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A Multi-generation Product Diffusion Model with Social Media Effects -Accerelating Effect of Social Media on Leapfrogs and Switches by the iPhone 6 Battery Problem 2016–2017 -

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Abstract

This paper proposes a multi-generational model that captures the direct and indirect effects of social media on product diffusion. Direct effects appear in the adoption rate function as covariates, and indirect effects are imbedded in hierarchical models connecting diffusion parameters to successive generations. Unlike previous multi-generational diffusion models, ours forecasts sales of new-generation products before launch using social media as the leading indicator and a hierarchical model connecting successive diffusion parameters. Empirical results show our model forecasts more precisely and reveals how social media influenced sales of smartphones, particularly leapfrogging and switching to other generations and competing products, as Apple contended with defective batteries in the iPhone 6 during 2016–2017.

1. Introduction

Frameworks of diffusion studies generally follow the Bass (1969) model that assumes a social network is homogeneous and fully connected and that new adopters enter the market influenced by innovators and producers' marketing communications. Studies traditionally attribute the internal effect to the influence of word of mouth.

Peres et al. (2010) trace how diffusion studies since 1990 have transitioned from interpersonal communication (Mahajan, Muller, and Bass, 1990; Mahajan and Wind, 2000) to more general interactions, including social interdependence (Goldberg and Lilien, 2010; Van den Burte and Lilien, 2001).

Diffusion of multi-generational products also has been investigated using the Bass model by connecting single-generation models sequentially. Since Norton and Bass (1987) proposed a diffusion model applicable to successive generations, many studies, including Mahajan and Muller (1996) and Jiang and Jain (2012), have extended it to multi-generational diffusions using sales and marketing mix data during the initial period after launch.

Alongside such data, this study incorporates data from consumers communicating over social media and proposes a multi-generational diffusion model that detects the effects of social media before and after a product launches. It advances the literature in several respects. First, our model uses data other than post-launch sales to track diffusion of new products. In addition to price and competitors featured in the extended Bass models in Von Bertalanffy (1957), Mahajan and Muller (1981), Eastingwood et al. (1983) and Bewley and Fiebig (1988), we construct covariates by extracting text features from social media and incorporating them as social media effects into the adoption rate function of a multi-generational diffusion model, implying the direct effect of social media.

Second, earlier diffusion models assume that key parameters of market potential and imitation rates are independent among generations when the innovator parameter is given. Our model structures generation-specific parameters hierarchically to depict the mechanism underlying parameter shifts between generations. Our structural model of parameter shift across generations facilitates forecasting sales of new-generation products before launch. The term of social media effect is incorporated in the hierarchical models and it affects indirectly to sales.

Third, our model measures the effects of social media on sales, particularly leapfrogs and switches, by the defective battery problem that plagued the iPhone 6 since early November of 2016 and it became a hot topic in social media until the late of 2017. This "*Battery Problem*" is explained in detail in section 5. To measure this effect, we propose a *labeled dynamic topic* model as a hybrid of the labeled topic model (Daniel et al., 2009) and the dynamic topic model (Blei and Lafferty, 2006). Our model employs "battery" *a priori* as the specific dynamic topic that captures the shift in distribution of topics over time. We show empirically that social media reactions to iPhone's battery problem accelerated switching and leapfrogging to later-generation iPhones and competing products.

Section 2 discusses the effects of social media on multi-generational diffusion. Section 3 explains our model. Section 4 reveals empirical results for sales of successive generations of iPhones and shows our model predicts sales of products pre-launch by recognizing social media effects. Section 5 shows the accelerating effects of the iPhone 6 battery problem on leapfrogging and switching to later-generation iPhones and competing phones. Section 6 concludes.

2. Social Media Effects and Diffusion

2.1 Multi-generational Diffusion Model

Research into the diffusion of product generations adopts the framework of multi-generational diffusion models. The main differences among earlier models are their assumptions about key parameters and their use of marketing mix variables. Among studies distinguished by their key parameters, Norton and Bass (1987) assume that market potential m for a generation of product depends on innovation and imitation parameters p and q are constant across generations. Mahajan and Muller (1996), Jun and Park (1999), Kim et al. (2000), Danaher et al. (2001), and Jiang and Jain (2012) assume the constancy of p across generations, generation-specific market potential, and imitation parameters. Jiang (2010) and Guo and Chen (2018) assume that all parameters vary across generations.

Among diffusion studies distinguished by marketing mix variables, Robinson and Lakhani (1975) insert a price effect term into the adoption rate function, and Bass (1980) introduces price effects. These studies show that incorporating price information improves model performance. Horsky and Simon (1983) incorporate an advertising variable directly into sales, not into adoption rate function, and Horsky (1990) incorporates jointly advertising and price information.

The adoption rate function in Bass et. al (1994) includes two marketing mix variables: rates of change in prices and advertising expenditures relative to expenditures at product launch. Doing so permits comparing the effects of marketing variables. Jiang and Jain (2012) extend this model to a multi-generational diffusion model.

2.2 Labeled Dynamic Topics on Social Media

Marketing studies consider the effects of social media by extracting latent topics and their features from large-volume documents and modeling them as covariates. Natural language processing is noteworthy in this regard, as indicated by the Latent Dirichlet Allocation (LDA) model of Blei et al. (2003).

Recent research uses topic models to analyze text data for marketing applications. The LDA model of Tirunillai and Tellis (2014) incorporates consumer reviews into five sets of marketing data to extract dimensions (topics) from UGC (User Generated Content) for comparison among markets. They find that some topics resonate across multiple markets and others only in certain markets. Through sentimental analysis they tag topics for better interpretation and show that multidimensional scaling via LDA captures the dynamics of brand positioning. Netzer et al. (2012) exploit the co-occurrence of words and semantic network analysis to derive market structure from online consumer forums. Their study highlights use of text mining to indicate marketing effectiveness and how marketers can affect brand position.

Li and Terui (2018) incorporate social media effects into a diffusion model to discuss how they influence market potential and internal parameters. Analyzing text data of SNS on mobile phones, they extract subjective and objective features by naïve Bayes and topic analysis respectively, then use them as covariates of time-varying market potential and imitation parameters. They show how social media affect single-generation diffusion of mobile phones and enhance forecasts. The supervised topic model of Ansari et al. (2018) identifies topics hidden in product reviews that reveal consumer preferences. Their stochastic variational Bayesian approach yields fast and scalable inferences from big data.

The dynamic topic model (DTM) is a generative model that reveals the evolution of topics hidden in collections of documents. Blei and Lafferty (2006) extend the LDA model to a dynamic model for handling sequential documents. They describe the dynamics of hyper parameters $h_{\theta_t,k}$ and $h_{\psi_t,k}$ respectively for topic distribution with parameters

$\left\{\theta_{t,k}, k=1,\ldots,K-1\right\}$ and vocabulary distribution with parameters $\left\{\psi_{t,k}, t=1,\ldots,K-1\right\}$ in a state space model so that they have a shift in the mean of Gaussian distribution when the previous state is given. That is, they assume that parameters shift as follows:

$$\begin{aligned} h_{\theta_t,k} \mid h_{\theta_{t-1},k} &\sim N\left(h_{\theta_{t-1},k}, \sigma_\theta^2\right), \\ h_{\psi_t} \mid h_{\psi_{t-1}} &\sim N\left(h_{\psi_{t-1}}, \sigma_\psi^2\right). \end{aligned} \tag{3.1}$$

Corresponding to latent topics in the DTM, our study identifies topics on social media and enters them into our multi-generational diffusion model to uncover changes in consumer's interests.

Next, focusing on the social media coverage following iPhone's battery problem, we extend the DTM by labeling one topic “battery.” Daniel et al. (2009) extend the LDA model by labeling topics to be estimated in advance. We extend their labeled topic model to a labeled dynamic topic model (LDTM), which is characterized as a hybrid of the labeled topic and DTMs. Its graphical representation appears in the Appendix.

We denote η as the hyper parameter for the prior probability that some specific words include “battery” among three topics. Then we define $\eta^{"battery"} = (1, 0, 0)$ to indicate all instances of “battery” are bound to the first topic when assigned in the MCMC procedure. Other words are allocated as per DTM.

3. Models

3.1 Direct Effects of Social Media on Multi-generational Diffusion

We employ the sales function in Srinivasan and Mason (1986) to define our multi-generational diffusion model. Norton and Bass (1987) proposed a successive-generations model by assuming constant p and q, but we employ the diffusion model generalized by

Jiang and Jain (2012) that assumes p is constant across generations but q as well as market potential m are generation-specific.

First, we denote $y_G(t)$ as the G-th generation adoptions at time t with launch time $\tau_G > 0$, and then following Jiang and Jain (2012), we define our model with additive noise for adoption of each generation as follows: Starting with the first generation launched at time 0, we have

$$\begin{cases} y_1(t) = m_1 f_1(t) + u_1(t), & t < \tau_2, \\ y_1(t) = m_1 f_1(t)\left(1 - F_2\left(t - \tau_2\right)\right) + u_1(t), & t \geq \tau_2, \end{cases}$$

$$\begin{cases} y_G(t) = I_G(t) + L_G(t) + S_G(t) + u_G(t), & \tau_G \leq t < \tau_{G+1},\ 1 < G \leq N, \\ \quad \equiv m_G f_G(t-\tau_G) F_G\left(t-\tau_G\right) + y_{G-1}(t) F_G\left(t-\tau_G\right) + Y_{G-1}(t) f_G(t-\tau_G) + u_G(t) & \\ y_G(t) = I_G(t) + L_G(t) + S_G(t) - L_{G+1}(t) + u_G(t), & t \geq \tau_{G+1},\ 1 < G \leq N, \\ \quad \equiv \left(m_G f_G(t-\tau_G) F_G\left(t-\tau_G\right) + y_{G-1}(t) F_G\left(t-\tau_G\right) + Y_{G-1}(t) f_G(t-\tau_G)\right) & \\ \quad \times\left(1 - F_{G+1}\left(t - \tau_{G+1}\right)\right) + u_G(t), & \end{cases} \tag{3.1}$$

where $I_G(t)$ is the independent adoption of the G-th generation, $Y_G(t)$ denotes cumulative adoptions (sales) of the G-th generation at time t, $f_G = F_G\left(t\right) - F_G\left(t-1\right)$, and $L_G(t)$ and $S_G(t)$ are adoptions from leapfrogging and switching, respectively.

In Eq.(3.1), the error term $u_G(t)$ is assumed to follow normal distribution $u_G(t) \sim N\left(0, \sigma^2\right)$ independently across generations and time. We assume constant variance across generations because our data do not identify sales by generation. The adoption rate function $F_G\left(t\right)$ for the G-th generation product accommodates not only marketing mix according to Jiang and Jain (2012) but also social media effects :

$$F_G\left(t\right) = \frac{1 - \exp\left(-\left(p + q_G\right) X_G(t)\right)}{1 + \left(q_G / p\right)\exp\left(-\left(p + q_G\right) X_G(t)\right)}, \tag{3.2}$$

where the covariate $X_G(t)$ is formulated by the hierarchical model as follows:

$$X_G(t) = t + \beta_G \log\left(V_G(t)/V_G(0)\right) + \boldsymbol{\alpha}_G\,'\boldsymbol{LTopic}_G(t-1) + e_x(t). \tag{3.3}$$

In the above, $V_G(t)$ is the price of the G-th generation product in period t and β_G is the coefficient of the price effect. $\boldsymbol{LTopic}_G(t-1)$ is a K dimensional vector with the element of $\log\left(Topic_{Gi}(t-1)/Topic_{Gi}(0)\right)$, where $Topic_{Gi}(t-1)$ denotes the frequency of words in the *i-th* topic at $t-1$, and $\boldsymbol{\alpha}_G$ is the corresponding coefficient vector. We assume that error term $e_x(t)$ follows $e_x(t) \sim N\left(0, \sigma_x^2\right)$.

We use $Topic_{Gi}(t-1)$ rather than $Topic_{Gi}(t)$ to acknowledge the leading property of the social media effect, i.e., *people chat before they act*. Social media variables usually are leading indicators of sales in related research (Li and Terui, 2018), and we show it is empirically confirmed for our data in section 4.

3.2 Indirect Effects of Social Media via Generation-Specific Parameters

We assume that diffusion in each generation persists from the previous generation. Specifically, generation-specific parameters are determined by those of previous generations, and topics $\boldsymbol{T_G}$ are defined as the vector of number of words allocated to each topic after the G-th generation launch for $t < \tau_G$. If we denote $\boldsymbol{Topic}_G(t) = \left(Topic_{G1}(t), \ldots, Topic_{GK}(t)\right)'$, then $\boldsymbol{T}_G$ is represented by $\sum_{t<\tau_G} \boldsymbol{Topic}_G(t)$.

Then we define the structural equations for m_G and q_G as

$$\begin{cases} m_G = \delta_{m0} + \delta_{m1} m_{G-1} + \boldsymbol{\delta}_{mT}\,'\boldsymbol{T}_G + \varepsilon_m \\ q_G = \delta_{q0} + \delta_{q1} q_{G-1} + \boldsymbol{\delta}_{qT}\,'\boldsymbol{T}_G + \varepsilon_q \\ \boldsymbol{\alpha}_G = \delta_{\alpha 0} + \boldsymbol{\delta}_{\alpha 1}\,'\boldsymbol{\alpha}_{G-1} + \boldsymbol{\delta}_{\alpha T}\,'\boldsymbol{T}_G + \varepsilon_\alpha \\ \beta_G = \delta_{\beta 0} + \delta_{\beta 1}\beta_{G-1} + \boldsymbol{\delta}_\beta\,'\boldsymbol{T}_G + \varepsilon_\beta, \end{cases} \tag{3.4}$$

where we assume $\varepsilon_i \sim N\left(0,\sigma_i^2\right)$ for the *i-th* equation and independence across equations. These models define prior distributions for the parameter and we constitute the posterior distributions when combined with the likelihood function derived by Eqs. (3.1)–(3.3).

Jiang and Jain (2012) use the data of number of units and the aggregate adoption sales with marketing variables. However, their empirical performance is undemonstrated because their marketing mix data are unavailable. We also find that estimated parameters of multi-generations are unstable when the number of generations increased. Furthermore, models by Jiang and Jain (2012) and Norton and Bass (1987) structurally do not allow forecasting next-generation sales before launch because they lack structure to connect generations for parameters. Our hierarchical model in Eq.(3.4) accommodates parameter shifts from previous generations and social media effects on parameters.

3.3 Posterior Density for Model Parameters

The joint posterior density of our model parameters is represented by

$$
\begin{aligned}
&p\left(\left\{m_G,q_G,p\right\},\sigma,\left\{\boldsymbol{\alpha}_G,\beta_G\right\},\sigma_x,\left\{\boldsymbol{\Delta}_\mathbf{m},\boldsymbol{\Delta}_\mathbf{q},\boldsymbol{\Delta}_\boldsymbol{\alpha},\boldsymbol{\Delta}_\boldsymbol{\beta}\right\},\left\{\sigma_m,\sigma_q,\boldsymbol{\sigma}_\boldsymbol{\alpha},\sigma_\beta\right\}\mid\left\{\boldsymbol{y}_G,t\right\},\left\{\boldsymbol{LTopic}_G,\boldsymbol{T}_G,\boldsymbol{V}_G\right\}\right)\\
&\quad = p\left(\left\{m_G,q_G,p\right\},\left\{X_G\right\}\mid\left\{\boldsymbol{y}_G,t\right\},\left\{\boldsymbol{\Delta}_\mathbf{m},\boldsymbol{\Delta}_\mathbf{q}\right\},\left\{\boldsymbol{\alpha}_G,\beta_G\right\},\sigma\right)p\left(\sigma\mid\left\{m_G,q_G,p\right\},\left\{\boldsymbol{y}_G,t,X_G\right\}\right)\\
&\quad \times p\left(\boldsymbol{\alpha}_G\mid\left\{X_G,t,\boldsymbol{LTopic}_G,\boldsymbol{V}_G\right\},\boldsymbol{\Delta}_\boldsymbol{\alpha},\sigma_x\right)\\
&\quad \times p\left(\beta_G\mid\left\{X_G,t,\boldsymbol{LTopic}_G,\boldsymbol{V}_G\right\},\boldsymbol{\Delta}_\boldsymbol{\beta},\sigma_x\right)p\left(\sigma_x\mid\left\{\boldsymbol{\alpha}_G,\beta_G\right\},\left\{X_G,t,\boldsymbol{LTopic}_G,\boldsymbol{V}_G\right\}\right)\\
&\quad \times p\left(\boldsymbol{\Delta}_\mathbf{m}\mid\left\{m_G,m_{G-1}\right\},\left\{\boldsymbol{T}_G\right\},\sigma_m\right)p\left(\sigma_m\mid\left\{m_G,m_{G-1}\right\},\left\{\boldsymbol{T}_G\right\},\boldsymbol{\Delta}_\mathbf{m}\right)\\
&\quad \times p\left(\boldsymbol{\Delta}_\mathbf{q}\mid\left\{q_G,q_{G-1}\right\},\left\{\boldsymbol{T}_G\right\},\sigma_q\right)p\left(\sigma_q\mid\left\{q_G,q_{G-1}\right\},\left\{\boldsymbol{T}_G\right\},\boldsymbol{\Delta}_\mathbf{q}\right)\\
&\quad \times p\left(\boldsymbol{\Delta}_\boldsymbol{\alpha}\mid\left\{\alpha_G,\alpha_{G-1}\right\},\left\{\boldsymbol{T}_G\right\},\boldsymbol{\sigma}_\boldsymbol{\alpha}\right)p\left(\boldsymbol{\sigma}_\boldsymbol{\alpha}\mid\left\{\alpha_G,\alpha_{G-1}\right\},\left\{\boldsymbol{T}_G\right\},\boldsymbol{\Delta}_\boldsymbol{\alpha}\right)\\
&\quad \times p\left(\boldsymbol{\Delta}_\boldsymbol{\beta}\mid\left\{\beta_G,\beta_{G-1}\right\},\left\{\boldsymbol{T}_G\right\},\sigma_\beta\right)p\left(\sigma_\beta\mid\left\{\beta_G,\beta_{G-1}\right\},\left\{\boldsymbol{T}_G\right\},\boldsymbol{\Delta}_\boldsymbol{\beta}\right)
\end{aligned}
\tag{3.5}
$$

where $\boldsymbol{\Delta}_\mathbf{i}$ is the coefficient of the vector of each equation in Eq. (3.4).

The second line of (3.5) captures the product of conditional posterior density for parameters in the diffusion model in Eqs. (3.1) and (3.2). The third and fourth lines are joint posterior density of parameters in the marketing mix in Eq. (3.3). The fifth to eighth lines define joint posterior density for parameters in the hierarchical structure connecting the (*G*-1) generation to the *G* generation in Eq.(3.4).

We employ MCMC to estimate parameters because the procedure for hierarchical models is well-established and some of necessary conditional posterior densities are available in closed form. The sampling scheme of MCMC for estimating the model is a hybrid of Metropolis-Hastings and Gibbs sampling for other parameters in hierarchical models. The algorithm appears in the Appendix.

3.4 Predictive Density

According to Eqs. (3.1) and (3.2), if we simulate sales for the second-generation product, we need at least one data point of that generation to estimate m_2 and q_2 even when p is assumed constant across generations. $\boldsymbol{\alpha_2}$ and β_2 also must be estimated using data for the second generation. The feature of their models hampers forecasting new generations without numerical data for this generation.

Fig. 3.1 Role of Social Media Pre- and Post-Launch

We note that the topic's total frequency vector ($\boldsymbol{T_G}$) in Eq. (3.4) differs from vector $\boldsymbol{LTopic}_G(t-1)$ in Eq. (3.3), as Fig 3.1 exemplifies. Before *G*-th generation launches, only social media provide information about a new generation of *G*-th product. Then, in terms of hierarchical models in Eq. (3.4) implying prior information, we forecast adoption of new generations when parameters are given. After *G*-th generation product launches, the marketing mix (Eq. (3.3)) enters the adoption rate function to generate the forecast. Two structures using social media make it possible to use text data fully and to compare social media effects before and after new generations launch. Forecasting is constituted formally by the predictive density.

$$
\begin{aligned}
& p\left(y_G(t) \mid m_G, q_G, p, X_G(t), \boldsymbol{T}_G, \boldsymbol{LTopic}_G(t-1), V_G(t)\right) \\
& = \int p\left(y_G(t) \mid m_G, q_G, p, t\right) p\left(m_G, q_G, p \mid m_{G-1}, q_{G-1}, \boldsymbol{T}_G\right) dm_G dq_G dp \quad \text{if } t \le \tau_G \\
& = \int p\left(y_G(t) \mid m_G, q_G, p, X_G(t)\right) p\left(m_G, q_G, p \mid m_{G-1}, q_{G-1}, \boldsymbol{T}_G\right) \\
& \times p\left(X_G(t) \mid \boldsymbol{\alpha}_G, \beta_G, t, \boldsymbol{LTopic}_G(t-1), V_G(t)\right) \\
& \times p\left(\boldsymbol{\alpha}_G, \beta_G \mid \boldsymbol{\alpha}_{G-1}, \beta_{G-1}, \boldsymbol{T}_G\right) dX_G(t) dm_G dq_G d\boldsymbol{\alpha}_G d\beta_G dp \quad \text{if } t > \tau_G
\end{aligned}
\tag{3.6}
$$

These integrations are numerically evaluated in the MCMC iterations as were applied to time series forecasting by Terui et al. (2010) and Terui and Ban (2014). The algorithm for Eq.(3.6) appears in the Appendix.

4. Empirical Results

4.1 Data

We use five generations of iPhone products with the same launching times between iPhones 5 through 7 as training data, .i.e., *G*1(iPhone 5), *G*2(iPhone 5s, 5c), *G*3(iPhone 6, 6 Plus), *G*4(IPhone 6s, 6s Plus), *G*5(iPhone 7, 7 Plus). The last generation *G*6(iPhone 8, 8 Plus, X) is reserved for test data. All data were obtained from Statista (www.statista.com).

Fig. 4.1 iPhone Sales

Sales data are total iPhone sales shown in Fig.4.1. There are no unit sales for each generation, making it hard to optimize all parameters for each generation only by total sales.

Text data were acquired from gsmarena (http://www.gsmarena.com/). This is a well-known BBS where users worldwide comment upon mobile phones. Through conventional preprocessing procedure taken in natural language processing, punctuations, stop words and low frequency words less than ten times were removed. Then we use remained 124,263 comments in total for all generations.

4.2 Dynamic Topics

The number of topics needs to be fixed to construct covariate of features in social media and we use the criterion of root mean squared error (RMSE) of forecasts for test data to choose the number of topics. This measure is shown in Fig 4.2 when the number of topics changes from one to five. T suggests three topics for our social media.

Fig. 4.2 Evaluating Number of Topics

LDTM lets us detect changes in each topic, implying potential changes in customer interest. The top words among topics appear in Table 4.1. We interpret the topics as follows.

1. Topic 1 (Property-battery): because the frequent words are “nfc” (iPhone5), “apps,” “battery” (all), “face recognition,” and “fingerprint” (iPhone X).
2. Topic 2 (Comparison): because competitors and their product names appear continually (s3, Samsung, Android) with the word “better.”
3. Topic 3 (Discussion): because of top words “don’t,” “u” (you), “buy,” and “im” (I

am).

4.3 Leading Property of Topics to Sales

Fig. 4.3 plots time series for sales and frequency (number of words) under three topics from 2012: Q4 to 2017: Q4. It shows that the frequency of each topic leads sales by one period.

Fig. 4.3 Time Series Plot of Sales and Topics

Table 4.2 shows cross-correlation of time lags of one and two periods with sales and frequency of three topics. They show all topics lead sales by one period with similar magnitudes of effect. There is no delayed relation after one period for Topics 1 and 2, although two lags could exist for Topic 3.

Table 4.1 Top Words within Each Topic

Table 4.2 Cross-correlation between Topics and Sales

Table 4.3 Comparative Models

4.4 Model Comparison

We now compare 10 models by covariate, treatment of generation-specific heterogeneity of parameters, and hierarchical structure of parameters connecting current and previous generations (Table 4.3).

Models 1–4.

Model 1 observes the formula in Norton-Bass (1987). Model 2 mirrors Jiang and Jain (2012). Models 3 and 4 include topic variables. All models lack the structure to connect generations except as an innovator parameter (i.e., no trans-generational memory). Thus we call them 0_{th}-order models.

Model 5 ~ Model 8.

Models 5–8 are structured for parameter shifts to the next generation as hierarchical models. Models 5 and 7 use only parameters of previous generations. Models 6 and 8 include additional variables for topics in their hierarchical models. These models can predict one step ahead for a new product even before launch. These are first-order models.

Model 9 ~ Model 10.

Models 9 and 10 also are first-order models. The difference from Models 5–8 is the homogeneous coefficients of marketing mix variables between generations. To improve model performance, all previous studies including marketing mix assume that it is heterogeneous. We assume marketing variables are homogeneous for all generations to achieve parsimony.

We compare the 10 models in three measures: log of marginal likelihood (LMD), deviance information criteria (DIC), and RMSEs of forecasts for training data (*G*1–*G*5) and test data (*G*6). We confirmed convergence for all models via Geweke's test (Geweke, 1992) at 95% significance in Table 4.4.

Table 4.4 Model Evaluations

0_{th}-order models (Models 1–4) cannot forecast sales of *G*6 as test data because they lack structure to accommodate the shift of (m_G, q_G). Then we evaluate all first-order models by RMSE(Train), log of marginal likelihood and DIC, and RMSE (Test).

Model 3 exhibits the best in-sample performance indicated by RMSE and log of marginal likelihood. Accompanying the marketing mix with social media information sharpens forecasting precision. RMSE of the holdout sample and DIC suggest Model 10 has the best performance. This finding shows the effectiveness of topic models. Also, our assumption of a homogeneous marketing mix yields better fit to the holdout sample, indicating heterogeneity may lead models to over-fit training data.

Comparing models among the first-order group shows that those with topic variables (Models 6, 8, and 10) out-perform those without topics (Models 5, 7, and 9). In general, model fit erodes for models with more restrictions, but even in the training data RMSE and LMD of first-order models approach those of Model 3 (best among the zero-order group). First-order Models 6 and 10 exhibit larger LMD and smaller DIC than Model 3. This finding implies hierarchical models enhance model fit.

After examining RMSE of test data and DIC criteria, we chose Model 10 and examine the results of estimation and forecasting. Model 10 features no price effect. That is consistent with iPhones having higher prices across generations than competing mobile phones and with adopters of iPhones showing themselves insensitive to price.

4.5 Parameter Estimates

Parameter estimates of Model 10 are in Table 4.5, where number indicates posterior means and the posterior standard deviation is in parentheses.

Table 4.5 Parameter Estimates

Columns m and show that estimates of m_G, q_G, and p are constant across generations. Coefficients $\alpha_i, i = 1, 2, 3$, are homogeneous among generations in response to covariates for Model 10.

Values of m_G, p, and q_G define the curve for the G-th generation's diffusion, and it is influenced by the topic feature of $\log\left(Topic_{Gi}(t-1)/Topic_{Gi}(0)\right)$.

Estimates of m_G imply that the market for $G1$ is potentially double that of other generations. The first generation faces its original potential market. After the $G2$ launches, the market divides into an original and an influenced segment. As a result, its market is smaller than for the market for $G1$.

Estimates for imitation parameter q_G increase slowly across all generations except $G2$. This finding suggests that consumers become imitators rather than innovators as generations proceed and await reviews before buying new-generation products.

Quasi-t values (posterior mean divided by standard deviation) are large for $\alpha_i, i = 1, 2, 3$, implying that all three topics are significant in this model. Topic 1(Property-battery) has expected negative correlations with sales. Topics 2(Competitor) and 3 (Discussion) have significant positive correlations with sales.

Table 4.6 examines the hierarchical model's estimates in Eq. (4.5).

Table 4.6 Estimates of Hierarchical Structure

We summarize results from Table 4.6 as follows.

(1) Estimated coefficients of m_{G-1} and q_{G-1} are nearly 1, indicating parameters for market size and imitation rate shift smoothly from previous generations. Estimates of m_G decline gradually and estimates for q_G increase across generations, expecting that

m_G declines because fewer new customers remain as the market matured and sales were more influenced by previous generations and their marketing. After several generations, q_G trends upward with the popularity of products.

(2) We interpret the indirect topic effect from the estimated hierarchical structure as follows. Topic 1(Property-battery) has significant positive correlations with m_G and q_G. This finding implies that market size and imitation rate swell as consumers talked more about the property(battery) before launch. Topic 2 (Competitor) has a negative correlation with m_G and a positive correlation on q_G. These findings mean that more communication on competitors attract imitators, but the risk to declining market size remains. Topic 3 (Discussion) has the effect opposite that of Topic 2.

(3) Indirect effects of topics in hierarchical models differ from direct effects in the adoption rate function as marketing mix variables. This finding implies different roles for social media pre- and post-launch. For instance, Topic 1(Property-battery) has a positive indirect effect on parameter changes in the hierarchical structure and a negative direct effect on adoption rate function. These findings show that customers who were interested in properties of new generations before launch may be disappointed with them post-launch.

4.6 Forecasting Sales of Unlaunched New Generation

Using the estimated hierarchical structure and diffusion model parameter estimates, we can forecast sales of new generations before launch. Fig 4.4 shows forecast (solid line) of new-generation $G6$ and its actual sales (dashed line) during 2017Q4 to 2018Q3. Forecasts are accurate even if products were launched without using social media and prior structure on the parameter shifts inferred from hierarchical model.

Fig. 4.4 Forecasting Sales of Unlaunched New-Generation Products

5. Influence of Social Media on the Battery Problem

5.1 What is the battery problem?

Since early November of 2016, increasing numbers of iPhone 6 owners worldwide have complained their phones shut down unexpectedly even when adequately charged. Apple admitted that some phones needed their batteries replaced and offered to replace them gratis. In late 2016, Apple noted that some phones shut down "under normal conditions in order for the iPhone to protect its electronics." Although Apple tried to solve the battery problem by updating the iPhone OS, consumers complained on social media that their devices slowed after updating the OS. This problem caused by the battery appeared prominently on social media until late 2017. We focus on how social media discussion of the battery problem affected sales, leapfrogging, and switching of iPhones to subsequent generations and competing products.

5.2 Acceleration Effect of Social Media on Leapfrogging to Competitor

Multi-generational diffusion models demonstrate the leapfrog effect, but discussions of it differ in the literature. Without mentioning leapfrog or switch effects explicitly, Norton and Bass (1987) distinguish independent from influenced markets. The former is the original or incremental market for current-generation products. The latter implies sales effects from previous generations.

Mahajan and Muller (1996) extend the Norton and Bass (1987) model to incorporate leapfrog and switch parameters into multi-generational diffusion of durable technological innovations. Jiang and Jain (2012) enlarge the model to consider how marketing mix shapes diffusion of each generation where leapfrog and switch effects are incorporated as in the Norton-Bass model.

The blue line in the left panel of Fig 5.1 shows iPhone sales. It is hard from this figure to identify the influence of social media on the battery problem. Thus, we collect data for sales and market share of competing Android smartphones. Sales of Androids and total units, (iPhone plus Android) appear on the left and market shares on the right in Fig 5.1.

Fig. 5.1 Smartphone Unit Shares and Marketing Share

We seek to detect leapfrogging to a later iPhone generation and switching to Android smartphones induced by social media. Following Jiang and Jain (2012), we calculate them as

$$L_2(t) = m_1 f_1(t) F_2(t-\tau_2) \tag{5.1}$$

$$S_2(t) = m_1 F_1(t) f_2(t-\tau_2)\,. \tag{5.2}$$

$L_2(t)$ and $S_2(t)$ are leapfroggers and switchers, respectively, from the first to the second generation. $L_2(t)$ is affected by the remaining fraction of first generation, and $S_2(t)$ is affected by the installed fraction of first generation. The model detects switching from a previous to a later generation and leapfrogging after generations but not leapfrogging from iPhones to Androids. We then discuss how social media affect leapfrogging to a competitor using additional Android sales data.

Denote $MS_q(t)$ as the market share of iPhones and $TS(t)$ as total unit sales of iPhones and Androids at time *t*. Then we define

$$D(t) = TS(t) \times \left[MS_G(t) - MS_G(t-1) \right], \qquad t \geq 2, \tag{5.3}$$

where $D(t)$ indicates the change in unit sales of iPhones corresponding to the total smartphone market. If we define $SL(t)$ as total leapfrogging from one iPhone to its later generation in period *t*, we derive the leapfrog effect from competitor $CL(t)$ by

$$CL(t) = D(t) - SL(t). \tag{5.4}$$

We compare two kinds of leapfrogging from iPhones and Androids via the ratio

$$rate(t) = \frac{SL(t)}{CL(t)}. \tag{5.5}$$

From 2016:Q4 through 2017:Q4, the period of the battery problem, iPhone unit sales did not decrease significantly (left of Fig. 5.1), but the market share of iPhones declined during the two preceding years. To detect how social media coverage of the battery problem affected sales, we compare two models—one including topics extracted from social media and one that ignores them. Fig. 5.2 shows a numerical example of the social media effect when one topic is included in the model.

Fig. 5.2 Social Media Effect

The left panel of Fig. 5.2 shows topic frequency. It implies the influence on sales by comparing the model that estimated α_G and that which estimated $\alpha_G = 0$. The "alpha" in the right panel of Fig. 5.2 depicts sales corresponding to three kinds of value, (α_G=0, 0.5, 1) when we fix the parameters as (*m*=10, *p*=0.1, *q*=0.5). $\alpha_G = 0$ indicates the model without social media effects. Social media become more influential as α_G increases.

Fig. 5.3 reveals leapfrogging ($SL(t)$) among iPhone generations with and without the social media effect. Social media exert greater influence for later generations, implying that consumers were more influenced as generations proceeded.

Fig 5.4 plots $rate(t)$ from Eq. (5.5). A negative value means more customers leapfrogged to iPhones than to Androids. A positive value means more leapfrogged to Androids. To evaluate $SL(t)$ in the model lacking social media effects, we enforced 0 as the value of $\boldsymbol{\alpha_G}$. Then we measured the effect of social media by the difference between these two leapfrogs estimates (Fig. 5.5).

Fig. 5.3 Leapfrog Effect of iPhone

Fig. 5.4 Comparison of Leapfrogging between iPhone and Android

Fig. 5.5 Difference in Leapfrogging

In Fig 5.5, over periods with positive unit sales, which implies social media were significant with respect to the battery problem, unit sales of iPhones decline gradually. Periods with negative sales mean social media had a negative effect on iPhones. Starting with iPhone 5, the social media effect slowly erodes unit sales of iPhones. When the battery problem was exposed in 2016:Q4, the social media effect was distinguished by leapfrogging to Android. Social media had a significant influence on product diffusion.

6. Conclusion

This study proposed a multi-generational diffusion model with social media effects that included a hierarchical structure connecting diffusion parameters of successive generations. Results show the effects of social media using a dynamic labeled topic model directly on the adoption rate function as effective marketing mix as well as indirectly on the parameter change described by hierarchical model on the shift of diffusion parameters to next generation.

Unlike previous multi-generational diffusion models, ours forecasts sales of new-generation products before launch using social media as the leading indicator and the hierarchical model.

Finally, our model featuring social media effects improves the precision of forecasting and shows how social media affected sales of smartphones via leapfrogging and switching to next-generation and competing products. We examined how iPhone's battery problem affected sales, leapfrogging, and switching to later-generation iPhones and competing Androids.

Further issues remain. We proposed the LDTM to extract dynamic features hidden in social media and confirmed that "people chat before they act." However, results of the model depend on prior LDTM settings. Determining the full robustness of dynamic text analysis awaits future studies.

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Appendix.

A. Labeled Dynamic Topic model

The DTM is base model for LDTM as detailed in Blei and Lafferty (2006). Based on DTM, we assume the word “battery” belongs to Topic 1 in this study. The algorithm of LDTM can be written as

```
For t = 1,2, …, T
  1. Draw Topics
```
$\beta_{t,k}|\beta_{t-1,k} \sim N(\beta_{t-1,k}, \sigma^2 I)$

```
  2.  Draw at
```
$\alpha_t|\alpha_{t-1} \sim N(\alpha_{t-1}, \delta^2 I)$

```
  For each document:
    3. Topic proportion
```
$\eta_{i,d}|\alpha_t \sim N(\alpha, a^2\mathbf{I})$

$\theta_{i,d}|\eta_{i,d} \sim \pi(\eta_{i,d})$

```
    For each word:
        if word == ‘battery’:
```
$z_{t,d,n}$ = 1

```
      else:
        4. Topic-word assignment
```
$z_{t,d,n}|\theta_{t,d} \sim Multinomial(\theta_{t,d})$

$w_{d,n}|z_{d,n}\{\beta_{t,k}\} \sim Multinomial\left(\pi(\beta_{t,z_{d,z}})\right)$

Here $\pi(\cdot)$ is a softmax function that can fix the sum of $\beta_{t,z_{d,z}}$ to 1.

B. MCMC method for the Bass model

B.1 Prior Settings for the Bass model

Parameter	Setting
$m_G \sim N(\mu_{m0}, {\tau_{m0}}^{-1})$	$\mu_{m0} = 0, \tau_{m0} = 0.1$
$q_G \sim N(\mu_{q0}, {\tau_{q0}}^{-1})$	$\mu_{q0} = 0, \tau_{q0} = 0.1$
$p \sim N(\mu_{p0}, {\tau_{p0}}^{-1})$	$\mu_{p0} = 0, \tau_{p0} = 0.1$
$X_G \sim N(\mu_X, {\tau_X}^{-1})$	$\mu_{X0} = 0, \tau_{X0} = 0.1$
$\sigma \sim IG(a, b)$	$\mathrm{a} = 3, \mathrm{b} = 10$
$\alpha_G \sim N(\mu_{\alpha 0}, {\tau_{\alpha 0}}^{-1})$	$\mu_{\alpha 0} = 0, \tau_{\alpha 0} = 0.1$
$\boldsymbol{\beta_G} \sim N(\mu_{\beta 0}, {\tau_{\beta 0}}^{-1})$	$\mu_{\beta 0} = 0, \tau_{\beta 0} = 0.1$
$\sigma_x \sim IG(a, b)$	$\mathrm{a} = 3, \mathrm{b} = 10$
$\boldsymbol{\Theta} \sim N(\mu_{\Theta 0}, {\tau_{\Theta 0}}^{-1})$	$\mu_{\Theta 0} = 0, \tau_{\Theta 0} = 0.1$
$\boldsymbol{\Xi} \sim IG(a, b)$	$\mathrm{a} = 3, \mathrm{b} = 10$

* $\boldsymbol{\delta}$ is the vector of $(\Delta_m, \Delta_q, \sigma_\alpha, \sigma_\beta)$, $\boldsymbol{\Xi}$ is the vector of $(\sigma_m, \sigma_q, \sigma_\alpha, \sigma_\beta)$.

B.2 Conditional Posterior Distributions

(1) $m_G \mid \{y_G, t, X_G\}\{p, q_G\}, \sigma$

For *iter* (=1, …, R) of MCMC iterations, we use Metropolis-Hastings with a random walk algorithm for each generation G,

$$m_G^{(iter)} = m_G^{(iter-1)} + \lambda_m;\ \lambda_m \sim N(0,0.05), \tag{B.1}$$

where the acceptance probability is

$$\alpha = \min\left(1, \frac{p\left(m_G^{(iter)} \mid \{y_G, t, X_G\}, \{p, q_G\}, \sigma\right)}{p\left(m_G^{(iter-1)} \mid \{y_G, t, X_G\}, \{p, q_G\}, \sigma\right)}\right), \tag{B.2}$$

where t = 1, …, N and G = 1, 2, …, 5.

(2) $p \mid \{y_G, t, X_G\}\{m_G, q_G\}, \sigma$

For p and q_G, we also use Metropolis-Hastings sampling, which is the same as m_G above.

$$p^{(iter)} = p^{(iter-1)} + \lambda_p ; \lambda_p \sim N\left(0, 0.05\right). \tag{B.3}$$

The probability of acceptance is

$$\min\left(1, \frac{p\left(p^{(iter)} \mid \{y_G, t, X_G\}, \{m_G, q_G\}, \sigma\right)}{p\left(p^{(iter-1)} \mid \{y_G, t, X_G\}, \{m_G, q_G\}, \sigma\right)}\right). \tag{B.4}$$

(3) $q_G \mid \{y_G, t, X_G\}\{m_G, p\}, \sigma$

$$q_G^{(iter)} = q_G^{(iter-1)} + \lambda_q ; \lambda_q \sim N(0,0.05). \tag{B.5}$$

The acceptation probability is

$$\min\left(1, \frac{p\left(q_G^{(iter)} \mid \{y_G, t, X_G\}, \{m_G, p\}, \sigma\right)}{p\left(q_G^{(iter-1)} \mid \{y_G, t, X_G\}, \{m_G, p\}, \sigma\right)}\right). \tag{B.6}$$

(4) $X_G \mid \{y_G, t\}\{m_G, q_G, p\}, \sigma$

$$X_G^{(iter)} = X_G^{(iter-1)} + \lambda_X ; \lambda_X \sim N(0,0.05). \tag{B.7}$$

The acceptation probability is

$$\min\left(1, \frac{p\left(X_G^{(iter)} \mid \{y_G, t, \}, \{m_G, q_G, p\}, \sigma\right)}{p\left(X_G^{(iter-1)} \mid \{y_G, t\}, \{m_G, q_G, p\}, \sigma\right)}\right). \tag{B.8}$$

(5) $\sigma \mid \{y_G, t, \boldsymbol{X_G}\}, \{m_G, q_G, p\}$

If we define estimated sales in period t for the *Gth* generation as

$$\hat{y}_{Gt} = f(\{y_G, t, X_G\}, \{m_G, q_G\}), \tag{B.9}$$

we can update σ by

$$IG\left(\mathrm{a} + \frac{\mathrm{n}}{2}, \mathrm{b} + \frac{\sum_{t=1}^{n}(\mathrm{y}_t - \sum_{G=1}^{M}\hat{y}_{Gt})^2}{2}\right), \tag{B.10}$$

M stands for number of generations in this equation.

Marketing Mix.

(6) $\boldsymbol{\alpha}_G \mid \left\{X_G, t, \boldsymbol{LTopic}_G, \boldsymbol{V}_G\right\}, \boldsymbol{\Delta}_{\boldsymbol{\alpha}}, \sigma_x$

For each topic *j* the posterior of coefficient α_{Gj} can be derived from a normal regression equation from

$$N\left((n\sigma_\alpha + \sigma_{\alpha 0})^{-1}\left(\sigma_\alpha \sum_{i=1}^{n}\left(X_{Gi} - \boldsymbol{V}_{\boldsymbol{Gi}} \cdot \beta_G - \sum_{k=1}^{T\neq j} Topic_{Gki} \cdot \alpha_{Gk}\right)Topic_{Gj}^{-1} + \mu_{\alpha 0}\sigma_{\alpha 0}\right), (n\sigma_\alpha + \sigma_{\alpha 0})^{-1}\right)$$

$$\equiv m_G f_G(t-\tau_G)F_G\left(t-\tau_G\right) + y_{G-1}(t)F_G\left(t-\tau_G\right) + Y_{G-1}(t) f_G(t-\tau_G)$$

$$(B.11)$$

(7) $\beta_G \mid \left\{X_G, t, \boldsymbol{LTopic}_G, \boldsymbol{V}_G\right\}, \boldsymbol{\Delta}_{\boldsymbol{\beta}}, \sigma_x$

$$N\left(\left(n\sigma_\beta + \sigma_{\beta 0}\right)^{-1}\left(\sigma_\alpha \sum_{i=1}^{n}(X_{Gi} - \boldsymbol{Topic}_{\boldsymbol{Gi}} \cdot \boldsymbol{\alpha}_{\boldsymbol{G}})V_G^{-1} + \mu_{\beta 0}\sigma_{\beta 0}\right), \left(n\sigma_\beta + \sigma_{\beta 0}\right)^{-1}\right) \quad (B.12)$$

(8) $\sigma_x \mid \left\{\boldsymbol{\alpha}_G, \beta_G\right\}, \left\{X_G, t, \boldsymbol{LTopic}_G, \boldsymbol{V}_G\right\}$

$$IG\left(\alpha + \frac{n}{2}, \beta + \frac{\sum_{i=1}^{n}(X_{Gi} - \boldsymbol{Topic}_{\boldsymbol{Gi}} \cdot \boldsymbol{\alpha}_{\boldsymbol{G}} - \boldsymbol{V}_{\boldsymbol{Gi}} \cdot \beta_G)^2}{2}\right). \quad (B.13)$$

Hierarchical Structure.

(9) $\boldsymbol{\Delta}_{\mathbf{m}} \mid \left\{m_G, m_{G-1}\right\}, \left\{\boldsymbol{T}_G\right\}, \sigma_m$

Assuming *D* as data matrix for hierarchical structure, we can derive the posterior of coefficient δ_{mj} by

$$N\left((n\sigma_m + \sigma_{m0})^{-1}\left(\sigma_m \sum_{G=1}^{M}\left(m_G - \sum_{z=1}^{K\neq j}(D_{Gz} \cdot \delta_{mz})\right)D_{Gj}^{-1} + \mu_{m0}\sigma_{m0}\right), (n\sigma_m + \sigma_{m0})^{-1}\right).$$

$$(B.14)$$

(10) $\sigma_m \mid \left\{m_G, m_{G-1}\right\}, \left\{\boldsymbol{T}_G\right\}, \boldsymbol{\Delta}_{\mathbf{m}}$

$$IG\left(\alpha+\frac{n}{2},\beta+\frac{\sum_{G=1}^{M}\left(m_G-\sum_{z=1}(D_{Gz}\cdot\delta_{mz})\right)^2}{2}\right). \quad (B.15)$$

As the sampling methods are the same among all the hierarchical structures, we can sample for other parameters ($\boldsymbol{\Delta_q}, \boldsymbol{\Delta_\alpha}$ and $\boldsymbol{\Delta_\beta}$) as well.

Forecasting

(10) $y(t)\,|\,m_G, q_G, p, X_G(t), \boldsymbol{T}_G, \boldsymbol{LTopic}_G(t-1), V_G(t)$

For $iter = 1, \ldots,$ ITER, after sampling all the parameters using MCMC, the forecasting unit sales for the G-th generation $y_G(t)$ can be written as

$$\begin{cases} y_1(t)^{(iter)} = m_1^{(iter)} f_1(t), & t<\tau_2, \\ y_1(t)^{(iter)} = m_1^{(iter)} f_1(t)\left(1-F_2\left(t-\tau_2\right)\right), & t\geq\tau_2, \end{cases}$$

$$\begin{cases} y_G(t)^{(iter)} = m_G^{(iter)} f_G(t-\tau_G)F_G\left(t-\tau_G\right) & \tau_G\leq t<\tau_{G+1},\ 1<G\leq N, \\ \qquad + y_{G-1}(t)^{(iter)} F_G\left(t-\tau_G\right) + Y_{G-1}(t)^{(iter)} f_G(t-\tau_G), & \\ y_G(t)^{(iter)} = (m_G^{(iter)} f_G(t-\tau_G)F_G\left(t-\tau_G\right) & t\geq\tau_{G+1},\ 1<G\leq N, \\ \qquad + y_{G-1}(t)^{(iter)} F_G\left(t-\tau_G\right) + Y_{G-1}(t)^{(iter)} f_G(t-\tau_G))\times\left(1-F_{G+1}\left(t-\tau_{G+1}\right)\right), & \end{cases}$$

(B.16)

where

$$F_G\left(t\,|\,p^{(iter)}, q_G^{(iter)}\right) = \frac{1-\exp\left(-\left(p^{(iter)}+q_G^{(iter)}\right)X_G(t)^{(iter)}\right)}{1+\left(q_G^{(iter)}/p^{(iter)}\right)\exp\left(-\left(p^{(iter)}+q_G^{(iter)}\right)X_G(t)^{(iter)}\right)}. \quad (B.17)$$

Note that

$$\begin{cases} X_G(t)^{(iter)} = t, & t<\tau_G \\ X_G(t)^{(iter)} = t+\beta_G^{(iter)}\log\left(V_G(t)/V_G(0)\right)+\boldsymbol{\alpha}_G^{(iter)\prime}\boldsymbol{LTopic}_G(t-1), & t\geq\tau_G \end{cases}$$

(B.18)

then total sales $y(t)^{(iter)}$ can be calculated by

$$y(t)^{(iter)} = \sum_G y_G(t)^{(iter)} . \quad (B.19)$$

Table 4.1 Cross-correlation between Topics and Sales

	Topic 1	Topic 2	Topic 3
t	-0.193	-0.271	-0.051
t-1	0.575	0.575	0.688
t-2	0.097	0.020	0.286

Table 4.2 Top Words of Each Topic

$\varphi_{1,v}^{t}$ **Property**

Topic 1	iPhone 5	iPhone 5s	iPhone 6	iPhone 6s	iPhone 7	iPhone X/8
1	apple	apple	phone	apple	apple	apple
2	time	**battery**	apple	**battery**	ios	face
3	new	ios	android	ios	phone	phone
4	iphone	use	ios	phone	android	**id**
5	market	apps	use	time	charging	charging
6	device	**problem**	time	back	back	**recognition**
7	**innovation**	phone	apps	charging	**battery**	**fingerprint**
8	**nfc**	update	**problem**	new	fast	like
9	apples	app	device	apps	time	screen
10	like	like	new	**jack**	wireless	**innovation**

$\varphi_{2,v}^{t}$ **Comparison**

Topic 2	iPhone 5	iPhone 5s	iPhone 6	iPhone 6s	iPhone 7	iPhone X/8
1	iphone	iphone	iphone	iphone	iphone	iphone
2	5	5s	6s	7	8	s8
3	**samsung**	**android**	6	plus	plus	**samsung**
4	better	better	better	better	7	x
5	**galaxy**	**samsung**	**samsung**	ram	x	8
6	4s	**s4**	camera	camera	better	display
7	s3	phone	**android**	**samsung**	5	screen
8	ios	5	ram	screen	s8	better
9	screen	camera	plus	6s	screen	**android**
10	**lumia**	good	s6	**s7**	display	ram

$\varphi_{3,v}^{t}$ **Discussion**

Topic 3	iPhone 5	iPhone 5s	iPhone 6	iPhone 6s	iPhone 7	iPhone X/8
1	iPhone 5	iphone	iphone	iphone	iphone	iphone
2	apple	phone	phone	phone	phone	phone
3	phone	**buy**	apple	**buy**	apple	apple
4	iphone	u	**dont**	apple	**dont**	x
5	**dont**	**dont**	**buy**	**dont**	**buy**	**dont**
6	u	5s	like	like	like	**buy**
7	**buy**	apple	people	u	people	like
8	like	one	best	phones	one	want
9	people	im	one	android	im	better
10	im	want	u	want	samsung	one

Table 4.3 Comparative Models

		Price $\boldsymbol{V_G}$	Topics $\boldsymbol{Topic_G}$	Heterogeneity for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$	Hierarchical Model	
Zeroth-Order Model	Model 1	-	-	○	-	
	Model 2	○	-	○	-	
	Model 3	-	○	○	-	
	Model 4	○	○	○	-	
First-Order Model	Model 5	○	○	○	$m_G \mid m_{G-1}$ $\alpha_G \mid \alpha_{G-1}$	$q_G \mid q_{G-1}$ $\beta_G \mid \beta_{G-1}$
	Model 6	○	○	○	$m_G \mid m_{G-1}, \boldsymbol{T_G}$ $\alpha_G \mid \alpha_{G-1}, \boldsymbol{T_G}$	$q_G \mid q_{G-1}, \boldsymbol{T_G}$ $\beta_G \mid \beta_{G-1}, \boldsymbol{T_G}$
	Model 7	-	○	○	$m_G \mid m_{G-1}$ $\alpha_G \mid \alpha_{G-1}$	$q_G \mid q_{G-1}$ $\beta_G \mid \beta_{G-1}$
	Model 8	-	○	○	$m_G \mid m_{G-1}, \boldsymbol{T_G}$ $\alpha_G \mid \alpha_{G-1}, \boldsymbol{T_G}$	$q_G \mid q_{G-1}, \boldsymbol{T_G}$ $\beta_G \mid \beta_{G-1}, \boldsymbol{T_G}$
	Model 9	-	○	-	$m_G \mid m_{G-1}$ –	$q_G \mid q_{G-1}$ –
	Model 10	-	○	-	$m_G \mid m_{G-1}, \boldsymbol{T_G}$ –	$q_G \mid q_{G-1}, \boldsymbol{T_G}$ –

Table 4.4 Model Evaluations

	Zeroth-Order				First-Order					
Model	1	2	3	4	5	6	7	8	9	10
RMSE(Train)	3.251	3.215	**2.883**	3.059	4.217	3.766	3.649	3.038	3.587	2.897
RMSE(Test)	-	-	-	-	9.097	5.091	8.881	4.987	3.552	**3.495**
log(ml)	-95.07	-90.57	**-75.6**	-75.9	-85.56	-74.48	-84.473	-75.993	-85.92	-75.911
DIC	258.4	250.5	207.18	208.88	269.52	211.57	271.43	205.55	241.58	**196.1**

Table 4.5 Parameter Estimates

	m	p	q	α_1	α_2	α_3
G1	18.391 (1.035)		0.900 (0.186)			
G2	9.916 (0.739)		0.285 (0.112)			
G3	10.182 (0.746)	0.098 (0.012)	1.002 (0.103)	-0.018 (0.001)	0.011 (0.001)	0.047 (0.000)
G4	10.839 (0.473)		1.061 (0.137)			
G5	10.106 (1.275)		1.108 (0.204)			

Table 4.6 Estimates of Hierarchical Structure

	$intercept$	T_1	T_2	T_3	$m(q)_{i-1}$
δ_m	0.037 (0.002)	0.031 (0.002)	-0.025 (0.001)	0.022 (0.001)	0.976 (0.001)
δ_q	-0.014 (0.000)	0.036 (0.001)	0.011 (0.002)	-0.049 (0.001)	1.010 (0.001)

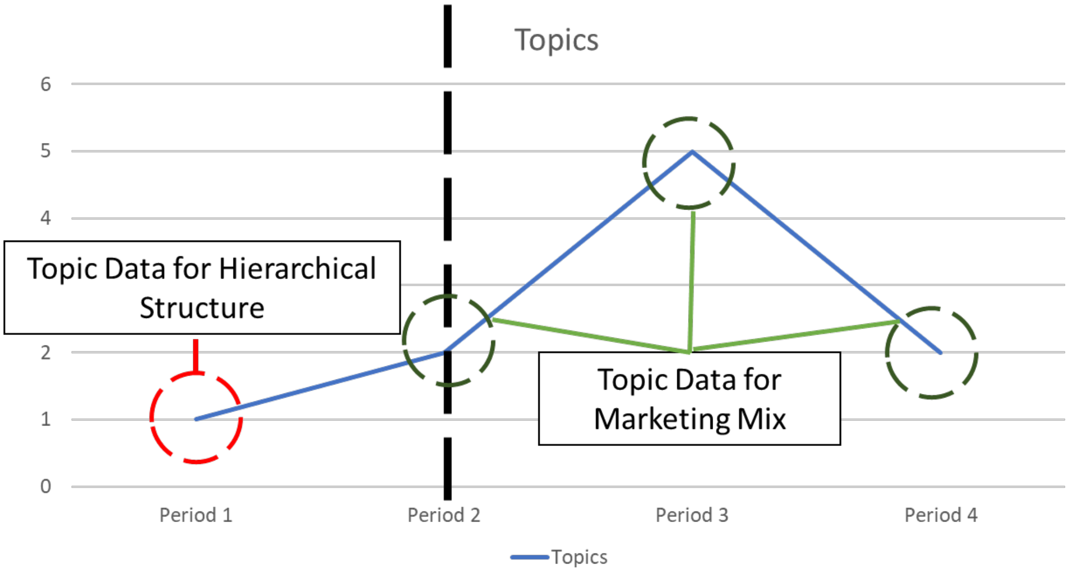

Fig. 3.1 Role of Social Media before and after Launch

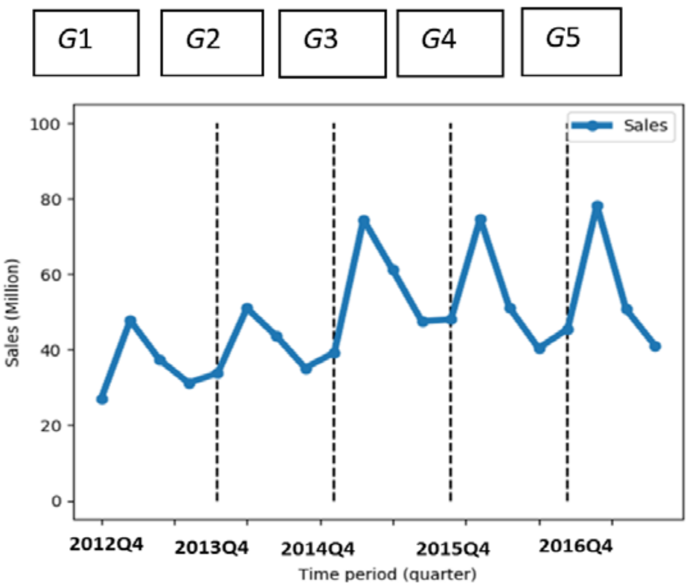

Fig. 4.1 Sales of iPhone

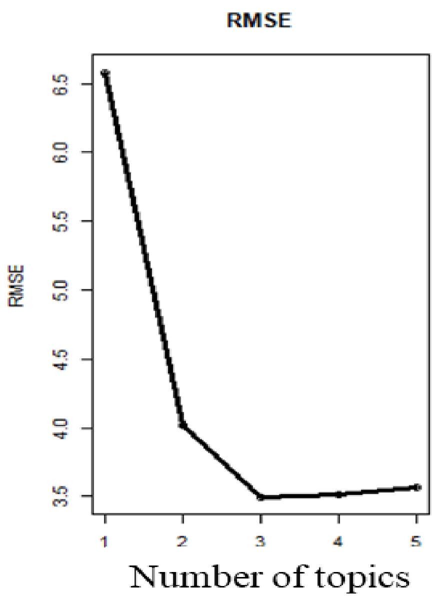

Fig. 4.2 Evaluating Number of Topics

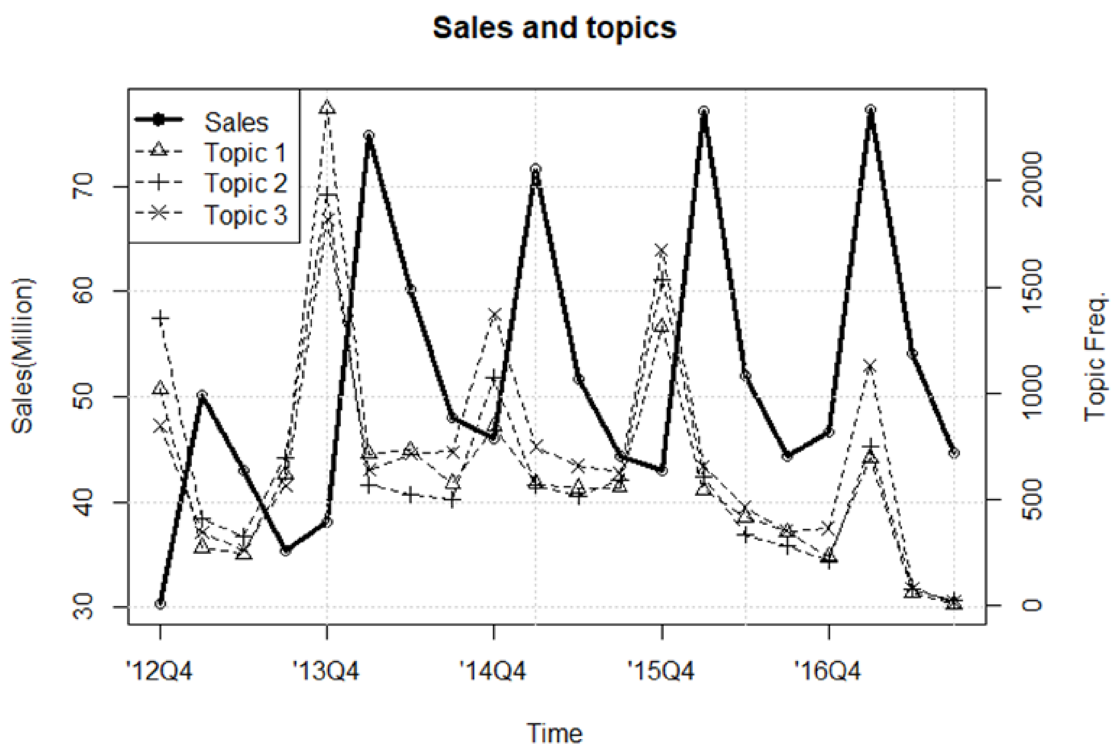

Fig. 4.3 Time Series Plot of Sales and Topics

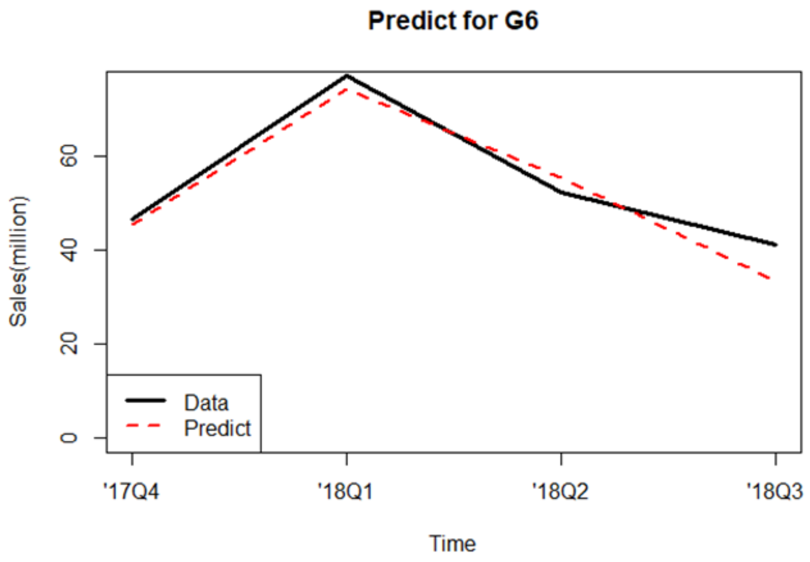

Fig. 4.4 Forecasting Sales of Unlaunched New Generation

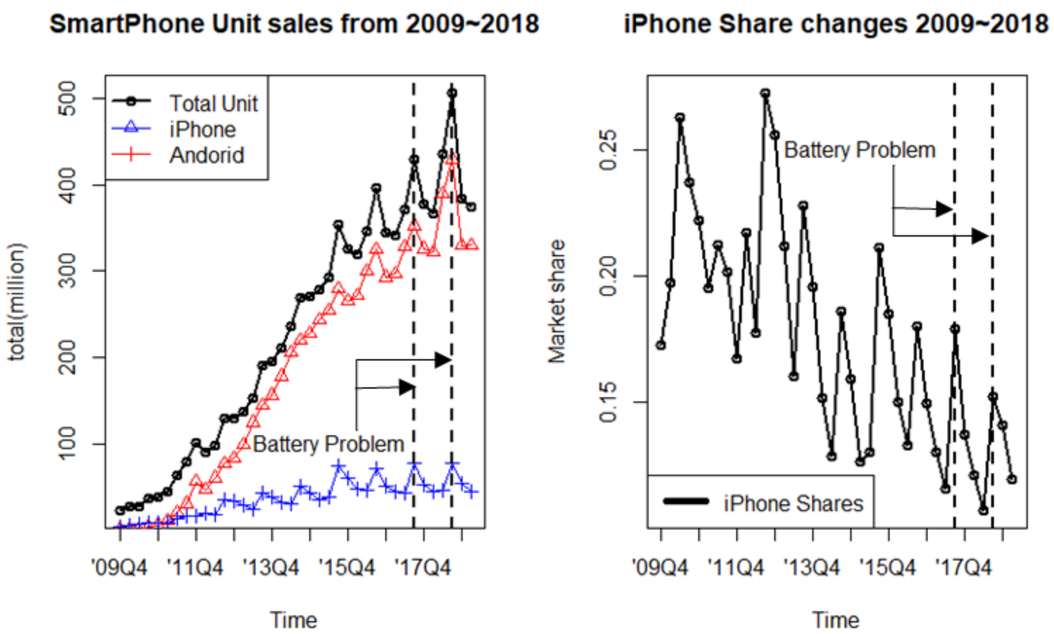

Fig. 5.1 Smartphone Unit Shares and Marketing Share

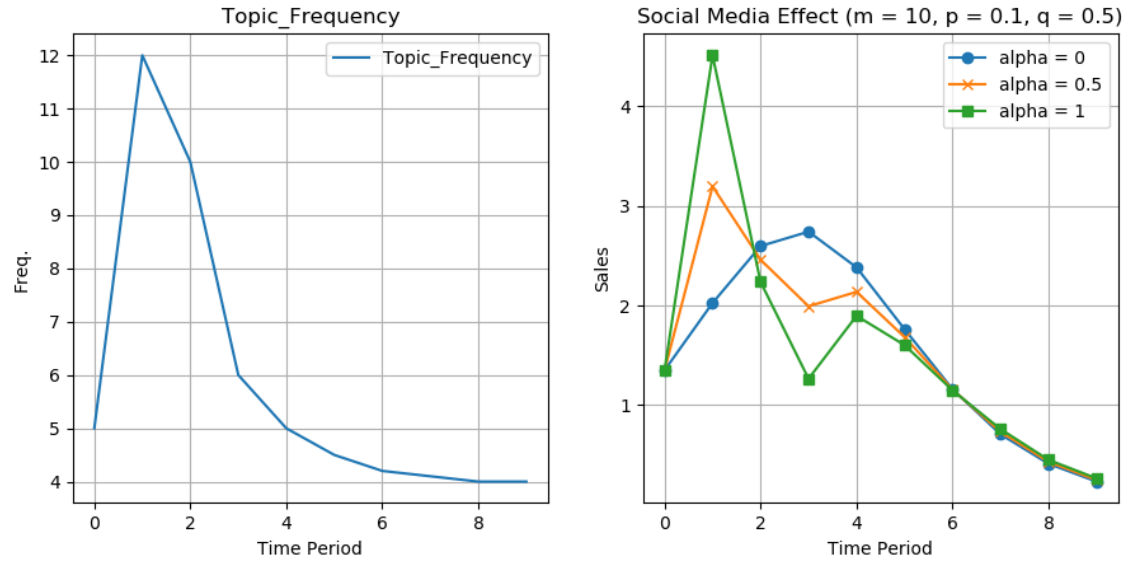

Fig. 5.2 Social Media Effect

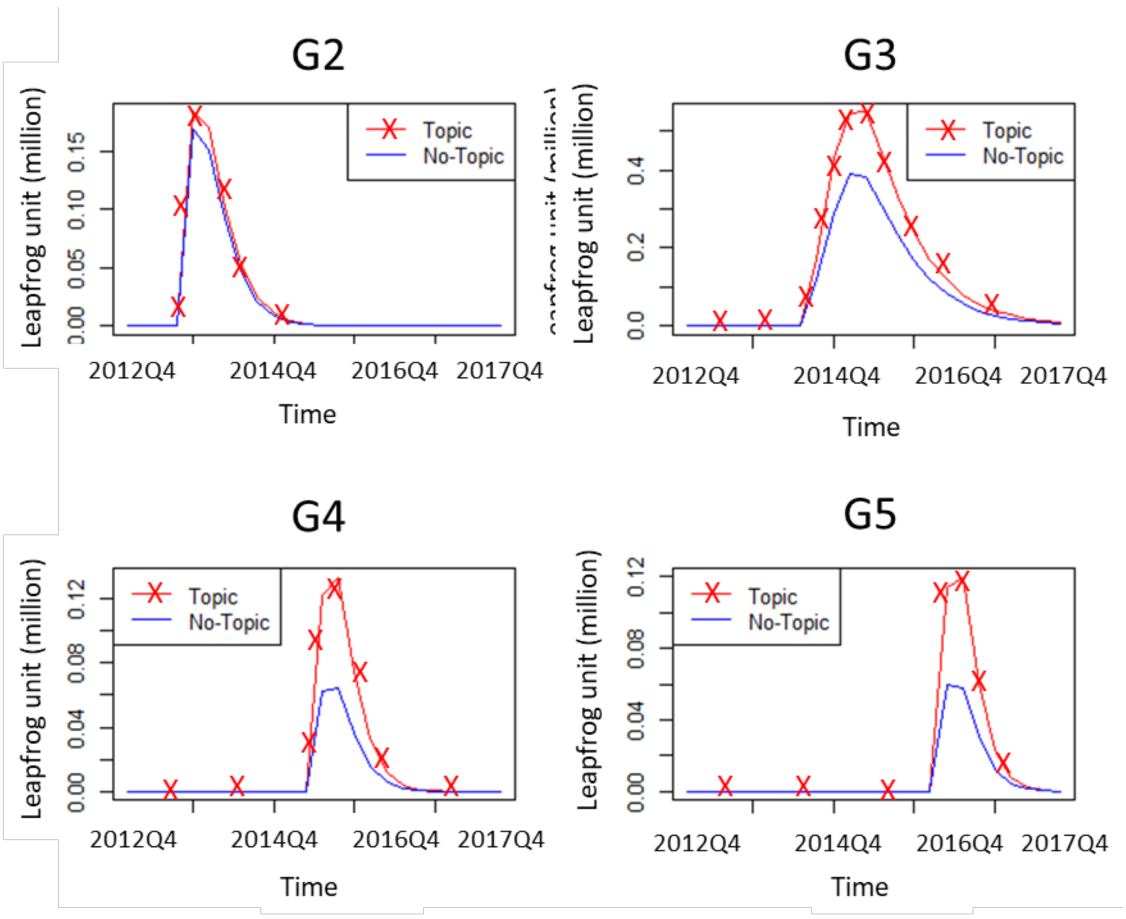

Fig. 5.3 Leapfrog Effect of iPhone

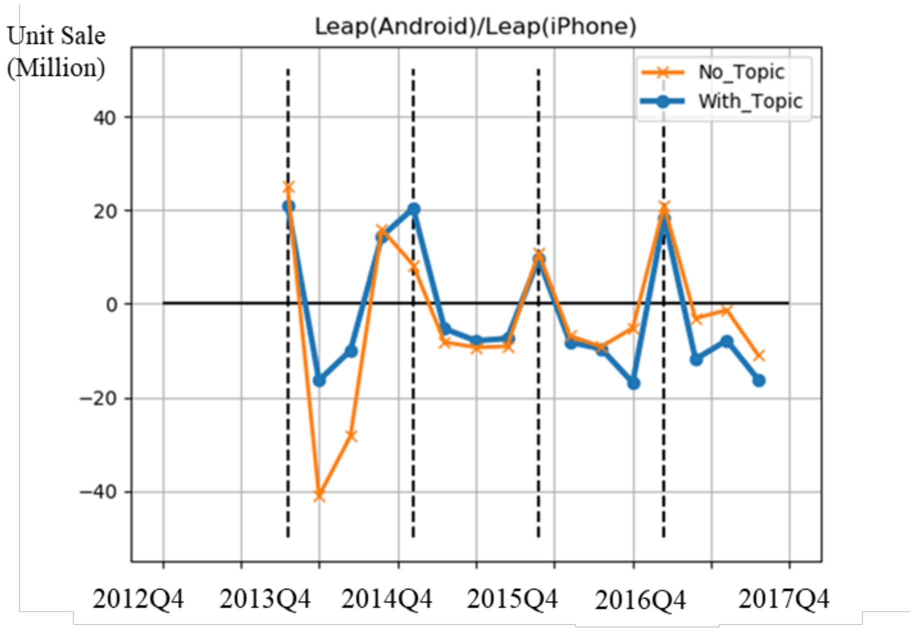

Fig. 5.4 Leapfrog Effect Comparison between iPhone and Android

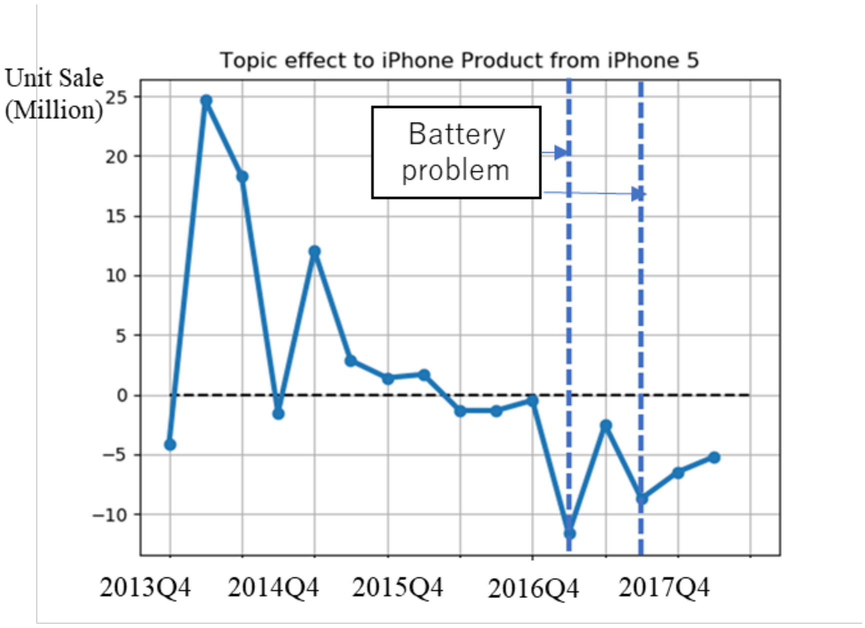

Fig. 5.5 Difference of Leapfrogging