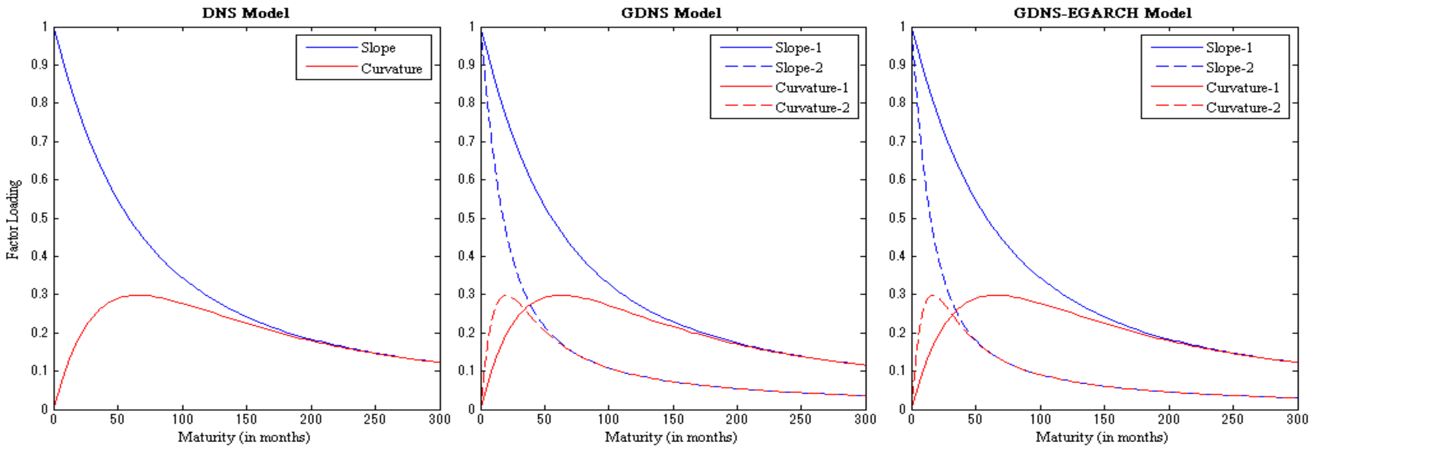

Figure 1. The slope and curvature factor loadings in the yield functions of the DNS, GDNS and GDNS-EGARCH models. The left-hand figure shows the slope and curvature factors loadings in the yield function of the DNS model with $\lambda = 0.0254$. The figure in middle shows the factor loadings of the two slopes and two curvatures in the yield function of the GDNS model with λ_1 and λ_2 equal to 0.0287 and 0.0919, respectively, while the right-hand figure shows the factor loadings of two slopes and two curvatures in the yield function of the GDNS-EGARCH model with λ_1 and λ_2 equal to 0.0271 and 0.1098, respectively. The λ values are set equal to the estimated values obtained in section 3.2, and the require maturity is measured in months.

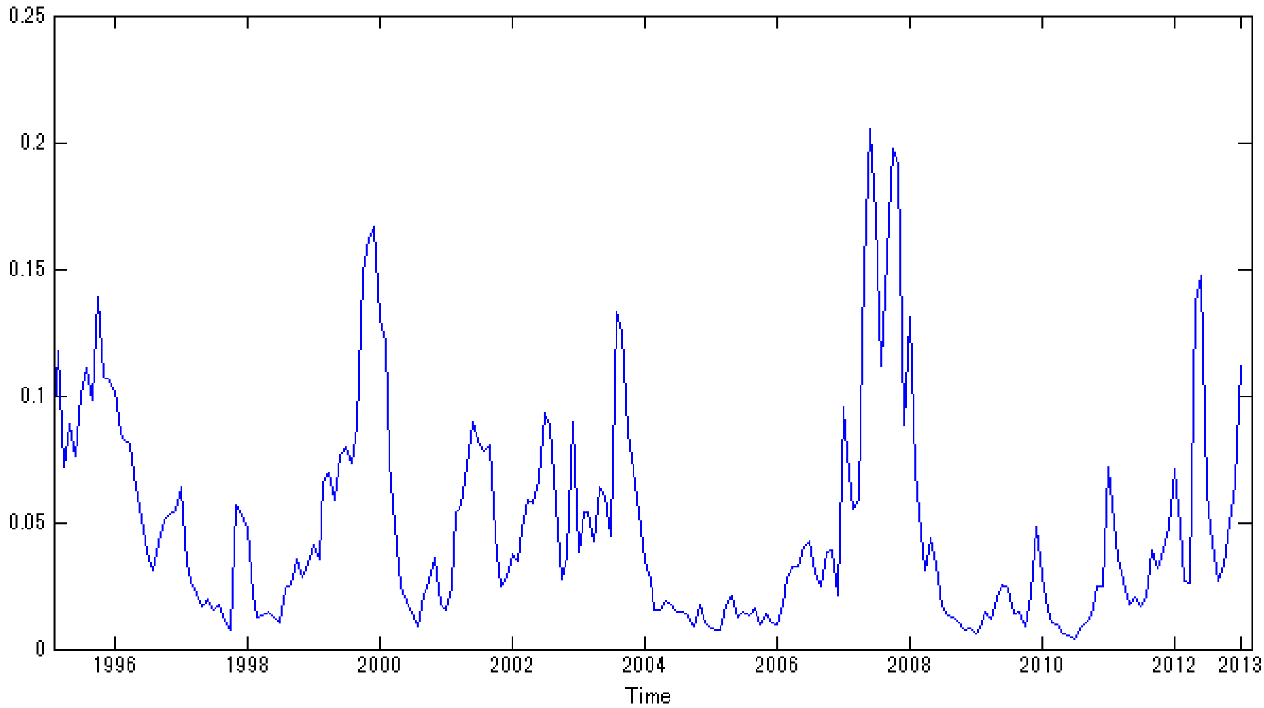

Figure 2. GDNS-EGARCH common volatility (h_t). The figure shows the plot of the volatility (h_t) of the common shock component (ε_t^*), which is modelled as EGARCH process, over time for the generalized dynamic Nelson-Siegel EGARCH (GDNS-EGARCH) model

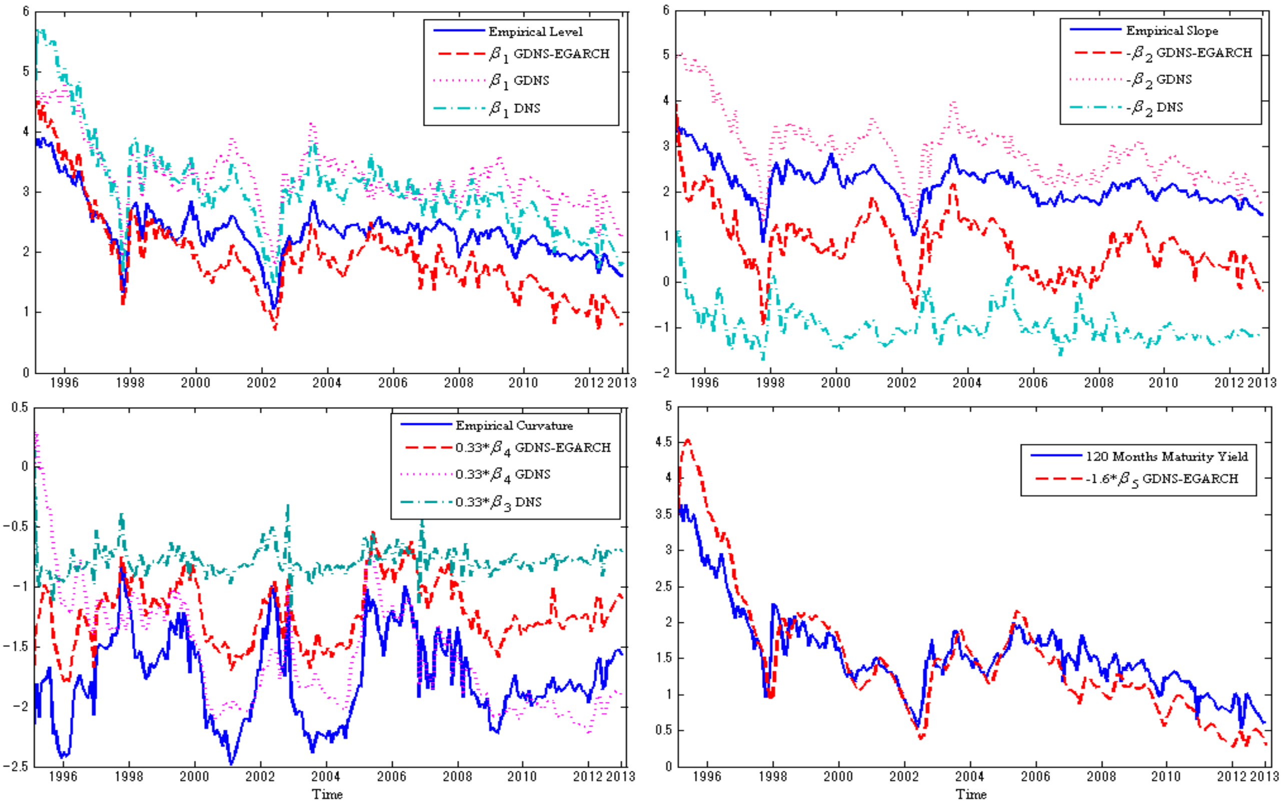

Figure 3. Time series plot of estimated factors and their empirical proxies. Model-based level, slope and curvature (i.e., estimated factors) vs. data-based level, slope and curvature (i.e., empirical proxies), where level is defined as the 25-year yield, slope as the difference between the 25-year and 3-month yields and curvature as two times the 2-year yield minus the sum of the 25-years and 3- month zero-coupon yields. Rescaling of estimated factors is based on Diebold and Li (2006).

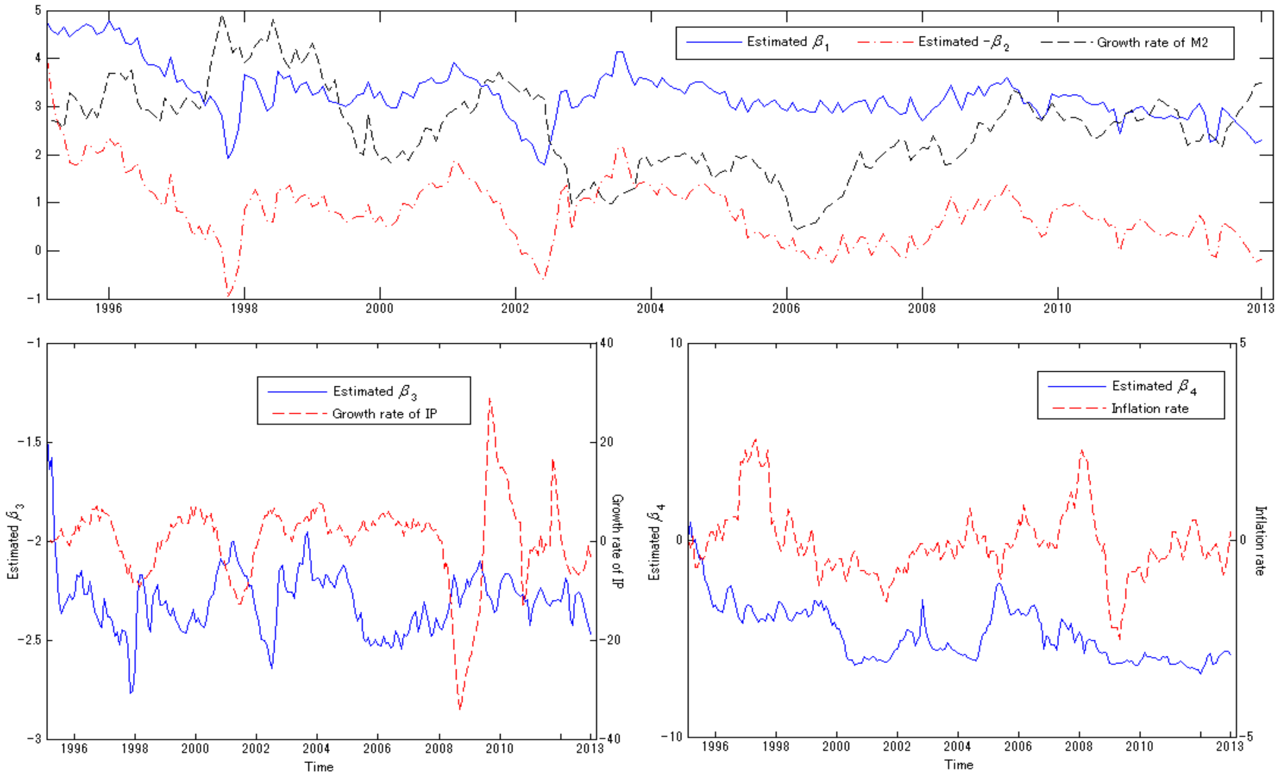

Figure 4. Time series plot of GDNS model estimated factors with macroeconomic variables. The estimated level and first slope factors ($\hat{\beta}_{1t}$and $-\hat{\beta}_{2t}$) are plotted vs. annual growth of the M2 (Money Supply) in the top figure. In the lower left pane, the second slope factor $(\hat{\beta}_{3t})$ is shown with the annual growth rate of industrial production (IP_t). The $\hat{\beta}_{3t}$ is scaled on the left y-axis, while IP_t is measured on the right y-axis. The lower right pane presents the first curvature factor estimate $(\hat{\beta}_{4t})$ against the annual inflation rate. The $\hat{\beta}_{4t}$ is scaled on the left y-axis, while INF_t is measured on the right y-axis. Inflation rate is the 12-month percent change in the consumer price index.

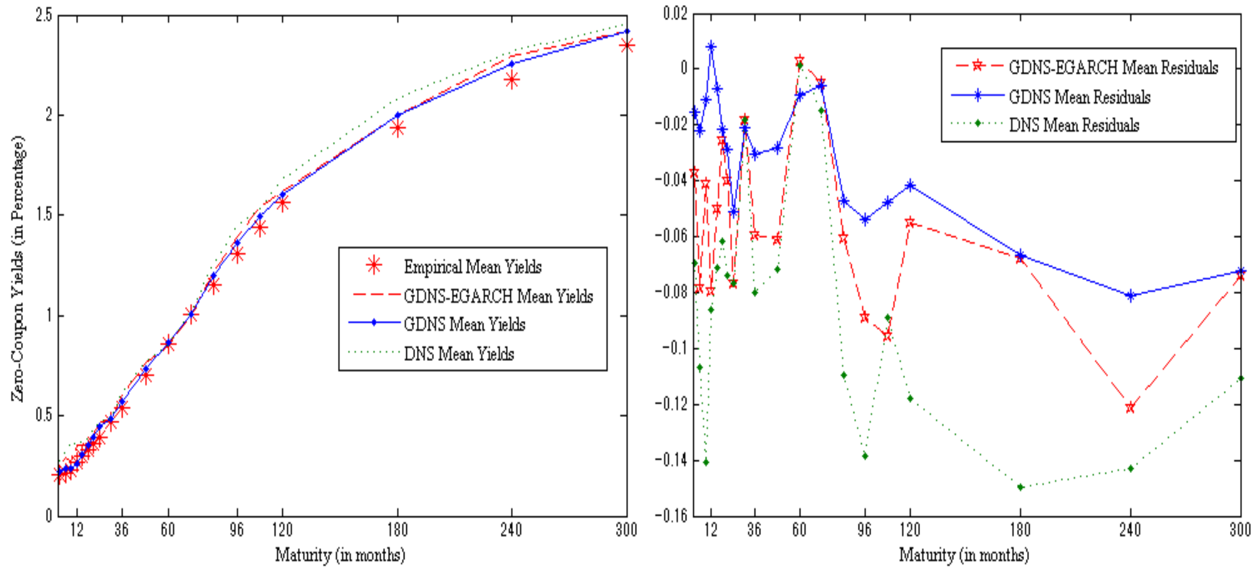

Figure 5. Mean yield curves and residuals. The figure shows the empirical yield curve and the mean estimated yield curves (mean fitted yields against maturities) and averaged residuals of the GDNS- EGARCH (five-factor model with time-varying volatility), GDNS (five-factor model) and DNS (three-factor model) yield curve models. The average fitted curves are computed by substituting the smoothed estimates of the yield curve factors and the estimated λs in the corresponding signal equations. The left pane shows the mean yields curves, while averaged residuals are shown in right pane. The averaged residual are computed as the mean of residuals across time for 20 distinct maturities.

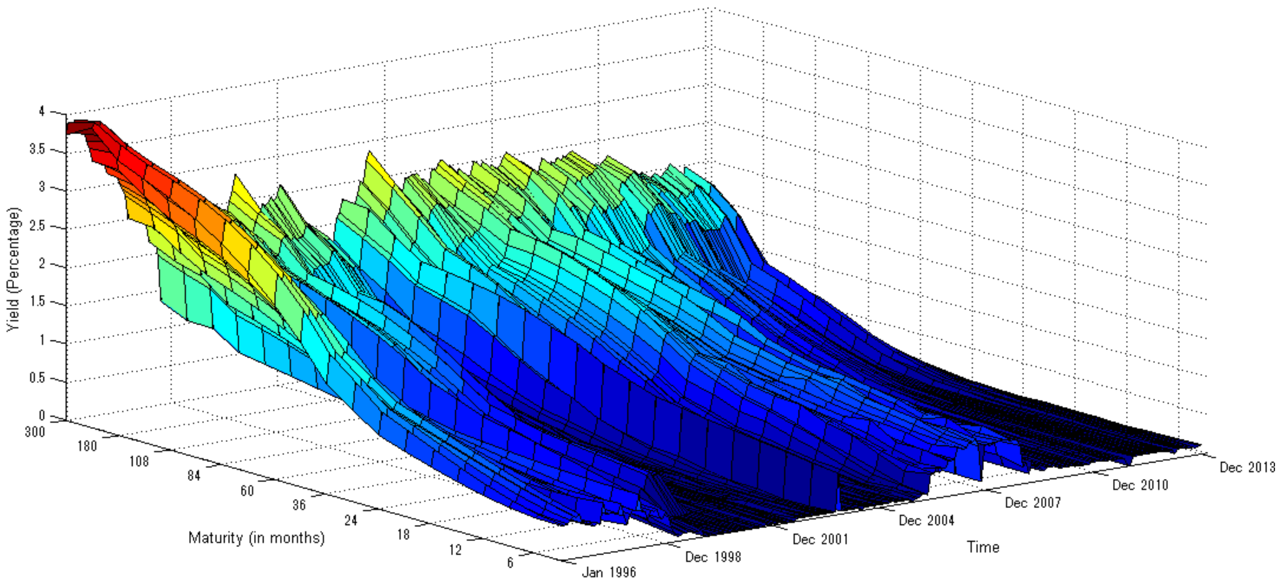

Figure A1. The figure shows the yield curves, 1996:01–2013:12. The sample consists of monthly yield data from January 1996 to December 2013 (216 months) for maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, 120, 180, 240 and 300 months (20 maturities).