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Abstract

This paper considers the long-run consequences of ranking job applicants on the basis of their unemployment durations by using a general equilibrium model in which statistical discrimination by firms against jobless workers may yield multiple stationary equilibria. Because the most inefficient equilibrium is supported by the belief that jobless workers have lost their employability, the government should dissuade firms from holding this extreme belief, thereby creating second chances for jobless workers. Moreover, by reducing the incomes of jobless workers through taxation, the government can create a new equilibrium in which job seekers can find new jobs without experiencing long-term unemployment.

Keywords: ranking, unemployment duration, efficiency wages, statistical discrimination.

JEL Classifications: E24, J64, J71.

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1 Introduction

Long-term unemployment has become one of the biggest problems facing developed economies. While details differ from one economy to another, there is one common feature found in all economies: the longer a worker is unemployed, the more difficult it becomes for him or her to find a job. There has been controversy over the cause of this phenomenon for more than three decades.[1] One group of researchers argue that the long-term unemployed cannot find a job simply because they are no longer employable. This argument is based on two presumptions. First, jobless workers gradually lose their employability as their unemployment duration becomes longer. Second, at job interviews, employers detect and reject job applicants who have lost their employability. It is self-evident that these presumptions jointly produce the above-mentioned observation. A second group of researchers does not accept this explanation, arguing that the long-term unemployed cannot find a job because employers "rank" job applicants by their unemployment durations and tend to hire those with shorter unemployment durations without checking whether they are really employable or not. Following their explanation, such a discriminatory practice, which Blanchard and Diamond (1994) call "ranking," makes it difficult for the long-term unemployed to find jobs, which produces the observed duration dependence. Tautological as it may sound, a worker is long-term unemployed because he or she is long-term unemployed.

Recently, a number of empirical and experimental studies have presented evidence that supports the second view. For example, Eriksson and Lagerström (2006, 2012) use an Internet-based CV database in Sweden to investigate empirically whether being unemployed reduces the probability of getting contacted by firms. Because all workers looking for new jobs are invited to submit their personal details to this database, the authors have access to exactly the same information as the firms, and they find that an unemployed applicant faces a lower contact probability than an otherwise identical employed applicant. Eriksson and Rooth (2011) use unique data from a field experiment in the Swedish labor market to investigate how past and contemporary unemployment affect a young worker's probability of being invited to a job interview; they find evidence that recruiting employers use not past but contemporary unemployment to sort workers. While these studies emphasize that being unemployed reduces the probability of getting contacted by firms, Oberholzer-Gee (2008), in a field experiment in Switzerland, documents that job market opportunities for unemployed workers diminish rapidly over time. According to this experiment, a person who has been without a job for 2.5 years is 51 percent less likely to be invited to an interview than an employed person. After 30 months, it makes little sense

[1] See Machin and Manning (1999) for a survey.

for an individual to keep applying for jobs because few firms will express an interest in hiring this person. The results of these studies strongly suggest that ranking is a common practice in the actual labor market.[2]

This paper considers the long-run consequences of ranking job applicants by their unemployment durations in the framework of a simple general equilibrium model with the following features. First, unemployment arises from the problem of nonverifiability rooted in the employer–employee relationship. Specifically, the model assumes not only that employers can only imperfectly observe the work efforts of their employees, as assumed in the model of Shapiro and Stiglitz (1984), but also that the work effort of any worker is unverifiable to third parties, including the law court. As argued by MacLeod and Malcomson (1989), when the work effort is unverifiable, backloading wage payment systems, such as performance pay, are ineffective in eliciting workers' effort because such a system gives employers an incentive to evade wage payment by asserting that the employee has not expended work efforts. This, in turn, induces the workers hired by an employer with that system to shirk in response to the prospective nonpayment of wages. To avoid such an unproductive situation, employers choose to adopt a non-backloading wage payment system as considered by Shapiro and Stiglitz, which leaves some workers in the jobless state. Second, statistical discrimination against the long-term unemployed arises from informational asymmetry between employers and jobless workers. The model assumes not only that employers can only imperfectly observe the employability of job candidates, but also that the employability of any worker is unverifiable to third parties. Under these assumptions, employers' beliefs about the employability of jobless workers can influence labor market performance based on the theory of statistical discrimination pioneered by Phelps (1972) and Arrow (1973). Specifically, employers believe that jobless workers who have been unemployed for at least a given length of time are not employable, and they set an admissible length of unemployment duration to qualify them for employment. Jobless workers, on the other hand, decide whether or not to preserve their employability, given such a ranking behavior and the wages they will receive after being rehired. This interaction between employers and jobless workers potentially gives rise to multiple stationary equilibria,

[2] In addition, it is worth mentioning the study by Ghayad and Dickens (2012) and a follow-up experiment performed by Ghayad. Their study shows that since June 2009, the Beveridge curve in the United States, which normally exhibits a downward relationship between job openings and unemployment, has broken down for the long-term unemployed in such a manner that a rising number of job openings does not reduce the number of the long-term unemployed. This implies the possibility that employers rank the long-term unemployed lower than other potential candidates. To test this possibility, Ghayad performs an experiment in which he sends out resumes describing the qualifications and employment history of 4,800 fictitious workers and finds that those who reported having been unemployed for six months or more received very few callbacks even when their qualifications were better than those of the workers who attracted employers' interest.

each of which supports a distinct belief of employers. Third, the number of employers is endogenously determined by free entry, which makes it possible to explore the implications of increased entry costs for labor market performance and allocative efficiency. As the entry cost takes a larger value, employers are forced to pay lower wages to their employees to keep investments in their firms profitable. When the entry cost is larger than a critical level, wages become too low to motivate employed workers, unless unemployment duration before finding a new job is expected to be sufficiently long. This necessitates a large scale of unemployment, which creates room for employers' belief to play a crucial role in selecting job applicants.

Theoretical and numerical analyses show that the number of stationary equilibria varies with the entry cost. When the entry cost is smaller than the above-mentioned critical level, there are two stationary equilibria in this model. In one of them, which we call the "no-second-chance equilibrium," or NSCE, employers believe that all jobless workers have lost their employability, and they fill their vacancies exclusively with new entrants into the labor force, who are employable by assumption. This hiring policy discourages jobless workers from maintaining their employability since it completely destroys their chances of being rehired. In the other equilibrium, which we call the "second-chance equilibrium," or SCE, employers believe that some jobless workers, whose unemployment durations do not exceed a given positive length of time, are still employable, and they fill their vacancies not only with new entrants, but also with the jobless workers who are qualified for employment. This effectively assures all jobless workers of finding a new job after experiencing one period of unemployment as long as they are employable, and thus, they choose to maintain their employability. Because the SCE facilitates rapid access by the unemployed to jobs, and because its allocation is far more efficient than that of the NSCE, the government in this case should guide the economy to the SCE by persuading employers to abandon the extreme belief that all jobless workers are unemployable. When the entry cost is larger than the above-mentioned critical level but smaller than another level, there are multiple SCEs and one NSCE in this model, although the number of SCEs decreases as the entry cost takes a larger value. Again, in this case, the NSCE is vastly inferior to the SCEs in employment and allocative efficiency. Thus, the government should persuade employers not to hold the belief that all jobless workers are unemployable, thereby preventing the realization of the NSCE. In addition, none of the existing SCEs attains the maximal level of employment, which implies that in those equilibria, jobless workers must experience more than one period of unemployment to find a new job. This malfunction of the labor market can be improved if the government can reduce the incomes of jobless workers by taxation. This policy coalesces the existing SCEs into a single SCE that attains the maximal level of employment, which works in favor of not only workers but also investors with stakes in firms operated by employers. When the entry cost is larger than the second critical level, there is no stationary equilibrium but a single NSCE in this model. In that equilibrium, no production occurs because wages that can motivate employed workers are too high for investors in a new firm to make a profit or break even, and thus, there is no labor demand in the economy. Again, in this case, the policy of reducing the incomes of jobless workers is effective in restoring production activities, as it creates one NSCE and one SCE, both of which attain a positive level of employment; the latter, in fact, attains its maximal level. However, this does not lead to an improvement in economic welfare since it brings a substantial welfare loss to workers (although it always makes investors better off). In this case, taxing jobless workers effectively curtails their income-earning opportunities other than employment, thereby forcing them to work for low wages.

The idea that the prolonged unemployment of some workers results from employers' discriminatory treatment of job applicants based on their observable record of unemployment has been formalized by Lockwood (1991), Blanchard and Diamond (1994), Acemoglu (1995), Kübler and Weizsäcker (2003), Kugler and Saint-Paul (2004), Eriksson and Gottfries (2005), and Eriksson (2006). Among these theoretical works, that of Acemoglu is most closely related to this paper, as he also uses statistical discrimination to explain long-term unemployment. Unlike in the model of this paper, unemployment occurs as a result of a mismatch in his model, but the models have two common assumptions. First, jobless workers have to incur a cost to maintain their skills (or employability). Second, in the current model, employers can only imperfectly observe whether the jobless workers have done so, whereas in Acemoglu's model, once employment begins, employers can perfectly observe whether their newly-recruited employees have skills. These assumptions jointly produce multiple equilibria, in one of which employers discriminate against the long-term unemployed, who, in response, allow their skills to atrophy. When the cost to maintain skills is sufficiently small and employers can fairly precisely observe whether the unemployed have incurred that cost, Acemoglu's model also has an equilibrium in which employers do not discriminate against the long-term unemployed, who, in response, decide to maintain their skills. As stated above, similar results are obtained in this paper, which is a natural result based on the assumptions that both models have in common.

Pioneering as it is, Acemoglu's analysis has several issues. First of all, it is not sufficiently detailed to explore the full implications of his model. For analytical simplicity, he has assumed that matching probabilities are constant, which markedly limits the scope of his model and possibly conceals some implications that would be evident if these probabilities were endogenized, especially those of entrepreneurial environments for employment. A more detailed analysis is needed, as the importance of entrepreneurship for job creation is now widely recognized. In addition, a well-known criticism against the theory of statistical discrimination is true of his model. Since, in his model, employers can perfectly observe whether their employees have skills after employment has begun, discrimination against the long-term unemployed is eliminated if employers adopt an employment system in which higher wages and tenure are given to the workers who have skills at the stage of non-tenured employment. In other words, the occurrence of discrimination in his model depends crucially on the assumption that labor contracts must be written before the employability of a job applicant becomes clear.

To advance the theory of long-term unemployment one step further in the direction indicated by Acemoglu, this paper extends the efficiency wage model of Shapiro and Stiglitz (1984) into a general equilibrium framework in which the number of employers is endogenously determined by free entry, thereby exploring the implications of entry costs for labor market performance and allocative efficiency. To avoid the criticism against the theory of statistical discrimination, the model assumes that employers can only imperfectly observe the employability of job candidates and that the employability of any worker is unverifiable to third parties, including the law court, as already stated. Under these assumptions, employers are reluctant to adopt the above-mentioned tenure-track system because the job openings of those who have adopted it are likely to be avoided by job applicants, who deeply suspect that those employers have an incentive to exploit that nonverifiability, firing their employees at the end of the non-tenured employment stage under the pretense that they have turned out to be unemployable, thereby avoiding the payment of higher wages. To dispel this suspicion, employers need to enter into contracts with workers before their employability becomes clear by screening job applicants based on their employment records before interviews.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 considers the labor contract between the employer and the worker under the assumed informational frictions, and then derives the profile of the aggregate labor demand. Section 4 derives profiles of the "aggregate incentive constraints," which provide the wage level employers would choose given the aggregate employment, although they are defined only when those wages are high enough to motivate the jobless workers whom employers believe to be employable to preserve their employability. Section 5 derives the stationary equilibria numerically and examines their welfare properties to obtain some policy implications. Section 6 concludes.

2 The Model

The analysis is based on a simple dynamic general equilibrium model with efficiency wages à la Shapiro and Stiglitz (1984). Time is discrete, extending from negative infinity to positive infinity.

2.1 Workers

At the beginning of each period, a continuum of workers of measure $[\theta/(1-\theta)]N$ is born, where θ and N are constants satisfying $\theta \in (0,1)$ and $N > 0$. Workers are mortal: they die in a given period with a probability θ, and therefore, their probability of surviving into the next period is $1-\theta$. This shock is idiosyncratic, and thus, the population of workers born s period prior decreases to $\theta(1-\theta)^{s-1}N$ at the beginning of the current period. In the first period of their lives, workers neither work nor consume, but devote the period to job search activities. Because they do not work until the second period of their lives, the population of the labor force in a given period, that is, the total number of workers who can work in that period, is

$$\sum_{s=1}^{\infty} \theta(1-\theta)^{s-1}N = N.$$

The constant population of the labor force is attained in such a manner that at the beginning of each period, the old workers of measure θN die and exit the labor force and, as a substitute, the same measure of new workers, who were born in the previous period, enter it.[3]

The lifetime utility of a worker born in a generic period t is given by

$$E_t \sum_{s=1}^{\infty} \left(\frac{1-\theta}{1+r}\right)^s (I_{t+s} - e_{t+s})$$

where E_t, r, I_{t+s}, and e_{t+s} represent, respectively, the conditional expectation evaluated at the beginning of period t, the discount rate that is positive and constant over time, the real income earned in period $t+s$, and the effort level chosen in that period. After entry into the labor force, a worker is either employed by a firm or unemployed in any period. She is paid wages by her firm when employed and endowed with some fixed amount ($\overline{w}$ units) of the consumption good when unemployed.[4] Thus, I_{t+s} can be written as

$$I_{t+s} = \begin{cases} w_{t+s} & \text{if employed in period } t+s \\ \overline{w} & \text{otherwise,} \end{cases}$$

where w_{t+s} denotes the real wage received in period $t+s$.

In each period, the worker sets the level of effort at either zero or a fixed positive, denoted by e, which affects her employer's production when employed and her own employability when unemployed. An employed worker contributes to her employer's production if and only if she is employable

[3] For any generation of workers, by assumption, those of measure $[\theta^2/(1-\theta)]N$ die before entering the labor force.

[4] This unemployment allows for workers that are self-employed or in low-paid jobs in the secondary labor market.

and expends e units of work effort. Every worker is employable when entering the labor force and costlessly maintains this ability during periods of employment. During periods of unemployment, in contrast, a worker must expend e units of effort in every period to maintain her employability. If she neglects this maintenance even in one period, her employability is lost and never restored.

2.2 Firms and Investors

In this economy, firms are also mortal: in every period, new firms are established, while some of the existing ones disappear.

Establishing a new firm requires a lump sum payment of F, which is funded by a continuum of investors who are assumed to live from the infinite past to the infinite future. In each period, they attempt to maximize a discounted sum of net income flows in which future incomes are discounted by the rate of r, as in the utility function of workers. They can borrow and lend as much as they like at a market interest rate, the level of which equals r in equilibrium and, thus, can establish as many firms as they want.

An established firm does not start production until its second period of existence, but in the first period, it recruits workers for production in the next period. In addition, at the end of each period except the first one, after having paid wages and dividends in that period, some firms experience with probability b such an idiosyncratic shock that they suddenly become unproductive. The firms that experience this shock are never productive again. To formalize this assumption, the technology that firms use to produce the consumption good is specified as

$$Y = z\tilde{L}^{\alpha},$$

where Y and $\tilde{L}$ denote, respectively, the output of the consumption good and the number of employees who are employable and have expended work efforts. The value of $\tilde{L}$ does not necessarily correspond to the total number of employees since some of them might be either "unemployable" or "employable but expending zero effort." Firms take α and z as given: the former is a common constant satisfying $\alpha \in (0, 1)$ and the latter is a firm-specific productivity that initially takes a unitary value but irreversibly switches to a zero value with probability b at the end of each period.

Let $\tilde{L}_{t+s}$, L^i_{t+s}, and w^i_{t+s} denote, respectively, the number of employees who are employable and expending efforts in period $t + s$, the number of employees who complete the ith period of service in that period, and the level of the wage paid to those employees in that period. The assumption on the production technology suggests that one of the tasks assigned to the management of a firm is to make the value of $\tilde{L}_{t+s}$ correspond to the total number of employees, $\sum_{i=1}^{s} L^i_{t+s}$, for any value of s (> 0) by motivating employable workers to expend work efforts while preventing unemployable ones from being added to the firm. This is not an easy task because of some informational frictions built into this economy; these will be discussed in the next subsection. In this subsection, we simply suppose that firms are able to do this task. Then, the net gain from establishing a new firm in period t is evaluated by investors as

$$-F + \frac{1}{1+r}\sum_{s=1}^{\infty}\left(\frac{1-b}{1+r}\right)^{s-1}\left[(\mathbf{1}_s \cdot \mathbf{L}_{t+s})^{\alpha} - \mathbf{w}_{t+s}\cdot \mathbf{L}_{t+s}\right], \tag{1}$$

where $\mathbf{1}_s$, $\mathbf{w}_{t+s}$, and $\mathbf{L}_{t+s}$ denote, respectively, an s-dimensional all-ones vector,

$$\mathbf{w}_{t+s} = (w^1_{t+s}, \cdots, w^s_{t+s}),$$

and

$$\mathbf{L}_{t+s} = (L^1_{t+s}, \cdots, L^s_{t+s}).$$

The firm maximizes (1) by optimally selecting a path of wages and employment, $\{(\mathbf{w}_{t+s}, \mathbf{L}_{t+s})\}_{s=1}^{\infty}$, since it is in the best interest of their investors.

2.3 Informational Frictions

In a given period t, the state of a worker who has already entered the labor force can be summarized by a pair (n_t, Q_t). The first variable, n_t, represents the worker's recent experience of (un)employment. Specifically, if n_t is a positive integer, this worker enters the n_tth period of service with the current firm in period t. If n_t is a negative integer, she enters the $-n_t$th period of continuing unemployment in period t. If $n_t = 0$, she is born in period t, and has not yet entered the labor force. Every job seeker has a non-positive value for n_t because workers are not allowed to hop directly from one firm to another. If a worker separates from the current firm, then she must experience at least one period of unemployment before being hired by the next firm. The second variable, Q_t, indicates her employability, and takes a unitary value if she is employable at the beginning of period t and zero otherwise. The following properties of this variable are easy to verify. First, $Q_t = 1$ for those who are born in period $t-1$ since every worker is employable when entering the labor force. Second, if $Q_t = 0$ for some t, then $Q_{t+s} = 0$ for all $s > 0$ since lost employability is never restored. Before applying for a job opening in period t, jobless workers decide whether or not to expend an employability-preserving effort, and thus, at the point of a job interview, their employability in the next period (i.e., the value of Q_{t+1} for each job applicant) is already determined.

Firms can observe an applicant's recent experience of employment (i.e., the value of n_t) perfectly, but evaluate her employability in the next period (i.e., the value of Q_{t+1}) only imperfectly. In job interviews, they can detect an unemployable applicant only with a fixed probability q ($\in (0,1)$), although they never mistake an employable applicant for an unemployable one. Even after having started an employment relationship with a worker, the firm can observe this employee's performance only imperfectly. Specifically, when the employee does not expend work efforts, the firm can catch her shirking only with probability q, although it never mistakes a non-shirking employee for a shirking one. Neither the firm nor the employee herself can verify the employee's employability or effort expenditure to a third party such as the law court. All they can verify is whether or not the employee has shown up at the office and whether or not the firm has paid the promised wage.

2.4 Sequence of Events within a Period

Events within a given period proceed as follows.

After the birth of new workers and the establishment of new firms, all firms present in that period announce both their plan for current and future hiring and wage levels for newly recruited and already-employed workers. Firms may also announce the admissible duration of unemployment for job applicants, which is used as a criterion to select the applicants to be interviewed. Given these announcements, jobless workers decide whether or not to continue maintaining their employability, and employed workers decide whether or not to expend work effort in the current period. Then, firms other than the newly established ones start production with the employees recruited in or before the previous period. After production activities are finished, wages are paid to employed workers. Then, a fraction b of operating firms become unproductive, and their employees lose their jobs. In addition, labor contracts with workers who have been caught shirking are terminated. Labor contracts with the other employed workers are renewed, although some of them die at the end of the current period. To replace them, the firms that survive into the next period post some job openings. Newly born and jobless workers apply to these openings, and the firms interview those who have passed the criteria, selecting new employees from among them.

3 Labor Contracts and Aggregate Labor Demand in Stationary Equilibria

The model constructed in the previous section potentially has a huge variety of equilibria that yield qualitatively different outcomes. Instead of fully characterizing them, we restrict our focus to a class of stationary equilibria in which the wage paid to a worker depends solely on her length of service with her current employer. That is, if w^n_{t+s} denotes the level of wages paid to the workers who are currently employed by a firm established in period t and who complete their nth period of service in period $t+s$, then, in such a stationary equilibrium, w^n_{t+s} takes a common value of w^n for any combination of t and s, although w^n may vary with n. In this section, based on the assumption that all employed workers are employable, we first characterize the labor contract written in such a stationary equilibrium, then use the free-entry condition to examine the determination of wages and employment at an individual firm, and finally profile the aggregate labor demand.

3.1 Labor Contracts

Under the informational frictions assumed in the previous section, a labor contract that is acceptable to workers must be such that in any period, wage payment is made if an employee shows up at the office, independent of her effort expenditure. Wage payment cannot be conditioned on the employee's effort expenditure since it is unverifiable. On the other hand, unconditional payment of wages gives employees an incentive to shirk. To prevent this moral hazard, the labor contract also stipulates that the wage level that the firm should pay is significantly higher than the employee's reservation, $\overline{w}$; that the term of the contract is one period; and that the contract terminates unless both of the contracting parties agree to renew it. These clauses jointly make termination of the contract highly damaging to the employee, thereby enabling the firm to punish a shirking employee through that termination.[5]

Such an "at-will" employment relationship renders it difficult for a pair of a firm and a worker to make the promise that the wage will increase whenever the labor contract is renewed. If the starting wage is lower than those supposed to be paid in and after the second period of service, the firm naturally has an incentive to save labor costs by terminating the contracts of current employees at the end of each period and recruiting new workers at a low starting wage with the empty promise of a future raise. Given the possibility of this moral hazard, newly recruited workers choose to expend zero effort if their current wages are lower than those they are supposed to receive in and after the next period because they regard such an upward-sloping wage profile as an informal notice that their contracts will be terminated at the end of the current period. To alleviate their suspicions and, thus, to elicit their work efforts, the firm needs to make the starting wage no lower than those paid in and after the second period of service.[6]

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[5] Because an employee's effort expenditure is unverifiable, it is impossible for a firm to discipline its employees by use of such a bonding system in which, prior to starting work, the employees post bond to a third party, such as the law court, which would be forfeited by that party in the case of being caught shirking.

[6] This moral hazard incentive is introduced to make this model immune to the criticism

The next proposition characterizes a profile of wages paid to an employee. It is "almost" flat in the sense that the employee receives a constant level of wages in and after the second period of service with her employer, although she is possibly better paid in the first period.

Proposition 1 (Wage Profile)**.** *Suppose that the economy is settled in a stationary equilibrium, and let* $V(n, Q)$ *and* w^n *denote the lifetime utility of a worker in the labor force whose current state is* (n, Q) *and the wage level an employed worker receives in the nth period of service with her employer, respectively. Moreover, define* w^{2+} *as*

$$w^{2+} \equiv \frac{r+\theta}{1+r}V(-1,1) + \frac{1+r-(1-\theta)(1-b)(1-q)}{(1-\theta)(1-b)q}e. \tag{2}$$

Then, wages are paid as $w^1 \geq w^{2+}$ *and* $w^n = w^{2+}$ *for* $\forall n \geq 2$.

Proof. See Appendix A. □

As long as wages are set as in Proposition 1, employed workers have no incentive to shirk. In the rest of this paper, w^1 and w^{2+} denote, respectively, the wages paid to newly recruited workers and those paid to other employed workers.

3.2 Aggregate Labor Demand

Our next task is to profile the aggregate labor demand, which is a relation between the aggregate employment and a weighted average of w^1 and w^{2+} that is implied by firms' optimizing behavior and the free-entry condition.

To this end, we first need to show that in any stationary equilibrium, all operating firms keep their number of employees constant over time at a common level.

Lemma 1. *Suppose that the economy is settled in a stationary equilibrium, and let* L_{t+s} *represent the total number of employees a firm established in*

by Carmichael (1985) of Shapiro-Stiglitz efficiency wage model. The point of his criticism is that even if firms are restricted to eliciting work efforts by paying efficiency wages, they can still sell their jobs and require newly recruited workers to accept a low starting wage level, or pay an entrance fee. He asserts that, contrary to the argument of Shapiro and Stiglitz, unemployment cannot be involuntary since the starting wage or entrance fee clears the job market in each period. In the present model, however, the starting wage may fail to clear the job market when its market-clearing level is sufficiently low. Firms are reluctant to set the starting wage at such a low level, being apprehensive that this may induce newly recruited workers to expend zero effort. To make the labor contract with them "credible," firms rather choose to set the starting wage at a sufficiently high level, which causes job rationing and involuntary unemployment in the job market. Similar arguments are made by Arvan and Esfahani (1993) and Ritter and Taylor (1994).

period t puts into production activities in period $t+s$. Then, for $\forall t$ and $\forall s$ (≥ 1),

$$L_{t+s} = L,$$

where the value of L is given by

$$\alpha L^{\alpha-1} = w^1 - \frac{(1-b)(1-\theta)(w^1 - w^{2+})}{1+r} \ (\equiv \hat{w}). \tag{3}$$

Proof. See Appendix A. □

This lemma might give a paradoxical impression since it asserts that new firms, which have just started production (thus having no option but to use only newly recruited workers) and old firms, which started production one or more periods before (thus being not so dependent on newly recruited workers) would choose the same number of employees even when $w^1 > w^{2+}$. This result is accounted for by the fact that in choosing the number of employees, firms only consider the expected sum of wages that will be paid to a marginally hired worker, which is given by

$$w^1 + \sum_{s=1}^{\infty} \left[\frac{(1-b)(1-\theta)}{1+r}\right]^s w^{2+} = \frac{1+r}{1+r-(1-\theta)(1-b)} \hat{w}.$$

As suggested by this equation, the expected wage payment can be averaged out to $\hat{w}$ per period. Regardless of their histories, all of the operating firms view $\hat{w}$ as the marginal cost of labor, and they thus choose the same number of employees.

In a stationary equilibrium in which a positive number of firms are operating, the net gain from investing in a new firm is reduced to zero by free entry. This uniquely determines the values of $\hat{w}$ and L, given α, b, r and F.

Lemma 2. *Suppose that the economy is settled in a stationary equilibrium, where the net gain from investing in a new firm is reduced to zero by free entry. Then, in that equilibrium, every operating firm sets $\hat{w}$ and L as*

$$\hat{w} = \left[\frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{(r+b)^{1-\alpha} F^{1-\alpha}}\right]^{1/\alpha} (\equiv w^*), \quad L = \left[\frac{(r+b)F}{1-\alpha}\right]^{1/\alpha} (\equiv L^*). \tag{4}$$

Proof. See Appendix A. □

Now we can state the following.

Proposition 2 (Aggregate Labor Demand). *Suppose that the economy is settled in a stationary equilibrium, and let E denote the aggregate employment, whereas $\hat{w}$ and w^* are defined by (3) and (4), respectively. Then, in that equilibrium, E and $\hat{w}$ satisfy*

$$(\hat{w} - w^*)E = 0, \quad E \geq 0 \quad \textit{and} \quad \hat{w} - w^* \geq 0. \tag{5}$$

Figure 1: Aggregate Labor Demand

Proof. See Appendix A. $\square$

Figure 1 depicts a locus of pairs of E and $\hat{w}$ satisfying (5). To see why the locus is L-shaped, we need to recall the definitions of $\hat{w}$ and w^*. As implied by (3), $\hat{w}$ is the marginal labor cost incurred by a firm when it hires an additional worker. To motivate this worker, the firm must, on average, pay $\hat{w}$ units of wage per period. On the other hand, w^* is the break-even level of $\hat{w}$, at which the net gain from establishing a new firm becomes zero. When $\hat{w} > w^*$, operating firms, if they exist, cannot discipline their employees without paying wages that are higher than their break-even levels. This deprives investors of the incentive to establish a new firm, which ultimately reduces the number of operating firms and, thus, the aggregate labor demand, to zero. When $\hat{w} < w^*$, in contrast, operating firms can discipline their employees by paying wages that are lower than their break-even levels. This gives investors a strong incentive to establish a new firm, which ultimately makes the number of operating firms and, thus, the aggregate labor demand, infinitely large. However, such a large labor demand cannot be met by any means, meaning that there is no such equilibrium with $\hat{w} < w^*$ in this model. Finally, when $\hat{w} = w^*$, the net gain from establishing a new firm becomes zero, which makes the number of operating firms and, thus, the aggregate labor demand, indeterminate.

Table 1: Model Parameters

Workers	r	Discount rate	0.01
	θ	Death rate	0.03
	$\overline{w}$	Income when unemployed	1.00
	e	Work effort	0.50
	N	Population of the labor force	1.00
Firms	α	Labor share in production	0.70
	b	Business closing rate	0.04
	q	Rate of detecting unemployable and shirking workers	0.90

4 Firms' Belief, Ranking, and Aggregate Incentive Constraints

To determine the equilibrium values of E and $\hat{w}$, we also have to derive the aggregate incentive constraints (AIC). A profile of an AIC provides the level of $\hat{w}$ firms would choose given the level of E, although it is defined only when that level of $\hat{w}$ is high enough to motivate the jobless workers whom firms believe to be employable to actually preserve their employability. As in other models of efficiency wages, firms choose a wage level with the goal of preventing their employees from shirking. In this model, moreover, these wages then unintentionally keep some jobless workers from becoming unemployable. Because our focus is on a stationary equilibrium in which firms' belief is fully supported, the AIC must be consistent with their belief about jobless workers' employability.

We index firms' belief about jobless workers' employability by a non-positive integer $\overline{n}$, which means that they believe that job applicants with $n \in [\overline{n}, 0]$, who have been unemployed for no more than $-\overline{n}$ periods, are still employable, while other applicants are unemployable. When firms have a belief indexed by $\overline{n}$, they naturally "rank" job applicants by their unemployment duration or, more specifically, set the admissible level of unemployment duration for job applicants at $\overline{n}$, thereby disqualifying applicants with $n < \overline{n}$. Obviously, this hiring policy discourages those who have been unemployed for more than $-\overline{n}$ periods from preserving their employability, which partially fulfills firms' belief. The question is whether this really motivates other jobless workers to preserve their employability, as firms believe. Checking whether or not this condition is met for qualified workers is such a detailed process that we cannot but resort to a numerical method to derive an AIC profile for each value of $\overline{n}$.

Figures 2 and 3 depict the profiles of the AICs that are obtained with this numerical method under the parameter configuration of Table 1.[7] For

[7]For the details of the derivation, see Appendix B. Since the expected length of a worker's stay in the labor force is given by $1/\theta$, we have effectively assumed that workers

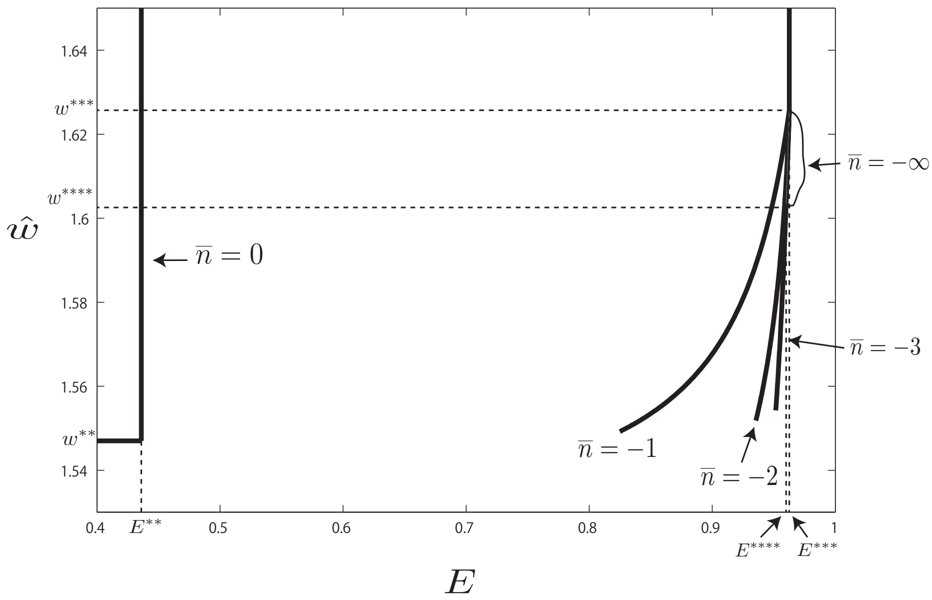

Figure 2: AICs for Various Values of $\overline{n}$

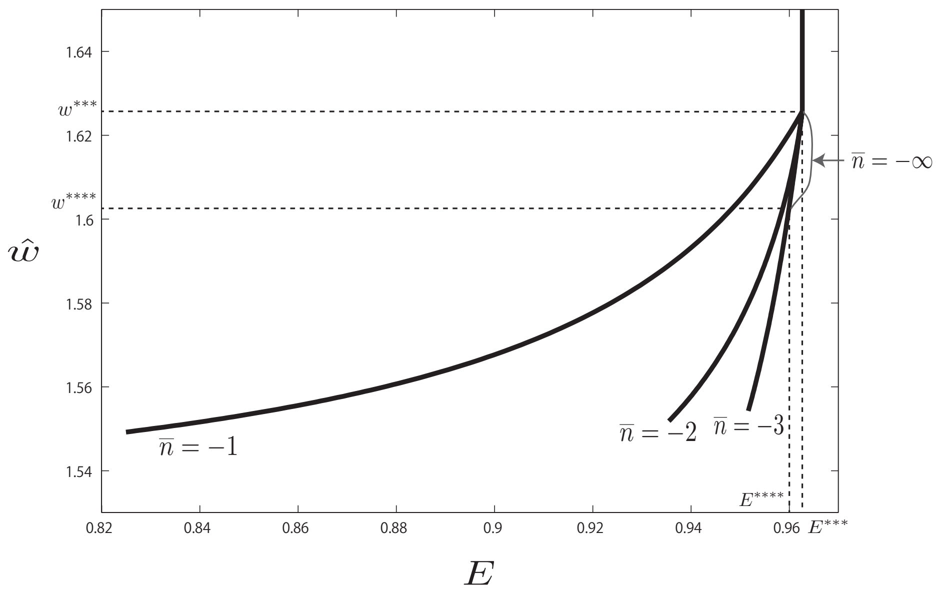

Figure 3: AICs for Various Values of $\overline{n}$ (Enlarged)

Table 2: New Notations

Notation	Definition	Implied Value
E^{**}	$[\theta/(\theta+b-\theta b)]N$	0.436
E^{***}	$[1/(1+b-\theta b)]N$	0.963
E^{****}	$(1-b)N$	0.960
w^{**}	$\overline{w}+\{[1+r-(1-\theta)(1-b)(1-q)]/(1-\theta)(1-b)q\}e$	1.547
w^{***}	$\overline{w}+\{[1+r+(1-\theta)b]/(1-\theta)(1-b)q\}e$	1.626
w^{****}	$\overline{w}+[(1+r)/(1-\theta)(1-b)q]e$	1.603

conciseness in expression, these figures use some new notations: E^{**}, E^{***}, E^{****}, w^{**}, w^{***}, and w^{****}. Their definitions and implied values under the above parameter configuration are summarized in Table 2.

As shown in Figure 2, the profile of the AIC for $\overline{n}=0$ is derived for relatively small values of E, while those of the AICs for $\overline{n}<0$ are derived for relatively large values of E. This contrast is mostly attributable to whether or not jobless workers are given "second chances," that is, chances to be rehired. When $\overline{n}=0$, firms fill their vacancies exclusively with new entrants into the labor force, and thus there is no second chance for jobless workers. Because of this exclusive hiring policy, even if all the new entrants are hired in every period, the aggregate employment cannot exceed E^{**}, which, under our parameter configuration, means that more than half of the labor force is left in the jobless state. When $\overline{n}<0$, firms fill their vacancies not only with new entrants but also with those who have been unemployed for $-\overline{n}$ periods or less, and thus, there are some second chances for jobless workers. Such an inclusive hiring policy makes it possible for the aggregate employment to be as large as E^{***}. When $E=E^{***}$, in particular, all workers but those who have just lost their jobs are employed; equivalently, every job seeker can find a job by experiencing only one period of unemployment. These imply that there are no workers who have been unemployed for two periods or more, and thus that, as long as $\overline{n}<0$, the difference in the value of $\overline{n}$ has no effect on equilibrium outcomes. This is why profiles of AICs for $\overline{n}<0$ have a common vertical part.

The reason profiles of the AICs for $\overline{n}<0$ cannot be derived for relatively small values of E is explained as follows. As E takes a smaller value, the job finding rate for jobless workers decreases its value, which enables firms to discipline their employees by paying lower wages. At the same time, the lower rate of job finding and lower wages after being rehired gradually reduce

spend an average of 33.3 periods in the labor force by setting the value of θ at 0.03. The life expectancy of a firm is $1/b=25$ periods, and the expected duration of an employment relationship is $1/(\theta+b-\theta b)\approx 14.5$ periods. Moreover, the probability of "lifelong employment," that is, the probability that a newly recruited worker will not experience unemployment until she dies, is $\theta/(\theta+b-\theta b)\approx 0.43$, which implies that more than 56% of newly recruited workers experience unemployment at least once during their lives.

the incentive of jobless workers to preserve their employability, eventually obliterating it. This runs counter to firms' belief of $\overline{n} < 0$, therefore, AICs cannot be defined for such small values of E.

This argument helps us understand why profiles of the AICs for $\overline{n} < 0$ have an upward-sloping part, while that of the AIC for $\overline{n} = 0$ has a horizontal part instead. As already explained, when $\overline{n} < 0$, a smaller value of E enables firms to discipline their employees by paying lower wages, which explains why the AICs for $\overline{n} < 0$ have an upward-sloping part. When $\overline{n} = 0$, on the other hand, there is no second chance for jobless workers, and thus, their job finding rate always takes the zero value. As a result, neither the incentive of employed workers to expend efforts nor firms' wage-setting behavior is affected by any change in E, which is the intuition behind the horizontal part of the AIC for $\overline{n} = 0$.

Figures 2 and 3 also show that a decrease in $\overline{n}$ rotates the upward-sloping part of the AIC for $\overline{n} < 0$ counterclockwise around its upper end, gradually shortening its length.[8] Such a decrease in $\overline{n}$ qualifies a larger set of jobless workers for the given vacant positions, thereby reducing the probability that a qualified worker will be rehired. From the viewpoint of currently employed workers, this effectively stiffens the penalty for shirking since, if they lose their current jobs under the smaller value of $\overline{n}$, they are likely to experience a longer duration of unemployment before finding a new job. This, in turn, enables firms to discipline their employees by paying lower wages in accordance with the level of E, which leads to the counterclockwise rotation of AIC. From the viewpoint of currently jobless workers, the same decrease in $\overline{n}$ effectively reduces the probability that they will be rehired since it enlarges the set of jobless workers who are qualified for employment. This discourages them from preserving their employability, which cannot be undone by anything but the prospect that they will receive sufficiently high wages after reemployment. This is why the upward-sloping part of AIC is shortened as $\overline{n}$ takes a smaller value.

5 Stationary Equilibria with and without Second Chances

We now are in a position to examine the stationary equilibria of this model. A stationary equilibrium in which firms embrace the belief of $\overline{n}$ is an intersection of the loci of the aggregate labor demand and the AIC for $\overline{n}$. Considering the qualitative difference between the AIC for $\overline{n} = 0$ and those

[8]In Figures 2 and 3, the upward-sloping parts of the AICs for $\overline{n} \leq -4$ are not depicted except for $\overline{n} = -\infty$ because they are hidden by that of the AIC for $\overline{n} = -3$. The only difference among the upward-sloping parts of the AICs for $\overline{n} \leq -3$ is their lengths: as $\overline{n}$ takes a smaller value, the length of the upward-sloping part becomes shorter.

Table 3: Critical Values of F

Notation	Definition	Implied Value
F_1	$(1-\alpha)(\alpha/w^{***})^{\alpha/(1-\alpha)}/(r+b)$	0.840
F_2	$(1-\alpha)(\alpha/w^{****})^{\alpha/(1-\alpha)}/(r+b)$	0.868
F_3	$(1-\alpha)(\alpha/w^{**})^{\alpha/(1-\alpha)}/(r+b)$	0.943

for $\overline{n} < 0$, we call the equilibrium with $\overline{n} = 0$ the "no-second-chance equilibrium" (NSCE), and those with $\overline{n} < 0$ "second-chance equilibria" (SCE).

5.1 Diagrammatic Expositions

In examining the NSCE and SCE, we should pay attention to the entry cost for new firms, F, since, as shown in (2), it is the primary determinant of the aggregate labor demand, w^*, defined by (4).

For analytical convenience, we define F_1, F_2, and F_3 as in Table 3, the third column of which reports their implied values under the parameter configuration of Table 1. As is easily verified, w^* is a decreasing function of F with

$$w^* = \begin{cases} w^{***} & \text{if} \quad F = F_1, \\ w^{****} & \text{if} \quad F = F_2, \\ w^{**} & \text{if} \quad F = F_3, \end{cases}$$

where w^{**}, w^{***}, and w^{****} are as defined in Table 2. Dependent on the amount of the entry cost, four patterns are conceivable for the existence of the SCE and NSCE.

Figure 4 depicts a case in which the entry cost is sufficiently small, $F \in [0, F_1]$, by configuring the parameters as in Table 1. In such a case, the model has a unique SCE and a unique NSCE, which suggests that whether firms believe that those who have experienced unemployment are no longer employable is critical in determining the equilibrium of this economy; other aspects of their belief have no effect on this determination. This can be explained as follows. Small costs of entry, which characterize the present cases, promote the establishment of new firms, thereby making the labor market so tight that all job seekers in each period can find jobs. In the SCE, in particular, every jobless worker can find a new job if she experiences only one period of unemployment. As a result, there are no workers who have been unemployed for more than one period, which makes all details of firms' belief except for whether $\overline{n} = 0$ is satisfied insignificant.

Figure 5 depicts a case in which the entry cost is not so small, $F \in (F_1, F_3)$, using the same configuration of parameters as in Figure 4. Again, there is a unique NSCE in this model. In addition, there are multiple SCEs,

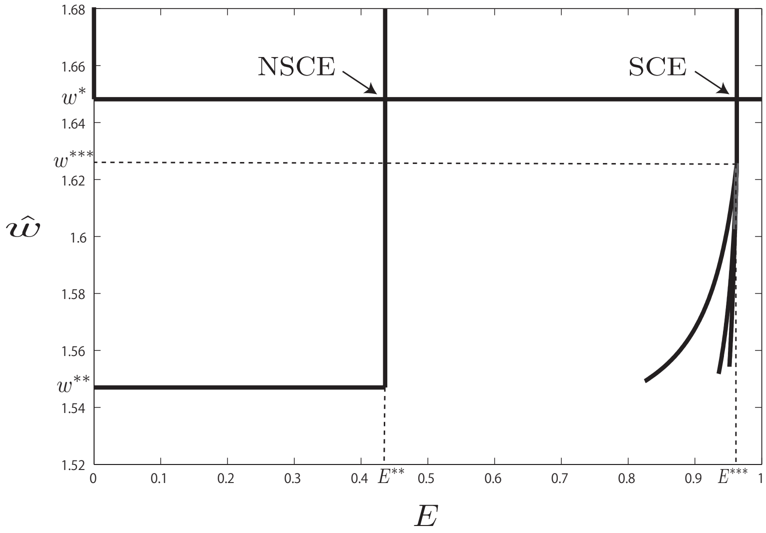

Figure 4: SCE and NSCE when $F \in [0, F_1]$

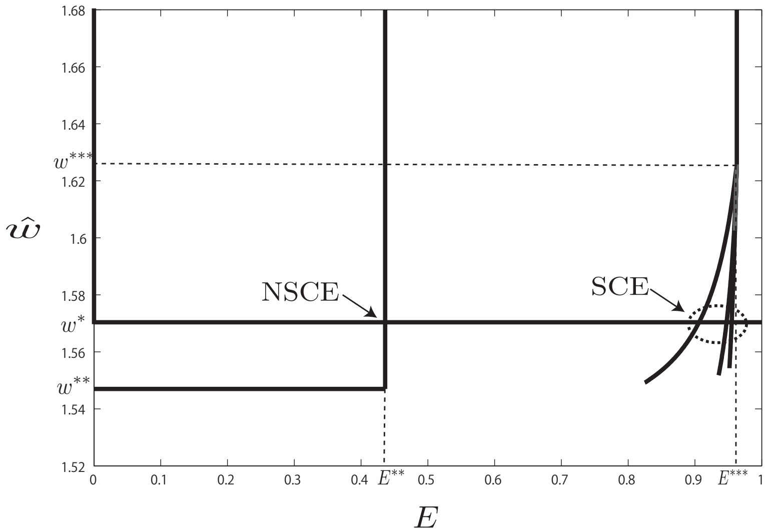

Figure 5: SCE and NSCE when $F \in (F_1, F_3)$

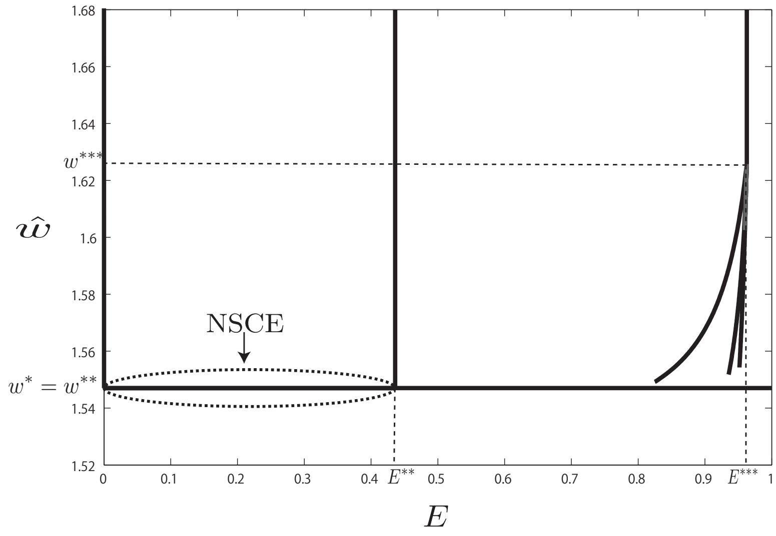

Figure 6: NSCE when $F = F_3$

as depicted in Figure 5. The reason SCEs are so diversified in this case will be considered in the next subsection.

Figures 6 and 7 depict cases in which the entry cost is large, $F \in [F_3, +\infty)$. In these cases, the SCE does not exist since any level of $\hat{w}$ at which investors can break even or profit is too low to motivate jobless workers to preserve their employability. In contrast, the NSCE still exists for these cases, but its status changes according to the value of F. When $F = F_3$, there is a continuum of NSCEs, which are differentiated from one another by E. Across them, the job finding rate increases its value from zero to unity as E increases from zero to $[\theta/(\theta + b - \theta b)]N$.[9] When $F \in (F_3, +\infty)$, there is a unique NSCE, the aggregate employment of which equals zero. There, w^{**}, which is the minimum level of $\hat{w}$ that can motivate employed workers, is too high for investors to break even or profit by investing in a new firm, and thus, no firm is established. As a result, there is no job offer to newly born workers, who then have no choice but to live by consuming their endowments in each period. Put another way, not only second chances but also first ones are completely destroyed in this case.

[9] Using welfare measures defined in the next subsection, we can even show that these equilibria can be Pareto-ranked by E; that is, an equilibrium indexed by a larger value of E dominates that by a smaller value in the Paretian sense. Intuitively, a larger value of E improves economic welfare by raising the job finding rate, as well as by increasing the total amount of the net profit payment, which is composed of the profits earned by currently operating firms minus the entry costs paid by investors for establishing new firms.

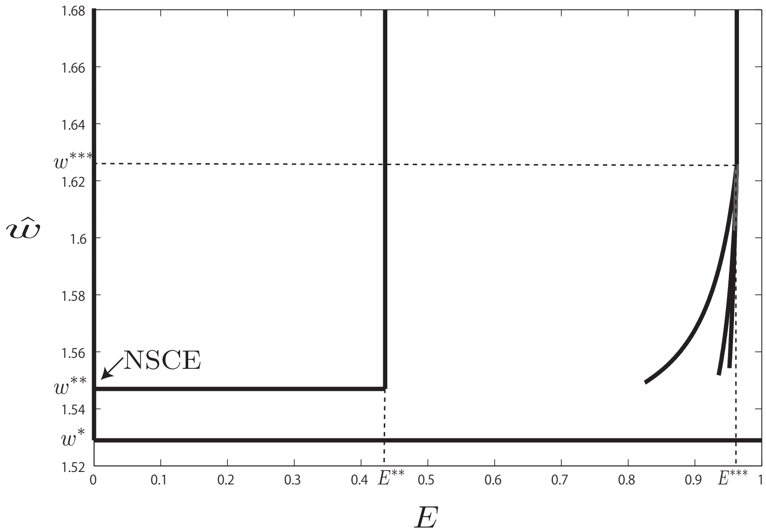

Figure 7: NSCE when $F \in (F_3, +\infty)$

5.2 Numerical Derivation of Stationary Equilibria

Despite its comprehensibility, diagrammatic expositions can only provide a sketch of the stationary equilibria that exist in this model. To take a close look at these equilibria, we will numerically derive the equilibrium values of $\hat{w}$, E, U_O, a, L^*, E/L^*, w^1, and w^{2+} for such values of $\overline{n}$ and F as

$$\overline{n} = 0, -1, -2, -3, -4, -5, -10, -20, -30, -\infty,$$

$$F = \begin{cases} 0.1i & \text{for } i = 1, 2, \cdots, 8 \\ F_1^{1-0.1j} F_3^{0.1j} & \text{for } j = 0, 1, 2, \cdots, 10 \end{cases} \quad \text{and} \quad F > F_3,$$

where U_O and a, respectively, represent the population of unemployable workers and the per-period job finding rate for employable workers.[10]

Tables 4–9 report the results of this numerical analysis. As suggested by these tables, when the value of F is increased from below to above F_1, the number of SCE suddenly explodes from one to positive infinity. To understand why, we need to recall that the equilibrium value of $\hat{w}$, w^*, decreases as F takes a larger value. As long as F is smaller than F_1, the level of w^* is high enough to motivate employed workers, even when finding a new job takes only one period of unemployment experience. If F takes a larger value than F_1, in contrast, the level of w^* becomes so low that it cannot motivate employed workers unless finding a new job takes more than one period of unemployment experience. As a consequence, only a subset of

[10]For the details of the derivation, see Appendix C.

Table 4: Aggregate Employment (E)

F	$\hat{w}$	$\overline{n}$: 0	-1	-2	-3	-4	-5	-10	-20	-30	$-\infty$
0.1	4.047	0.436	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963
0.2	3.007	0.436	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963
0.3	2.527	0.436	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963
0.4	2.234	0.436	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963
0.5	2.031	0.436	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963
0.6	1.878	0.436	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963
0.7	1.758	0.436	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963
0.8	1.660	0.436	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963
0.840 (F_1)	1.626 (w^{***})	0.436	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963	0.963
0.850	1.618	0.436	0.958	0.962	0.962	0.962	0.962	0.962	0.962	0.962	0.962
0.860	1.610	0.436	0.954	0.960	0.961	0.961	0.961	0.961	0.961	0.961	0.961
0.870	1.602	0.436	0.948	0.958	0.960	0.960	0.960	0.960	0.960	0.960	NA
0.880	1.594	0.436	0.941	0.956	0.959	0.959	0.959	0.959	0.959	NA	NA
0.890	1.586	0.436	0.932	0.954	0.958	0.958	0.958	0.958	NA	NA	NA
0.900	1.578	0.436	0.921	0.951	0.956	0.957	0.957	0.957	NA	NA	NA
0.911	1.570	0.436	0.906	0.947	0.955	0.956	0.956	NA	NA	NA	NA
0.922	1.562	0.436	0.886	0.943	0.953	0.955	0.955	NA	NA	NA	NA
0.932	1.555	0.436	0.856	0.938	0.952	NA	NA	NA	NA	NA	NA
0.943 (F_3)	1.547 (w^{**})	0.436	NA	NA	NA	NA	NA	NA	NA	NA	NA
$F > F_3$	1.547 (w^{**})	0	NA	NA	NA	NA	NA	NA	NA	NA	NA

Note: The largest possible value is reported when $(F, \overline{n}) = (F_3, 0)$.

Table 5: Population of Unemployable Workers (U_O)

F	$\overline{n}$: 0	-1	-2	-3	-4	-5	-10	-20	-30	$-\infty$
0.1	0.564	0	0	0	0	0	0	0	0	0
0.2	0.564	0	0	0	0	0	0	0	0	0
0.3	0.564	0	0	0	0	0	0	0	0	0
0.4	0.564	0	0	0	0	0	0	0	0	0
0.5	0.564	0	0	0	0	0	0	0	0	0
0.6	0.564	0	0	0	0	0	0	0	0	0
0.7	0.564	0	0	0	0	0	0	0	0	0
0.8	0.564	0	0	0	0	0	0	0	0	0
0.840 (F_1)	0.564	0	0	0	0	0	0	0	0	0
0.850	0.564	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0
0.860	0.564	0.009	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0
0.870	0.564	0.015	0.002	0.000	0.000	0.000	0.000	0.000	0.000	NA
0.880	0.564	0.022	0.003	0.000	0.000	0.000	0.000	0.000	NA	NA
0.890	0.564	0.031	0.005	0.000	0.000	0.000	0.000	NA	NA	NA
0.900	0.564	0.043	0.007	0.001	0.000	0.000	0.000	NA	NA	NA
0.911	0.564	0.057	0.010	0.001	0.000	0.000	NA	NA	NA	NA
0.922	0.564	0.078	0.013	0.002	0.000	0.000	NA	NA	NA	NA
0.932	0.564	0.108	0.018	0.002	NA	NA	NA	NA	NA	NA
0.943 (F_3)	0.564	NA	NA	NA	NA	NA	NA	NA	NA	NA
$F > F_3$	1	NA	NA	NA	NA	NA	NA	NA	NA	NA

Note: The smallest possible value is reported when $(F, \overline{n}) = (F_3, 0)$.

Table 6: Per-Period Job Finding Rate for Employable Workers (a)

	$\overline{n}$									
F	0	-1	-2	-3	-4	-5	-10	-20	-30	$-\infty$
0.1	1	1	1	1	1	1	1	1	1	1
0.2	1	1	1	1	1	1	1	1	1	1
0.3	1	1	1	1	1	1	1	1	1	1
0.4	1	1	1	1	1	1	1	1	1	1
0.5	1	1	1	1	1	1	1	1	1	1
0.6	1	1	1	1	1	1	1	1	1	1
0.7	1	1	1	1	1	1	1	1	1	1
0.8	1	1	1	1	1	1	1	1	1	1
0.840 (F_1)	1	1	1	1	1	1	1	1	1	1
0.850	1	0.997	0.987	0.986	0.986	0.986	0.986	0.986	0.986	0.986
0.860	1	0.992	0.974	0.972	0.972	0.972	0.972	0.972	0.972	0.972
0.870	1	0.987	0.962	0.959	0.959	0.959	0.959	0.959	0.959	NA
0.880	1	0.981	0.949	0.945	0.945	0.945	0.945	0.945	NA	NA
0.890	1	0.974	0.937	0.932	0.931	0.931	0.931	NA	NA	NA
0.900	1	0.964	0.924	0.918	0.918	0.918	0.918	NA	NA	NA
0.911	1	0.951	0.911	0.905	0.904	0.904	NA	NA	NA	NA
0.922	1	0.934	0.897	0.891	0.891	0.891	NA	NA	NA	NA
0.932	1	0.907	0.881	0.878	NA	NA	NA	NA	NA	NA
0.943 (F_3)	1	NA	NA	NA	NA	NA	NA	NA	NA	NA
$F > F_3$	0	NA	NA	NA	NA	NA	NA	NA	NA	NA

Note: The largest possible value is reported when $(F, \overline{n}) = (F_3, 0)$.

Table 7: Number of Operating Firms (E/L^*)

		$\overline{n}$									
F	L^*	0	-1	-2	-3	-4	-5	-10	-20	-30	$-\infty$
0.1	0.003	151.27	333.95	333.95	333.95	333.95	333.95	333.95	333.95	333.95	333.95
0.2	0.008	56.20	124.06	124.06	124.06	124.06	124.06	124.06	124.06	124.06	124.06
0.3	0.014	31.49	69.52	69.52	69.52	69.52	69.52	69.52	69.52	69.52	69.52
0.4	0.021	20.88	46.09	46.09	46.09	46.09	46.09	46.09	46.09	46.09	46.09
0.5	0.029	15.18	33.51	33.51	33.51	33.51	33.51	33.51	33.51	33.51	33.51
0.6	0.037	11.70	25.82	25.82	25.82	25.82	25.82	25.82	25.82	25.82	25.82
0.7	0.046	9.39	20.72	20.72	20.72	20.72	20.72	20.72	20.72	20.72	20.72
0.8	0.056	7.76	17.12	17.12	17.12	17.12	17.12	17.12	17.12	17.12	17.12
0.840 (F_1)	0.060	7.23	15.97	15.97	15.97	15.97	15.97	15.97	15.97	15.97	15.97
0.850	0.061	7.11	15.64	15.69	15.69	15.69	15.69	15.69	15.69	15.69	15.69
0.860	0.062	7.00	15.30	15.41	15.42	15.42	15.42	15.42	15.42	15.42	15.42
0.870	0.063	6.88	14.96	15.13	15.15	15.15	15.15	15.15	15.15	15.15	NA
0.880	0.064	6.77	14.60	14.85	14.89	14.89	14.89	14.89	14.89	NA	NA
0.890	0.065	6.66	14.23	14.57	14.63	14.63	14.63	14.63	NA	NA	NA
0.900	0.067	6.55	13.83	14.28	14.37	14.37	14.38	14.38	NA	NA	NA
0.911	0.068	6.44	13.38	14.00	14.11	14.12	14.12	NA	NA	NA	NA
0.922	0.069	6.34	12.87	13.71	13.86	13.88	13.88	NA	NA	NA	NA
0.932	0.070	6.23	12.24	13.40	13.61	NA	NA	NA	NA	NA	NA
0.943 (F_3)	0.071	6.13	NA	NA	NA	NA	NA	NA	NA	NA	NA
$F > F_3$	0.071	0	NA	NA	NA	NA	NA	NA	NA	NA	NA

Note: The largest possible value is reported when $(F, \overline{n}) = (F_3, 0)$.

Table 8: Starting Wage (w^1)

		$\overline{n}$									
F	$\hat{w}$	0	-1	-2	-3	-4	-5	-10	-20	-30	$-\infty$
0.1	4.047	33.594	6.197	6.197	6.197	6.197	6.197	6.197	6.197	6.197	6.197
0.2	3.007	20.262	4.234	4.234	4.234	4.234	4.234	4.234	4.234	4.234	4.234
0.3	2.527	14.114	3.328	3.328	3.328	3.328	3.328	3.328	3.328	3.328	3.328
0.4	2.234	10.356	2.775	2.775	2.775	2.775	2.775	2.775	2.775	2.775	2.775
0.5	2.031	7.744	2.390	2.390	2.390	2.390	2.390	2.390	2.390	2.390	2.390
0.6	1.878	5.788	2.102	2.102	2.102	2.102	2.102	2.102	2.102	2.102	2.102
0.7	1.758	4.249	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875	1.875
0.8	1.660	2.996	1.691	1.691	1.691	1.691	1.691	1.691	1.691	1.691	1.691
0.840 (F_1)	1.626 (w^{***})	2.556	1.626	1.626	1.626	1.626	1.626	1.626	1.626	1.626	1.626
0.850	1.618	2.453	1.618	1.618	1.618	1.618	1.618	1.618	1.618	1.618	1.618
0.860	1.610	2.350	1.610	1.610	1.610	1.610	1.610	1.618	1.610	1.610	1.610
0.870	1.602	2.248	1.602	1.602	1.602	1.602	1.602	1.602	1.602	1.602	NA
0.880	1.594	2.146	1.594	1.594	1.594	1.594	1.594	1.594	1.594	NA	NA
0.890	1.586	2.045	1.586	1.586	1.586	1.586	1.586	1.586	NA	NA	NA
0.900	1.578	1.945	1.578	1.578	1.578	1.578	1.578	1.578	NA	NA	NA
0.911	1.570	1.844	1.570	1.570	1.570	1.570	1.570	NA	NA	NA	NA
0.922	1.562	1.745	1.562	1.562	1.562	1.562	1.562	NA	NA	NA	NA
0.932	1.555	1.646	1.555	1.555	1.555	NA	NA	NA	NA	NA	NA
0.943 (F_3)	1.547 (w^{**})	1.547	NA	NA	NA	NA	NA	NA	NA	NA	NA
$F > F_3$	1.547 (w^{**})	1.547	NA	NA	NA	NA	NA	NA	NA	NA	NA

Table 9: Wage Paid in and after the Second Period of Service (w^{2+})

		$\overline{n}$									
F	$\hat{w}$	0	-1	-2	-3	-4	-5	-10	-20	-30	$-\infty$
0.1	4.047	1.547	3.865	3.865	3.865	3.865	3.865	3.865	3.865	3.865	3.865
0.2	3.007	1.547	2.903	2.903	2.903	2.903	2.903	2.903	2.903	2.903	2.903
0.3	2.527	1.547	2.460	2.460	2.460	2.460	2.460	2.460	2.460	2.460	2.460
0.4	2.234	1.547	2.189	2.189	2.189	2.189	2.189	2.189	2.189	2.189	2.189
0.5	2.031	1.547	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
0.6	1.878	1.547	1.859	1.859	1.859	1.859	1.859	1.859	1.859	1.859	1.859
0.7	1.758	1.547	1.748	1.748	1.748	1.748	1.748	1.748	1.748	1.748	1.748
0.8	1.660	1.547	1.657	1.657	1.657	1.657	1.657	1.657	1.657	1.657	1.657
0.840 (F_1)	1.626 (w^{***})	1.547	1.626	1.626	1.626	1.626	1.626	1.626	1.626	1.626	1.626
0.850	1.618	1.547	1.618	1.618	1.618	1.618	1.618	1.618	1.618	1.618	1.618
0.860	1.610	1.547	1.610	1.610	1.610	1.610	1.610	1.610	1.610	1.610	1.610
0.870	1.602	1.547	1.602	1.602	1.602	1.602	1.602	1.602	1.602	1.602	NA
0.880	1.594	1.547	1.594	1.594	1.594	1.594	1.594	1.594	1.594	NA	NA
0.890	1.586	1.547	1.586	1.586	1.586	1.586	1.586	1.586	NA	NA	NA
0.900	1.578	1.547	1.578	1.578	1.578	1.578	1.578	1.578	NA	NA	NA
0.911	1.570	1.547	1.570	1.570	1.570	1.570	1.570	NA	NA	NA	NA
0.922	1.562	1.547	1.562	1.562	1.562	1.562	1.562	NA	NA	NA	NA
0.932	1.555	1.547	1.555	1.555	1.555	NA	NA	NA	NA	NA	NA
0.943 (F_3)	1.547 (w^{**})	1.547	NA	NA	NA	NA	NA	NA	NA	NA	NA
$F > F_3$	1.547 (w^{**})	1.547	NA	NA	NA	NA	NA	NA	NA	NA	NA

job seekers are hired in equilibrium, which creates room for firms' belief to play a crucial role in selecting job applicants, thereby diversifying the SCEs.

All of the diversified SCEs, each of which corresponds to a single belief such that $\overline{n} < 0$, remain in existence as long as $F \in (F_1, F_2]$. However, if F exceeds F_2, they start to decrease in number, and cease to exist before F reaches F_3. As shown in the tables, an SCE characterized by a smaller $\overline{n}$ ceases to exist earlier than those by larger $\overline{n}$s. A larger value of F and its resulting lower value of w^* weaken the incentive of jobless workers to preserve their employability, thereby making it more difficult for an SCE to satisfy all of its EPCs, the number of which is equal to the absolute value of $\overline{n}$. When F is sufficiently close to F_3, even the SCE with $\overline{n} = -1$ cannot satisfy its single EPC. Thus, the last SCE vanishes from the economy.

5.3 Welfare Comparison among the Existing Equilibria

To evaluate the economic welfare of the existing equilibria, we construct three measures of economic welfare: workers' surplus (WS), investors' surplus (IS), and their sum, which we call total surplus (TS), as follows.

$$\begin{aligned}
\text{WS} &\equiv \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left[\tilde{w}E + \overline{w}(N-E) - e(N-U_O)\right], \\
\text{IS} &\equiv \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \cdot \frac{E}{L^*} \cdot [(L^*)^\alpha - \tilde{w}L^* - bF], \\
\text{TS} &\equiv \text{WS+IS} \\
&= \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left[E(L^*)^{\alpha-1} + \overline{w}(N-E) - \frac{bFE}{L^*} - e(N-U_O)\right],
\end{aligned}$$

where $\tilde{w} \equiv w^1 - (1-\theta)(1-b)(w^1 - w^{2+})$.[11] As defined above, the WS is the discounted sum of workers' incomes minus their effort expenditures. In each period, employed workers receive, on average, $\tilde{w}$ units of labor income, and jobless ones receive $\overline{w}$ units of endowment; these add up to $\tilde{w}E + \overline{w}(N-E)$. Moreover, employable workers, the population of which is equal to $N - U_O$, expend e units of effort. Thus, the workers constituting the labor force receive, on aggregate, $\tilde{w}E + \overline{w}(N-E) - e(N-U_O)$ units of surplus, which

[11] We can also express $\tilde{w}$ as the average of average labor costs. While marginal labor costs are common across all firms and equal to $\hat{w}$ defined by (3), the average labor costs of new firms, which have just started production, may differ from those of old firms, which started production one or more periods before. On the one hand, the average labor costs of new firms, the number of which is b, are given by w^1 since all of their employees are newly recruited workers. On the other hand, the average labor costs of old firms, the number of which is $1-b$, is given by $\theta w^1 + (1-\theta)w^{2+}$ since newly recruited workers only account for a fraction θ of their employees. By averaging out these average costs over all operating firms, we can obtain the definition of $\tilde{w}$.

Table 10: Workers' Surplus

F	$\overline{n}$: 0	-1	-2	-3	-4	-5	-10	-20	-30	$-\infty$
0.1	200.17	344.69	344.69	344.69	344.69	344.69	344.69	344.69	344.69	344.69
0.2	159.78	244.45	244.45	244.45	244.45	244.45	244.45	244.45	244.45	244.45
0.3	141.15	198.23	198.23	198.23	198.23	198.23	198.23	198.23	198.23	198.23
0.4	129.76	169.98	169.98	169.98	169.98	169.98	169.98	169.98	169.98	169.98
0.5	121.85	150.34	150.34	150.34	150.34	150.34	150.34	150.34	150.34	150.34
0.6	115.92	135.64	135.64	135.64	135.64	135.64	135.64	135.64	135.64	135.64
0.7	111.26	124.07	124.07	124.07	124.07	124.07	124.07	124.07	124.07	124.07
0.8	107.46	114.65	114.65	114.65	114.65	114.65	114.65	114.65	114.65	114.65
0.840 (F_1)	106.13	111.34	111.34	111.34	111.34	111.34	111.34	111.34	111.34	111.34
0.850	105.81	110.51	110.50	110.50	110.50	110.50	110.50	110.50	110.50	110.50
0.860	105.50	109.68	109.67	109.66	109.66	109.66	109.66	109.66	109.66	109.66
0.870	105.19	108.86	108.84	108.83	108.83	108.83	108.83	108.83	108.83	NA
0.880	104.89	108.04	108.01	108.01	108.01	108.01	108.01	108.01	NA	NA
0.890	104.58	107.22	107.19	107.19	107.19	107.19	107.19	NA	NA	NA
0.900	104.27	106.40	106.37	106.37	106.37	106.37	106.37	NA	NA	NA
0.911	103.97	105.58	105.55	105.55	105.55	105.55	NA	NA	NA	NA
0.922	103.67	104.75	104.74	104.74	104.74	104.74	NA	NA	NA	NA
0.932	103.37	103.91	103.93	103.94	NA	NA	NA	NA	NA	NA
0.943 (F_3)	103.07	NA	NA	NA	NA	NA	NA	NA	NA	NA
$F > F_3$	101	NA	NA	NA	NA	NA	NA	NA	NA	NA

Note: The largest possible value is reported when $(F, \overline{n}) = (F_3, 0)$.

is the summand without the discount factor in the defining equation of the WS. The IS is a discounted sum of "net profit flows" from the firm sector to the investors, that is, profits earned by currently operating firms minus the entry costs needed to establish new firms. In each period, operating firms distribute their profits to investors, which add up to $(E/L^*)[(L^*)^\alpha - \tilde{w}L^*]$. On the other hand, the investors pay an aggregate of $b(E/L^{**})F$ to fund new firms. Thus, on balance, $(E/L^{**})[(L^{**})^\alpha - \tilde{w}L^{**} - bF]$ units of net profit flow from the firm sector to the investors. This is the summand without the discount factor in the defining equation of the IS. The TS is the discounted sum of the net output of this economy, that is, the sum of the produced and endowed consumption goods minus investment in new firms and effort expenditures.[12]

Tables 10–12 report, respectively, the values of the WS, IS, and TS, which are computed under the parameter configuration of Table 1. As shown in these tables, for a given entry cost level, the decrease in $\overline{n}$ from 0 to -1 produces significant welfare gains, whereas other decreases in $\overline{n}$ produce only small ones. To understand why, we must note that a decrease in $\overline{n}$ has two conflicting effects on TS. On the one hand, it increases the aggregate employment, thereby enhancing the total amounts of labor incomes and net profit flows, which contributes to an enhancement of the TS. On the other

[12] In fact, WS and IS can be interpreted as, respectively, the discounted sum of the lifetime utilities of the workers constituting the labor force in the present and future periods and the discounted sum of expected gains from past, current, and future investments. For the details, see Appendix D.

Table 11: Investors' Surplus

	$\overline{n}$									
F	0	-1	-2	-3	-4	-5	-10	-20	-30	$-\infty$
0.1	28.29	35.82	35.82	35.82	35.82	35.82	35.82	35.82	35.82	35.82
0.2	18.95	26.25	26.25	26.25	26.25	26.25	26.25	26.25	26.25	26.25
0.3	14.64	21.84	21.84	21.84	21.84	21.84	21.84	21.84	21.84	21.84
0.4	12.01	19.15	19.15	19.15	19.15	19.15	19.15	19.15	19.15	19.15
0.5	10.18	17.27	17.27	17.27	17.27	17.27	17.27	17.27	17.27	17.27
0.6	8.81	15.87	15.87	15.87	15.87	15.87	15.87	15.87	15.87	15.87
0.7	7.73	14.76	14.76	14.76	14.76	14.76	14.76	14.76	14.76	14.76
0.8	6.85	13.86	13.86	13.86	13.86	13.86	13.86	13.86	13.86	13.86
0.840 (F_1)	6.55	13.55	13.55	13.55	13.55	13.55	13.55	13.55	13.55	13.55
0.850	6.47	13.42	13.47	13.47	13.47	13.47	13.47	13.47	13.47	13.47
0.860	6.40	13.29	13.38	13.39	13.39	13.39	13.39	13.39	13.39	13.39
0.870	6.33	13.14	13.29	13.31	13.31	13.31	13.31	13.31	13.31	NA
0.880	6.26	12.98	13.20	13.23	13.23	13.23	13.23	13.23	NA	NA
0.890	6.19	12.79	13.10	13.15	13.15	13.15	13.15	NA	NA	NA
0.900	6.12	12.58	12.99	13.07	13.07	13.07	13.07	NA	NA	NA
0.911	6.05	12.31	12.88	12.98	12.99	12.99	NA	NA	NA	NA
0.922	5.98	11.98	12.76	12.90	12.91	12.92	NA	NA	NA	NA
0.932	5.91	11.52	12.62	12.81	NA	NA	NA	NA	NA	NA
0.943 (F_3)	5.84	NA	NA	NA	NA	NA	NA	NA	NA	NA
$F > F_3$	0	NA	NA	NA	NA	NA	NA	NA	NA	NA

Note: The largest possible value is reported when $(F, \overline{n}) = (F_3, 0)$.

Table 12: Total Surplus

	$\overline{n}$									
F	0	-1	-2	-3	-4	-5	-10	-20	-30	$-\infty$
0.1	228.46	380.51	380.51	380.51	380.51	380.51	380.51	380.51	380.51	380.51
0.2	178.73	270.71	270.71	270.71	270.71	270.71	270.71	270.71	270.71	270.71
0.3	155.79	220.07	220.07	220.07	220.07	220.07	220.07	220.07	220.07	220.07
0.4	141.77	189.13	189.13	189.13	189.13	189.13	189.13	189.13	189.13	189.13
0.5	132.03	167.62	167.62	167.62	167.62	167.62	167.62	167.62	167.62	167.62
0.6	124.73	151.50	151.50	151.50	151.50	151.50	151.50	151.50	151.50	151.50
0.7	118.99	138.83	138.83	138.83	138.83	138.83	138.83	138.83	138.83	138.83
0.8	114.32	128.51	128.51	128.51	128.51	128.51	128.51	128.51	128.51	128.51
0.840 (F_1)	112.67	124.89	124.89	124.89	124.89	124.89	124.89	124.89	124.89	124.89
0.850	112.29	123.93	123.97	123.97	123.97	123.97	123.97	123.97	123.97	123.97
0.860	111.91	122.96	123.04	123.05	123.05	123.05	123.05	123.05	123.05	123.05
0.870	111.52	122.00	122.13	122.14	122.14	122.14	122.14	122.14	122.14	NA
0.880	111.15	121.01	121.21	121.24	121.24	121.24	121.24	121.24	NA	NA
0.890	110.77	120.01	120.28	120.33	120.34	120.34	120.34	NA	NA	NA
0.900	110.39	118.98	119.36	119.43	119.44	119.44	119.44	NA	NA	NA
0.911	110.02	117.89	118.43	118.54	118.55	118.55	NA	NA	NA	NA
0.922	109.65	116.73	117.50	117.64	117.66	117.66	NA	NA	NA	NA
0.932	109.28	115.43	116.55	116.75	NA	NA	NA	NA	NA	NA
0.943 (F_3)	108.91	NA	NA	NA	NA	NA	NA	NA	NA	NA
$F > F_3$	101	NA	NA	NA	NA	NA	NA	NA	NA	NA

Note: The largest possible value is reported when $(F, \overline{n}) = (F_3, 0)$.

hand, it increases the number of employable workers, thereby increasing the total expenditure of efforts, which contributes to a reduction in the TS. Therefore, the effects of a decrease in $\overline{n}$ on the TS are theoretically inconclusive since it always increases IS and has mixed effects on the WS.

When $\overline{n}$ is decreased from 0 to -1, both IS and WS increase their values, and therefore, TS also exhibits a substantial enhancement. This decrease in $\overline{n}$ substantially increases the populations of employed and employable workers. As stated above, the increase in employed workers enhances IS and WS, while the increase in employable workers reduces WS. Despite this friction, WS increases its value in this case since the positive contribution through the increased labor income, $(1 + r^{-1})(\tilde{w} - \overline{w})\Delta E$, dominates the negative contribution through the increased effort expenditure, $(1 + r^{-1})e\Delta U_O$.

In contrast, when $\overline{n}$ is decreased from -1 or a smaller value to a further smaller one, none of WS, IS, or TS exhibit a substantial change for any level of the entry cost. When F is no larger than F_1, the existing SCEs are coalesced into one equilibrium, and thus, any decrease in $\overline{n}$ has no marginal effect on WS, IS, or TS. When F takes a value between F_1 and F_3 but is not so close to F_3, the existing SCEs are so close to one another that any decrease in $\overline{n}$ can hardly change the values of E and U_O, thus producing only small changes in WS, IS, and TS at best.[13] When F takes a value between F_1 and F_3 that is sufficiently close to F_3, a decrease in $\overline{n}$ may lead to a substantial increase in E and a substantial decrease in U_O. For example, when $F = 0.932$, the decrease in $\overline{n}$ from -1 to -2 increases E by 0.082 and reduces U_O by 0.090 (See Tables 4 and 5). However, these changes do not lead to a substantial enhancement of WS because the positive contribution through the increased labor income, $(1 + r^{-1})(\tilde{w} - \overline{w})\Delta E \approx 4.58$, is mostly offset by the negative contribution through the increased effort expenditure, $(1+r^{-1})e\Delta U_O \approx -4.56$. On the other hand, the increased employment also enhances IS by 1.10, which explains the greater part of the enhancement of TS observed in this case.

5.4 Policy Implications

The theoretical and numerical analyses made thus far yield the following policy implications. First, second chances should be given to jobless workers. It is clear from the second and third columns of Table 12, which summarize the total surpluses for the cases of $\overline{n} = 0$ and $\overline{n} = -1$, respectively, that the absence of second chances causes a substantial loss of economic welfare by discouraging all jobless workers from preserving their employability. Unless

[13] Nevertheless, we should note that the decrease in $\overline{n}$ may reduce WS for some values of F. As shown in Table 10, the decrease in $\overline{n}$ from -1 to -2 reduces WS if F ranges from 0.850 to 0.922, and the decrease in $\overline{n}$ from -2 to -3 also reduces WS if F equals either 0.860 or 0.870. In these cases, the increment of labor income is dominated by that of effort expenditure, which causes the observed reduction in WS.

Table 13: Changes in Workers' Surplus

		$\overline{n}$										
		0										
F	$\overline{w}$	NSCE	SCE	-1	-2	-3	-4	-5	-10	-20	-30	$-\infty$
0.840 (F_1)	1	0	5.21	0	0	0	0	0	0	0	0	0
0.850	0.992	-0.50	4.71	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
0.860	0.984	-1.00	4.21	0.03	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.870	0.976	-1.49	3.72	0.05	0.07	0.08	0.08	0.08	0.08	0.08	0.08	NA
0.880	0.968	-1.99	3.22	0.07	0.10	0.10	0.10	0.10	0.10	0.10	NA	NA
0.890	0.960	-2.48	2.73	0.10	0.12	0.13	0.13	0.13	0.13	NA	NA	NA
0.900	0.952	-2.96	2.25	0.12	0.15	0.15	0.15	0.15	0.15	NA	NA	NA
0.911	0.944	-3.45	1.76	0.16	0.18	0.18	0.18	0.18	NA	NA	NA	NA
0.922	0.937	-3.93	1.28	0.20	0.20	0.20	0.20	0.20	NA	NA	NA	NA
0.932	0.929	-4.41	0.80	0.25	0.23	0.23	NA	NA	NA	NA	NA	NA
0.943 (F_3)	0.921	-4.89	0.32	NA	NA	NA	NA	NA	NA	NA	NA	NA
1	0.883	-6.70	-1.49	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	0.495	-45.86	-40.65	NA	NA	NA	NA	NA	NA	NA	NA	NA

Note: The smallest possible changes are reported when $(F, \overline{n}) = (F_3, 0)$.

Table 14: Changes in Investors' Surplus

		$\overline{n}$										
		0										
F	$\overline{w}$	NSCE	SCE	-1	-2	-3	-4	-5	-10	-20	-30	$-\infty$
0.840 (F_1)	1	0	7.00	0	0	0	0	0	0	0	0	0
0.850	0.992	0.04	7.01	0.06	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.860	0.984	0.08	7.01	0.13	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
0.870	0.976	0.13	7.02	0.21	0.06	0.04	0.04	0.04	0.04	0.04	0.04	NA
0.880	0.968	0.17	7.02	0.31	0.09	0.05	0.05	0.05	0.05	0.05	NA	NA
0.890	0.960	0.21	7.03	0.42	0.12	0.07	0.06	0.06	0.06	NA	NA	NA
0.900	0.952	0.25	7.03	0.57	0.16	0.09	0.08	0.08	0.08	NA	NA	NA
0.911	0.944	0.29	7.04	0.77	0.21	0.10	0.09	0.09	NA	NA	NA	NA
0.922	0.937	0.33	7.04	1.04	0.27	0.12	0.11	0.11	NA	NA	NA	NA
0.932	0.929	0.37	7.05	1.44	0.33	0.15	NA	NA	NA	NA	NA	NA
0.943 (F_3)	0.921	0.41	7.05	NA	NA	NA	NA	NA	NA	NA	NA	NA
1	0.883	6.10	12.57	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	0.495	4.64	9.34	NA	NA	NA	NA	NA	NA	NA	NA	NA

Note: The smallest possible changes are reported when $(F, \overline{n}) = (F_3, 0)$.

the entry cost is so high that no stationary equilibrium but the NSCE can exist in this economy, the government should make every effort to persuade firms not to hold the extreme view that all jobless workers have already lost their employability. This would create second chances for those who have been unemployed for a relatively short span of time and would lead to an extensive improvement in the aggregate employment. As shown in Figures 2 and 3, the aggregate employment cannot be larger than 0.436 when second chances are absent, that is, $\overline{n} = 0$, but it never falls below 0.820 when these chances are present, that is, $\overline{n} < 0$.

Second, if the entry cost takes such a value that multiple SCEs can exist in this economy, taxing jobless workers may lead to an improvement in economic welfare in the sense that it enhances both WS and IS. When the value

Table 15: Changes in Total Surplus

F	$\overline{w}$	$\overline{n}$ 0 NSCE	0 SCE	-1	-2	-3	-4	-5	-10	-20	-30	$-\infty$
0.840 (F_1)	1	0	12.21	0	0	0	0	0	0	0	0	0
0.850	0.992	−0.46	11.72	0.07	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
0.860	0.984	−0.91	11.22	0.16	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
0.870	0.976	−1.37	10.73	0.26	0.13	0.12	0.11	0.11	0.11	0.11	0.11	NA
0.880	0.968	−1.82	10.25	0.38	0.19	0.16	0.15	0.15	0.15	0.15	NA	NA
0.890	0.960	−2.27	9.76	0.52	0.24	0.20	0.19	0.19	0.19	NA	NA	NA
0.900	0.952	−2.72	9.28	0.70	0.31	0.24	0.23	0.23	0.23	NA	NA	NA
0.911	0.944	−3.16	8.80	0.93	0.38	0.28	0.27	0.27	NA	NA	NA	NA
0.922	0.937	−3.60	8.32	1.24	0.47	0.33	0.31	0.31	NA	NA	NA	NA
0.932	0.929	−4.04	7.84	1.69	0.57	0.37	NA	NA	NA	NA	NA	NA
0.943 (F_3)	0.921	−4.48	7.37	NA	NA	NA	NA	NA	NA	NA	NA	NA
1	0.883	−0.59	11.09	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	0.495	−41.22	−31.30	NA	NA	NA	NA	NA	NA	NA	NA	NA

Note: The smallest possible changes are reported when $(F, \overline{n}) = (F_3, 0)$.

of $\overline{w}$ is reduced by taxation, workers are more drawn to the status of employment, which enables firms to motivate their employees by paying lower wages than before. This, in turn, shifts the profiles of the AICs downward. If $\overline{w}$ is reduced to such a level that the profiles of the AICs for $\overline{n} < 0$ can intersect with that of the aggregate labor demand at the upper end of their upward-sloping parts, then, possible stationary equilibria are restricted to one SCE and one NSCE, which we call the "new SCE" and the "new NSCE," respectively. In the new SCE, E and $\hat{w}$ are determined as $E = E^{***}$ and $\hat{w} = w^* = w^{***}$, and, regardless of firms' belief, jobless workers can find new jobs by experiencing only one period of unemployment. In the new NSCE, E and $\hat{w}$ are determined as $E = E^{**}$ and $\hat{w} = w^* = w^{***}$. Since the new SCE attains the maximum employment, it is quite natural to guess that the government can enhance both WS and IS by guiding the economy from the current equilibrium to the new SCE.

Tables 13–15 report the results of a numerical experiment conducted to judge the validity of this guess, with all effects evaluated under the parameter configuration of Table 1.[14] There are thirteen columns in each table; the first lists the values of F, all of which, except for F_1, give rise to either multiple or no SCEs before $\overline{w}$ is reduced; the second reports the maximal value of $\overline{w}$ that can support the new SCE for a given value of F; the third reports the surplus gains from moving the economy from the old NSCE, which exists before $\overline{w}$ is reduced, to the new NSCE; the fourth reports the surplus gains from moving the economy from the old NSCE to the new SCE; and the fifth to thirteenth report the surplus gains from moving the economy from an old SCE, which exists before $\overline{w}$ is reduced, to the new one. The case in which $F = F_1$ serves as a benchmark, since in this case, the profile of the aggregate

[14]For the details of the derivation, see Appendix E.

labor demand runs through the upper end of the upward-sloping parts of the AICs for $\overline{n} < 0$, although $\overline{w}$ is not reduced. When $F \in (F_1, F_3)$, the reduction in $\overline{w}$ coalesces the multiple old SCEs into the new one, in which E is increased to E^{***}, U_O is decreased to zero, but the levels of w^1 and w^{2+}, and thus, that of $\tilde{w}$ remain unchanged. It also creates the new NSCE, in which E, U_O, and $\hat{w}$ take the same values as in the old one, but w^1 and w^{2+} have changed their values in such a manner that $\tilde{w}$ is reduced. As shown in Tables 13–15, the resulting changes in WS, IS, and TS are positive if the economy moves to the new SCE, implying that in these cases, guiding the economy from the current equilibrium to the new SCE surely improves economic welfare. When $F = F_3$, the reduction in $\overline{w}$ coalesces the multiple NSCEs into the one with $E = E^{**}$ and $w^{2+} < \tilde{w} < \hat{w} = w^* = w^{***} < w^1$ while creating from scratch the new SCE with $E = E^{***}$ and $w^{2+} = \tilde{w} = \hat{w} = w^* = w^{***} = w^1$. Again, in this case, guiding the economy from an old NSCE to the new SCE improves economic welfare, as implied by Tables 13–15. When $F > F_3$, the situation changes drastically. Before $\overline{w}$ is reduced, there is a single NSCE, in which no production occurs, and the reduction in $\overline{w}$ restarts production activities in the economy by creating the new SCE and the new NSCE, in both of which a positive population of workers is hired. However, this does not lead to an improvement in economic welfare since it decreases WS even if the economy is guided to the new SCE, as shown in Table 13. In these cases, firms need to pay low wages to their employees to generate profit flows that are large enough to compensate for a large entry cost. Such low wages significantly erode the attraction of employment and completely eliminate the incentive of workers to expend efforts unless $\overline{w}$ is substantially reduced. Put another way, when the entry cost is substantially large, the reduction in $\overline{w}$ effectively curtails workers' income-earning opportunities other than employment, thereby forcing them to work for low wages. As shown in Table 13, when F is as large as two, it decreases TS even if the economy is guided to the new SCE, which implies that production activities are no longer socially desirable. To sum up this argument, taxing jobless workers can improve economic welfare only when the entry cost takes such an intermediate value that multiple SCEs can exist in this economy. When the entry cost is substantially large, such taxation makes only investors better off at the expense of workers' welfare.

6 Conclusion

This paper has examined the long-run consequences of ranking job applicants by their unemployment durations by using a simple general equilibrium model with efficiency wages, in which the statistical discrimination of firms against jobless workers has a distorting effect on labor market performance. When the cost of establishing a new firm takes an intermediate value, the model exhibits multiple stationary equilibria, which differ from one another in allocative efficiency, and the most inefficient one is realized by the belief of firms that jobless workers have already lost their employability. Thus the government should persuade firms not to hold such an extreme belief, thereby creating second chances for jobless workers. Moreover, if the government can reduce the incomes of jobless workers through taxation, the multiple equilibria, in which a subset of jobless workers are given second chances, are coalesced into a more efficient one, in which every job seeker can find a new job by experiencing one period of unemployment. These policies effectively nullify the distorting effect of statistical discrimination considered here.

Appendix A: Proofs (Not for Publication)

Proof of Proposition 1

Proof. Let $V(n, Q)$ denote the lifetime utility of a worker in the labor force whose current state is (n, Q). If the path of wages paid to an employed worker is optimally chosen, then she finds it optimal to expend work effort in every period, and thus, for $\forall n \geq 1$, the following relations hold true:

$$\begin{aligned} V(n,1) &= w^n - e + \frac{1-\theta}{1+r}\left[(1-b)V(n+1,1) + bV(-1,1)\right] \\ &\geq w^n + \frac{1-\theta}{1+r}\left[\begin{array}{l} (1-b)(1-q)V(n+1,1) \\ +(b+q-bq)V(-1,1) \end{array}\right]. \end{aligned} \tag{6}$$

The larger side of (6) measures the lifetime utility when this worker expends e units of work effort in the nth period. If she makes this choice, then, in the next period, she will either keep this job with probability $1 - b$, in which case her lifetime utility changes from $V(n, 1)$ to $V(n + 1, 1)$, or lose it with probability b, in which case her lifetime utility changes from $V(n, 1)$ to $V(-1, 1)$. The smaller side, on the other hand, measures the lifetime utility when this agent expends zero effort in the nth period. If she makes this choice, then, in the next period, she will either keep this job with probability $(1 - b)(1 - q)$, in which case her lifetime utility changes from $V(n, 1)$ to $V(n + 1, 1)$, or lose it with probability $b + q - bq$, in which case her lifetime utility changes from $V(n, 1)$ to $V(-1, 1)$. By rearranging both sides of (6), we can obtain

$$\forall n \geq 1, \quad V(n+1,1) \geq V(-1,1) + \frac{1+r}{(1-\theta)(1-b)q}e. \tag{7}$$

The firm sets wages paid in and after the second period of service at such a level that the equalities of (7) hold true, thereby minimizing labor costs. Because workers are not allowed to hop directly from one firm to another, the firm has no incentive to pay employees recruited in or before the previous period higher wages than those necessary to elicit their work efforts. Using the equalities of (7) to eliminate $V(n,1)$ and $V(n+1,1)$ from (6), we can obtain

$$\forall n \geq 2,\ w^n = \frac{r+\theta}{1+r}V(-1,1) + \frac{1+r-(1-\theta)(1-b)(1-q)}{(1-\theta)(1-b)q}e = w^{2+}. \quad (8)$$

To elicit work efforts from newly recruited workers or, equivalently, to convince them that their labor contracts will be renewed at the end of the first period of service, the starting wage, w^1, cannot be lower than those paid in and after the second period of service, w^{2+}. When $w^1 \geq w^{2+}$, the workers are convinced that unless they shirk, their contracts will be renewed since their wages decrease from w^1 to w^{2+} in the next period, which gives their firm a strong incentive to continue the employment relationship with them. When $w^1 < w^{2+}$, in contrast, the workers are so skeptical about the renewal of their contracts that they choose to expend zero effort. If new workers can be recruited at a starting wage that is lower than w^{2+}, the firm finds it profitable to terminate its contracts with the current employees, replacing them with new ones and, thereby, saving labor costs. From the viewpoint of newly recruited workers, however, the starting wage lower than w^{2+} is an informal notice that their contracts will terminate at the end of the first period of service. Thus, if such a low starting wage is offered, they will choose to expend zero efforts while receiving that wage, which is paid independent of their effort expenditure. We must also note that the cost minimization of the firm cannot preclude the possibility of $w^1 > w^{2+}$. Unlike employed workers, job applicants can apply to multiple job openings, which makes it possible for starting wages to take a larger value than w^{2+}. When the aggregate demand for labor is sufficiently large, firms may not hire a desired number of new workers without making the level of w^1 higher than w^{2+}. □

Proof of Lemma 1

Consider a firm established in period t, and let L^n_{t+s} be the number of its employees who complete the nth period of service in period $t+s$. Obviously, L_{t+s} and $\{L^n_{t+s}\}^s_{n=1}$ satisfy

$$L_{t+s} = \sum_{n=1}^{s} L^n_{t+s}. \quad (9)$$

As shown in Proposition 1, if the firm sets the wage levels for the newly recruited and other employed workers at w^1 and w^{2+}, respectively, then, all employees expend work efforts. Hence, the expected profits of this firm can be written as

$$\sum_{s=1}^{\infty}\left(\frac{1-b}{1+r}\right)^{s-1}\left[\left(\sum_{n=1}^{s}L_{t+s}^{n}\right)^{\alpha}-\sum_{n=1}^{s}w^{n}L_{t+s}^{n}\right]$$

$$= L_{t+1}^{\alpha}-w^{1}L_{t+1}$$

$$+\sum_{s=2}^{\infty}\left(\frac{1-b}{1+r}\right)^{s-1}\left[\left(\sum_{n=1}^{s}L_{t+s}^{n}\right)^{\alpha}-w^{1}L_{t+s}^{1}-w^{2+}\sum_{n=2}^{s}L_{t+s}^{n}\right] \qquad (10)$$

$$= L_{t+1}^{\alpha}-w^{1}L_{t+1}$$

$$+\sum_{s=2}^{\infty}\left(\frac{1-b}{1+r}\right)^{s-1}\left\{\begin{array}{l} L_{t+s}^{\alpha}-w^{2+}(1-\theta)L_{t+s-1} \\ -w^{1}[L_{t+s}-(1-\theta)L_{t+s-1}]\end{array}\right\},$$

the first equality of which is obtained from the fact that $w^n = w^{2+}$ for $\forall n \geq 2$, and the second from (9) and the fact that only a fraction $(1-\theta)$ of current employees survive into the next period:

$$\forall t, \quad \forall s \geq 1, \quad \forall n = 1, \ldots, s, \qquad L_{t+s+1}^{n+1} = (1-\theta)L_{t+s}^{n}.$$

If $\{L_{t+s}\}_{s=1}^{\infty}$ is chosen to maximize (10), then

$$\forall t, \quad \forall s \geq 1, \quad \alpha L_{t+s}^{\alpha-1} = w^1 - \frac{(1-b)(1-\theta)(w^1-w^{2+})}{1+r},$$

which implies our desired result.

Proof of Lemma 2

Proposition 1 and Lemma 1 jointly imply that the net gain can be rewritten as

$$-F+\frac{1}{1+r}\sum_{s=1}^{\infty}\left(\frac{1-b}{1+r}\right)^{s-1}\left(\tilde{L}_{t+s}^{\alpha}-\mathbf{w}_{t+s}\cdot\mathbf{L}_{t+s}\right)$$

$$=-F+\frac{zL^{\alpha}-w^{1}L}{1+r}+\sum_{s=2}^{\infty}\left(\frac{1-b}{1+r}\right)^{s-1}\frac{L^{\alpha}-[\theta w^{1}+(1-\theta)w^{2+}]L}{1+r}$$

$$=-F+\frac{zL^{\alpha}-w^{1}L}{1+r}+\frac{1-b}{r+b}\cdot\frac{L^{\alpha}-[\theta w^{1}+(1-\theta)w^{2+}]L}{1+r}$$

$$=-F+\frac{1}{r+b}\left[L^{\alpha}-w^{1}L+\frac{(1-\theta)(1-b)(w^{1}-w^{2+})}{1+r}L\right]$$

$$=-F+\frac{L^{\alpha}-\hat{w}L}{r+b},$$

the second line of which is obtained from the assertions of Proposition 1 and Lemma 1 that every firm sets the wage for the employees in the nth period of service at a constant level as in (8); that such a wage profile induces employees to expend work efforts, thereby making each firm's effective labor input coincide with its total number of employees, $\tilde{L}_{t+s} = L_{t+s}$; and that each firm keeps its total number of employees constant over time, $L_{t+s} = L$. Also note that the last line of this equation is obtained from the definition of $\hat{w}$, (3). Thus, if the net gain equals zero, the following must hold:

$$-F + (L^{\alpha} - \hat{w}L)/(r + b) = 0. \tag{11}$$

By solving (3) and (11) with respect to $\hat{w}$ and L, we can obtain our desired results.

Proof of Proposition 2

Let m and L denote, respectively, the number of firms operating in such an equilibrium and the number of employees each firm hires. Then, m, L, E, and $\hat{w}$ satisfy (3),

$$mL = E,$$

and

$$-F + (L^{\alpha} - \hat{w}L)/(r + b) \leq 0. \tag{12}$$

Condition (12) means that the net gain from setting up a new firm cannot be positive in equilibrium. To see why, suppose that the net gain is positive,

$$-F + (L^{\alpha} - \hat{w}L)/(r + b) > 0,$$

which, combined with (3), implies that

$$\hat{w} - w^{*} < 0. \tag{13}$$

Because the number of firms is increasing as long as the net gain is positive, there are infinitely many firms operating in equilibrium, $m = +\infty$. We must also note that the aggregate employment (i.e., E) takes a finite value since the number of workers in the labor force equals N in any period. These jointly suggest that the number of employees chosen by each operating firm must be infinitesimally small, $L = 0$. According to (3), firms find it optimal to select such a small employment size if and only if $\hat{w} = +\infty$, which obviously contradicts (13). Thus, we can safely say that (12), and thus, $\hat{w} - w^{*} \geq 0$, are true in equilibrium. When the equality of (12) is valid, operating firms set their levels of $\hat{w}$ and L as in (4), as already shown in Lemma 2. On the other hand, their number cannot be determined since any non-negative value of m is consistent with investors' rational decisions.

Thus, in this case, E can take any non-negative value. When the inequality of (12) is valid, conditions (3) and (12) jointly imply that firms would set the level of $\hat{w}$ as $\hat{w} - w^* > 0$ if they were operating in equilibrium. However, in equilibrium, no firm is established because $m = 0$, and thus, E must be equal to zero.

Appendix B: Derivation of Aggregate Incentive Constraints (Not for Publication)

When $\overline{n} = 0$ and $\overline{n} = -\infty$, equilibrium conditions are so simple that we can analytically derive the profiles of the AIC for these cases. When $\overline{n} \in (-\infty, 0)$, equilibrium conditions are so complicated that we must resort to a numerical method to derive the profiles of the AIC for these cases.

AIC for $\overline{n} = 0$

Let us begin by deriving the AIC for the case of $\overline{n} = 0$. This case is special in that firms' belief is unconditionally self-fulfilling. Specifically, firms strongly believe that jobless workers have already lost their employability, disqualifying currently jobless workers for employment and thereby nullifying the risk of hiring an unemployable worker. In response to this hiring policy, jobless workers stop maintaining their employability, which completely fulfills firms' belief.

Lemma 3. *Suppose that there is a stationary equilibrium with $\overline{n} = 0$. Then, in that equilibrium, (a) jobless workers, who have negative values of n, expend zero effort, and their lifetime utilities are determined as*

$$\forall n \leq -1, \quad \forall Q = 0, 1, \quad V(n, Q) = [(1 + r)/(r + \theta)]\overline{w},$$

and (b) the level of w^{2+} is determined as

$$w^{2+} = \overline{w} + \frac{1 + r - (1 - \theta)(1 - b)(1 - q)}{(1 - \theta)(1 - b)q} e \ (\equiv w^{**}). \tag{14}$$

Proof. When $\overline{n} = 0$, job openings are limited to newly born workers in each period, and thus the lifetime utility of a worker unemployed at the beginning of period t can be written as

$$\sum_{s=0}^{\infty} \left(\frac{1 - \theta}{1 + r} \right)^s (\overline{w} - e_{t+s}).$$

Obviously, this utility attains its maximum, $[(1 + r)/(r + \theta)]\overline{w}$, at $e_{t+s} = 0$ for $\forall s \geq 0$, which establishes part (a). Part (a) implies that in this case,

$$V(-1, 1) = [(1 + r)/(r + \theta)]\overline{w}. \tag{15}$$

Substituting (15) into (2) produces (14). $\square$

Because every worker dies with probability θ, and because every job is destroyed and re-created with probability b, there are $[(\theta+b-\theta b)/(1-\theta)]E$ job openings in each period.[15] When firms fill their vacancies exclusively with new entrants into the labor force, then, the following must hold:

$$a[\theta/(1-\theta)]N = [(\theta+b-\theta b)/(1-\theta)]E, \tag{16}$$

where $a \in [0,1]$ denotes the job finding rate for workers believed to be employable, who are the new entrants in this case. The value of a is not exogenously given, but endogenously determined in equilibrium.

Proposition 3 (AIC for $\overline{n}=0$)**.** *Suppose that there is a stationary equilibrium with $\overline{n}=0$, and let E, $\hat{w}$, and w^{**} represent the aggregate employment, the average wage defined by (3), and the constant defined by (14), respectively. Moreover, define E^{**} as*

$$E^{**} \equiv [\theta/(\theta+b-\theta b)]N.$$

Then, in that equilibrium, E and $\hat{w}$ satisfy

$$(E^{**}-E)(\hat{w}-w^{**}) = 0, \tag{17}$$

$$E^{**}-E \geq 0 \tag{18}$$

and

$$\hat{w}-w^{**} \geq 0. \tag{19}$$

Proof. When the aggregate labor demand is so large that all newly born workers can find jobs, $a=1$, their starting wages w^1 and the aggregate employment E must satisfy

$$E = [\theta/(\theta+b-\theta b)]N(\equiv E^{**}), \qquad w^1 \geq w^{**}, \tag{20}$$

where w^{**} is as defined in (14). When the aggregate labor demand is not large enough to ensure jobs for all newly born workers, $a<1$, the starting wages and the aggregate employment must satisfy

$$E < E^{**}, \qquad w^1 = w^{**}. \tag{21}$$

Conditions (20) and (21) can be merged as

$$\begin{aligned} (E^{**}-E)(w^1-w^{**}) = 0; \\ E^{**}-E \geq 0; \\ w^1-w^{**} \geq 0, \end{aligned}$$

which are equivalent to (17)-(19) since the sign of w^1-w^{**} always coincides with that of $\hat{w}-w^{**}$. □

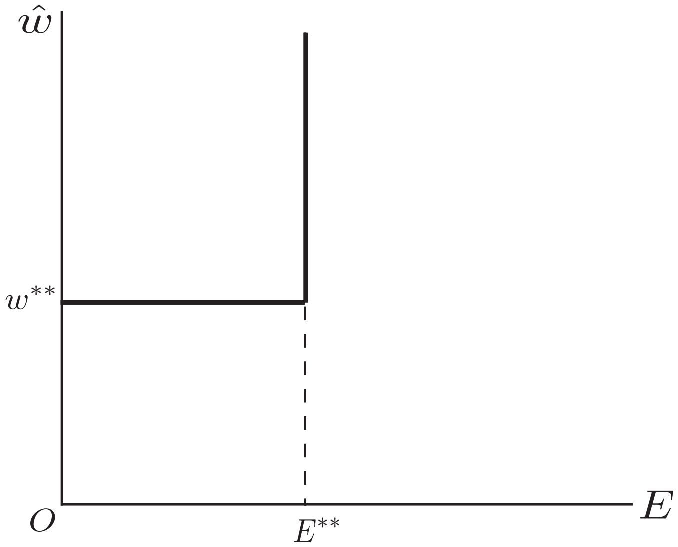

Figure 8: AIC for $\overline{n} = 0$

Figure 8 depicts the AIC for $\overline{n} = 0$. As shown in that figure, when firms embrace the belief of $\overline{n} = 0$, it is impossible to attain a level of aggregate employment that is larger than E^{**}. Under this belief, all vacant positions must be filled with new entrants, which necessitates that the aggregate employment be no larger than E^{**}. When $E = E^{**}$, the labor market is so tight that all newly born workers can find a job, $a = 1$, and thus, wages satisfy

$$w^1 \geq \hat{w} \geq w^{2+} = w^{**}.$$

When $E < E^{**}$, the labor market is so loose that some of the newly born workers cannot find a job, $a < 1$, and thus, wages satisfy

$$w^1 = w^{2+} = \hat{w} = w^{**}.$$

AIC for $\overline{n} = -\infty$

When $\overline{n} = -\infty$, firms do not care about an applicant's record of unemployment because they believe that unemployed workers retain their employability. However, this belief is not sufficient to ensure the employability of unemployed workers. Even when firms share such a belief, unemployed workers may find it optimal to stop preserving their employability if they

[15]Note that the total number of job openings is larger than that of the vacancies that are expected to appear, $(\theta + b - \theta b)E$. This is because firms recruit extra workers, anticipating that a fraction θ of their recruited workers die at the end of the current period.

expect to experience a sufficiently long duration of unemployment before being rehired or if they expect to receive sufficiently low wages after being rehired. Put another way, some additional conditions must be met for the self-fulfillment of firms' belief.

Lemma 4. *Suppose that there is a stationary equilibrium with $\overline{n} = -\infty$. Then, in that equilibrium, (a) the following conditions must hold:*

$$\begin{aligned}\forall n \leq -1, \qquad & \overline{w} - e + \frac{1-\theta}{1+r}[aV(1,1) + (1-a)V(n-1,1)] \\ & \geq \overline{w} + \frac{1-\theta}{1+r}[a(1-q)V(1,0) + (1-a+aq)V(n-1,0)],\end{aligned} \tag{22}$$

and (b) the lifetime utilities of employed and unemployed workers are determined as

$$V(n,Q) = \begin{cases} V_{DU} & \textit{if } n \leq -1 \textit{ and } Q = 0 \\ V_{DE}^1 & \textit{if } n = 1 \textit{ and } Q = 0 \\ V_{DE} & \textit{if } n \geq 2 \textit{ and } Q = 0 \\ V_U & \textit{if } n \leq -1 \textit{ and } Q = 1 \\ V_E^1 & \textit{if } n = 1 \textit{ and } Q = 1 \\ V_E & \textit{if } n \geq 2 \textit{ and } Q = 1 \end{cases},$$

where the values of V_{DU}, V_{DE}^1, V_{DE}, V_U, V_E^1, and V_E are given by

$$V_{DU} = \overline{w} + \frac{1-\theta}{1+r}\left[a(1-q)V_{DE}^1 + (1-a+aq)V_{DU}\right], \tag{23}$$

$$V_{DE}^1 = w^1 + \frac{1-\theta}{1+r}\left[(1-b)(1-q)V_{DE} + (b+q-bq)V_{DU}\right], \tag{24}$$

$$V_{DE} = w^{2+} + \frac{1-\theta}{1+r}\left[(1-b)(1-q)V_{DE} + (b+q-bq)V_{DU}\right], \tag{25}$$

$$V_U = \overline{w} - e + \frac{1-\theta}{1+r}\left[aV_E^1 + (1-a)V_U\right], \tag{26}$$

$$V_E^1 = w^1 - e + \frac{1-\theta}{1+r}\left[(1-b)V_E + bV_U\right], \tag{27}$$

and

$$V_E = w^{2+} - e + \frac{1-\theta}{1+r}\left[(1-b)V_E + bV_U\right]. \tag{28}$$

Proof. Part (a): Consider an employable worker who is currently experiencing her $-n$th period of unemployment. If this worker preserves her employability in the current period, then at the end of that period, she will either be hired by a firm with probability a, in which case her lifetime utility evaluated at the beginning of the next period is given by $V(1,1)$, or remain unemployed with probability $1-a$, in which case her lifetime utility evaluated at the beginning of the next period is given by $V(n-1,1)$. Thus, in this case, her lifetime utility evaluated at the beginning of the current period can be expressed as

$$\overline{w} - e + \frac{1-\theta}{1+r}\left[aV(1,1) + (1-a)V(n-1,1)\right]. \tag{29}$$

If this worker stops preserving her employability in the current period, then at the end of that period, she will either be hired by a firm with probability $a(1-q)$, in which case her lifetime utility evaluated at the beginning of the next period is given by $V(1,0)$, or remain unemployed with probability $1-a+aq$, in which case her lifetime utility evaluated at the beginning of the next period is given by $V(n-1,0)$. Thus, in this case, her lifetime utility evaluated at the beginning of the current period can be expressed as

$$\overline{w} + \frac{1-\theta}{1+r}\left[a(1-q)V(1,0) + (1-a+aq)V(n-1,0)\right]. \tag{30}$$

The worker finds it optimal to preserve her employability if and only if the value of (29) is no smaller than that of (30). Therefore, (22) is necessary for the existence of a stationary equilibrium with $\overline{n} = -\infty$.

Part (b): When $\overline{n} = -\infty$, the lifetime utilities of workers who have already lost their employability satisfy

$$\forall n \leq -1, \quad V(n,0) = \overline{w} + \frac{1-\theta}{1+r}\left[\begin{array}{l} a(1-q)V(1,0) \\ +(1-a+aq)V(n-1,0) \end{array}\right], \tag{31}$$

$$V(1,0) = w^1 + \frac{1-\theta}{1+r}\left[\begin{array}{l} (1-b)(1-q)V(2,0) \\ +(b+q-bq)V(-1,0) \end{array}\right], \tag{32}$$

and

$$\forall n \geq 2, \quad V(n,0) = w^{2+} + \frac{1-\theta}{1+r}\left[\begin{array}{l} (1-b)(1-q)V(n+1,0) \\ +(b+q-bq)V(-1,0) \end{array}\right]. \tag{33}$$

To understand why these conditions hold, consider an unemployable worker who is currently experiencing her $-n$th period of unemployment. At the end of the current period, she will either be hired by a firm with probability $a(1-q)$, in which case her lifetime utility changes from $V(n,0)$ to $V(1,0)$, or remain unemployed with probability $1-a+aq$, in which case her lifetime utility changes from $V(n,0)$ to $V(n-1,0)$. Alternatively, consider an unemployable worker who is experiencing her nth period of service with her current firm. Her employment will either continue into the next period with probability $(1-b)(1-q)$, in which case her lifetime utility changes from $V(n,0)$ to $V(n+1,0)$, or terminate at the end of the current period with probability $b+q-bq$, in which case her lifetime utility changes from $V(n,0)$ to $V(-1,0)$. Condition (31) implies that $V(n,0)$ takes a constant value,

$$\forall n \leq -1, \quad V(n,0) = V(n-1,0).$$

Otherwise, it diverges toward either positive or negative infinity as n approaches negative infinity. Likewise, condition (33) implies that $V(n,0)$ takes a constant value,

$$\forall n \geq 2, \quad V(n,0) = V(n+1,0).$$

Otherwise, it diverges toward either positive or negative infinity as n approaches positive infinity. These results allow us to rewrite (31)–(33) as (23)–(25). Next, consider the workers who are still employable. Their lifetime utilities satisfy

$$\forall n \leq -1, \quad V(n,1) = \overline{w} - e + \frac{1-\theta}{1+r}\left[\begin{array}{l} aV(1,1) \\ +(1-a)V(n-1,1) \end{array}\right], \tag{34}$$

$$V(1,1) = w^1 - e + \frac{1-\theta}{1+r}\left[\begin{array}{l} (1-b)V(2,1) \\ +bV(-1,1) \end{array}\right], \tag{35}$$

and

$$\forall n \geq 2, \quad V(n,1) = w^{2+} - e + \frac{1-\theta}{1+r}\left[\begin{array}{l} (1-b)V(n+1,1) \\ +bV(-1,1) \end{array}\right]. \tag{36}$$

To understand why these conditions hold, consider an employable worker who is currently experiencing her $-n$th period of unemployment. Since (22) is true, this worker finds it optimal to expend e units of effort to preserve her employability in the current period. At the end of that period, she will either be hired by a firm with probability a, in which case her lifetime utility changes from $V(n,1)$ to $V(1,1)$, or remain unemployed with probability $1-a$, in which case her lifetime utility changes from $V(n,1)$ to $V(n-1,1)$. Alternatively, consider an employable worker who is experiencing her nth period of service with her current firm. As shown in Lemma 1, the wage profile is optimally designed such that this worker chooses to expend e units of work effort in the current period. Her employment will either continue into the next period with probability $1-b$, in which case her lifetime utility changes from $V(n,1)$ to $V(n+1,1)$, or terminate at the end of the current period with probability b, in which case her lifetime utility changes from

$V(n,1)$ to $V(-1,1)$. Condition (34) implies that $V(n,1)$ takes a constant value,

$$\forall n \leq -1, \quad V(n,1) = V(n-1,1).$$

Otherwise, it diverges toward either positive or negative infinity as n approaches negative infinity. Likewise, condition (36) implies that $V(n,1)$ takes a constant value,

$$\forall n \geq 2, \quad V(n,1) = V(n+1,1).$$

Otherwise, it diverges toward either positive or negative infinity as n approaches positive infinity. These results allow us to rewrite (34)–(36) as (26)–(28). $\square$

As shown in this lemma, all of the unemployed workers choose to preserve their employability if and only if condition (22) is true. Otherwise, some unemployed workers stop preserving their employability, which makes it difficult for firms to embrace the belief that all of the unemployed workers are still employable. We call (22) the employability preserving condition (EPC) for $\overline{n} = -\infty$.

Because both new entrants and currently unemployed workers are believed to be employable in this case, the total number of job seekers is given by $[\theta/(1-\theta)]N + N - E$, whereas $[(\theta + b - \theta b)/(1-\theta)]E$ jobs become open in each period, as in the case of $\overline{n} = 0$. Thus, in equilibrium, the following must hold:

$$a\{[\theta/(1-\theta)]N + N - E\} = [(\theta + b - \theta b)/(1-\theta)]E. \tag{37}$$

Then we can state the following.

Proposition 4 (AIC for $\overline{n} = -\infty$)**.** *Suppose that there is a stationary equilibrium with $\overline{n} = -\infty$. Then, in that equilibrium, E and $\hat{w}$ must satisfy*

$$\begin{aligned} \left\{ \hat{w} - \overline{w} - \left[\frac{r + \theta + (1-\theta)b}{(1-\theta)(1-b)q} + \frac{\theta + b - \theta b}{(1-b)q} \cdot \frac{E}{N - (1-\theta)E} \right] e \right\} & \\ \times \left(\frac{1}{1 + b - \theta b} N - E \right) = 0, & \end{aligned} \tag{38}$$

$$\hat{w} - \overline{w} - \left[\frac{r + \theta + (1-\theta)b}{(1-\theta)(1-b)q} + \frac{\theta + b - \theta b}{(1-b)q} \cdot \frac{E}{N - (1-\theta)E} \right] e \geq 0, \tag{39}$$

$$\frac{1}{1 + b - \theta b} N - E \geq 0, \tag{40}$$

and

$$E - (1-b)N \geq 0. \tag{41}$$

Proof. From (23)–(28), we can derive V_{DU} and V_U as

$$V_{DU} = \frac{1+r}{(r+\theta)[1+r-(1-\theta)(1-q)(1-a-b)]} \times \left\{ \begin{array}{l} [1+r-(1-\theta)(1-q)(1-b)]\overline{w} \\ \\ +\dfrac{(1-\theta)(1-q)a[1+r-(1-\theta)(1-q)(1-b)]}{1+r-(1-\theta)(1-b)}\hat{w} \\ \\ -\dfrac{(1-\theta)^2(1-q)qa(1-b)}{1+r-(1-\theta)(1-b)}w^{2+} \end{array} \right\} \tag{42}$$

and

$$\begin{aligned} V_U &= \frac{(1+r)[1+r-(1-\theta)(1-b)]}{(r+\theta)[1+r-(1-\theta)(1-a-b)]}(\overline{w}-e) \\ &+ \frac{(1+r)(1-\theta)a}{(r+\theta)[1+r-(1-\theta)(1-a-b)]}(\hat{w}-e). \end{aligned} \tag{43}$$

Lemma 4 implies that (22) can be rewritten as $V_U \geq V_{DU}$. Thus, by substituting the values of V_{DU} and V_U into this inequality, we can obtain another expression of the EPC:

$$\begin{aligned} &\frac{[1+r-(1-\theta)(1-b)]\overline{w}+(1-\theta)a\hat{w}}{1+r-(1-\theta)(1-a-b)} - e \\ &\geq [1+r-(1-\theta)(1-q)(1-a-b)]^{-1} \\ &\times \left\{ \begin{array}{l} [1+r-(1-\theta)(1-q)(1-b)]\overline{w} \\ \\ +\dfrac{(1-\theta)(1-q)a[1+r-(1-\theta)(1-q)(1-b)]}{1+r-(1-\theta)(1-b)}\hat{w} \\ \\ -\dfrac{(1-\theta)^2(1-q)qa(1-b)}{1+r-(1-\theta)(1-b)}w^{2+} \end{array} \right\}. \end{aligned} \tag{44}$$

Next, we will derive the relation between $\hat{w}$ and a implied by firms' optimal wage setting behavior. According to Lemma 1, firms set the wages paid in and after the second period of service as

$$\begin{aligned} w^{2+} &= \frac{r+\theta}{1+r}V_U + \frac{1+r-(1-\theta)(1-b)(1-q)}{(1-\theta)(1-b)q}e \\ &= \frac{[1+r-(1-\theta)(1-b)]\overline{w}+(1-\theta)a\hat{w}}{1+r-(1-\theta)(1-a-b)} \\ &\quad +\frac{1+r-(1-\theta)(1-b)}{(1-\theta)(1-b)q}e, \end{aligned} \tag{45}$$

the second equality of which is obtained from (43). As is easily verified, condition (37) can be reduced to

$$a = \frac{(\theta + b - \theta b)E}{N - (1-\theta)E}. \tag{46}$$

When $a = 1$, this condition can be further reduced to

$$E = [1/(1 + b - \theta b)]N.$$

In this case, the labor market is so tight that firms may not hire their desired number of new employees without making starting wages higher than those paid in and after the second period of service,

$$w^{2+} \leq \hat{w} \leq w^1,$$

which, combined with (45), implies that

$$\hat{w} \geq \overline{w} + \frac{1 + r + (1-\theta)b}{(1-\theta)(1-b)q}e. \tag{47}$$

When $a < 1$, condition (46) implies that

$$E < [1/(1 + b - \theta b)]N.$$

In this case, the labor market is so loose that firms can hire their desired number of new employees by setting starting wages at the same level as those paid in and after the second period of service,

$$w^1 = w^{2+} = \hat{w}, \tag{48}$$

which, combined with (45), implies that

$$\hat{w} = \overline{w} + \frac{1 + r - (1-\theta)(1-a-b)}{(1-\theta)(1-b)q}e. \tag{49}$$

These results are summarized by

$$\left[\hat{w} - \overline{w} - \frac{1 + r - (1-\theta)(1-a-b)}{(1-\theta)(1-b)q}e\right] \times (1-a) = 0, \tag{50}$$

$$\hat{w} - \overline{w} - \frac{1 + r - (1-\theta)(1-a-b)}{(1-\theta)(1-b)q}e \geq 0, \tag{51}$$

and

$$1 - a \geq 0. \tag{52}$$

We also need to examine whether optimal wages satisfy the EPC. When $a = 1$, the EPC, that is, (44), is reduced to

$$\frac{[1+r-(1-\theta)(1-b)]\overline{w}+(1-\theta)\hat{w}}{1+r+(1-\theta)b} - e$$
$$\geq [1+r+(1-\theta)(1-q)b]^{-1}$$
$$\times \left\{ \begin{array}{l} [1+r-(1-\theta)(1-q)(1-b)]\overline{w} \\ \\ +\dfrac{(1-\theta)(1-q)[1+r-(1-\theta)(1-q)(1-b)]}{1+r-(1-\theta)(1-b)}\hat{w} \\ \\ -\dfrac{(1-\theta)^2(1-q)q(1-b)}{1+r-(1-\theta)(1-b)}w^{2+} \end{array} \right\}. \tag{53}$$

Using (45) to eliminate w^{2+} from (53), we can obtain a simplified version of the EPC,

$$\hat{w} - \overline{w} \geq \frac{1+r+(1-\theta)b}{(1-\theta)q}e. \tag{54}$$

This condition is true for any value of $\hat{w}$ implied by optimal wage setting behavior since such a value satisfies (47). When $a < 1$, condition (48) is true, and thus, the EPC, that is, (44), is reduced to

$$\hat{w} - \overline{w} \geq$$
$$\frac{[1+r-(1-\theta)(1-a-b)][1+r-(1-q)(1-\theta)(1-a-b)]}{(1+r)(1-\theta)aq}e. \tag{55}$$

In addition, $\hat{w}$ satisfies (49) in this case. Using (49) to eliminate $\hat{w}-\overline{w}$ from (55) and rearranging it, we can obtain

$$a \geq 1 - b, \tag{56}$$

which is the necessary and sufficient condition for a value of $\hat{w}$ implied by optimal wage setting behavior to satisfy the EPC. Finally, using (46), we can rewrite (50)–(52) and (56) as (38)–(41). □

Figure 9 depicts the AIC for $\overline{n} = -\infty$ as well as that for $\overline{n} = 0$.[16]

[16] This figure is depicted under the condition that $\theta < (1-\theta)(1-b)$. If this condition is not met, the AICs for $\overline{n} = 0$ and $\overline{n} = -\infty$ have one intersection, creating the possibility that the aggregate employment attained under the belief that all unemployed workers are unemployable dominates that attained under the belief that they are employable. This possibility need not be seriously considered because its realization requires either b or θ to take an unrealistically high value. Because labor contracts are renewed every period in our model, the length of one period should be considered to be one year or shorter, and thus, it is reasonable to assume that both b and θ are smaller than 0.1, which suffices to make the above condition true.

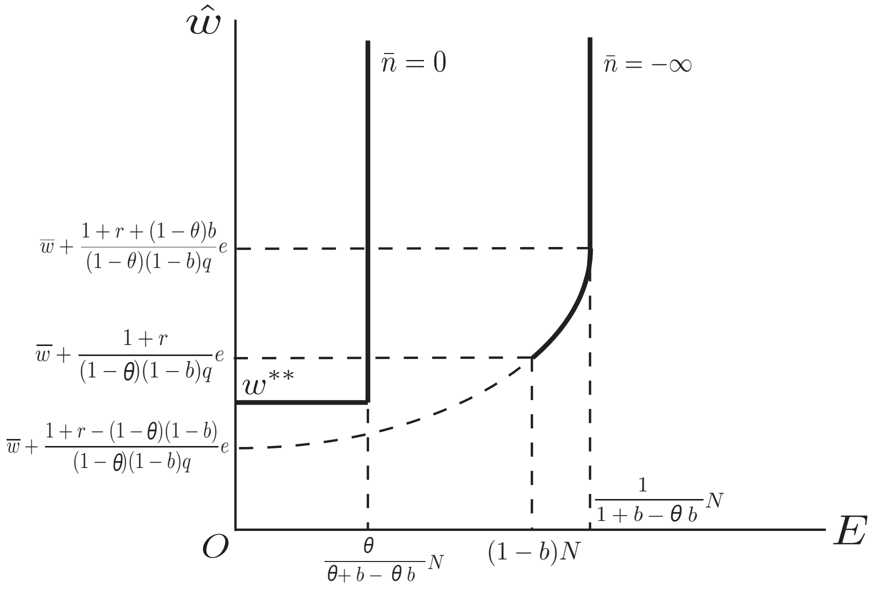

Figure 9: AICs for $\overline{n} = 0$ and $\overline{n} = -\infty$

As shown in that figure, the AIC for $\overline{n} = -\infty$ has vertical and upward-sloping parts. On the vertical part, the labor market is so tight that all job seekers can find a job, $a = 1$, and thus wages satisfy $w^1 \geq \hat{w} \geq w^2$. On the upward-sloping part, the labor market is so loose that some job seekers cannot find a job, $a < 1$, and thus, wages satisfy $w^1 = w^2 = \hat{w}$. The AIC for this case has an upward-sloping part rather than a horizontal one because it provides unemployed workers with second chances. In the presence of second chances, the level of wages required to elicit work efforts from employed workers increases with the aggregate employment. As the aggregate employment increases, the probability that an unemployed worker will be rehired improves, which effectively eases the penalty for shirking behavior by currently employed workers. To keep their employees from shirking, firms need to pay higher wages, thereby rendering the loss of their current job sufficiently costly to them. Also note that the wages that are set so as to discipline employees can also motivate unemployed workers if and only if the aggregate employment is no smaller than $(1-b)N$. In other cases, some unemployed workers stop preserving their employability, either because their prospects of being rehired are slim or because the wages they will receive after reemployment are too low. This is why the AIC for this case cannot be obtained when the aggregate employment is smaller than $(1-b)N$.

AIC for $\overline{n} \in (-\infty, 0)$

When $\overline{n} < 0$, the jobless workers whose unemployment durations are $\overline{n}$ periods or shorter are given "second chances," that is, chances of being rehired. The AIC for such a negative value of $\overline{n}$ requires not only that all of the employed workers spend work efforts, but also that the jobless workers who are given second chances preserve their employability (whereas other jobless ones stop preserving their employability).

Lemma 5. *Suppose that there is a stationary equilibrium with $\overline{n} < 0$, and let a be the job-finding rate for the jobless workers who are believed by firms to be employable. Then, in that equilibrium, (a) the following conditions must hold:*

$$\forall n \in [\overline{n}, -1], \qquad \overline{w} - e + \frac{1-\theta}{1+r}[aV(1,1) + (1-a)V(n-1,1)] \\ \geq \overline{w} + \frac{1-\theta}{1+r}[a(1-q)V(1,0) + (1-a+aq)V(n-1,0)], \tag{57}$$

and (b) the lifetime utilities of employed and jobless workers are determined as

$$V(n,Q) = \begin{cases} V_O & \text{if } n \leq \overline{n} - 1 \\ V_{DU}^{n} & \text{if } n \in [\overline{n}, -1] \text{ and } Q = 0 \\ V_{DE}^{1} & \text{if } n = 1 \text{ and } Q = 0 \\ V_{DE} & \text{if } n \geq 2 \text{ and } Q = 0 \\ V_{U}^{n} & \text{if } n \in [\overline{n}, -1] \text{ and } Q = 1 \\ V_{E}^{1} & \text{if } n = 1 \text{ and } Q = 1 \\ V_{E} & \text{if } n \geq 2 \text{ and } Q = 1 \end{cases},$$

where the values of V_O, V_{DU}^{n}, V_{DE}^{1}, V_{DE}, V_{U}^{n}, V_{E}^{1}, and V_E are given by

$$V_O = \overline{w} + \frac{1-\theta}{1+r}V_O, \tag{58}$$

$$V_{DU}^{\overline{n}} = \overline{w} + \frac{1-\theta}{1+r}\left[a(1-q)V_{DE}^{1} + (1-a+aq)V_O\right], \tag{59}$$

$$V_{DE}^{1} = w^{1} + \frac{1-\theta}{1+r}\left[(1-b)(1-q)V_{DE} + (b+q-bq)V_{DU}^{-1}\right], \tag{60}$$

$$V_{DE} = w^{2+} + \frac{1-\theta}{1+r}\left[(1-b)(1-q)V_{DE} + (b+q-bq)V_{DU}^{-1}\right], \tag{61}$$

$$V_{U}^{\overline{n}} = \overline{w} - e + \frac{1-\theta}{1+r}\left[aV_{E}^{1} + (1-a)V_O\right], \tag{62}$$

$$V_E^1 = w^1 - e + \frac{1-\theta}{1+r}\left[(1-b)V_E + bV_U^{-1}\right], \tag{63}$$

$$V_E = w^{2+} - e + \frac{1-\theta}{1+r}\left[(1-b)V_E + bV_U^{-1}\right], \tag{64}$$

and if $\overline{n} \leq -2$, *then, for* $n = \overline{n}+1, \cdots, -1$,

$$V_{DU}^n = \overline{w} + \frac{1-\theta}{1+r}\left[a(1-q)V_{DE}^1 + (1-a+aq)V_{DU}^{n-1}\right], \tag{65}$$

$$V_U^n = \overline{w} - e + \frac{1-\theta}{1+r}\left[aV_E^1 + (1-a)V_U^{n-1}\right]. \tag{66}$$

Proof. Part (a): Note that condition (57) is the same as condition (22), except that the former must be satisfied for $\forall n \in [\overline{n}, -1]$, whereas the latter for $\forall n \in (-\infty, -1]$. As already explained in the proof of Lemma 4, the larger side of (57) measures the lifetime utility of an employable worker who is currently experiencing her $-n$th period of unemployment in the case that she chooses to preserve her employability in the current period, whereas the smaller side measures her lifetime utility in the case that she chooses to stop preserving her employability. If condition (57) is not met for some $n \in [\overline{n}, -1]$, then, those who have been unemployed for $-n$ periods find it optimal to stop preserving their employability, which contradicts firms' belief. Therefore, condition (57) is necessary for the existence of a stationary equilibrium with $\overline{n} \in (-\infty, 0)$.

Part (b): When $\overline{n} \in (-\infty, 0)$, those who have been unemployed for more than $-\overline{n}$ periods will never be rehired, and thus, their lifetime utilities are determined as

$$\forall n \leq \overline{n}-1, \quad \forall Q = 0,1, \quad V(n,Q) = [(1+r)/(r+\theta)]\overline{w} \ (\equiv V_O). \tag{67}$$

The lifetime utilities of unemployable workers with $n \geq \overline{n}$ $(n \neq 0)$ satisfy

$$\forall n \in [\overline{n}, -1], \quad V(n,0) = \overline{w} + \frac{1-\theta}{1+r}\left[\begin{array}{l} a(1-q)V(1,0) \\ +(1-a+aq)V(n-1,0) \end{array}\right], \tag{68}$$

$$V(1,0) = w^1 + \frac{1-\theta}{1+r}\left[\begin{array}{l} (1-b)(1-q)V(2,0) \\ +(b+q-bq)V(-1,0) \end{array}\right], \tag{69}$$

and

$$\forall n \geq 2, \quad V(n,0) = w^{2+} + \frac{1-\theta}{1+r}\left[\begin{array}{l} (1-b)(1-q)V(n+1,0) \\ +(b+q-bq)V(-1,0) \end{array}\right]. \tag{70}$$

Condition (70) implies that $V(n,0)$ takes a constant value,

$$\forall n \geq 2, \quad V(n,0) = V(n+1,0).$$

Otherwise, it diverges toward either positive or negative infinity as n approaches positive infinity. These results allow us to rewrite (68)–(70) as (59)–(61) and (65). Next, consider the workers who are still employable. When unemployed, such a worker finds it optimal to expend e units of effort to preserve her employability since (57) is true. When employed, such a worker finds it optimal to expend e units of work effort since the wage profile is designed as shown in Lemma 1. Thus, their lifetime utilities satisfy

$$\forall n \in [\overline{n}, -1], \quad V(n,1) = \overline{w} - e + \frac{1-\theta}{1+r}\begin{bmatrix} aV(1,1) \\ +(1-a)V(n-1,1) \end{bmatrix}, \tag{71}$$

$$V(1,1) = w^1 - e + \frac{1-\theta}{1+r}\begin{bmatrix} (1-b)V(2,1) \\ +bV(-1,1) \end{bmatrix}, \tag{72}$$

and

$$\forall n \geq 2, \quad V(n,1) = w^{2+} - e + \frac{1-\theta}{1+r}\begin{bmatrix} (1-b)V(n+1,1) \\ +bV(-1,1) \end{bmatrix}. \tag{73}$$

Condition (73) implies that $V(n,1)$ takes a constant value,

$$\forall n \geq 2, \quad V(n,1) = V(n+1,1).$$

Otherwise, it diverges toward either positive or negative infinity as n approaches positive infinity. These results allow us to rewrite (71)–(73) as (62)–(64) and (66). □

Note that the EPC is modified as (57) since the equilibrium considered here requires that only unemployed workers with $n \in [\overline{n}, -1]$ choose to preserve their employability. In this equilibrium, moreover, those who have been unemployed for more than $-\overline{n}$ periods will be never rehired, regardless of their employability. They have no choice but to live by consuming their endowments in each period, which makes their lifetime utilities as low as $[(1+r)/(r+\theta)]\overline{w}$.

We can also state the following.

Lemma 6. *Suppose that there is a stationary equilibrium with $\overline{n} < 0$, and let E, U_n and U_O denote, respectively, the populations of employed workers, that of those who have been unemployed for $-n$ ($\leq -\overline{n}$) periods, and that of those who have been unemployed for more than $-\overline{n}$ periods. Further, the job-finding rate for the jobless workers who are believed by firms to be employable is a. Then, in this equilibrium, the values of E, U_n, U_O, and a must satisfy*

$$E = \frac{\theta a\{1 - [(1-\theta)(1-a)]^{-\overline{n}+1}\}}{(1-\theta)^2 ab[(1-\theta)(1-a)]^{-\overline{n}} + \theta[1-(1-\theta)(1-a-b)]}N, \tag{74}$$

$$U_O = \frac{[(1-a)\theta + b(1-\theta)][1-(1-\theta)(1-a)][(1-\theta)(1-a)]^{-\overline{n}}}{(1-\theta)^2 ab[(1-\theta)(1-a)]^{-\overline{n}} + \theta[1-(1-\theta)(1-a-b)]} N, \quad (75)$$

and for $n = \overline{n}, \cdots, -1$,

$$U_n = \frac{\theta[(1-a)\theta + b(1-\theta)][1-(1-\theta)(1-a)][(1-\theta)(1-a)]^{-n-1}}{(1-\theta)^2 ab[(1-\theta)(1-a)]^{-\overline{n}} + \theta[1-(1-\theta)(1-a-b)]} N. \quad (76)$$

Proof. When $\overline{n} < 0$, the population measures E, U_n, and U_O satisfy the following conditions:

$$(1-a)\theta N + b(1-\theta)E = U_{-1}, \quad (77)$$

$$\forall n = \overline{n}+1, \cdots, -1, \quad (1-\theta)(1-a)U_n = U_{n-1}, \quad (78)$$

$$(1-\theta)(1-a)U_{\overline{n}} = \theta U_O, \quad (79)$$

and

$$N = E + \sum_{n=\overline{n}}^{-1} U_n + U_O. \quad (80)$$

Condition (77) is required by the fact that those who are currently experiencing their first period of unemployment were either born in or working for some firm in the previous period. Condition (78) is required by the fact that those who are currently experiencing their $-n$th period of unemployment were experiencing their $(-n-1)$th period of unemployment in the previous period. Condition (79) is required by the fact that the population of those who have been unemployed for more than $-\overline{n}$ periods is constant over time. Condition (80) is required by the fact that any worker in the labor force is either employed or unemployed. By solving (77)–(80) with respect to E, U_n, and U_O, we can obtain (74)–(76). □

Conditions (74)–(76) are comparable to (37) in the case of $\overline{n} = 0$, as they keep the populations of employed and jobless workers constant in a stationary equilibrium. Using Lemmas 5 and 6, we can show that the AICs for $\overline{n} < 0$ have a common vertical part.

Proposition 5. *When* $a = 1$, *the AICs for* $\overline{n} < 0$ *are identical and derived as*

$$E = \frac{1}{1+b-\theta b} N \ (\equiv E^{***}), \quad \hat{w} \geq \overline{w} + \frac{1+r+(1-\theta)b}{(1-\theta)(1-b)q} e \ (\equiv w^{***}). \quad (81)$$

Proof. When $a = 1$, conditions (74)–(76) are reduced to

$$
\begin{aligned}
E &= [1/(1+b-\theta b)]N, & (82)\\
U_{-1} &= [(1-\theta)b/(1+b-\theta b)]N, & (83)\\
U_{-2} &= \cdots = U_{\overline{n}} = U_O = 0. & (84)
\end{aligned}
$$

Moreover, conditions (62)–(64) and (66) are reduced to

$$
\begin{aligned}
V_U^{-1} &= \overline{w} - e + \frac{1-\theta}{1+r} V_E^1, & (85)\\
V_E^1 &= w^1 - e + \frac{1-\theta}{1+r}\left[(1-b)V_E + bV_U^{-1}\right], & (86)\\
V_E &= w^{2+} - e + \frac{1-\theta}{1+r}\left[(1-b)V_E + bV_U^{-1}\right]. & (87)
\end{aligned}
$$

By solving (85)–(87) with respect to V_U^{-1}, V_E^1, and V_E, we can obtain

$$
V_U^{-1} = \left(\frac{1+r}{r+\theta}\right)\left[\frac{1+r-(1-\theta)(1-b)}{1+r+(1-\theta)b}\overline{w} + \frac{1-\theta}{1+r+(1-\theta)b}\hat{w} - e\right]. \quad (88)
$$

Proposition 1 implies that when $a = 1$, the following are true:

$$
\begin{aligned}
\hat{w} &\geq w^{2+}\\
&= \frac{r+\theta}{1+r}V_U^{-1} + \frac{1+r-(1-\theta)(1-b)(1-q)}{(1-\theta)(1-b)q}e\\
&= \frac{1+r-(1-\theta)(1-b)}{1+r+(1-\theta)b}\overline{w} + \frac{1-\theta}{1+r+(1-\theta)b}\hat{w}\\
&\quad + \frac{1+r-(1-\theta)(1-b)}{(1-\theta)(1-b)q}e,
\end{aligned}
$$

the last equality of which is obtained from (88). By rearranging it, we can obtain

$$
\hat{w} \geq \overline{w} + \frac{1+r+(1-\theta)b}{(1-\theta)(1-b)q}e = w^{***}. \quad (89)
$$

Clearly, (82) and (89) are our desired results. □

This result is not surprising. When $a = 1$, the labor market is so tight that every job seeker can find a job with one period of unemployment experience, and thus, the beliefs satisfying $\overline{n} < 0$ have no effect on labor market performance. In other words, if the differences among such beliefs affect labor market performance, the job finding rate must satisfy $a < 1$.

When $\overline{n} \in (+\infty, 0)$, however, it is difficult to analytically derive the AICs for the case of $a < 1$. Hence, we will derive the AICs for $\overline{n} \in (-\infty, 0)$, resorting to the following numerical method. First, we will increase the value of a from 0 to 1 in steps of 0.001 and, for each value of a, compute the values of V_O, V_{DU}^n, V_{DE}^1, V_{DE}, V_U^n, V_E^1, V_E, $\hat{w}$, E, U_n, and U_O satisfying (58)–(76) and

$$w^1 = w^{2+} = \hat{w} = \frac{r+\theta}{1+r} V_U^{-1} + \frac{1+r-(1-\theta)(1-b)(1-q)}{(1-\theta)(1-b)q} e.$$

The last condition is the firm's optimal wage-setting policy, that is, (2). Since $a < 1$, we can assume that $w^1 = w^{2+} = \hat{w}$. In computing these values, we take one period as one year, and we configure the values of r, α, θ, b, $\overline{w}$, e, q, and N as in Table 1. Combined with these parameter values, a given value of a uniquely determines a set of the values of V_O, V_{DU}^n, V_{DE}^1, V_{DE}, V_U^n, V_E^1, V_E, $\hat{w}$, E, U_n, and U_O. Thus, this process yields 1,001 sets of these values. For each of the obtained sets, we will examine whether it satisfies the EPC,

$$\forall n = \overline{n}, \cdots, -1, \quad V_U^n \geq V_{DU}^n.$$

If a set satisfies the EPC, it will be kept. Otherwise, it will be abandoned. In doing so, we effectively pick up pairs of E and $\hat{w}$ that can constitute an AIC. By depicting the locus of such pairs of E and $\hat{w}$ on the $(E, \hat{w})$ plane, we obtain Figures 2 and 3.

Appendix C: Derivation of NSCE and SCEs (Not for Publication)

As shown in Appendix B, the NSCE is always derived analytically. Moreover, when $\hat{w} \geq w^{***}$, the SCE is also derived analytically. Thus, in these cases, all we have to do to derive the SCE and NSCE is substitute parameter values into the closed form of E, U_O, a, E/L^*, w^1, and w^{2+}. When $\hat{w} < w^{***}$, we fix the value of $\overline{n}$ and then compare the values of $\hat{w}$, which are obtained in deriving the profiles of the AICs, with w^{**}, choosing the one nearest to w^*. We regard the value of a that gives this nearest value of w^* as the equilibrium value of a. Once the equilibrium value of a is obtained, it gives the equilibrium values of E, U_O, and E/L^*, as in Appendix B.

Appendix D: Alternative Interpretations of Workers' and Investors' Surpluses (Not for Publication)

Workers' Surplus

We can show that the WS equals the discounted sum of the lifetime utilities of the workers constituting the labor force in the present and future periods,

$$\begin{aligned} WS &= E[(\theta + b - \theta b)V_E^1 + (1-\theta)(1-b)V_E] + \sum_{n=\overline{n}}^{-1} U_n V_U^n + U_O V_O \\ &\quad + \frac{\theta N}{1-\theta} \sum_{t=0}^{\infty} \frac{V(0,1)}{(1+r)^t}, \end{aligned} \tag{90}$$

where $V(0,1)$ is the lifetime utility of newly born workers and V_E^1, V_E, V_U^n, V_O, E, U_n, and U_O are defined as in Appendix B.

Proposition 6. *The RHS of (90) can be expressed as*

$$RHS = \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \left[\tilde{w}E + \overline{w}(N-E) - e(N-U_O) \right], \tag{91}$$

where

$$\tilde{w} \equiv w^1 - (1-\theta)(1-b)(w^1 - w^{2+}). \tag{92}$$

Proof. Since V_O, V_U^n, V_E^1, V_E, E, U_O and U_n satisfy (58),(62)–(64) and (66), and since $V(0,1)$ is given by

$$V(0,1) = \frac{1-\theta}{1+r}[aV_E^1 + (1-a)V_U^{-1}],$$

(90) can be rewritten as

$$\begin{aligned}
RHS &= E[(\theta + b - \theta b)V_E^1 + (1-\theta)(1-b)V_E] + \sum_{n=\overline{n}}^{-1} U_n V_U^n + U_O V_O \\
&\quad + \frac{\theta N}{1-\theta}\sum_{t=0}^{\infty}\frac{V(0,1)}{(1+r)^t}, \\
&= (\theta + b - \theta b)E\left\{w^1 - e + \frac{1-\theta}{1+r}[(1-b)V_E + bV_U^{-1}]\right\} \\
&\quad +(1-\theta)(1-b)E\left\{w^{2+} - e + \frac{1-\theta}{1+r}[(1-b)V_E + bV_U^{-1}]\right\} \\
&\quad +U_{\overline{n}}\left\{\overline{w} - e + \frac{1-\theta}{1+r}[aV_E^1 + (1-a)V_O]\right\} \\
&\quad + \sum_{n=\overline{n}+1}^{-1} U_n\left\{\overline{w} - e + \frac{1-\theta}{1+r}[aV_E^1 + (1-a)V_U^{n-1}]\right\} \\
&\quad +U_O\left(\overline{w} + \frac{1-\theta}{1+r}V_O\right) + \frac{\theta N}{1+r}[aV_E^1 + (1-a)V_U^{-1}] \\
&\quad + \frac{\theta N}{1-\theta}\sum_{t=1}^{\infty}\frac{V(0,1)}{(1+r)^t}.
\end{aligned}$$

Using $\tilde{w}$ to rearrange the above equation, we can obtain

$$\begin{aligned}
RHS &= \tilde{w}E + \overline{w}(N-E) - e(N-U_O) + (1+r)^{-1} \\
&\quad \times \left\{\begin{array}{l}
a\left[(1-\theta)\sum_{n=\overline{n}}^{-1} U_n + \theta N\right] V_E^1 \\
+(1-\theta)(1-b)EV_E \\
+\left[(1-\theta)bE + \theta(1-a)N\right] V_U^{-1} \\
+\sum_{n=\overline{n}}^{-2}(1-\theta)(1-a)U_{n+1}V_U^n \\
+\left[(1-\theta)(1-a)U_{\overline{n}} + (1-\theta)U_O\right] V_O \\
+[\theta N/(1-\theta)]\sum_{t=0}^{\infty} V(0,1)/(1+r)^t
\end{array}\right\} \\
&= \tilde{w}E + \overline{w}(N-E) - e(N-U_O) + (1+r)^{-1} \\
&\quad \times \left\{\begin{array}{l}
E[(\theta + b - \theta b)V_E^1 + (1-\theta)(1-b)V_E] \\
+\sum_{n=\overline{n}}^{-1} U_n V_U^n + U_O V_O \\
+[\theta N/(1-\theta)]\sum_{t=0}^{\infty} V(0,1)/(1+r)^t
\end{array}\right\} \\
&= \tilde{w}E + \overline{w}(N-E) - e(N-U_O) + (1+r)^{-1}RHS,
\end{aligned}$$

the second equality of which is obtained from (77)–(80). The obtained result implies that WS must satisfy

$$WS = \frac{1+r}{r}\left[\tilde{w}E + \overline{w}(N-E) - e(N-U_O)\right],$$

which is equivalent to (91). □

Investors' Surplus

We can also show that the IS equals the discounted sum of investors' expected gains from past, current, and future investments,

$$\begin{aligned} IS = & \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t \cdot \frac{bE}{L^*}\cdot\left[-F+\frac{(L^*)^\alpha - w^*L^*}{r+b}\right] + \frac{E}{L^*}[(L^*)^\alpha - \tilde{w}L^*] \\ & +\sum_{t=1}^{\infty}\left(\frac{1-b}{1+r}\right)^t \cdot \frac{E}{L^*}\left\{(L^*)^\alpha - [\theta w^1 + (1-\theta)w^{2+}]L^*\right\}, \end{aligned} \tag{93}$$

where w^*, L^*, and $\tilde{w}$ are defined by (4) and (92). The first term on the RHS of (93) is the expected sum of net gains from establishing new firms in and after the current period. As already seen in the proof of Lemma 2, this term equals zero under free entry. The second and third terms are the expected sum of the profits that currently operating firms will distribute to their investors in future periods.

Proposition 7. *The RHS of (93) can be expressed as*

$$RHS = \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t \cdot \frac{E}{L^*}\cdot[(L^*)^\alpha - \tilde{w}L^* - bF]. \tag{94}$$

Proof. The second and third terms on the RHS of (93) can be reduced to

$$\frac{1+r}{r+b}\cdot\frac{E}{L^*}\cdot[(L^*)^\alpha - \tilde{w}L^* + b(w^1-w^*)L^*] \ (\equiv X).$$

Since the first term equals zero, we can safely say that X gives the value of the RHS. Note that

$$\begin{aligned} 0 &= b\left[-F+\frac{(L^*)^\alpha - w^*L^*}{r+b}\right] \\ &= (L^*)^\alpha - \tilde{w}L^* - bF + \frac{-r[(L^*)^\alpha - \tilde{w}L^*] + b(\tilde{w}-w^*)L^*}{r+b} \\ &= (L^*)^\alpha - \tilde{w}L^* - bF - \frac{r[(L^*)^\alpha - \tilde{w}L^* + b(w^1-w^*)L^*]}{r+b} \\ &= (L^*)^\alpha - \tilde{w}L^* - bF - \frac{r}{1+r}\cdot\frac{L^*}{E}X, \end{aligned}$$

the third equality of which is obtained from the fact that

$$\begin{aligned} \tilde{w}-w^* &= \tilde{w}-\hat{w} \\ &= -[r/(1+r)](1-\theta)(1-b)(w^1-w^{2+}) \\ &= -r(w^1-\hat{w}) \\ &= -r(w^1-w^*), \end{aligned}$$

where $\hat{w}$ is as defined in (3). The obtained result implies that the RHS must satisfy

$$RHS = \frac{1+r}{r} \cdot \frac{E}{L^*} \cdot [(L^*)^\alpha - \tilde{w}L^* - bF],$$

which is equivalent to (94). $\square$

Appendix E: Computation of Tables 13–15 (Not for Publication)

Let $\overline{w}(F)$ be a reduced level of $\overline{w}$, at which the profile of the aggregate labor demand runs through the upper end of the upward-sloping parts of the AICs for $\overline{n} < 0$, given the value of F. When $\overline{w}$ is reduced to $\overline{w}(F)$, the following must be true:

$$w^* = w^{***} = \overline{w}(F) + \frac{1+r+(1-\theta)b}{(1-\theta)(1-b)q}e,$$

which can be rearranged as

$$\begin{aligned} \overline{w}(F) &= w^* - \frac{1+r+(1-\theta)b}{(1-\theta)(1-b)q}e \\ &= \left[\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{(r+b)^{1-\alpha}F^{1-\alpha}}\right]^{1/\alpha} - \frac{1+r+(1-\theta)b}{(1-\theta)(1-b)q}e, \end{aligned} \tag{95}$$

the last line of which is obtained from (4). By substituting thirteen values of F from 0.840 to 2 and other parameter values configured as in Table 1 into (95), we obtain the second column of Tables 13–15.

The reduction in $\overline{w}$ newly produces one NSCE and one SCE. In both equilibrium, WS, IS, and TS can be expressed as

$$\text{WS} = \left(\frac{1+r}{r}\right)\left[\tilde{w}E + \overline{w}(F)(N-E) - e(N-U_O)\right], \tag{96}$$

$$\text{IS} = \left(\frac{1+r}{r}\right) \cdot \frac{E}{L^*} \cdot [(L^*)^\alpha - \tilde{w}L^* - bF], \tag{97}$$

$$\text{TS} = \left(\frac{1+r}{r}\right)\left[E(L^*)^{\alpha-1} + \overline{w}(F)(N-E) - \frac{bFE}{L^*} - e(N-U_O)\right] \tag{98}$$

where $L^* = [(r+b)F/(1-\alpha)]^{1/\alpha}$. Thus, to evaluate WS, IS, and TS in the new NSCE or the new SCE, we need to compute the values of E, U_O, and $\tilde{w}$ for each value of F at respective equilibria.

In the new NSCE, E, U_O, w^{2+}, w^1, and $\tilde{w}$ are determined as

$$E = E^{**} = \frac{\theta}{(\theta + b - \theta b)} N, \tag{99}$$

$$U_O = N - E^{**}, \tag{100}$$

$$w^{2+} = \overline{w}(F) + \frac{1 + r - (1-\theta)(1-b)(1-q)}{(1-\theta)(1-b)q} e, \tag{101}$$

$$w^1 = \frac{(1+r)w^* - (1-b)(1-\theta)w^{2+}}{1 + r - (1-b)(1-\theta)}, \tag{102}$$

$$\tilde{w} = w^1 - (1-\theta)(1-b)(w^1 - w^{2+}). \tag{103}$$

By substituting the thirteen values of F and other parameter values into (99)–(103), we compute the equilibrium values of E, U_O, w^{2+}, w^1, and $\tilde{w}$ for each value of F. Then, substituting the equilibrium values of E, U_O, and $\tilde{w}$ and other parameter values into (96)–(98), we evaluate the values of WS, IS, and TS in the NSCE for each value of F. Finally, taking the differences between the values of WS (resp. IS, TS) in the new NSCE and their counterparts in the old one, which are reported in the second column of Table 10 (resp. Table 11, Table 12), we obtain the third column of Table 13 (resp. Table 14, Table 15).

In the new SCE, E, U_O, w^{2+}, w^1, and $\tilde{w}$ are determined as

$$E = E^{***} = \frac{1}{(1 + b - \theta b)} N, \tag{104}$$

$$U_O = 0, \tag{105}$$

$$w^{2+} = w^1 = \tilde{w} = w^*. \tag{106}$$

By substituting the thirteen values of F and other parameter values into (104)–(106), we compute the equilibrium values of E, U_O, w^{2+}, w^1, and $\tilde{w}$ for each value of F. Then, substituting the equilibrium values of E, U_O, and $\tilde{w}$ and other parameter values into (96)–(98), we evaluate the values of WS, IS, and TS in the new SCE for each value of F. Finally, taking the differences between the values of WS (resp. IS, TS) in the new SCE and their counterparts in the current equilibrium, which are reported in the second to eleventh columns of Table 10 (resp. Table 11, Table 12), we obtain the fourth to thirteenth columns of Table 13 (resp. Table 14, Table 15).

References

Acemoglu, D. (1995): "Public Policy in a Model of Long-Term Unemployment," *Economica*, 62(246), 161–178.

Arrow, K. J. (1973): "The Theory of Discrimination," in *Discrimination in Labor Markets*, ed. by O. Ashenfelter. Princeton University Press.

Arvan, L., and H. S. Esfahani (1993): "A Model of Efficiency Wages as a Signal of Firm Value," *International Economic Review*, 34(3), 503–524.

Blanchard, O. J., and P. A. Diamond (1994): "Ranking, Unemployment Duration, and Wages," *Review of Economic Studies*, 61(3), 417–434.

Carmichael, H. L. (1985): "Can Unemployment Be Involuntary? Comment," *American Economic Review*, 75(5), 1213–1214.

Eriksson, S. (2006): "Skill Loss, Ranking of Job Applicants and the Dynamics of Unemployment," *German Economic Review*, 7(3), 265–296.

Eriksson, S., and N. Gottfries (2005): "Ranking of Job Applicants, On-the-Job Search, and Persistent Unemployment," *Labour Economics*, 12(3), 407–428.

Eriksson, S., and J. Lagerström (2006): "Competition between Employed and Unemployed Job Applicants: Swedish Evidence," *Scandinavian Journal of Economics*, 108(3), 373–396.

——— (2012): "Detecting Discrimination in the Hiring Process: Evidence from an Internet-Based Search Channel," *Empirical Economics*, 43(2), 537–563.

Eriksson, S., and D.-O. Rooth (2011): "Do Employers Use Unemployment as a Sorting Criterion When Hiring? Evidence from a Field Experiment," IZA Discussion Paper No. 6265.

Ghayad, R., and W. Dickens (2012): "What Can We Learn by Disaggregating the Unemployment-Vacancy Relationship?," Public Policy Briefs No.12-3, Federal Reserve Bank of Boston.

Kübler, D., and G. Weizsäcker (2003): "Information Cascades in the Labor Market," *Journal of Economics*, 80(3), 211–229.

Kugler, A. D., and G. Saint-Paul (2004): "How Do Firing Costs Affect Worker Flows in a World with Adverse Selection?," *Journal of Labor Economics*, 22, 553–584.

Lockwood, B. (1991): "Information Externalities in the Labour Market and the Duration of Unemployment," *Review of Economic Studies*, 58(4), 733–753.

Machin, S., and A. Manning (1999): "The Causes and Consequences of Longterm Unemployment in Europe," in *Handbook of Labor Economics*, ed. by O. Ashenfelter, and D. Card, vol. 3, chap. 47, pp. 3085–3139. Elsevier Science B.V.

MacLeod, W. B., and J. M. Malcomson (1989): "Implicit Contracts, Incentive Compatibility, and Involuntary Unemployment," *Econometrica*, 57(2), 447–480.

Oberholzer-Gee, F. (2008): "Nonemployment Stigma as Rational Herding: A Field Experiment," *Journal of Economic Behavior and Organization*, 65(1), 30–40.

Phelps, E. S. (1972): "The Statistical Theory of Racism and Sexism," *American Economic Review*, 62(4), 659–661.

Ritter, J. A., and L. J. Taylor (1994): "Workers as Creditors: Performance Bonds and Efficiency Wages," *American Economic Review*, 84(3), 694–704.

Shapiro, C., and J. E. Stiglitz (1984): "Equilibrium Unemployment as a Worker Discipline Device," *American Economic Review*, 74(3), 433–444.